Ultracold atoms

ICFP Master Course

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Master ICFP, Ultracold atoms

January 29, 2019

Lecture slides, exercise classes and other material available at

http://www.phys.ens.fr/ nascimbene/enseignement.php

References on atomic physics

- Claude Cohen-Tannoudji and David Guéry-Odelin, *Advances In Atomic Physics: An Overview*, World Scientific.
- Claude Cohen-Tannoudji, Jacques Dupont-Roc and Gilbert Grynberg, *Processus d'interaction entre photons et atomes*, CNRS Editions.

More details on laser cooling (in French): lectures notes by Jean Dalibard (http://www.phys.ens.fr/ dalibard/)

- Unpublished Master lecture notes : *Atomes ultrafroids*
- Collège de France lectures 2015: *Une brève histoire des atomes froids*
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Quantum gases:

- Wolfgang Ketterle *et al.*, arXiv:cond-mat/9904034 (1999) and arXiv:0801.2500 (2008)
- Sandro Stringari and Lev Pitaevskii, *Bose-Einstein condensation*, Oxford University Press.

A brief history of atomic physics

Atomic physics born with spectroscopy at the end of the 19th century.

Progressed hand-in-hand with quantum mechanics in the years 1900-1930.

AMO -Atomic, Molecular and Optical Physics : *dilute* gases (as opposed to *dense* liquids and solids).

Common view in the early 50's was that AMO physics was essentially understood, with little left to discover. Sixty years later, this view has been proven wrong.

AMO Physics underwent a serie of revolutions, each leading to the next one :

- the 1960's : the laser
- the 1970's : laser spectroscopy
- the 1980's : laser cooling and trapping of atoms and ions
- the 1990's : quantum degenerate atomic gases (Bose-Einstein condensates and Fermi gases)
- the 2000's : femtosecond frequency combs

Control over the quantum state of an atom

The quantum mechanical description of an atom introduces several quantum numbers to describe its state :

- internal quantum numbers to describing the relative motion of electrons with respect to the nuclei, *e.g.* electronic angular momentum $\hat{\boldsymbol{J}}$,
- external quantum numbers, *e.g.* center of mass position $\hat{\boldsymbol{R}}$.

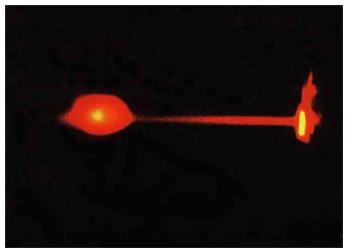
In spectroscopy, electromagnetic fields are used to probe the structure of internal states. Extensions of the same techniques developped for spectroscopy allow one to control the internal degrees of freedom coherently.

Laser cooling and trapping techniques allow one to do the same with the external degrees of freedom of the atom.

Milestones in ultracold atom physics

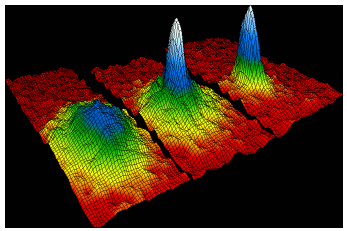
- first deflection of an atomic beam observed as early as 1933 (O. Frisch)
- revival of study of radiative forces in the lates 1970's; first proposals for laser cooling of neutral atoms (Hänsch – Dehmelt) and ions (Itano – Wineland)
  Why ? **Rise of the LASER**

Laser cooling and trapping:

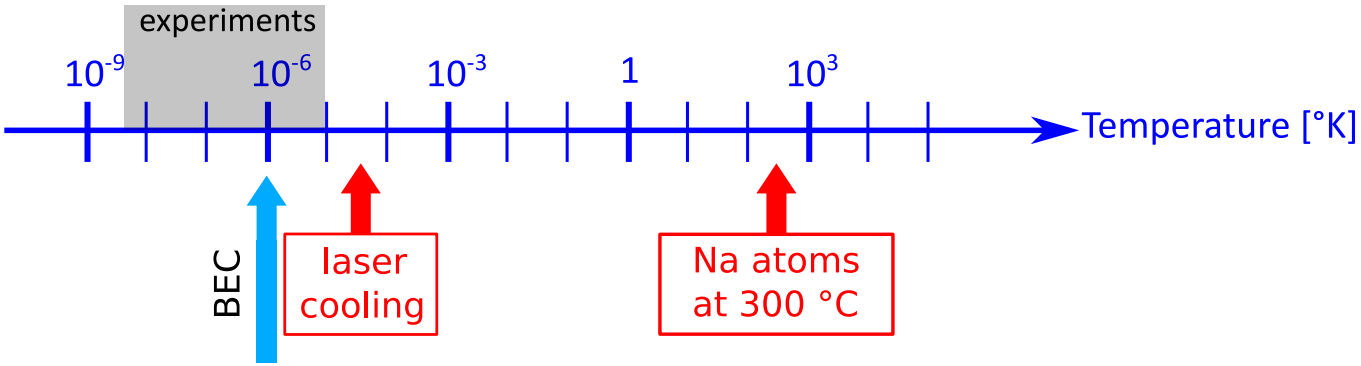


- 1980 : Slowing and bringing an atomic beam to rest
- 1985 : Optical molasses
- 1988 : magneto-optical traps , sub-Doppler cooling
- 1997 : Nobel Prize for Chu, Cohen-Tannoudji, Phillips

Quantum degenerate gases:

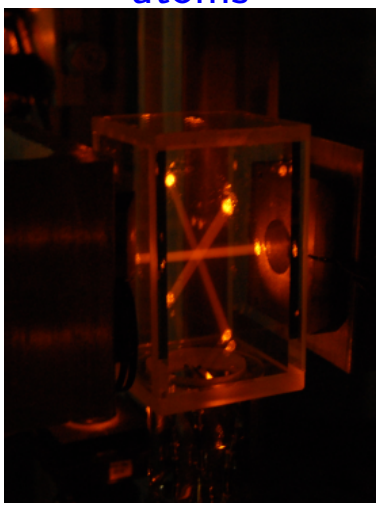


- Bose-Einstein condensation in 1995 [Cornell, Wieman, Ketterle : Nobel 2001]
- Degenerate Fermi gases in 2001 [JILA]

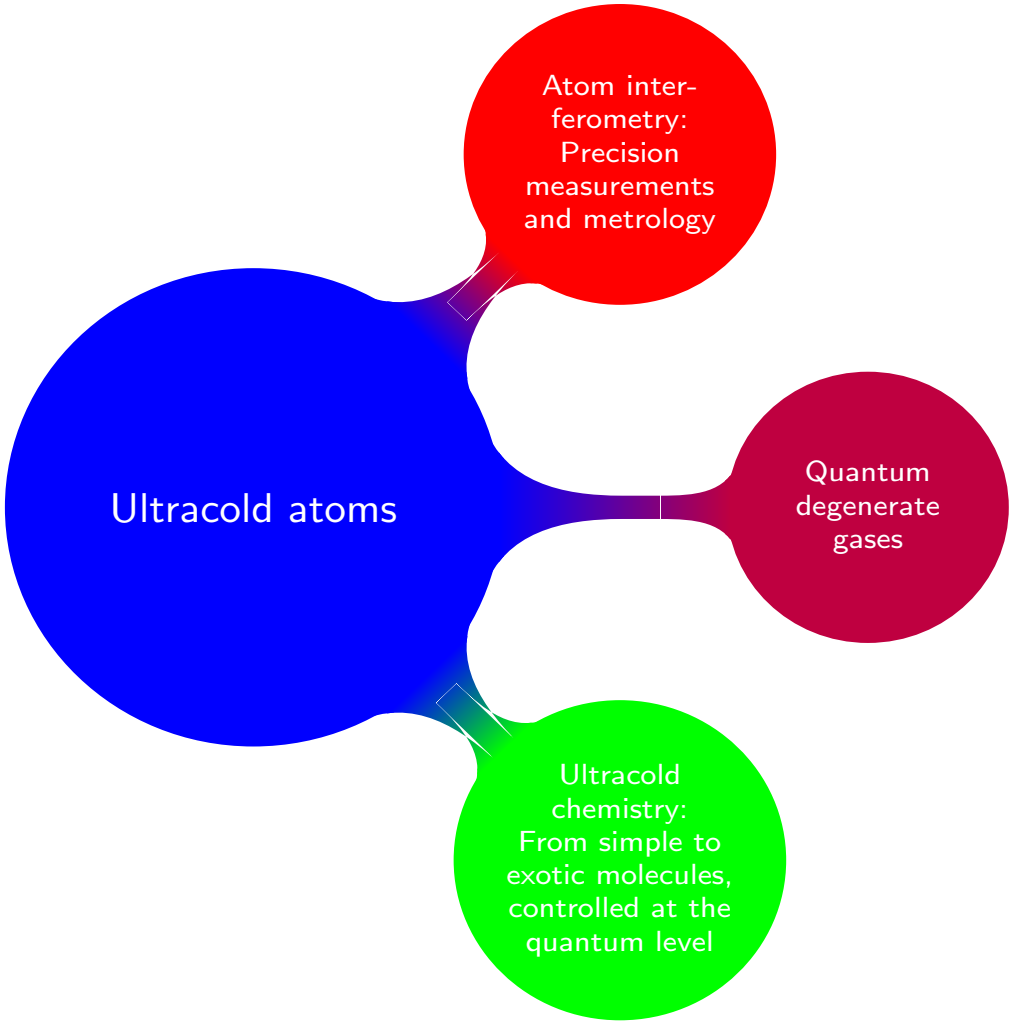


Why are ultracold atoms (and molecules) still interesting today ?

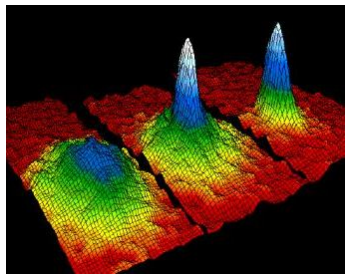
Laser cooling and trapping of neutral atoms



Nobel Prize 1997 :
S. Chu,
C. Cohen-Tannoudji,
W. D. Phillips

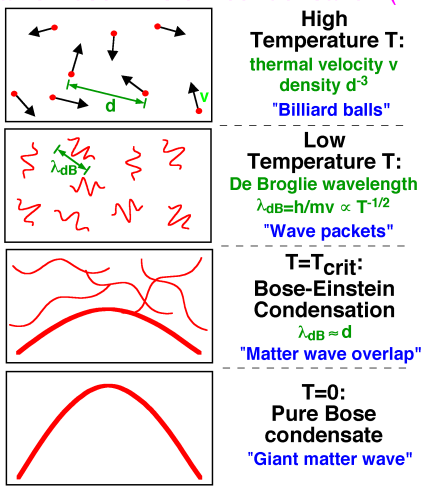


Quantum gases :
Bose-Einstein condensation (1995)
Degenerate Fermi gases (1999)



Nobel Prize 2001 :
E. Cornell,
W. Ketterle,
C. Wieman

What is Bose-Einstein condensation (BEC)?



Quantum degeneracy : phase space density $\lambda_{\mathrm{dB}} \gtrsim n^{-1/3}$

$\lambda_{\mathrm{dB}} = \sqrt{\frac{2\pi\hbar^2}{m k_B T}}$: thermal De Broglie wavelength

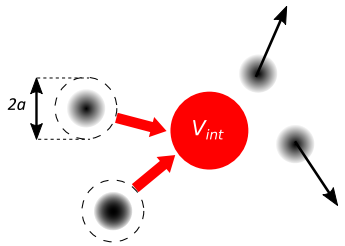
n: spatial density $\qquad$ $n^{-1/3}$: mean interparticle distance

From W. Ketterle group website, http://www.cua.mit.edu/

Interacting atoms, but dilute gas: $na^3 \ll 1$

a : scattering length for $s-$wave interactions

$8\pi a^2$: scattering cross-section (bosons)

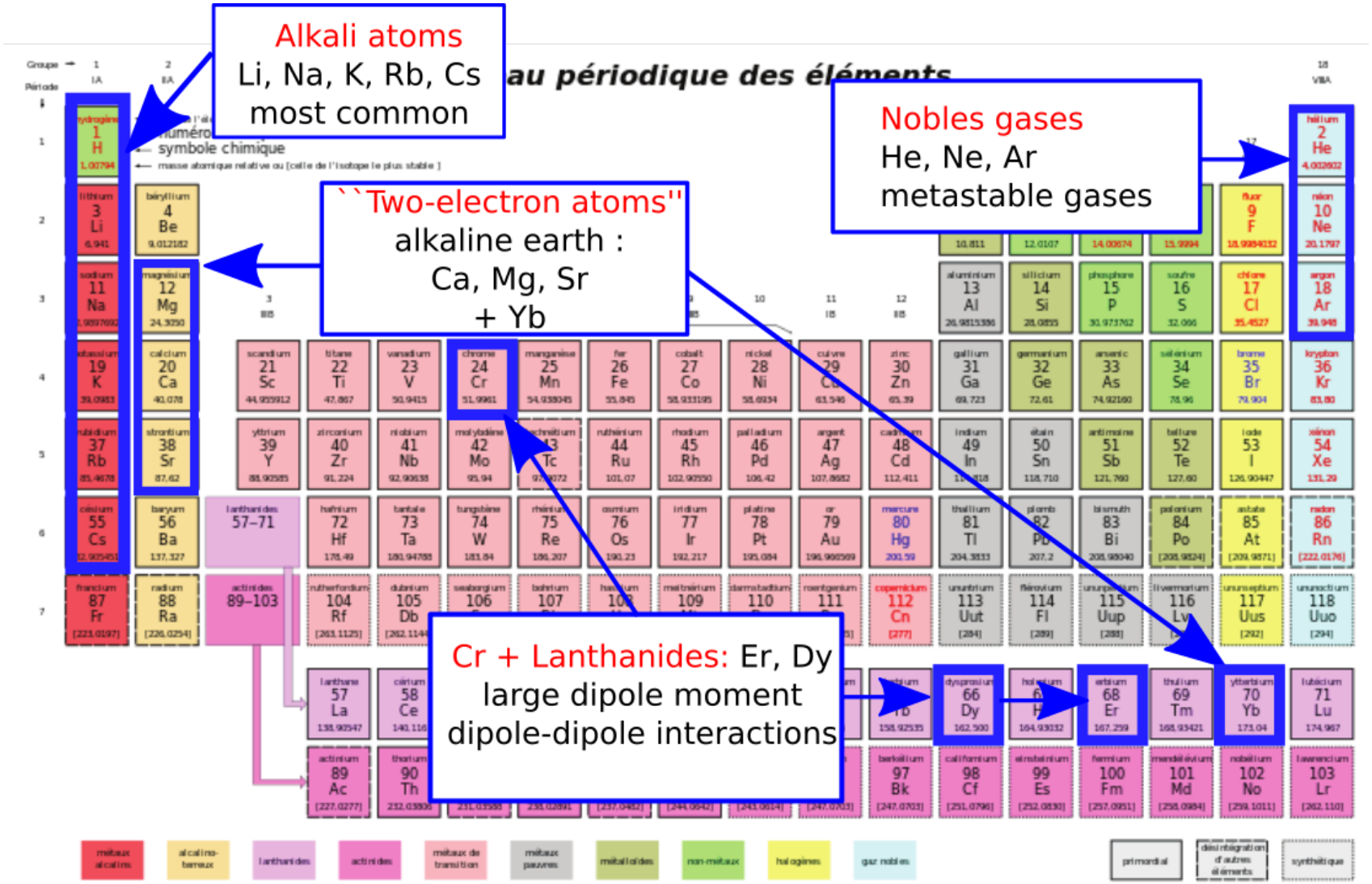


$$a \ll n^{-1/3} \ll \lambda_{\mathrm{dB}}$$

Typical values (BEC of ^{23}Na atoms) :

$$a \sim 2\,\mathrm{nm}$$
$$n^{-1/3} \sim 100\,\mathrm{nm}$$
$$\lambda_{\mathrm{dB}} \sim 1\,\mu\mathrm{m} \text{ at } T = 100\,\mathrm{nK}$$

Which atomic species ?



