Contagion - Diffusion

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Contagion

"Behavioral epidemics"

Same family of models as those used to model an epidemic propagation (such as the one of the covid-19):

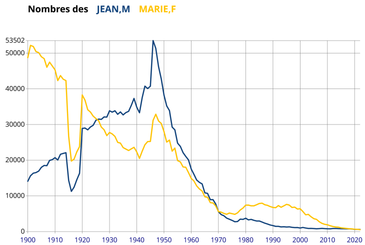
- Fashion first names
  B. Coulmont, V. Supervie and R. Breban, 2016
- Diffusion of innovations, Dynamics of sales of new products
  F. M. Bass, 1969, A New Product Growth for Model Consumer Durables, Managagement Science, reprinted in 2004 as one of the "10 most influential titles of this journal".
- Riot contagion
  S. L. Burbeck, W. J. Raine, and M. A. Stark, 1978;
  L. Bonnasse-Gahot et al, 2018;
  Caroca Soto *et al*, 2020

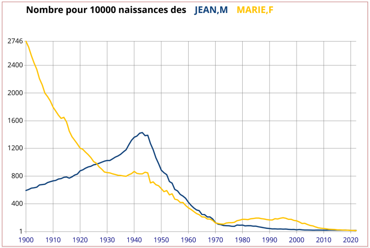
Fashion first names

INSEE data
https://www.insee.fr/fr/statistiques/3532172

Fashion first names

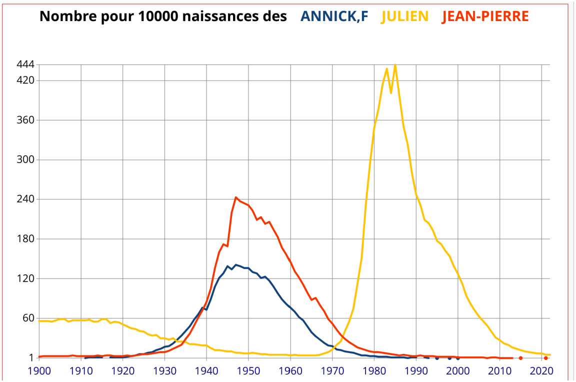
Fashion waves





Number vs. number per 1000, a firstname is given each year in France. INSEE data.

Fashion first names
Fashion waves



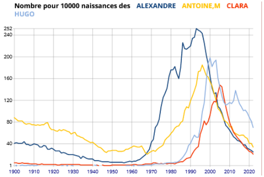
Number of times (per 1000) a firstname is given each year in France. INSEE data.

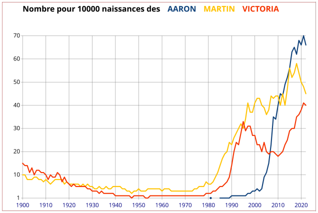
Fashion first names
Your names

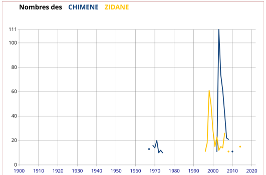
Aaron
Alexandre
Antoine
Clara
Fengyi
Hugo
Jimena
Manel
Martin
Pamela
Victoria
Yaelle

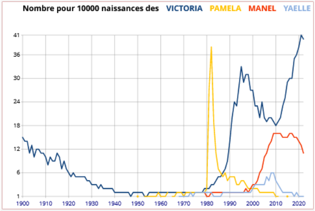
Fashion first names

Fashion waves









Zidane: externally driven fashion - France win the 1998 football world championship

Fashion first names

B. Coulmont, V. Supervie and R. Breban, 2016

Model inspired by models describing: the diffusion of innovation (the *Bass model*, see later), population dynamics in ecology, epidemics propagation (see later).

$$\frac{dN}{dt} = q\frac{N}{K}(K - N)$$

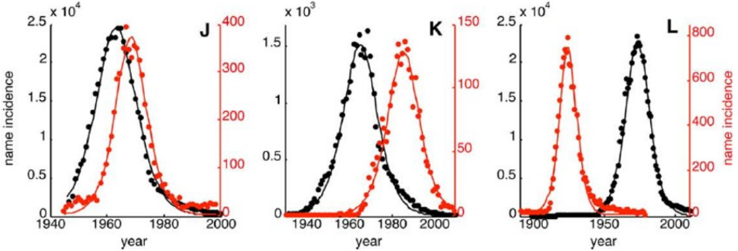
N = nber of adopters – here individuals with the considered first name

q = imitation coefficient

K = maximum number of bearers (carrying capacity)