Outline of the course

1. Introduction and overview of experimental techniques (2 lectures),
   - *Atomic structure; Magneto-optical trap*
   - *Atomic interferometry*
2. Non-interacting quantum gases (1 lecture),
3. Atom-atom scattering in the ultracold limit (1/2 lecture),
   - *Study of a model potential; scattering length*
4. Bose-Einstein condensates (1/2 lecture),
   - *Collective modes and free expansion of a Bose-Einstein condensate*
   - *Vortices in rotating Bose-Einstein condensates*
5. Coherence of quantum gases (1 lecture),
   - *First-order correlation function of a Bose gas*
   - *Hanbury-Brown-Twiss effect with cold atoms*

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6. Ultracold fermions ; BEC-BCS crossover (2 lectures),
   - *Superfluid hydrodynamics; boson-fermion mixtures*
7. Optical lattices ; superfluid-Mott insulator transition (3 lectures).
   - *Bloch oscillations*
   - *Particle-hole excitations of a Mott insulator*

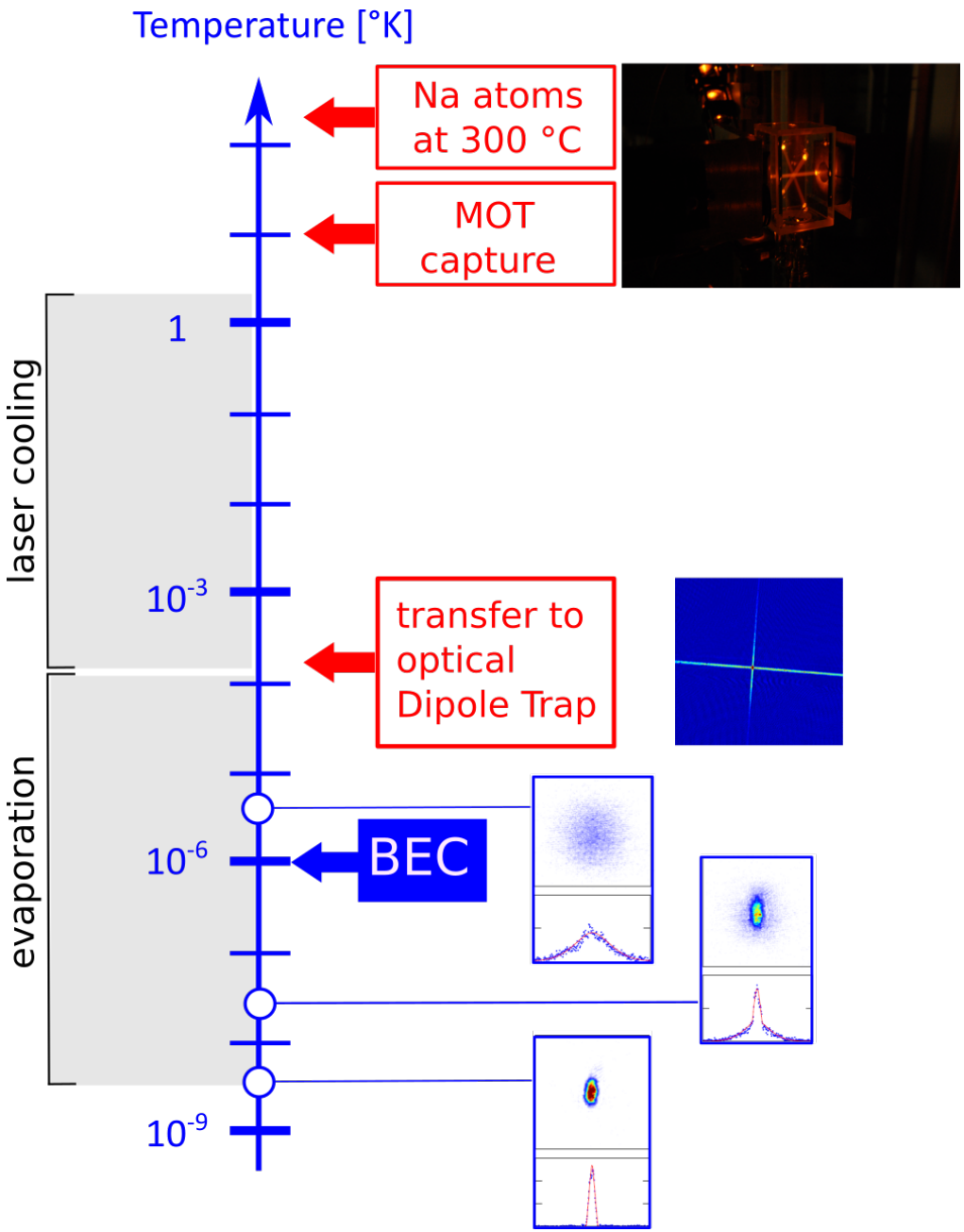
The experimental path to quantum degerate gases

Typical experimental sequence :

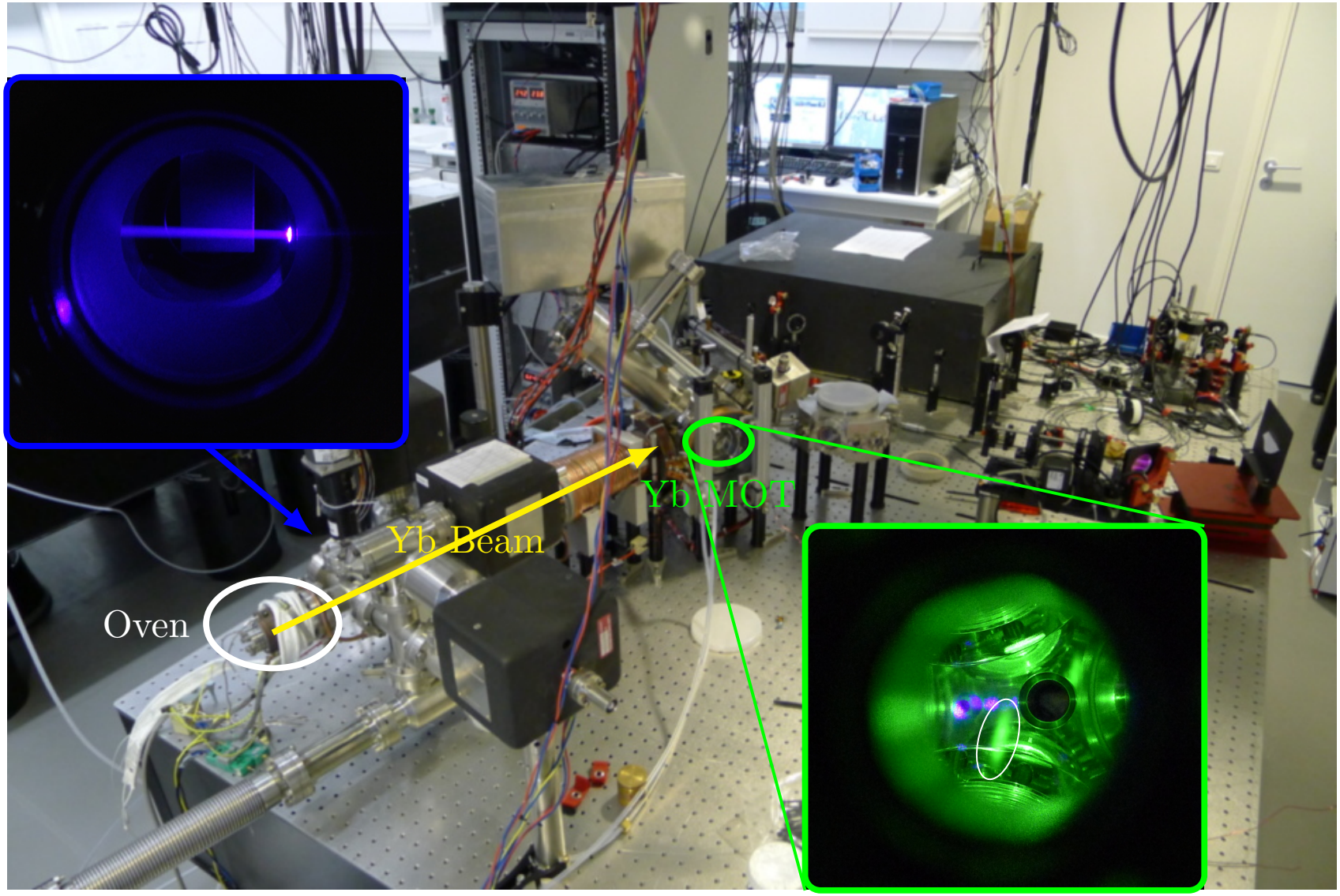
- catch atoms in a magneto-optical trap
- laser cooling to $\sim 50\,\mu$K
- transfer to conservative trap (no resonant light): optical trap or magnetic trap
- evaporative cooling to BEC

Take a picture of the cloud

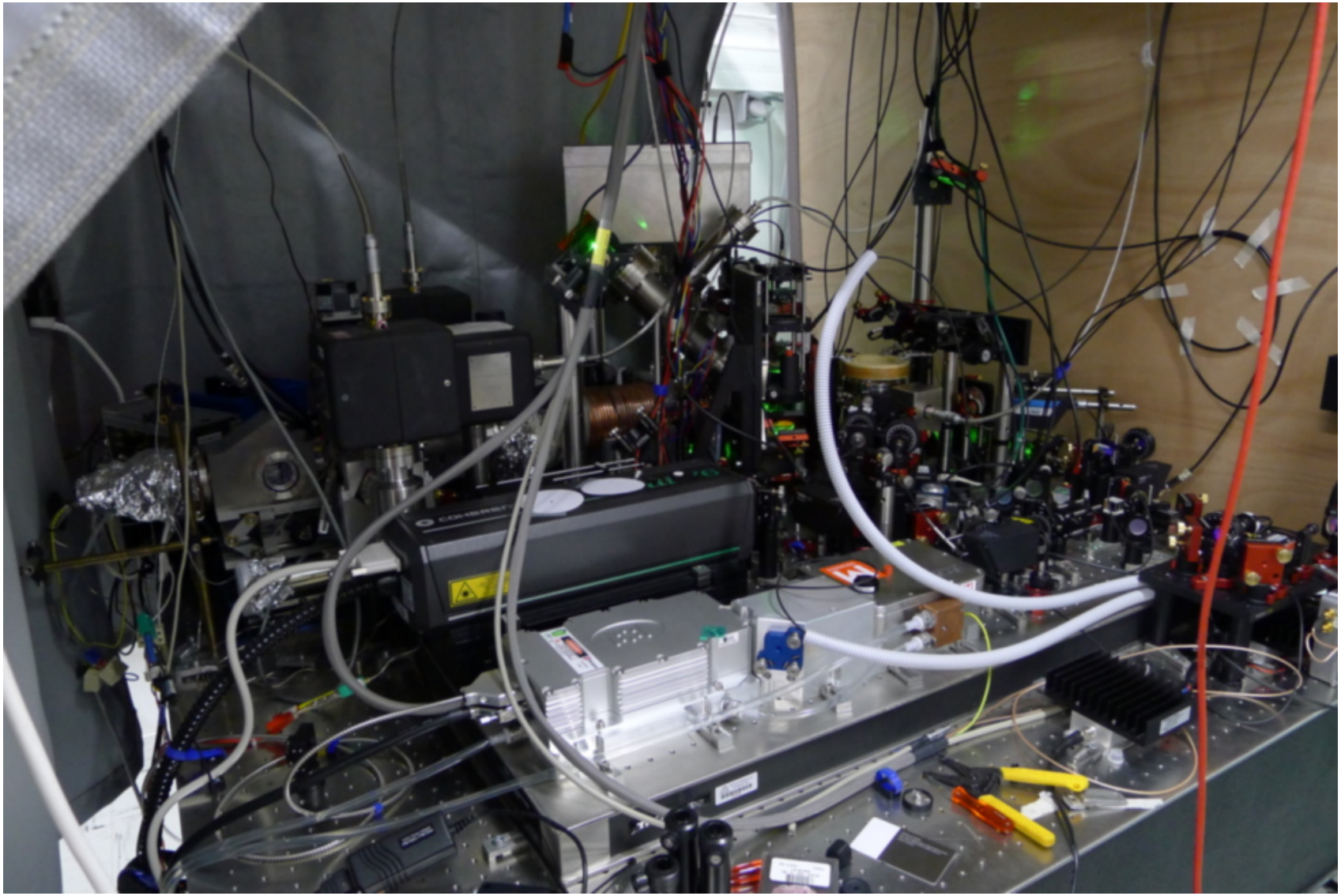
Repeat



Ytterbium BEC experiment at LKB



Ytterbium BEC experiment at LKB



Why is a two-step sequence necessary ?

Quantum gases : Phase-space density $\mathcal{D} = n\lambda_{\rm th}^3 \geq 1$

- Step 1: laser cooling in a magneto-optical trap
- Step 2 : evaporative cooling in a conservative trap

Laser cooling relies on the interaction between the atoms and a near-resonant laser. Spontaneous emission of photons is :

- essential to provide necessary dissipative mechanism to cool the motional degrees of freedom of the atoms,
- but also intrinsically random. This randomness prevents to cool the atoms below a certain limiting temperature !

Typical MOT of Yb :

$n \sim 10^{10} - 10^{11}\,\text{at/cm}^3, \qquad T \sim 10\,\mu\text{K}, \qquad \lambda_{\rm dB} \sim 40\,\text{nm}, \qquad \mathcal{D} \sim 10^{-6} - 10^{-5}$

To overcome the limitations of laser cooling, all experiments (with one exception) follow the same path :

- trapping in a *conservative trap*: optical trap or magnetic trap,
- evaporative cooling to quantum degeneracy.

1. Atom-photon interactions
   - Optical Bloch equations for a two-level atom
2. Radiative forces
   - Semi-classical theory
   - Radiation pressure; Zeeman slowing of an atomic beam
   - Optical molasses
   - Fluctuations of radiative forces : momentum diffusion
   - Sub-Doppler cooling
3. Imaging cold atoms
4. The magneto-optical trap
5. Optical dipole traps
6. Evaporative cooling

Photons :

Modes of the electomagnetic field, *e.g.* plane waves with

- momentum $\hbar \boldsymbol{k}_\mu$
- energy $\hbar\omega = \hbar c|\boldsymbol{k}_\mu|$
- polarisation $\boldsymbol{\epsilon_\mu} \perp \boldsymbol{k}_\mu$

Quantized electric field $\hat{\boldsymbol{E}}$

$$\hat{\boldsymbol{E}} = \sum_{\boldsymbol{k}_\mu, \boldsymbol{\epsilon}_\mu \perp \boldsymbol{k}_\mu} E_\mu \boldsymbol{\epsilon_\mu} \hat{a}_{\boldsymbol{k}_\mu, \boldsymbol{\epsilon}_\mu} + \text{ h.c.}$$

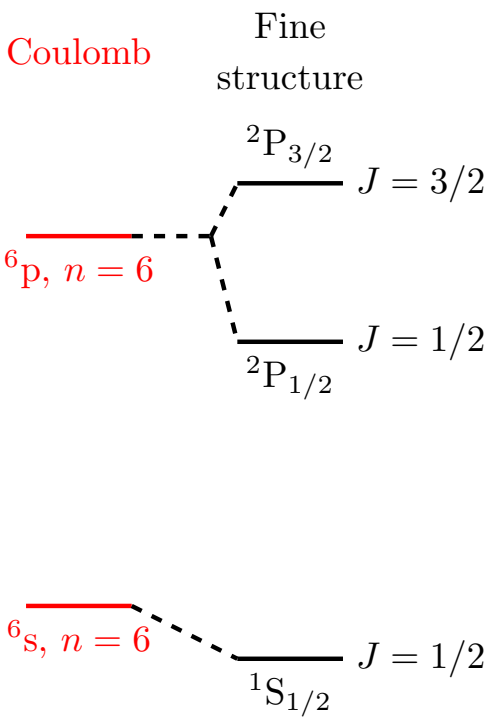
$$E_\mu = i\sqrt{\frac{\hbar c|\boldsymbol{k}_\mu|}{2\epsilon_0 L^3}} e^{i\boldsymbol{k}_\mu \cdot \boldsymbol{r}}$$

$\hat{a}_{\boldsymbol{k}_\mu, \boldsymbol{\epsilon}_\mu}$: annihilation operator of a photon in mode μ

Atoms :

- center-of mass variables
- internal variables, *e.g.* electronic angular momentum $|J, m_J\rangle$

Example : ^{133}Cs (or other alkali atoms)



Dipolar electric approximation :

$$\hat{V}_{A-\mathrm{em}} = -\hat{\boldsymbol{d}} \cdot \hat{\boldsymbol{E}}(\hat{\boldsymbol{R}})$$

- $\hat{\boldsymbol{d}}$: electric dipole of the atom (internal variables)
- $\hat{\boldsymbol{R}}$: center of mass of the atom (external variable)
- Approximation valid if atomic dimensions ($\sim$ Å) $\ll$ wavelength (UV, visible, IR, ...)
- Higher-order terms in the multipolar expansion smaller by a factor $\sim \alpha^2$

Electric dipole operator in the atomic basis:

$$\hat{\boldsymbol{d}} = \sum_{g_i, e_j} \boldsymbol{d}_{ij} |e_j\rangle\langle g_i| + \mathrm{h.c.}, \qquad \boldsymbol{d}_{ij} = \langle e_j|\hat{\boldsymbol{d}}|g_i\rangle$$

$\{|g_i\rangle\}_i, \{|e_j\rangle\}_j$ denotes the ground state and excited states manifolds (internal variables).

Dipolar electric Hamiltonian :

$$\hat{V}_{A-\mathrm{em}} = -\sum_{\boldsymbol{k}_\mu, \boldsymbol{\epsilon}_\mu \perp \boldsymbol{k}_\mu} \left(\hat{\boldsymbol{d}} \cdot \boldsymbol{\epsilon_\mu}\right) E_\mu \hat{a}_{\boldsymbol{k}_\mu, \boldsymbol{\epsilon}_\mu} + \mathrm{h.c.}$$

Consider a two level atoms with ground state g, excited state e, initially in a momentum eigenstate. We denote by $\omega_{eg} = \frac{E_e - E_g}{\hbar}$ the Bohr frequency of the transition.

Absorption of a photon $(\boldsymbol{q}, \boldsymbol{\epsilon})$: atomic state changes from $|g, \boldsymbol{k}_i\rangle$ to $|e, \boldsymbol{k}_f\rangle$

Conservation of momentum : $\boldsymbol{k}_f = \boldsymbol{k}_i + \boldsymbol{q}$

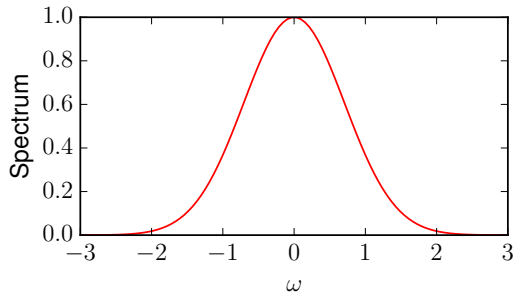
Conservation of energy : $\omega = \omega_{eg} + \frac{\hbar q^2}{2M} + \frac{\hbar}{M}\boldsymbol{q}\cdot\boldsymbol{k}_i$

$E_R = \frac{\hbar^2 q^2}{2M}$: recoil energy, $\quad \frac{\hbar}{M}\boldsymbol{q}\cdot\boldsymbol{k}_i = \boldsymbol{q}\cdot\boldsymbol{v}_i$: Doppler shift

$\omega_R = \frac{E_R}{\hbar}$: recoil frequency

Gas in thermal equilibrium : $P(\boldsymbol{v}) \propto e^{-\frac{M\boldsymbol{v}^2}{2k_B T}}$

Doppler broadening : spectral width $\Delta\omega = q\sqrt{\frac{8k_B T}{\pi M}}$



Laser polarisation $\boldsymbol{\varepsilon}$: $\boldsymbol{\varepsilon} \cdot \boldsymbol{k}_L = 0$

Decomposition in the standard basis $\boldsymbol{\varepsilon} = \sum_{q=0,\pm1} \varepsilon_q \boldsymbol{e}_q$

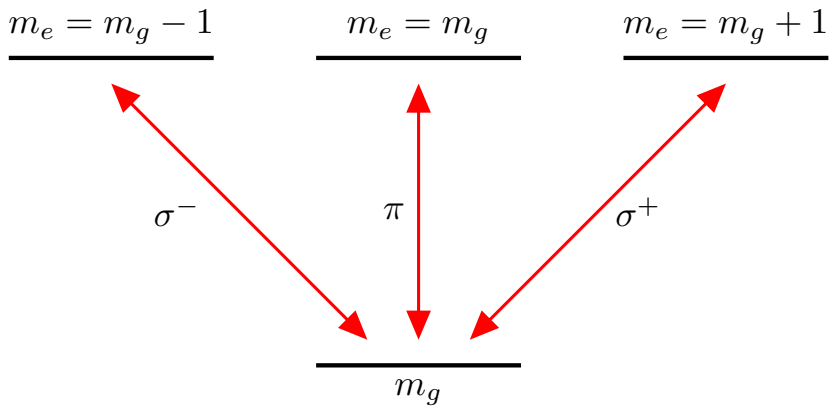
$$\boldsymbol{e}_+ = \frac{-1}{\sqrt{2}} (\boldsymbol{e}_x + i\boldsymbol{e}_y), \qquad \boldsymbol{e}_- = \frac{1}{\sqrt{2}i} (\boldsymbol{e}_x - i\boldsymbol{e}_y), \qquad , \boldsymbol{e}_0 = \boldsymbol{e}_z$$

$$\sigma_\pm\text{-polarized light: } \boldsymbol{\varepsilon} = \boldsymbol{e}_\pm, \qquad \pi\text{-polarized light: } \boldsymbol{\varepsilon} = \boldsymbol{e}_z$$

N.B. in atomic physics one usually chooses z along the direction of a static magnetic field (if any), not necessarily along the laser propagation direction $\boldsymbol{k}_L$.

Selection rules for allowed transitions :

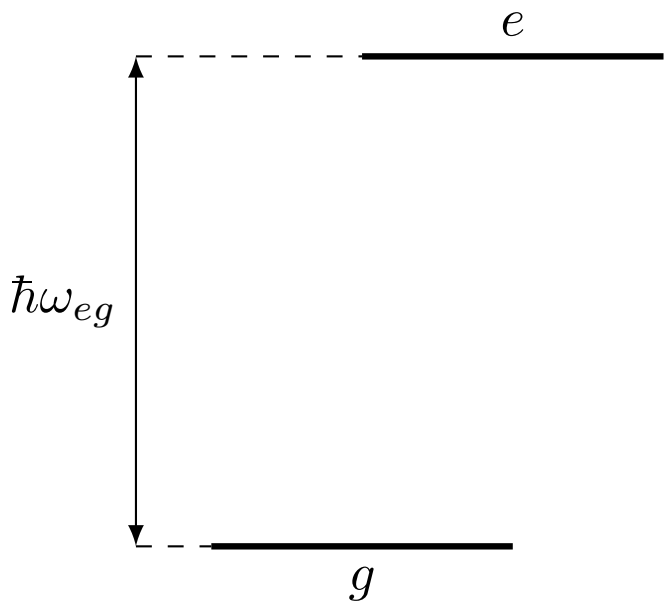
$$\Delta J = 0, \pm 1$$
$$\Delta m_J = 0, \pm 1$$



Atom in free space:

$$H_{\rm at} = \frac{\hat{\boldsymbol{P}}^2}{2M} + \hbar\omega_{eg}|e\rangle\langle e|$$

- center-of-mass momentum/position $\hat{P}/\hat{R}$
- two internal states g/e



- Bohr frequency $\omega_{eg} = \frac{E_e - E_g}{\hbar}$

Dipole operator :

$$\hat{d} = d\hat{D}^+ + d\hat{D}^-$$

$$\hat{D}^+ = |e\rangle\langle g| = \left(\hat{D}^-\right)^\dagger$$

Master equation for the atomic density operator

Atom and field are entangled : the proper quantum mechanical labeling of a state is $|\phi\rangle_{\text{atom}} \otimes |\alpha\rangle_L \otimes |\eta\rangle_{m\neq L}$. We need a purely atomic description that erases all information about the electromagnetic field, except for its effect on the atomic variables.

Density matrix formalism : $\hat{\sigma}$ describes the state of the composite atom+field system

- $\text{Tr}(\hat{\sigma}) = 1$
- $\text{Tr}(\hat{\sigma}^2) \leq 1$
- $\langle \hat{O} \rangle = \text{Tr}(\hat{\sigma}\hat{O})$
- $i\hbar \frac{d}{dt}\hat{\sigma} = [\hat{H}_{\text{atom+field}}, \hat{\sigma}]$

Reduced atomic density matrix :

$$\hat{\rho} = \text{Tr}_{\text{field}}\left[\hat{\sigma}\right] = \begin{pmatrix} \hat{\rho}_{gg} & \hat{\rho}_{ge} \\ \hat{\rho}_{eg} & \hat{\rho}_{ee} \end{pmatrix} : \text{ purely atomic operator}$$

N.B. $\hat{\rho}_{\alpha\beta} = \langle\alpha|\hat{\rho}|\beta\rangle$ is still an operator in general with respect to the external degrees of freedom of the atom.

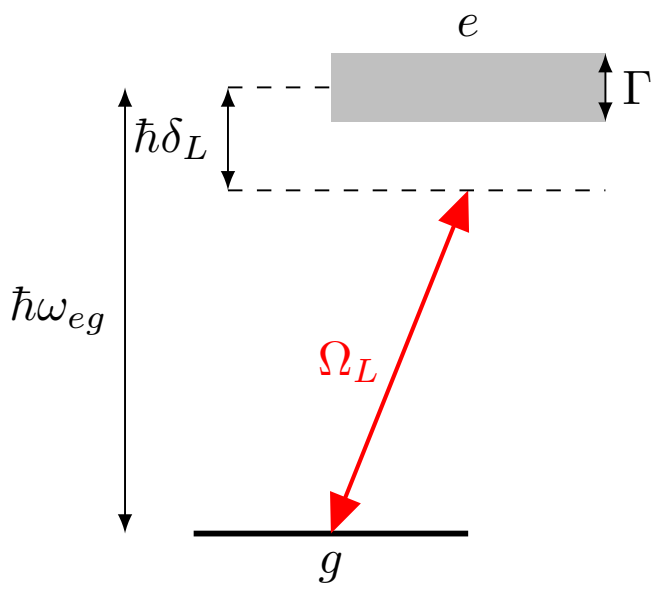
$$i\hbar \frac{d}{dt}\hat{\rho} = [\hat{H}_{\text{at}} + \hat{V}_{A-L}, \hat{\rho}] + \text{Tr}_{\text{field}}\left[\hat{V}_{A-V}, \hat{\sigma}_{\text{atom+field}}\right]$$

$\hat{V}_{A-L}$ corresponds to coherent driving by the classical Laser:
$\boldsymbol{E}_L = \boldsymbol{\epsilon}_L E_L \cos(\omega_L t + \varphi_L)$

$\hat{V}_{A-V}$ corresponds to the coupling to the electromagnetic field vacuum (spontaneous emission): Dissipation.

Optical Bloch equations :

$$\frac{d}{dt}\rho_{ee} = -\Gamma\rho_{ee} - \frac{\Omega_L}{2i}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$
$$\frac{d}{dt}\rho_{gg} = \Gamma\rho_{ee} + \frac{\Omega_L}{2i}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$
$$\frac{d}{dt}\tilde{\rho}_{eg} = \left(i\delta_L - \frac{\Gamma}{2}\right)\tilde{\rho}_{eg} + i\frac{\Omega_L}{2}\left(\rho_{ee} - \rho_{gg}\right)$$



We defined: $\tilde{\rho}_{ge} = e^{-i(\omega_L t+\varphi_L)}\rho_{ge}$, $\tilde{\rho}_{eg} = e^{i(\omega_L t+\varphi_L)}\rho_{eg}$.

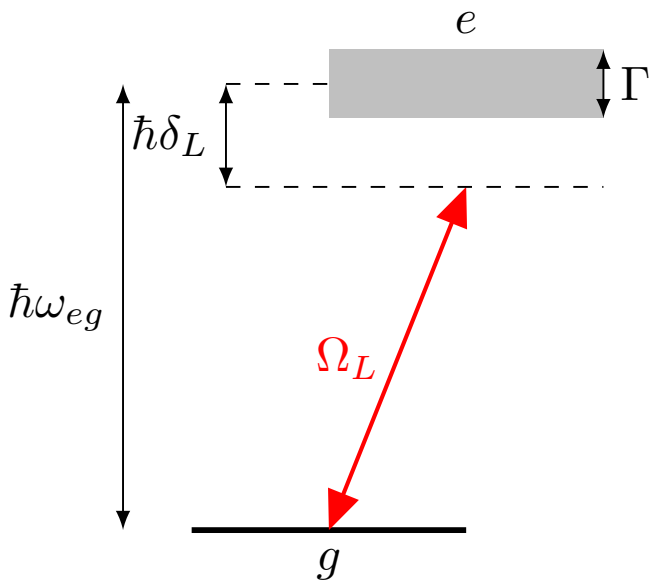
Natural linewidth of the transition: $\Gamma = \frac{\omega_{eg}^3 d^2}{6\pi\epsilon_0\hbar c^2}$

Laser electric field: $\boldsymbol{E}_L = \boldsymbol{\epsilon}_L E_L \cos(\omega_L t + \varphi_L)$

Rabi frequency: $\boldsymbol{\Omega}_L = \frac{dE_L}{2}$

Optical Bloch equations :

$$\frac{d}{dt}\rho_{ee} = -\Gamma\rho_{ee} - \frac{\Omega_L}{2i}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$
$$\frac{d}{dt}\rho_{gg} = \Gamma\rho_{ee} + \frac{\Omega_L}{2i}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$
$$\frac{d}{dt}\tilde{\rho}_{eg} = \left(i\delta_L - \frac{\Gamma}{2}\right)\tilde{\rho}_{eg} + i\frac{\Omega_L}{2}\left(\rho_{ee} - \rho_{gg}\right)$$



Define:

$$u = \frac{1}{2}\left(\tilde{\rho}_{ge} + \tilde{\rho}_{eg}\right),$$
$$v = \frac{1}{2i}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$

$$\dot{u} = -\frac{\Gamma}{2}u + \delta_L v,$$
$$\dot{v} = -\frac{\Gamma}{2}v - \delta_L u - \Omega_L\frac{\rho_{ee} - \rho_{gg}}{2}$$

Interpretation of u and v :

$$\langle\hat{d}\rangle = 2d\left[u\cos(\omega_L t + \varphi_L) - v\sin(\omega_L t + \varphi_L)\right]$$

- u : component of the average dipole in-phase with the field
- v : component of the average dipole out-of-phase with the field