→ 'logistic model' $\frac{dN}{dt} = qN(1 - \frac{N}{K})$

Fashion first names



J: French names Philippe (black) and Francisco (red);

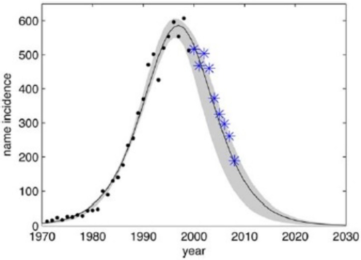
K: Dutch names Ingrid (black) and Moniek (red);

L: American names Diane (black) and Seymour (red).

Fashion first names

Predictive power of the model

Fit of the model from data up to the year 1999. The prediction for the following years is very precise.



The logistic model has predictive power. We illustrate the case of the French name Florine. The data up to year 1999 (black dots) were used to obtain the fit parameters; their 95% confidence intervals were estimated through wild bootstrap. The black line represents the best fit and the grey region the uncertainty generated by bootstrap. The data on the period 2000–2008, shown as blue stars, are well predicted by the logistic fit of the data on the previous period.

B. Coulmont, V. Supervie and R. Breban, 2016

Fashion first names

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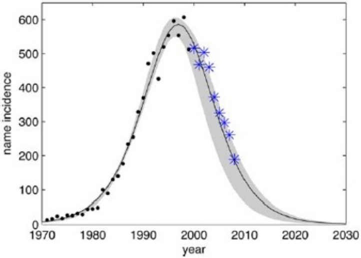

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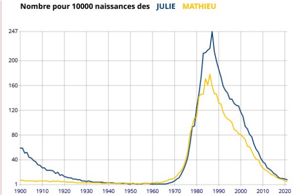
B. Coulmont, V. Supervie and R. Breban, 2016

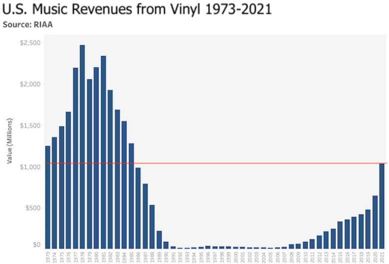
However, fitting the data up to few years before the peak would lead to a high uncertainty on when and how high will be the peak (not shown in the article).

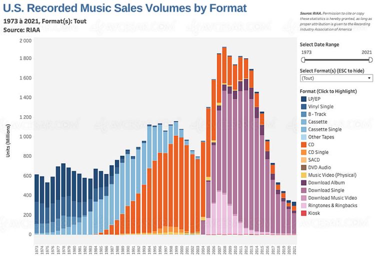


See the 'Julie' and 'Mathieu' waves: almost identical initial exponential phase but very different peaks.

Diffusion of innovations

Diffusion of innovations





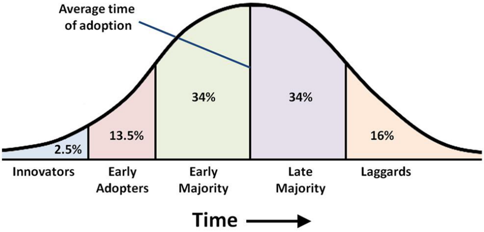
dissemination of a technology —— fashion

waves of adoptions

Diffusion of innovations
Diffusion of Innovation Theory

Diffusion of Innovation Theory: E.M. Rogers, 1962

E. M. Rogers, *Diffusion of Innovations*, Free Press of Glencoe, 1962; 5th edition, Simon & Schuster 2003.



socio-cognitive theory on the individial motivations for adopting or not adopting an innovation.

Diffusion of innovations
The Bass and Norton-Bass models

F. M. Bass, 1969, A New Product Growth for Model Consumer Durables, Managagement Science
reprinted in 2004 as one of the "10 most influential titles of this journal', with a comment from F. M. Bass.