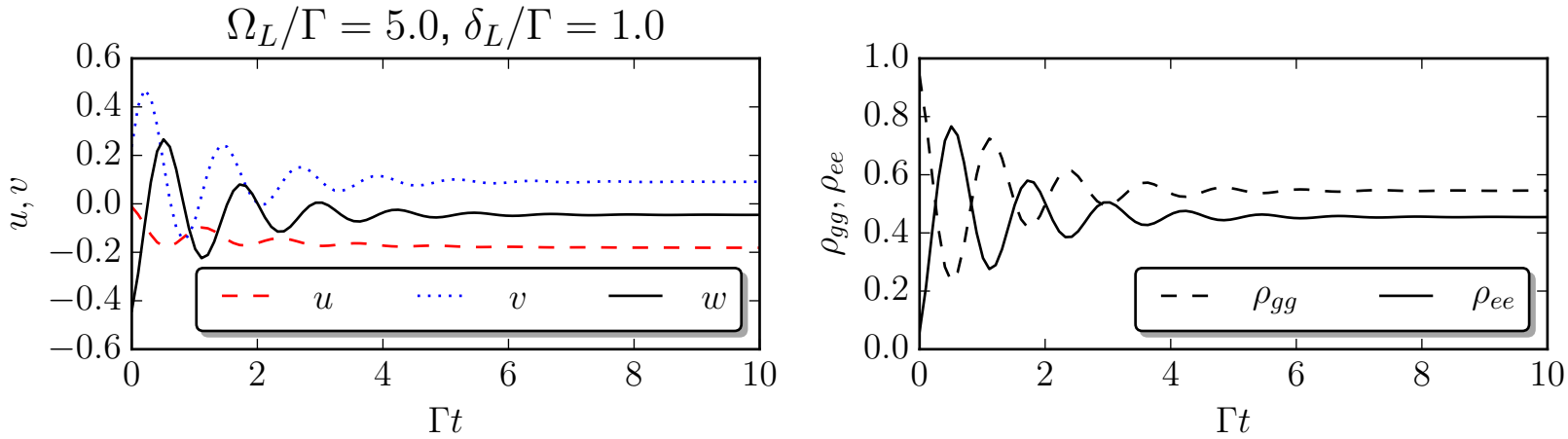
Stationary solutions :

$$u_{\mathrm{st}} = \frac{\delta_L}{\Omega_L}\frac{s}{1+s}$$
$$v_{\mathrm{st}} = \frac{\Gamma}{2\Omega_L}\frac{s}{1+s}$$
$$\rho_{ee,\mathrm{st}} = \frac{1}{2}\frac{s}{1+s}$$

Saturation parameter :

$$s = \frac{\frac{\Omega_L^2}{2}}{\delta_L^2 + \frac{\Gamma^2}{4}}$$

Numerical integration with $w(0) = -1/2$, $u(0) = v(0) = 0$:



Damped Rabi oscillations, damping to steady state after a few Γ^{-1}

Low and high intensity limits

Stationary solutions :

$$u_{\rm st} = \frac{\delta_L}{\Omega_L}\frac{s}{1+s}$$
$$v_{\rm st} = \frac{\Gamma}{2\Omega_L}\frac{s}{1+s}$$
$$\rho_{ee,\rm st} = \frac{1}{2}\frac{s}{1+s}$$

Saturation parameter :

$$s = \frac{\frac{\Omega_L^2}{2}}{\delta_L^2 + \frac{\Gamma^2}{4}}$$

High intensity $s \gg 1$: $\Omega_L^2 \gg \delta_L^2 + \frac{\Gamma^2}{4}$

$$u_{\rm st} \approx \frac{\delta_L}{\Omega_L} \to 0$$
$$v_{\rm st} \approx \frac{\Gamma}{2\Omega_L} \to 0$$
$$\rho_{ee,\rm st} \approx \frac{1}{2}$$

saturation of the resonant optical response of the atom

Low intensity $s \ll 1$: $\Omega_L^2 \ll \delta_L^2 + \frac{\Gamma^2}{4}$

$$u_{\rm st} \approx \frac{\delta_L}{\Omega_L} s$$
$$v_{\rm st} \approx \frac{\Gamma}{2\Omega_L} s$$
$$\rho_{ee,\rm st} \approx \frac{s}{2}$$

Aside: optical intensity and saturation intensity

Optical intensity :

$$I = \frac{\text{Optical power}}{\text{Area}} = \frac{\epsilon_0 c}{2} E_L^2,$$

$$\Omega_L^2 = \frac{2d^2 I}{\hbar^2 \epsilon_0 c}$$

Saturation intensity : I_{sat} defined such that $s = 1$ on resonance ($\delta_L = 0$)

$$I_{\text{sat}} = \frac{\hbar \Gamma c k_L^3}{12\pi} \sim \text{mW/cm}^2$$

$$\Omega_L^2 = \frac{\Gamma^2}{2} \frac{I}{I_{\text{sat}}}$$

$$s = \frac{\frac{\Omega_L^2}{2}}{\delta_L^2 + \frac{\Gamma^2}{4}} = \frac{\frac{I}{I_{\text{sat}}}}{\left(\frac{2\delta_L}{\Gamma}\right)^2 + 1},$$

$$\frac{s}{1+s} = \frac{\frac{I}{I_{\text{sat}}}}{\left(\frac{2\delta_L}{\Gamma}\right)^2 + 1 + \frac{I}{I_{\text{sat}}}}$$

Low intensity : linear susceptibility

Linear response to a harmonic perturbation: $\hat{H} = \hat{H}_0 - \hat{d}E_L \cos(\omega_L t + \phi_L)$

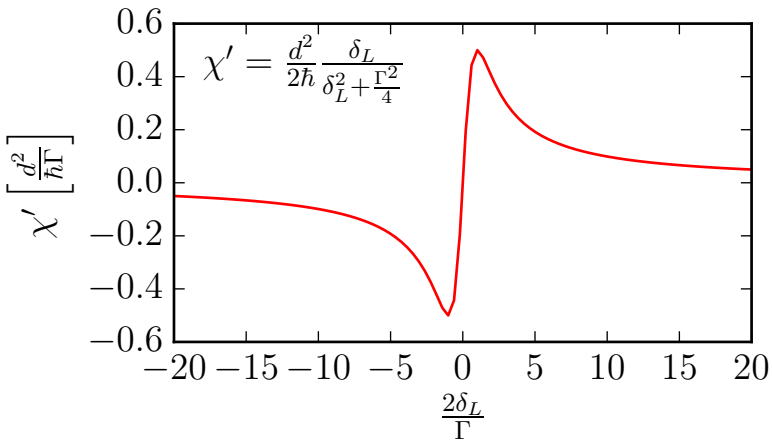
Complex susceptibility : $\chi = \chi' + i\chi''$

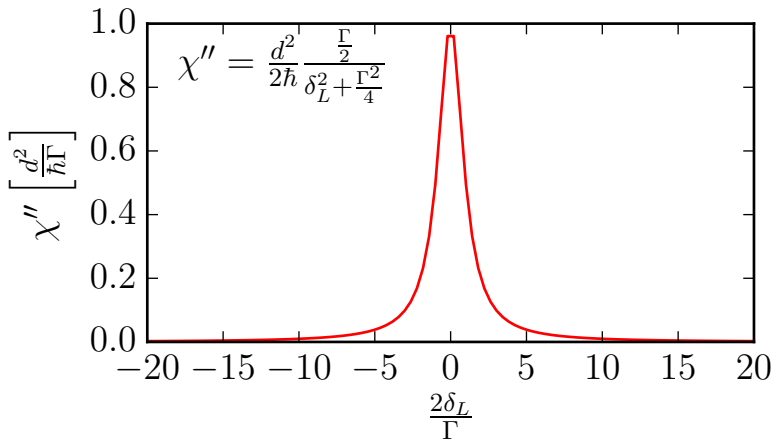
$$\begin{aligned}\langle \hat{d} \rangle &= \left(\chi E_L e^{i(\omega_L t + \phi_L)} + \chi^* E_L e^{-i(\omega_L t + \phi_L)} \right) \\ &= \underbrace{(\chi + \chi^*)}_{=2\chi'} E_L \cos(\omega_L t + \phi_L) - \underbrace{(\chi - \chi^*)}_{=2\chi''} E_L \sin(\omega_L t + \phi_L)\end{aligned}$$

Compare with : $\langle \hat{d} \rangle = 2d \left[u \cos(\omega_L t + \phi_L) - v \sin(\omega_L t + \phi_L) \right]$
In the limit where $s \ll 1$, we have

$$\chi' = \frac{d u_{\mathrm{st}}}{E_L} = \frac{d^2}{\hbar} \frac{\delta_L s}{\Omega_L^2} = \frac{d^2}{2\hbar} \frac{\delta_L}{\delta_L^2 + \frac{\Gamma^2}{4}}, \qquad \chi'' = \frac{d v_{\mathrm{st}}}{E_L} \approx \frac{d^2}{\hbar} \frac{\Gamma s}{2\Omega_L^2} = \frac{d^2}{2\hbar} \frac{\frac{\Gamma}{2}}{\delta_L^2 + \frac{\Gamma^2}{4}}$$

Linear response of a driven electric dipole.





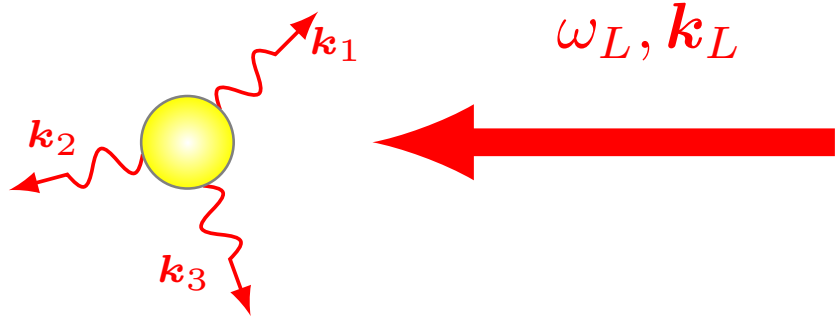
1. Atom-photon interactions
   - Optical Bloch equations for a two-level atom
2. Radiative forces
   - Semi-classical theory
   - Radiation pressure; Zeeman slowing of an atomic beam
   - Optical molasses
   - Fluctuations of radiative forces : momentum diffusion
   - Sub-Doppler cooling
3. Imaging cold atoms
4. The magneto-optical trap
5. Optical dipole traps
6. Evaporative cooling

Two-level atom irradiated by a near-resonant laser.

In a period of duration Δt, the atom undergoes $N_{\rm abs}$ fluorescence cycles (absorption followed by spontaneous emission).

Atomic momentum changes by

$$\Delta \boldsymbol{p}_{\rm at} = N_{\rm abs} \hbar \boldsymbol{k}_L - \sum_i \hbar \boldsymbol{k}_i^{\rm (sp)}$$



On average :

$$\overline{N_{\rm abs}} = \Gamma_{\rm sp} \Delta t$$

$$\sum_i \overline{\hbar \boldsymbol{k}_i^{\rm (sp)}} = 0$$

$\Gamma_{\rm sp}$: spontaneous emission rate = net absorption rate

Average force on the atom :

$$\overline{\boldsymbol{F}} = \frac{\Delta \boldsymbol{p}_{\rm at}}{\Delta t} = \Gamma_{\rm sp} \hbar \boldsymbol{k}_L$$

Very large forces! $\frac{F_{\rm max}}{M} \sim 10^5 - 10^6\, g$ for most atoms.

Atomic Hamiltonian :

$$\hat{H}_{\rm at} = \hbar\omega_{eg}|e\rangle\langle e| + \frac{\hat{\boldsymbol{P}}^2}{2M} + \hat{V}_{\text{A-L}}(\hat{\boldsymbol{R}}), \qquad \hat{V}_{\text{A-L}}(\hat{\boldsymbol{R}}) = -\hat{\boldsymbol{d}}\cdot\mathcal{E}_L(\hat{\boldsymbol{R}})$$

So far we have ignored the atomic center of mass position. How to treat it in a simple way ?

Ehrenfest theorem :

$$\frac{d}{dt}\langle\hat{\boldsymbol{R}}\rangle = \frac{1}{i\hbar}\left\langle[\hat{\boldsymbol{R}},\hat{H}]\right\rangle = \frac{\langle\hat{\boldsymbol{P}}\rangle}{M}$$
$$\frac{d}{dt}\langle\hat{\boldsymbol{P}}\rangle = \frac{1}{i\hbar}\left\langle[\hat{\boldsymbol{P}},\hat{H}]\right\rangle = -\left\langle\boldsymbol{\nabla}\hat{V}_{\text{A-L}}(\hat{\boldsymbol{R}})\right\rangle - \left\langle\boldsymbol{\nabla}\hat{V}_{\text{A-V}}(\hat{\boldsymbol{R}})\right\rangle$$

As seen from the qualitative description in the beginning, one has $\left\langle\boldsymbol{\nabla}\hat{V}_{\text{A-V}}(\hat{\boldsymbol{R}})\right\rangle$ (no net force from the electromagnetic vacuum). However, the mechanical contribution of the vacuum is NOT identically zero: only the mean force vanishes, not the fluctuations around the mean (to be discussed later).

Semi-classical approximation : replaces the full quantum dynamics of the atomic wavepacket by the trajectory defined by $\langle\hat{\boldsymbol{R}}\rangle, \langle\hat{\boldsymbol{P}}\rangle$:

$$\frac{d}{dt}\langle\hat{\boldsymbol{R}}\rangle = \frac{\langle\hat{\boldsymbol{P}}\rangle}{M}, \qquad \frac{d}{dt}\langle\hat{\boldsymbol{P}}\rangle = \boldsymbol{F}_{\rm rad}\left(\langle\hat{\boldsymbol{R}}\rangle\right) = \sum_{q=\sigma_\pm,\pi}\langle\hat{d}_q\rangle\boldsymbol{\nabla}\mathcal{E}_{L,q}\left(\langle\hat{\boldsymbol{R}}\rangle\right)$$

Validity of the semi-classical treatment: the broad line condition

Treat the atom as a wavepacket of extension Δx in real space and Δp in momentum space:

- well localized in real space if $\Delta x \ll \lambda_L$,
- well localized in momentum space if $\frac{k_L \Delta p}{M} \ll \Gamma$.

The second condition states that all relevant momentum components interact with the laser in the same manner.

The conditions are compatible if

$$\Delta x \Delta p \geq \hbar \leftrightarrow \Delta x \geq \frac{\hbar}{\Delta p} \gg \frac{\hbar k_L}{M\Gamma}$$

This leads to the broad line condition:

$$E_R = \frac{\hbar^2 k_L^2}{2M} \ll \hbar\Gamma$$

Separation of time scales :

- $\omega_R = E_R/\hbar$: typical rate of evolution of external variables,
- Γ : typical rate of evolution of internal variables.

The fast internal variables adapt almost instantaneously to the slower external ones.

Average radiative force: general expression for an atom at rest

We write the laser electric field as

$$\mathcal{E}_L(\boldsymbol{r}) = \boldsymbol{\varepsilon}_L E_L(\boldsymbol{r}) \cos[\omega_L t + \varphi_L(\boldsymbol{r})].$$

The laser polarization $\boldsymbol{\varepsilon}_L$ is assumed to be uniform (for now).
We found from the semi-classical assumption that:

$$\boldsymbol{F}_{\text{rad}}(\boldsymbol{r}) = \langle \hat{d} \rangle \left(\cos[\omega_L t + \varphi_L(\boldsymbol{r})] \, \boldsymbol{\nabla} E_L(\boldsymbol{r}) - E_L(\boldsymbol{r}) \sin[\omega_L t + \varphi_L(\boldsymbol{r})] \, \boldsymbol{\nabla} \varphi_L(\boldsymbol{r}) \right).$$

where $\boldsymbol{r} = \langle \hat{\boldsymbol{R}} \rangle$ and $\langle \hat{d} \rangle = \langle \hat{\boldsymbol{d}} \rangle \cdot \boldsymbol{\varepsilon}_L$.

Using the separation of time scales, the average dipole is given to a good approximation by the stationary solution of the optical Bloch equations,

$$\langle \hat{d} \rangle \approx \langle \hat{d} \rangle_{\text{st}} = 2d \left[u_{\text{st}} \cos(\omega_L t + \varphi_L) - v_{\text{st}} \sin(\omega_L t + \varphi_L) \right].$$

Putting everything together, one gets a mean radiative force (averaged in time over an optical period)

$$\boldsymbol{F}_{\text{rad}}(\boldsymbol{r}) = -d u_{\text{st}} \hbar \boldsymbol{\nabla} \Omega_L(\boldsymbol{r}) - d v_{\text{st}} \hbar \Omega_L(\boldsymbol{r}) \boldsymbol{\nabla} \phi_L$$

with the Rabi frequency $\hbar \Omega_L(\boldsymbol{r}) = -d E_L(\boldsymbol{r})$.

The two components of the mean radiative force are called

- **dipole force** $\propto u_{\text{st}}$: linked to the reactive response of the atom (dispersion), and to the gradient of amplitude,
- **radiation pressure** $\propto v_{\text{st}}$: linked to the dissipative response of the atom (absorption), and to the gradient of phase.

$$\boldsymbol{F}_{\rm rp}\left(\boldsymbol{r}\right) = -v_{\rm st}\hbar\Omega_L(\boldsymbol{r})\boldsymbol{\nabla}\phi_L, \qquad v_{\rm st} = \frac{\Gamma}{2\Omega_L}\frac{s}{1+s}$$

For a plane wave : $E_L = \text{cst}$, $\varphi_L = -\boldsymbol{k}_L \cdot \boldsymbol{r}$, and the radiation pressure force is given by

$$\boldsymbol{F}_{\rm rp}(\boldsymbol{r}) = \underbrace{\frac{\Gamma s}{2(1+s)}}_{=\Gamma(\rho_{ee})_{\rm st}} \hbar\boldsymbol{k}_L = \Gamma_{\rm sp}\hbar\boldsymbol{k}_L$$

We recover the same result obtained with the heuristic reasoning at the beginning, with the rate of spontaneous emission given by $\Gamma_{\rm sp} = \Gamma(\rho_{ee})_{\rm st}$.

Properties of the radiation pressure force :

- maximum on resonance
- saturates to $F_{\rm sat} = \frac{1}{2}\Gamma\hbar\boldsymbol{k}_L$ for high intensities
- very large ! $\frac{F_{\rm sat}}{M} \sim 10^5 - 10^6\, g$ for most atoms.

Detuning for an atom moving at velocity $\boldsymbol{v}$:

$$\delta_L = \omega_L - \omega_{eg} - \boldsymbol{k}_L \cdot \boldsymbol{v}$$

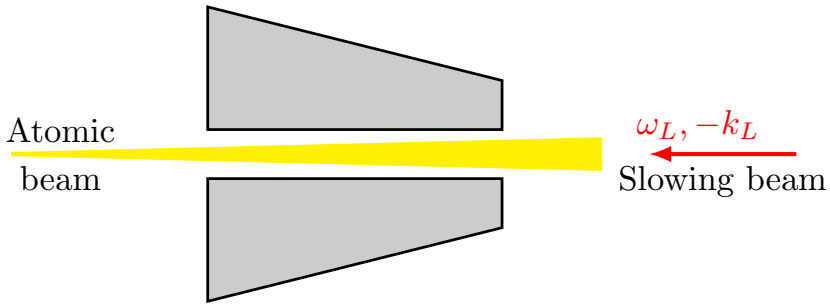
Issues arise when working with a beam with a thermal distribution :

- all velocity class in a thermal beam (Boltzmann distribution) are not addressed
- if atom with an initial velocity v_0 are decelerated, they move out of resonance due to the Doppler shift.

Solution:
Adjust detuning with an inhomogeneous magnetic field:

$$\delta_L = \omega_L - \omega_{eg} + k_L v_z + \mu B(z)$$

Use the Zeeman effect to counter the Doppler effect.
μ: difference of magnetic moments in the excited and ground state.



Phillips and Metcalf, Phys. Rev. Lett. 48, 596 (1982)

Design of the magnetic field profile : Maximum force, $F_{\max} = \frac{\Gamma \hbar \boldsymbol{k}_L}{2} \frac{s_0}{1+s_0}$, with s_0 the saturation parameter for $\delta_L = 0$.

$B(z)$ such that an atom entering with a velocity v_i remains on resonance as it is decelerated to $v_f \approx 0$ with a constant force $F_{\max}$.

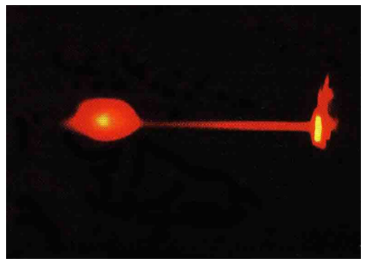
$$B(z) = \frac{\hbar \delta_L}{\mu} - \frac{\hbar k_L v_i}{\mu} \sqrt{1 - \frac{z}{L}}, L = \frac{v_i^2}{a_{\max}}$$

Optical molasses

First optical molasse demonstrated in 1985 (S. Chu's group, Bell labs)

3 pairs of counter-propagating beams to provide 3D cooling

- a few thousand Sodium atoms caught for about 1 second
- cooling transition in the visible ($\lambda_L = 589\,\text{nm}$): visible by naked eye !



S. Chu *et al.*, Phys. Rev. Lett. 55, 48 (1985)

Temperature measurement by time of flight :

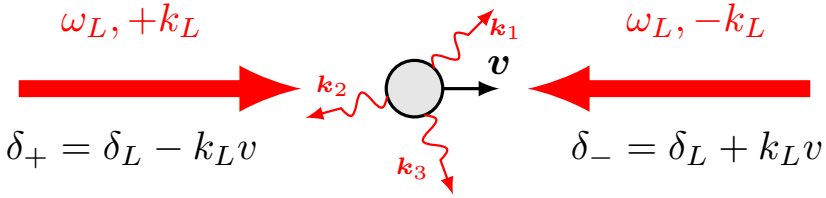
An atom initially at $\boldsymbol{r}_0$ with velocity $\boldsymbol{v}_0$ follows a ballistic trajectory, $\boldsymbol{r}(t) = \boldsymbol{r}_0 + \boldsymbol{v}_0 t$. At long time of flight, the density distribution reflects the initial velocity distribution.

Ballistic expansion, cloud radius : $R(t) = \sqrt{\frac{k_B T}{M}} t$

Result of the measurement : $T \approx 200\,\mu\text{K}$

Cooling configuration in one dimension :

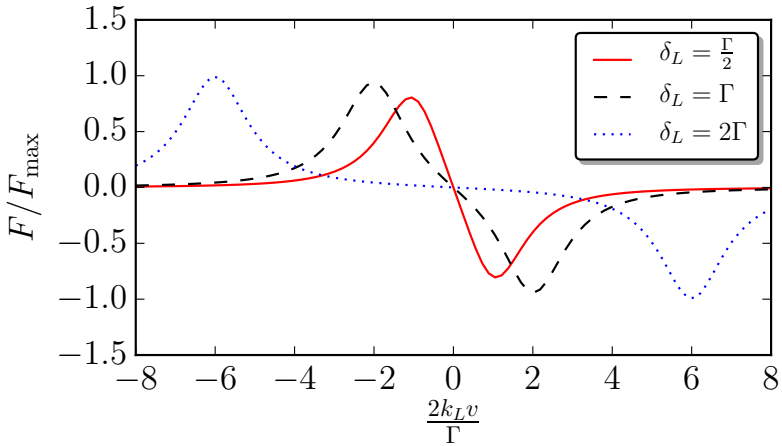
- Two counter-propagating beams (frequency ω_L, wavevector k_L)
- moving atoms with velocity $\boldsymbol{v}$



Weak saturation $s \ll 1$: $F_\pm \approx \frac{\Gamma\hbar k_L}{2} s_\pm = \frac{\Gamma\hbar k_L}{4} \frac{\Omega_L^2}{\delta_\pm^2 + \frac{\Gamma^2}{4}}$

In this limit (linear response regime), one can add independently the forces exerted by each beam to obtain a total *average* force

$$F_{\text{tot}} = F_+ + F_- = \frac{\Gamma\hbar k_L}{2} \frac{\Omega_L^2}{2} \left(\frac{1}{\delta_+^2 + \frac{\Gamma^2}{4}} - \frac{1}{\delta_-^2 + \frac{\Gamma^2}{4}} \right)$$



Maximum for $k_L v = |\delta_L|$,
$F_{\max} = \frac{\Gamma\hbar k_L}{2} \frac{I}{I_{\text{sat}}}$

For low velocities $k_L v \ll \Gamma$, we have

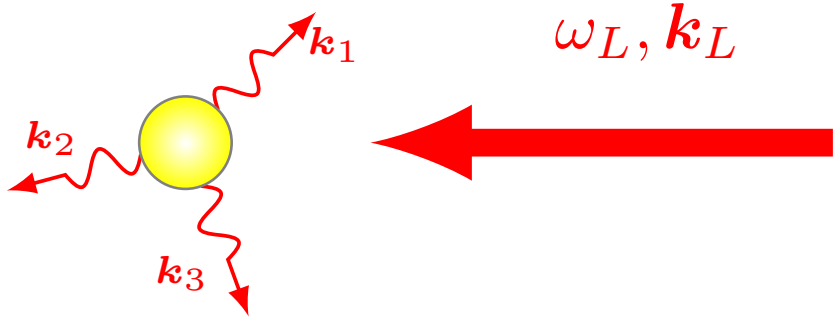
$$F_{\text{tot}} = -M\alpha v, \qquad \alpha = -\frac{\Gamma\hbar k_L^2}{M} \frac{2s\delta_L}{\delta_L^2 + \frac{\Gamma^2}{4}}$$

Viscous damping for $\delta_L < 0$ (red detuning), with a friction coefficient α (units: T^{-1})

So far we have computed the *average* radiative force $F_{\rm rad}$ exerted by the laser on the atom. Fluctuations around these average are also present (due to the stochastic character of the atom-light interaction). Recall our basic picture :

Atomic momentum changes by

$$\Delta \boldsymbol{p}_{\rm at} = N_{\rm abs} \hbar \boldsymbol{k}_L - \sum_i \hbar \boldsymbol{k}_i^{(\rm sp)}$$



The radiation pressure force fluctuates around its average $\boldsymbol{F}_{\rm rp}$ because of two factors :

- random momentum kicks due to the random direction of emission,
- random number of fluorescence cycles during Δt.

Equation of motion of a Brownian particle submitted to a deterministic force $F_{\rm rad}$ and to a fluctuating force $\delta F(t)$. We assume here that the fluctuating part has a short memory time, and is characterized by a correlation function

$$\langle \delta F(t')\delta F(t'')\rangle = 2D_p\delta(t'-t'').$$

The equation of motion of the Brownian particle is

$$M\ddot{x} = F_{\rm rad} + \delta F(t) \implies v(t) = v(0) + \frac{1}{M}\int_0^t dt' \left(F_{\rm rad}(t') + \frac{1}{M}\delta F(t')\right)$$

Averaging over many realizations of the fluctuating force ($\langle \delta F(t)\rangle = 0$):

$$\langle v\rangle = v(0) + \frac{1}{M}\int_0^t dt', F_{\rm rad}(t')$$

$$\Delta v^2 = \langle v^2\rangle - \langle v\rangle^2 = \frac{1}{M^2}\int_0^t dt' \int_0^t dt'' \langle \delta F(t')\delta F(t'')\rangle$$

This gives

$$\Delta p^2 = M^2\Delta v^2 = 2D_p t$$

Fluctuations of the force lead to a brownian motion in momentum space characterized by a diffusion coefficient D_p.

NB: $2D_p = \int d\tau \langle \delta F(t+\tau)\delta F(t)\rangle$ if δF stationary random process.

Fluctuations of the radiation pressure :

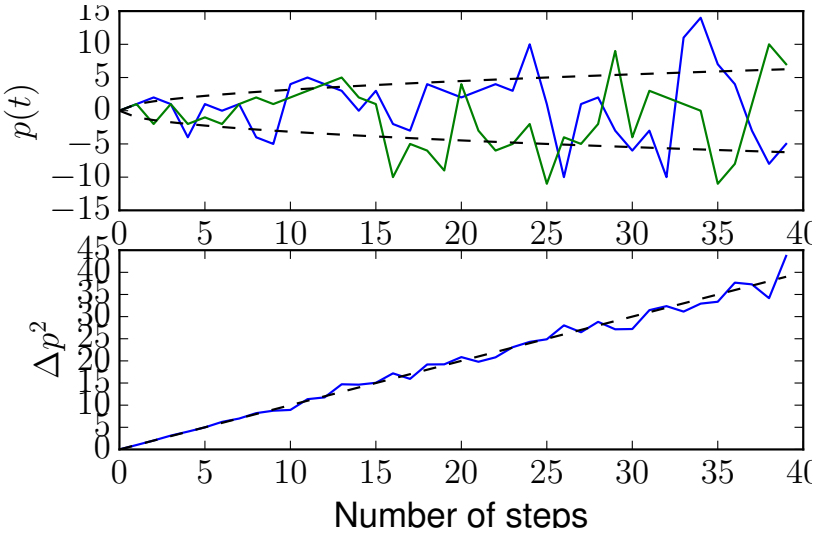
Random momentum kicks for each spontaneous emission.

Photons spontaneously emitted at a rate $\Gamma_{\rm sp} = \Gamma(\rho_{ee})_{\rm st}$.

Random walk of step size $\pm\hbar k_L$ in momentum space.

$$\left\langle \frac{\Delta p^2}{(\hbar k_L)^2} \right\rangle = N_{\rm step}$$

On average: $N_{\rm step} = \Gamma_{\rm sp}\Delta t$ steps per time interval $\Delta t \gg \Gamma^{-1}$



$$\Delta p^2 = (\hbar k_L)^2 \Gamma_{\rm sp} \Delta t = 2 \underbrace{\frac{\Gamma(\hbar k_L)^2}{4} \frac{s}{1+s}}_{=(Dp)_{\rm rec}} \Delta t$$

Fluctuations of the radiation pressure :

Random walk of step size $\pm\hbar k_L$ in momentum space, with (on average) $\Gamma_{\rm sp}\Delta t$ steps per time interval $\Delta t \gg \Gamma^{-1}$

$$(Dp)_{\rm rec} = \frac{\Gamma(\hbar k_L)^2}{4}\frac{s}{1+s}$$

Fluctuations of the number of fluorescence cycles per time interval :

Fluctuations in the number of absorbed photons during Δt.

Poisson statistics : $\Delta N_{\rm abs} = \overline{N}_{\rm abs} = \Gamma(\rho_{ee})_{\rm st}\Delta t$, $\Delta p^2 = \Delta N_{\rm abs}(\hbar k_L)^2$, so

$$(Dp)_{\rm abs} = \frac{\Gamma(\hbar k_L)^2}{4}\frac{s}{1+s}$$

NB: The probability distribution of the incident photons is Poissonian. However there are corrections in resonance fluorescence, expressed by the so-called *Mandel Q-factor* such that $\Delta N_{\rm abs} = \overline{N}_{\rm abs} = (1+Q)$. For resonant fluorescence, $Q < 0$: after absorbing a photon, an atom cannot absorb another one for a time $\sim \Gamma^{-1}$ (*i.e.* until the first photon has been reemitted).

Fluctuations of the radiation pressure :

Random walk of step size $\pm\hbar k_L$ in momentum space, with (on average) $\Gamma_{\rm sp}\Delta t$ steps per time interval $\Delta t \gg \Gamma^{-1}$

$$(Dp)_{\rm rec} = \frac{\Gamma(\hbar k_L)^2}{4}\frac{s}{1+s}$$

Fluctuations of the number of fluorescence cycles per time interval :

Fluctuations in the number of absorbed photons during Δt.

$$(Dp)_{\rm abs} = (Dp)_{\rm rec} = \frac{\Gamma(\hbar k_L)^2}{4}\frac{s}{1+s}$$

Momentum diffusion coefficient:

$$D_p = (Dp)_{\rm rec} + (Dp)_{\rm abs} = \frac{\Gamma(\hbar k_L)^2}{2}\frac{s}{1+s}$$

Optical molasses steady-state result from a balance between the cooling provided by the mean force and the heating (momentum diffusion) associated with the fluctuations.