Extension: J. A. Norton and F. M. Bass, 1987

Diffusion of innovations
The Bass model

$N(t) =$ nber of adopters at time t
$K =$ maximum number of bearers
$S(t) = K - N(t) =$ nber of individuals who have not yet adopted

Let $h(t) =$ probability (per unit of time) to adopt at time t given than this has not been done before
thus, between time t and $t + dt$, the number of new adopters is:

$$N(t + dt) - N(t) = S(t)\, h(t)\, dt$$

Limit $dt \to 0$:

$$\frac{dN}{dt} = h(t)\,(K - N(t))$$

Diffusion of innovations

The Bass model

$N(t)$ = nber of adopters at time t
K = maximum number of bearers
$S(t) = K - N(t)$ = nber of individuals who have not yet adopted

$h(t)$ = probability (per unit of time) to adopt at time t given than this has not been done before ("hazard rate").

$$\frac{dN}{dt} = h(t)\,(K - N(t))$$

Hypothesis (Bass model):

$$h(t) = p + q\frac{N(t)}{K}$$

p = innovation coefficient – likelihood that someone will make the choice because of, e.g., media coverage .
q = imitation coefficient

Diffusion of innovations
The Bass model

The **Bass model**:

$$\frac{dN}{dt} = (p + q\frac{N}{K})(K - N)$$

$N =$ nber of adopters

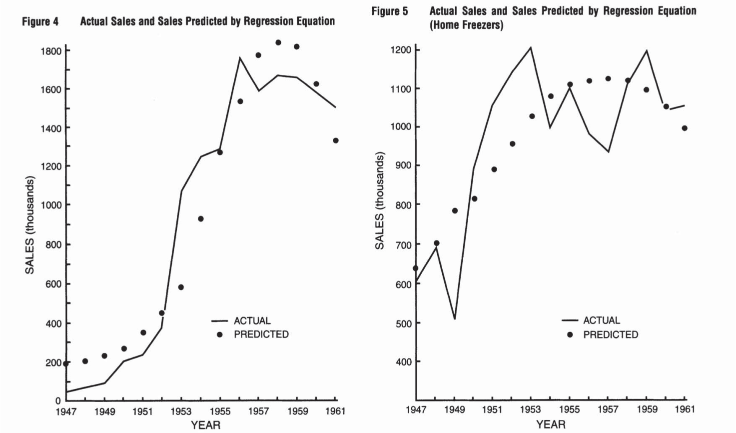
$p =$ innovation coefficient – likelihood that someone will make the choice because of media coverage .

$q =$ imitation coefficient

$K =$ maximum number of bearers (carrying capacity)

Similar to population dynamics in ecology.
Case of fashion first-names: $p = 0$.

Diffusion of innovations
The Bass model



Diffusion of innovations
The Bass model

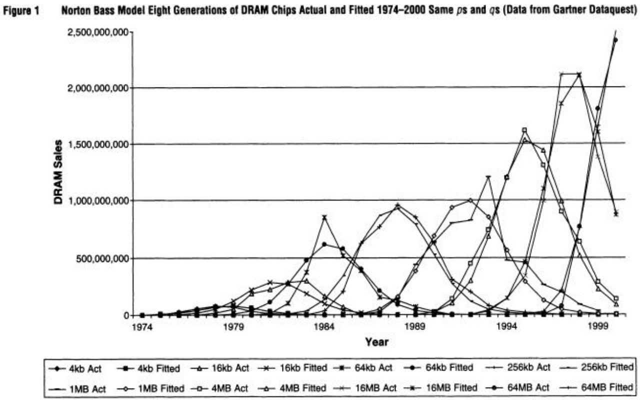
Table 3 Forecasting Accuracy of the Model for Eleven Consumer Durable Products

Product	Period of forecast	r^2
Electric refrigerators	1926–1940	0.762
Home freezers	1947–1961	0.473
Black & white television	1949–1961	0.077*
Water softeners	1950–1961	0.920
Room air conditioners	1950–1961	0.900
Clothes dryers	1950–1961	0.858
Power lawnmowers	1949–1961	0.898
Electric bed coverings	1950–1961	0.934
Automatic coffee makers	1951–1961	0.690
Steam irons	1950–1961	0.730
Recover players	1953–1958	0.953

* The low "explained" variance for this product is accounted for by extreme deviation from trend in two periods. Actually, the model provides a fairly good description of the growth rate, as indicated in Figure 9.