Langevin equation :

$$\dot{v} = -\alpha v + \frac{\delta F(t)}{M} \implies v(t) = v(0)e^{-\alpha t} + \frac{1}{M}\int_0^t \delta F(t')e^{-\alpha(t-t')}dt'$$

Average over different realizations of the fluctuating force :

$$\langle v \rangle = v(0)e^{-\alpha t}$$

$$\Delta v^2 = \langle v^2 \rangle - v(0)^2 e^{-2\alpha t} = \frac{1}{M^2}\int_0^t dt' \int_0^t dt'' \langle \delta F(t')\delta F(t'')\rangle e^{-\alpha(2t-t'-t'')}$$

Short memory : $\langle \delta F(t')\delta F(t'')\rangle = 2D_p\delta(t'-t'')$

$$\Delta v^2 = \frac{D_p}{M^2\alpha}\left(1-\exp^{-2\alpha t}\right) = \begin{cases} \frac{2D_p t}{M^2}, & t \ll \alpha^{-1} \\ \\ \frac{D_p}{M^2\alpha}, & t \gg \alpha^{-1} \end{cases}$$

We define a temperature (in the long time limit) by

$$\frac{k_B T}{2} = \frac{1}{2} M \langle \Delta v^2 \rangle \implies k_B T = \frac{D_p}{M\alpha} \quad \text{Einstein's relation}$$

$$\frac{D_p}{M} = \frac{\Gamma s (\hbar k_L)^2}{2M}$$

$$s \ll 1$$

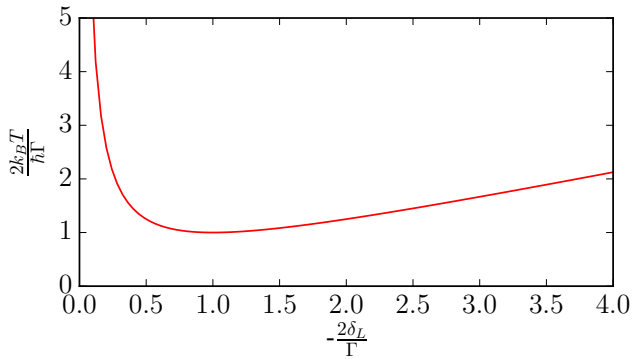
$$\alpha = \frac{\Gamma s (\hbar k_L)^2}{M} \frac{2|\delta_L|}{\delta_L^2 + \frac{\Gamma^2}{4}}$$

$$\delta_L < 0$$

Doppler temperature :

$$k_B T_{\mathrm{D}} = \frac{\hbar\Gamma}{4}\left(\frac{2|\delta_L|}{\Gamma} + \frac{\Gamma}{2|\delta_L|}\right)$$

Minimum for $\delta_L = -\frac{\Gamma}{2}$, $k_B T_{\mathrm{D,min}} = \frac{\hbar\Gamma}{2}$.



NB: More sophisticated approaches (Fokker-Planck equation for the atomic distribution) show that the velocity distribution is Gaussian, as expected for a Maxwell-Boltzmann distribution.

First optical molasses at Bell Labs

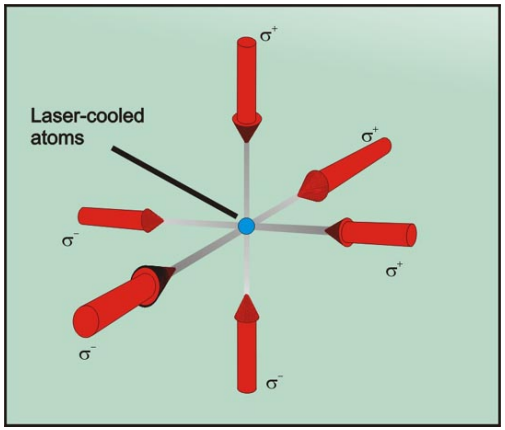
Results from S. Chu's group:

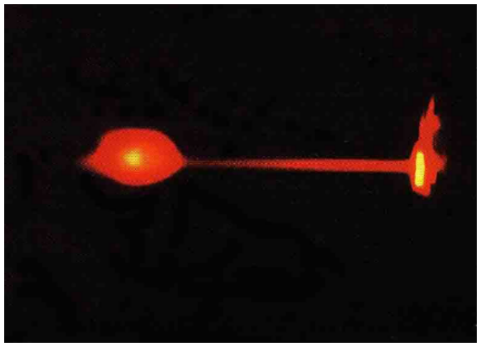
Sodium atom : $\lambda_L = 589\,\text{nm}$,
$\Gamma = 2\pi \times 10\,\text{MHz}$

$\frac{E_r}{h} \approx 25\,\text{KHz}$: broad line condition fulfilled

- A few thousand Sodium atoms caught for about 1 second:
  $D_x = \frac{k_B T_D}{M\alpha} \approx 2\,\text{mm}^2/\text{s}$ $(\frac{I}{I_{\text{sat}}} = 0.1)$

  Beam size $w \sim 1\,\text{cm} \implies$ Catching time $\sim \frac{w}{\sqrt{D_x}} \sim 6\,\text{s}$





S. Chu *et al.*, Phys. Rev. Lett. 55, 48 (1985)

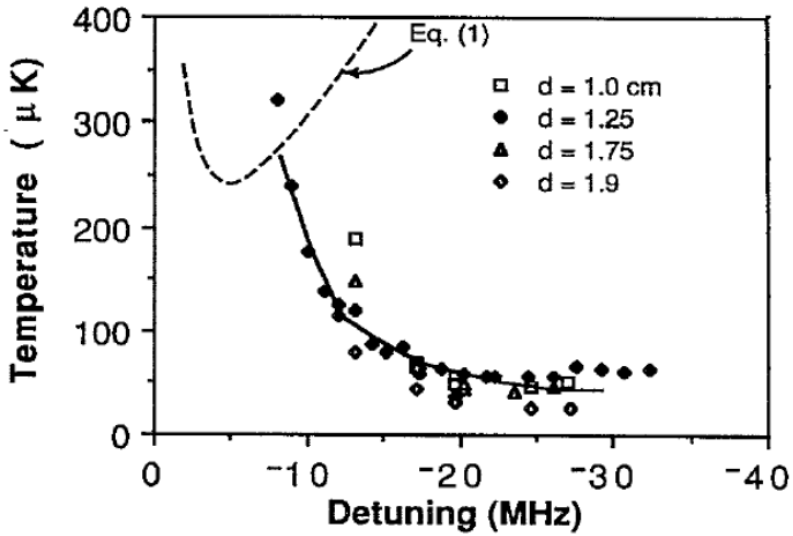
$\alpha^{-1} \approx 30\,\mu\text{s}$ for $\delta_L = -\Gamma/2$, $I = 0.1 I_{\text{sat}}$

- Experimental temperature: $T \approx 200\,\mu\text{K}$

  Doppler theory : $T_D \approx 200\,\mu\text{K}$

Experimental results for Sodium atoms

A couple of years after the first molasses, more careful measurements (Phillips' group at NIST) show that atoms in optical molasses are much colder than expected from Doppler theory !



P. Lett *et al.*, PRL (1998)

Dashed line : Doppler theory

Measurements done on Sodium atoms
$\Gamma = 2\pi \times 10\,\mathrm{MHz}$

- wrong minimal temperature
- wrong dependence with detuning

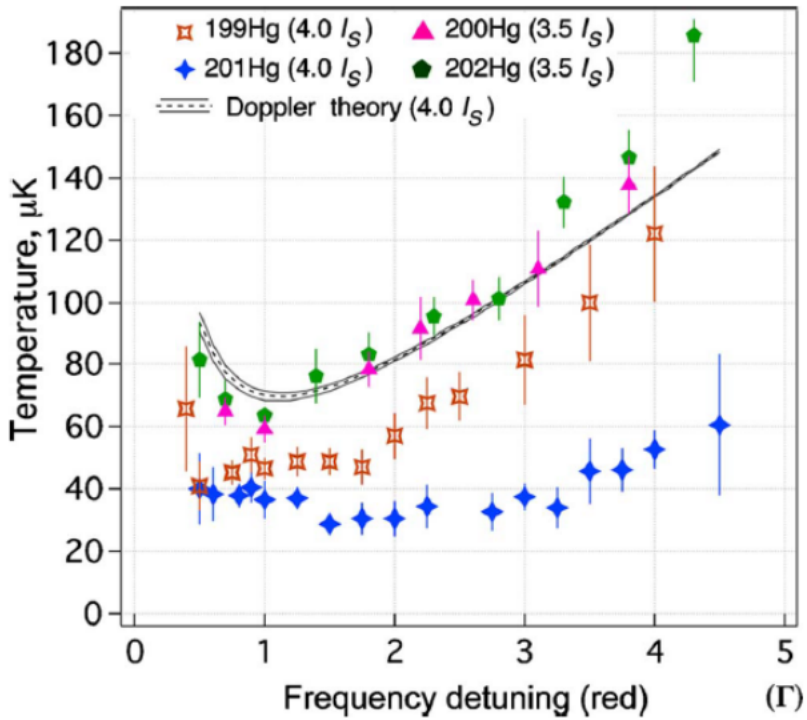
Doppler theory works fine for atoms with a simple internal structure ($J = 0 \to J = 1$).

It does not work for alkali atoms because of additional effects linked with their more complex internal structure.

Experimental results for Mercury atoms

$\Gamma = 2\pi \times 1.3\,\mathrm{MHz},\ \lambda_{eg} = 254\,\mathrm{nm}$

Shaded curve: Doppler theory (saturation included heuristically)



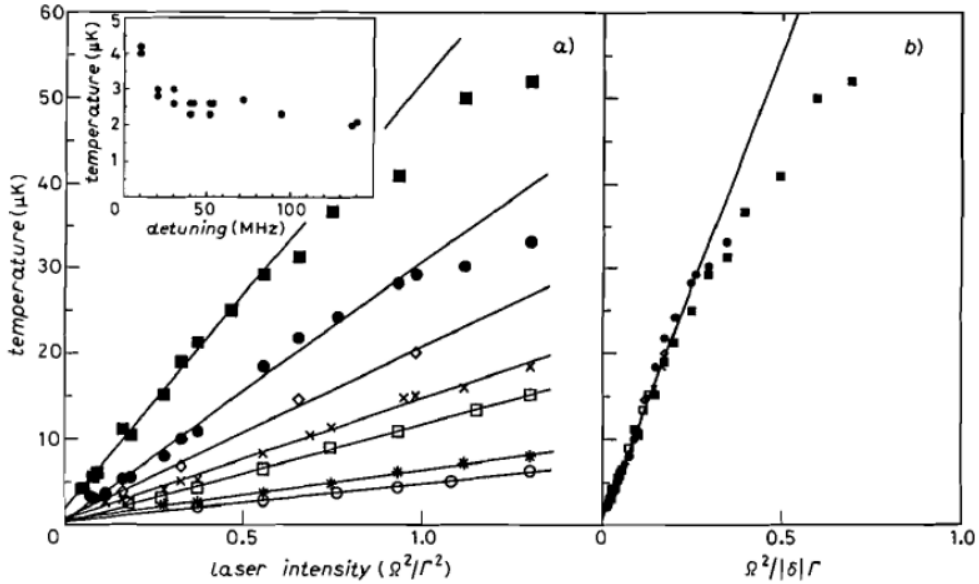
McFerran *et al.*, Optics letters 35 (18), 3078-3080 (2010)

Temperature:

$$T_{\mathrm{SD}} \geq \# E_R,$$

$\#$ is a pure number, frequently a few tens ($k_B T \sim 3\,\mu$K for the above experiment).

$$T_{\mathrm{SD}} \propto \frac{\hbar \Omega_L^2}{|\delta_L|},$$

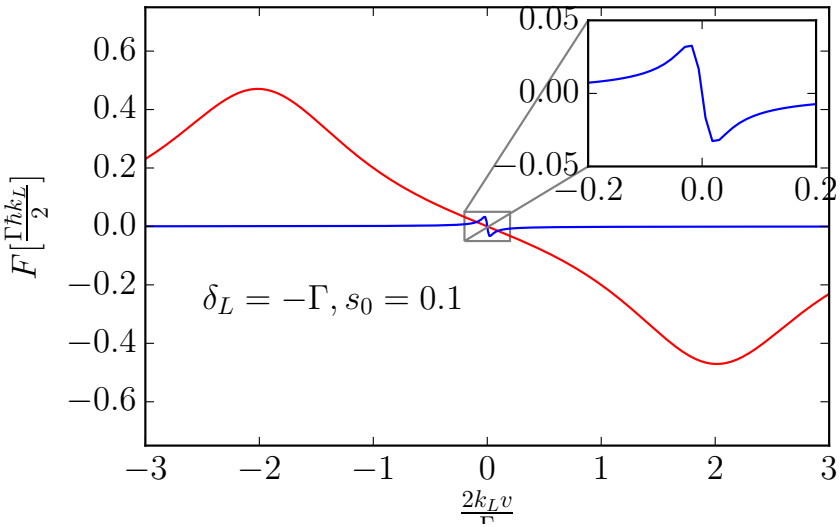


[Salomon *et al.*, EuroPhysics Letters 1990]

For Cesium atoms : $T_D \approx 130\,\mu$K, $E_R/k_B \approx 100\,$nK, $T_{\mathrm{SD}} \approx 6\,\mu$K for $\delta_L = \Gamma$, $s_0 = 0.1$

Explained by the interplay between the polarization of light and the internal structure of the atoms.

Conclusion on laser cooling



Laser cooling : two successive mechanisms leading to ultralow temperatures

- Doppler cooling : catches atoms around $v \sim \frac{\Gamma}{2k_L}$, cools atoms down to T_{D},
- Polarization gradient cooling : leads to sub-Doppler temperatures (several recoil temperatures in practice), active around $v \sim \frac{\Gamma s_0}{2k_L}$.

Only Doppler cooling for atoms lacking an internal structure (alkaline-earth atoms, like Sr, Hg, Ca; or Yb).