Diffusion of innovations
The Norton-Bass model

J. A. Norton and F. M. Bass, 1987



Mathematical modelling of an epidemics

Mathematical modelling of an epidemics

simplest case: **SIR** model, local epidemics, no spatial diffusion

exemple : measles or covid-19 outbreak in a school, where every child may come into contact with any other child

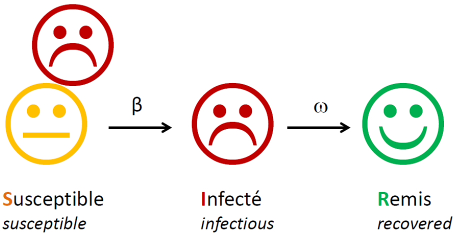
Susceptible
susceptible

Infecté
infectious

Remis
recovered

Mathematical modelling of an epidemics

simplest case: **SIR** model, local epidemics, no spatial diffusion



here, the recovered are immune: no return to the 'susceptible' state.

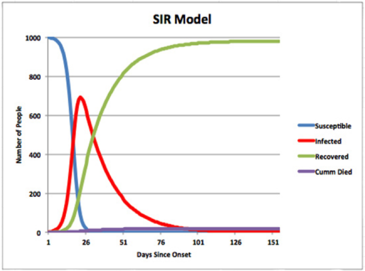
parameters:
β: susceptibility
(probability of contamination if one comes into contact with an infectious person.)
ω: recovery rate (or death + recovery rate)
t_0 : date of the first contamination
S_0: size of the population of susceptible individuals

Mathematical modelling of an epidemics
simplest case: SIR model

→ Mathematical equations giving the evolution over time of the numbers of susceptible, infected and recovered individuals.

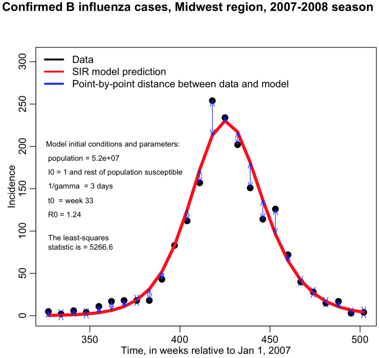
Numerical (computer) resolution of the equations

blue: susceptible; green: recovered; red: infected

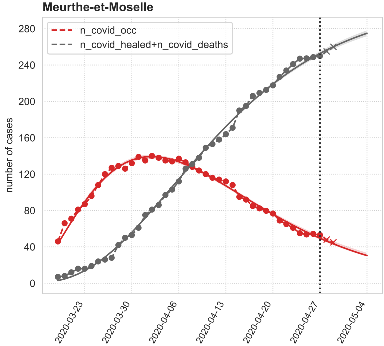


number of days since the beginning of the epidemic

Mathematical modelling of an epidemics
Influenza 2007-2008



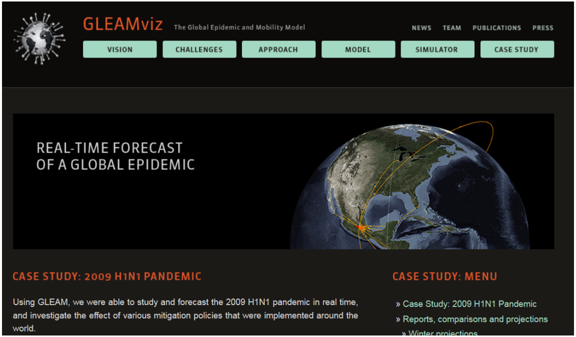
Influenza 2007-2008
(S Towers, 2013)



covid-19
(L Bonnasse-Gahot & ICUBAM, 2020)

Mathematical modelling of an epidemics

Pandemics



https://www.gleamviz.org/

Much more advanced models coupling epidemic and mobility models.

Innovation contagion, Fashion and epidemics

Bass model vs. SI model

Case no recovered (or dead) individuals ($\omega = 0$):

$\equiv$ Bass model with $p = 0$ $\equiv$ fashion first names model

$\rightarrow$ 'logistic model'

$$\frac{dI(t)}{dt} = q\, I(t)\,(1 - \frac{I(t)}{S_0})$$

where
$I(t) =$ number of infected individuals at time t,
$S_0 =$ total (initial) number of susceptible individuals,
$q = \beta / S_0$.

Urban riots

Behavioral epidemics

See specific file - or below, short presentation.