No trapping in optical molasses: atoms slowly diffuse away from the trapping region (intersection volume of all beams in a real, 3D configuration)

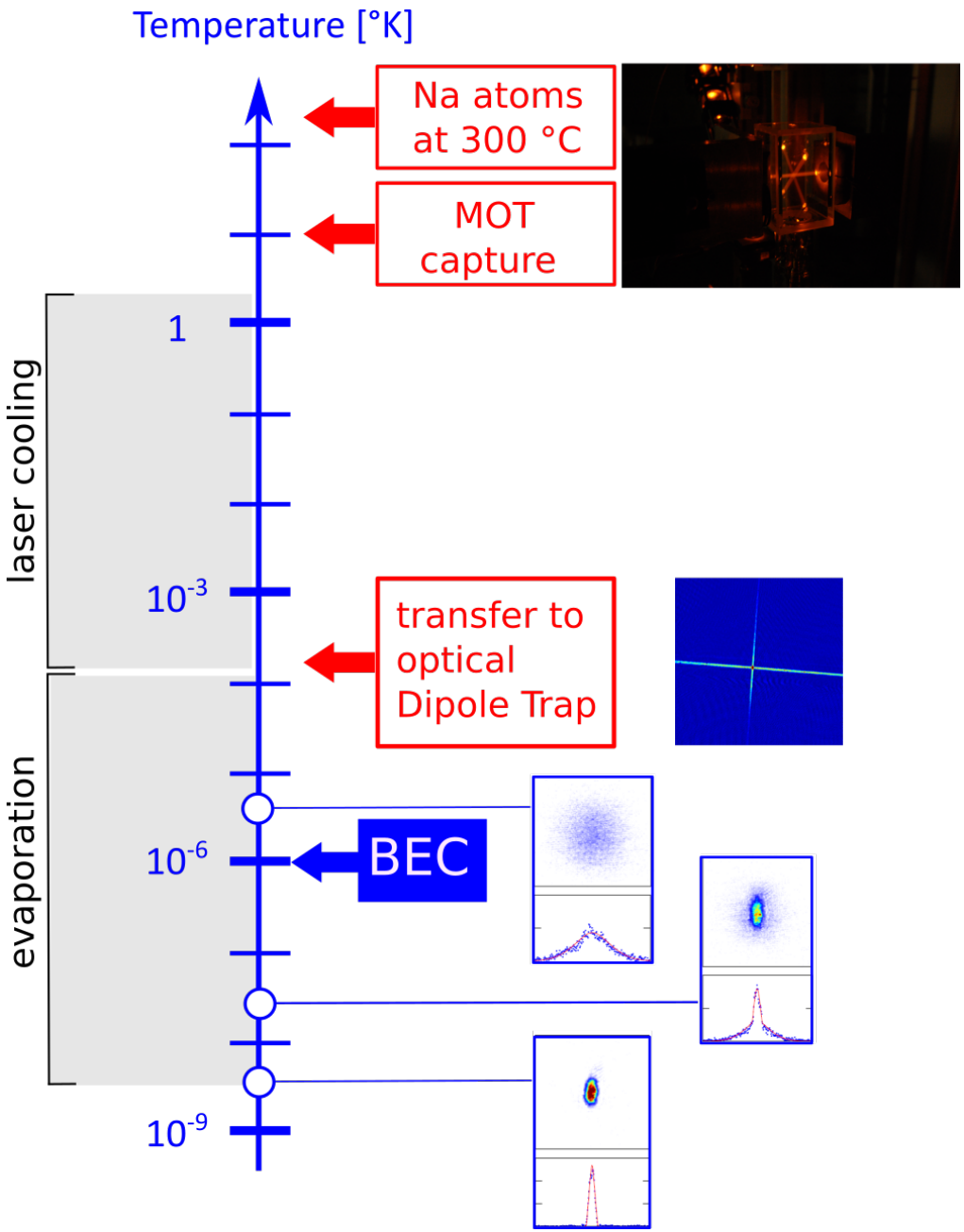
The experimental path to quantum degerate gases

Typical experimental sequence :

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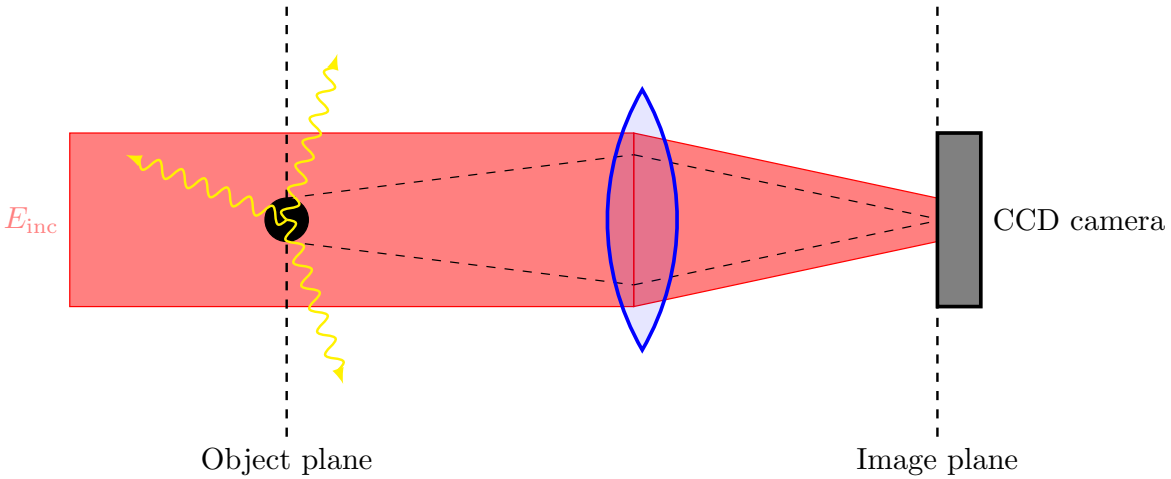
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Basic setup :



Camera placed in an image plane optically conjugate to the "plane of the atoms"
What are the properties of the transmitted light ?

What informations on the trapped atoms are recorded on the camera ?

Propagation in a dielectric medium :

$$\boldsymbol{\nabla}^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \boldsymbol{P}}{\partial t^2}$$

Electric polarization density:

$$\boldsymbol{P} = \epsilon_0 \chi \boldsymbol{E} = n_{\mathrm{at}} \langle \hat{\boldsymbol{d}} \rangle$$

$\chi = \chi' + i\chi''$: dipole susceptibility, $n_{\mathrm{at}}(\boldsymbol{r})$: atomic density

Writing $E(\boldsymbol{r}, t) = \frac{1}{2}\mathcal{E} e^{i\varphi_L} e^{i(k_L z - \omega_L t)} + \mathrm{c.c.}$, and invoking a slowly-varying envelope approximation for $\mathcal{E}$ and ϕ_L (terms $\propto \Delta\mathcal{E}, \Delta\varphi_L$ neglected) :

$$\frac{d\mathcal{E}}{dz} = -\frac{k_L n_{\mathrm{at}}}{2}\chi''\mathcal{E}(z), \ : \text{Beer-Lambert law}$$

$$\frac{d\varphi_L}{dz} = \frac{k_L n_{\mathrm{at}}}{2}\chi'\varphi_L(z) : \text{dephasing}$$

NB : the atomic density must also vary smoothly *along* z on the scale λ_L.

We rewrite the equation for $\mathcal{E}$ in terms of the laser intensity: $I = \frac{\epsilon_0 c}{2}|\mathcal{E}|^2$

$$\frac{dI}{dz} = -\kappa I(z) : \text{Beer-Lambert law}, \qquad \kappa = \frac{k_L n_{\text{at}}}{2}\chi'' = \frac{3\lambda_L^2}{2\pi}\frac{n_{\text{at}}}{\left(\frac{2\delta_L}{\Gamma}\right)^2 + 1}$$

Interpretation in terms of scattered photons :

$d(\text{Photon flux}) = -\left(n_{\text{at}} dz dA\right)\sigma \times (\text{Photon flux})$, with a scattering cross-section

$$\sigma = \frac{\sigma_0}{\left(\frac{2\delta_L}{\Gamma}\right)^2 + 1}, \quad \sigma_0 = \frac{3\lambda_L^2}{2\pi}, \quad \kappa = \sigma n_{\text{at}}$$

Maximum on resonance ($\sigma = \sigma_0$ for $\delta_L \approx 0$).

The intensity on the camera (assuming that the focal depth of the imaging system is $\gg$ cloud size) gives a magnified version of the transmitted intensity,

$$I_t(x,y) = I_{\text{inc}}(x,y)e^{-\sigma \int n_{\text{at}}(x,y,z)dz}$$

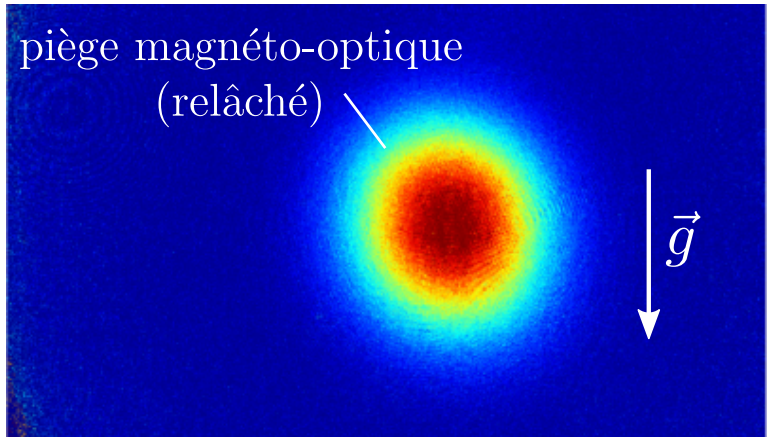
One calls $\tilde{n}(x,y) = \int n_{\text{at}}(x,y,z)dz$ the *column density* and $OD(x,y) = \sigma\tilde{n}(x,y)$ the *optical depth*.

Absorption signal :

$$\tilde{n}(x,y) = \frac{1}{\sigma}\ln\left(\frac{I_{\text{inc}}(x,y)}{I_t(x,y)}\right)$$

Absorption imaging of a MOT

Absorption image of a MOT of Yb atoms after a time of flight of $t \sim 10\,\text{ms}$, during which atom are released from the trap and expand freely.



Why do we need to take pictures after a time of flight ?

- To reduce absorption. Often the optical depth is too large: $OD \sim 5 - 10$ for a MOT, $OD \sim 100$ or more for a BEC. Images are "pitch black" and dominated by noise.
- To avoid photon reabsorption and multiple scattering, which makes the above description invalid and the interpretation of images difficult.

In-situ imaging are also possible, though easier with dispersive techniques than with absorption imaging.

Time-of-flight (t.o.f.) experiment :

- suddenly switch off the trap potential at $t = 0$,
- let the cloud expand for a time t.

The spatial distribution for long times is proportional to the initial *momentum distribution* $\mathcal{P}_0$ evaluated at $\boldsymbol{p} = \frac{M\boldsymbol{r}}{t}$:

$$\boxed{n_{\rm at}(\boldsymbol{r}, t) \longrightarrow_{t\to\infty} \left(\frac{M}{t}\right)^3 \mathcal{P}_0\left(\boldsymbol{p} = \frac{M\boldsymbol{r}}{t}\right)}$$

This results holds for a classical or a quantum gase, indistinctively, *provided one can neglect the role of interactions during the expansion*.

For non-degenerate gases, temperature can be inferred from the cloud size after t.o.f.

For a Boltzmann gas,

$$\frac{\langle p_x^2 \rangle}{2M} = \frac{k_B T}{2}$$

and suitable generalizations for Bose or Fermi gases.

Proof for a classical gas

A classical gas is described by its phase-space density $f(\mathbf{r}, \boldsymbol{p})$, such that

$$\mathcal{P}(\boldsymbol{p}) = \int d^3\boldsymbol{r}\, f(\mathbf{r}, \boldsymbol{p}) \ : \text{momentum distribution}$$

$$n_{\text{at}}(\boldsymbol{r}) = \int d^3\boldsymbol{p}\, f(\mathbf{r}, \boldsymbol{p}) \ : \text{spatial distribution.}$$

Liouville's theorem: a phase-space element of volume $d^3\boldsymbol{r}_0 d^3\boldsymbol{p}_0$ centered on $(\boldsymbol{r}_0, \boldsymbol{p}_0)$ is conserved along the classical trajectories, $f(\mathbf{r}, \boldsymbol{p}, t) = f_0(\mathbf{r}(t), \boldsymbol{p}_0)$ with f_0 the initial distribution.
Here, the particles undergo ballistic flight for $t > 0$ and the classical trajectory evolving from $(\boldsymbol{r}_0, \boldsymbol{p}_0)$ is $\boldsymbol{r}(t) = \boldsymbol{r}_0 + \frac{\boldsymbol{p}_0}{M} t$, $\boldsymbol{p}(t) = \boldsymbol{p}_0$.
The spatial density reads

$$n_{\text{at}}(\boldsymbol{r}, t) = \int d^3\boldsymbol{p}\, f(\mathbf{r}, \boldsymbol{p}, t) = \int d^3\boldsymbol{r}_0 \int d^3\boldsymbol{p}_0\, f_0(\mathbf{r}_0, \boldsymbol{p}_0) \delta\left(\boldsymbol{r}_0 + \frac{\boldsymbol{p}_0}{M} t - \boldsymbol{r}\right)$$

For long times, the terms $\propto t$ in the δ function dominate and

$$\begin{aligned} n_{\text{at}}(\boldsymbol{r}, t) &\approx \int d^3\boldsymbol{r}_0 \int d^3\boldsymbol{p}_0\, f_0(\mathbf{r}_0, \boldsymbol{p}_0) \delta\left(\frac{\boldsymbol{p}_0}{M} t - \boldsymbol{r}\right) \\ &\approx \left(\frac{M}{t}\right)^3 \int d^3\boldsymbol{r}_0\, f_0\left(\mathbf{r}_0, \frac{M\mathbf{r}}{\hbar t}\right) = \left(\frac{M}{t}\right)^3 \mathcal{P}_0\left(\frac{M\mathbf{r}}{t}\right). \end{aligned}$$

Validity: let Δr_0 and Δp_0 be the extent of f_0 in position and momentum, respectively. The asymptotic regime is reached when $\Delta p_0 t/M \gg \Delta r_0$.

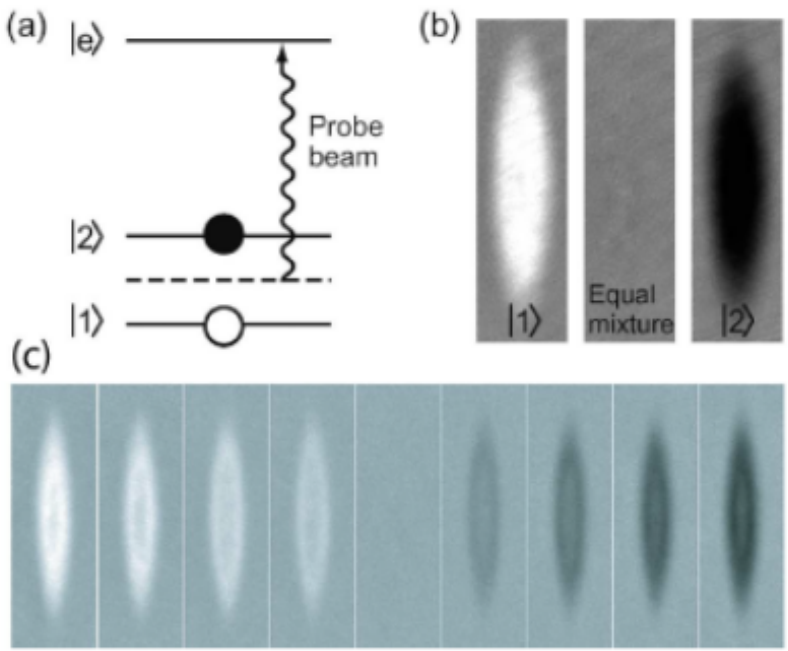
Dispersive imaging :

The phase $\phi_L = \delta\phi_L + k_L z$ of the transmitted wave also carries information on $\tilde{n}$:

$$\frac{d\varphi_L}{dz} = (n-1)k_L = \frac{1}{2}\sigma n_{\mathrm{at}} \left(\frac{2\delta_L}{\Gamma}\right)$$

n : optical index

By interfering the transmitted wave with the incident wave, this information can be retrieved (*e.g.* in *phase-contrast imaging*). Operated off resonance ($\delta_L \gg \Gamma$) to minimize absorption.



W. Ketterle and M. Zwierlein, arxiv0801.2500 (2008)

Phase-contrast images of a two-component Fermi gas

- a: level scheme
- b: $\phi_L > 0$ (< 0) for state 1 (2),
- c: different mixtures with $N_2 - N_1$ increasing from left to right.

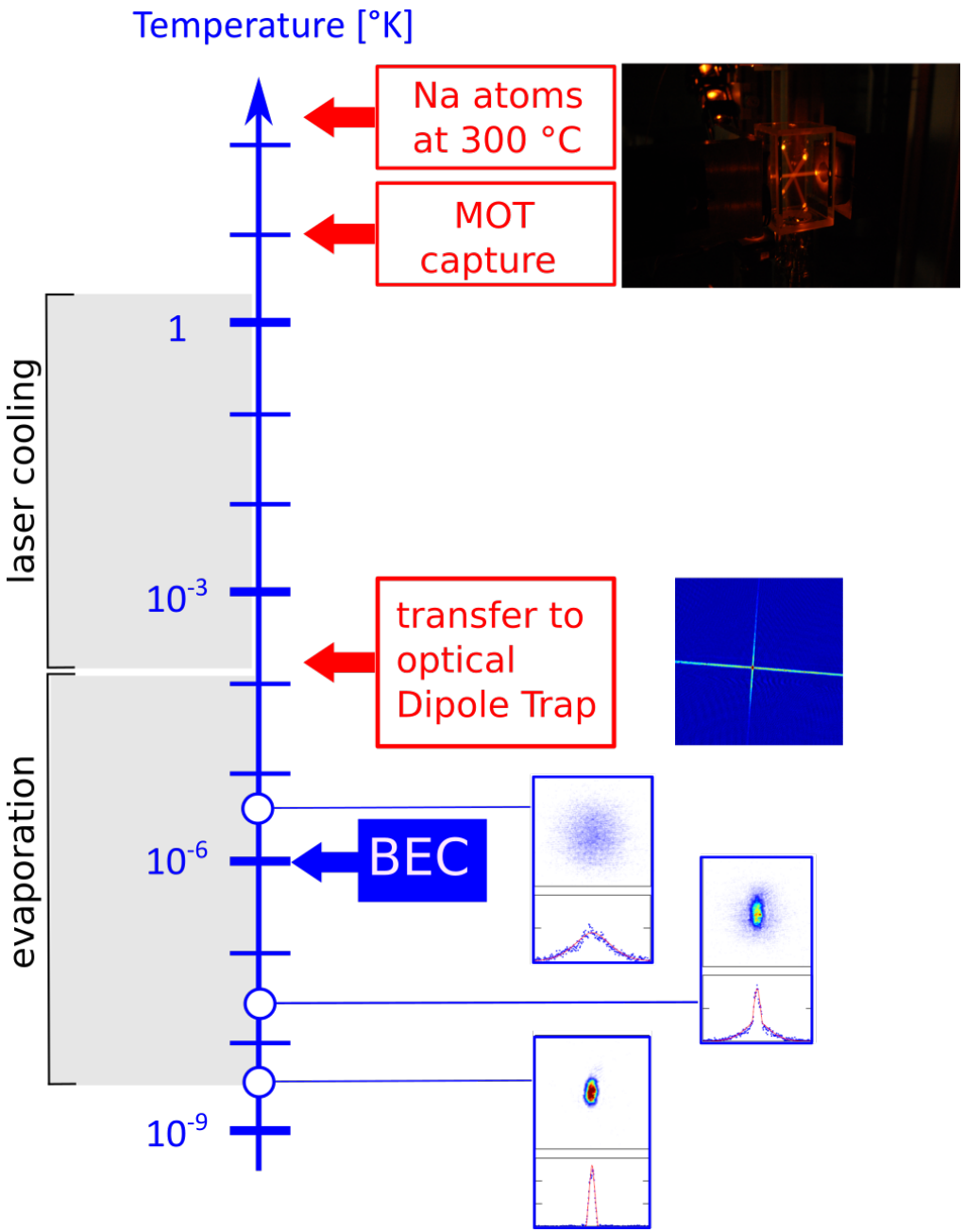
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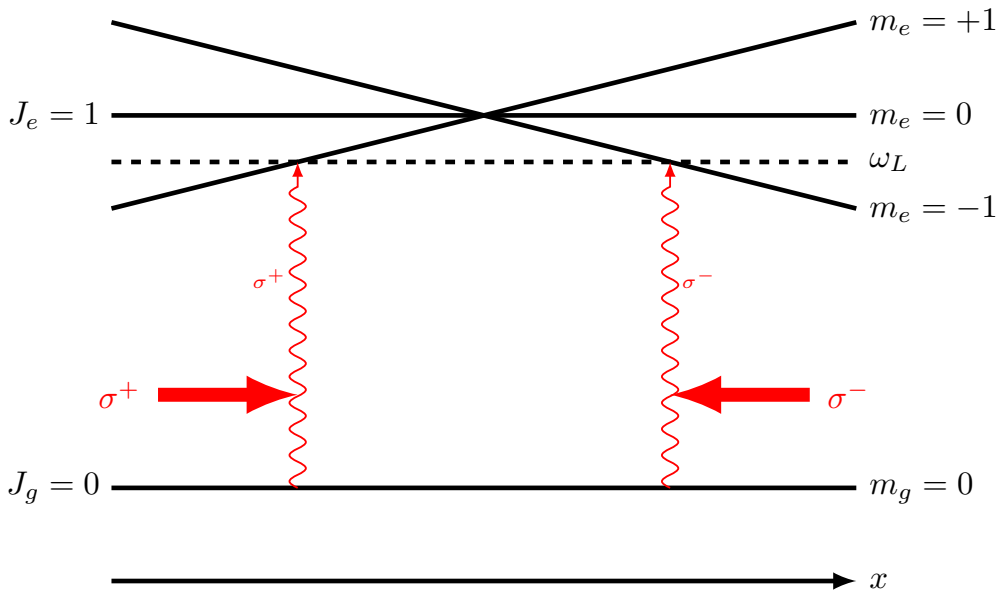
The magneto-optical trap : general principle

Atomic structure :

- $J_g = 0 \to J_e = 1$ transition
- magnetic moment μ in state e

Cooling + trapping in one dimension :

- Two counter-propagating beams with frequency ω_L, wavevector $\mp k_L$, polarizations $\sigma^{\pm}$
- inhomogeneous magnetic field $B(x) = b'x$



Weak saturation $s \ll 1$: the $\sigma^{\pm}$-polarized beam (with wavevector $\pm k_L$, respectively) exerts a radiation pressure force on the atom, with a detuning

$$\delta_{\pm k_L} = \delta_L \mp \left(k_L v + \frac{\mu b' x}{\hbar} \right).$$

For weak saturation $s \ll 1$, we add the forces exerted by each beam independently (as done for optical molasses),

$$F_{\text{tot}} = \frac{\Gamma \hbar k_L}{2} \frac{\Omega_L^2}{2} \left(\frac{1}{\delta_{+k_L}^2 + \frac{\Gamma^2}{4}} - \frac{1}{\delta_{-k_L}^2 + \frac{\Gamma^2}{4}} \right)$$

For $k_L v + \mu b' x \ll |\Delta_L|$, we have

$$F_{\text{tot}} \approx -M\alpha v - M\omega_{\text{MOT}}^2 x$$

$$\alpha = \frac{\Gamma s_0 \hbar k_L^2}{M} \frac{-2\delta_L}{\delta_L^2 + \frac{\Gamma^2}{4}} \qquad \omega_{\text{MOT}}^2 = \frac{\Gamma s_0 \hbar k_L^2}{M} \frac{-2\delta_L}{\delta_L^2 + \frac{\Gamma^2}{4}} \frac{\mu b'}{\hbar k_L}$$

- Viscous damping for $\delta_L < 0$ (red detuning), with a friction coefficient α

  Doppler (and sub-Doppler) cooling exactly as in optical molasses (near the center, where the Zeeman shift is small)

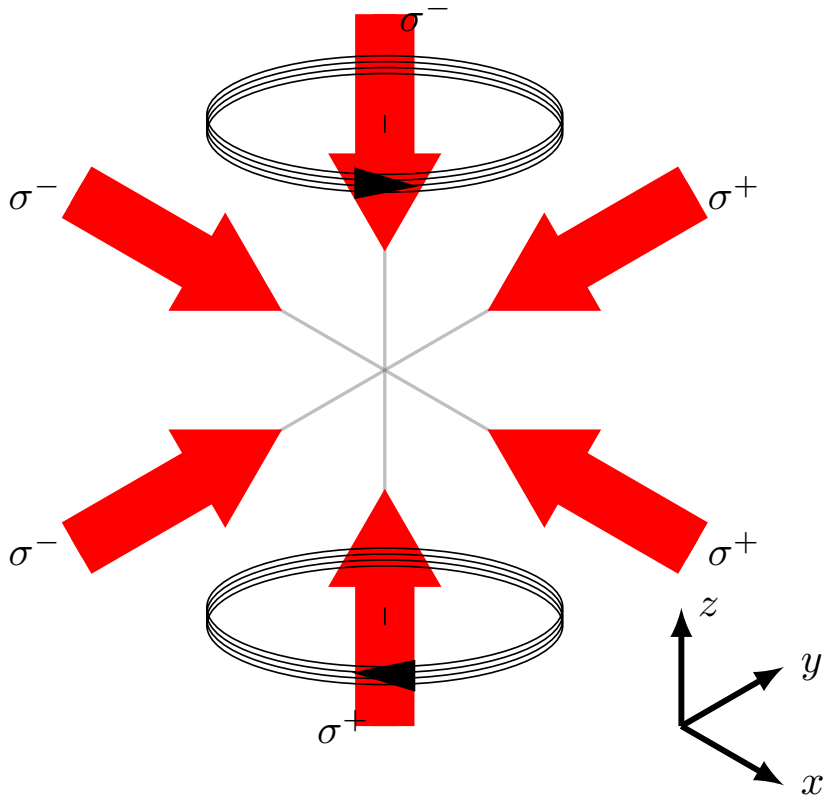
- Harmonic trapping for $\delta_L < 0$ (red detuning), with a frequency ω_{MOT}

The magneto-optical trap in 3D

MOT configuration in three dimensions :

- Three pairs of counter-propagating beams with frequency ω_L, wavevector $\mp k_L$, polarizations $\sigma^{\pm}$
- inhomogeneous magnetic field created by two coils with opposite current

$$\boldsymbol{B} \approx \begin{pmatrix} -b'x \\ -b'y \\ 2b'z \end{pmatrix}$$



Spatial distribution:

$$n(x) = n_0 e^{-\frac{x^2}{2R_{\mathrm{MOT}}^2}}, \qquad R_{\mathrm{MOT}} = \sqrt{\frac{k_B T}{M\omega_{\mathrm{MOT}}^2}}$$

This predicts $R \sim 50\,\mu$m, much smaller than what is experimentally observed for large MOTs ($N > 10^7$ atoms).

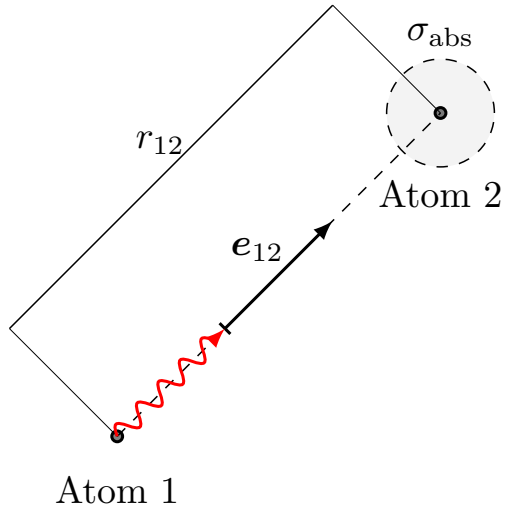
Reabsorption of spontaneously emitted photons :

A photon emitted spontaneously by atom (say 1) can be be reabsorbed by another one (say 2), with probability $\frac{d\sigma_{\mathrm{reabs}}}{d\boldsymbol{\Omega}} = \frac{\sigma_{\mathrm{reabs}}}{4\pi r_{12}^2}$ [with σ_{reabs} is the reabsorption cross-section].

Mean momentum transfer during a time $\Delta t \gg \Gamma^{-1}$:

$$\Delta \boldsymbol{p} = \boldsymbol{F}_{1\to 2}\Delta t = \Gamma_{\mathrm{sp}}\Delta t \frac{\sigma_{\mathrm{abs}}}{4\pi r_{12}^2}\hbar k_L \boldsymbol{e}_{12}$$

Long-range force mediated by spontaneous photons.



$$\boldsymbol{F}_{1\to 2} = \Gamma_{\rm sp}\hbar k_L \frac{\sigma_{\rm abs}}{4\pi r_{12}^2}\boldsymbol{e}_{12}$$

Equivalent to a Coulomb-like force between two charges $q_1 = q_2 = q$, with

$$\frac{q^2}{\epsilon_0} \to \hbar\Gamma_{\rm sp} k_L \sigma_{\rm abs}.$$

T. Walker, D. Sesko, C. Wieman, PRL 64, 408 (1990).
A. Steane, M. Chowdhury and C. Foot, J. Opt. Soc. Am. B 9, 2142 (1992).

Electrostatic analogy in the limit of a continuous medium.

- Long-range forces also strongly affect the MOT dynamics, which is non-linear. Analogies with plasmas : self-sustained oscillations, Coulomb explosion after release [L. Pruvost *et al.*, PRA 61, 063408 (2000)] ...
- Absorption from the MOT beams also play a role: it adds another contribution to the radiation pressure with the same form as the reabsorption force but opposite in sign [J. Dalibard, Optics Comm. 68, 203 (1988)]. The overall force is normally dominated by the reabsorption part [Walker et al., PRL 1990].

Equilibrium shape of a MOT in the regime of large reabsorption:

Assume uniform density in a sphere of radius R. The problem is then mapped to the one of a uniformly charged sphere with charge density $\rho = qn$.

Gauss theorem:

$$\boldsymbol{E}(r) = \frac{\rho r}{3\epsilon_0}\boldsymbol{e}_r$$

Coulomb force :

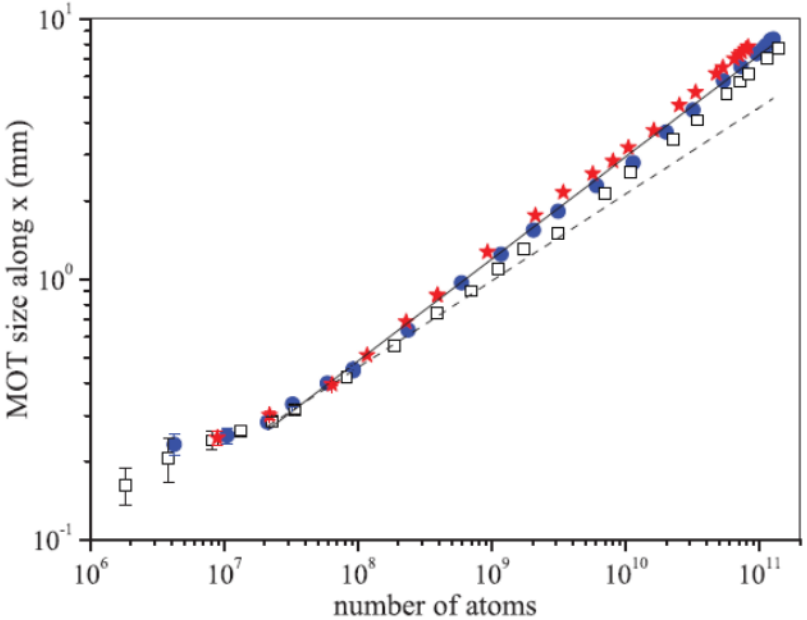
$$\boldsymbol{F}_{\text{Coulomb}} = \frac{q\rho r}{3\epsilon_0}\boldsymbol{e}_r = \frac{n\Gamma_{\text{sp}}\hbar k_L \sigma_{\text{abs}} r}{3}\boldsymbol{e}_r = nCr\boldsymbol{e}_r$$

Equilibrium, neglecting the viscous force :

$$\boldsymbol{F}_{\text{Coulomb}} + \boldsymbol{F}_{\text{MOT}} = 0$$

$$\Longrightarrow \quad n = \frac{m\omega_{\text{MOT}}^2}{C} = \frac{2|\delta_L|\Gamma}{\delta_L^2 + \frac{\Gamma^2}{4}} \frac{\mu b'}{\hbar\Gamma\sigma_{\text{abs}}}$$

Typical values :
$n \sim 10^{10}$ at/cm^3, $R \sim$ mm



A. Camarra, R. Kaiser, G. Labeyrie, PRA 90, 063404 (2014)

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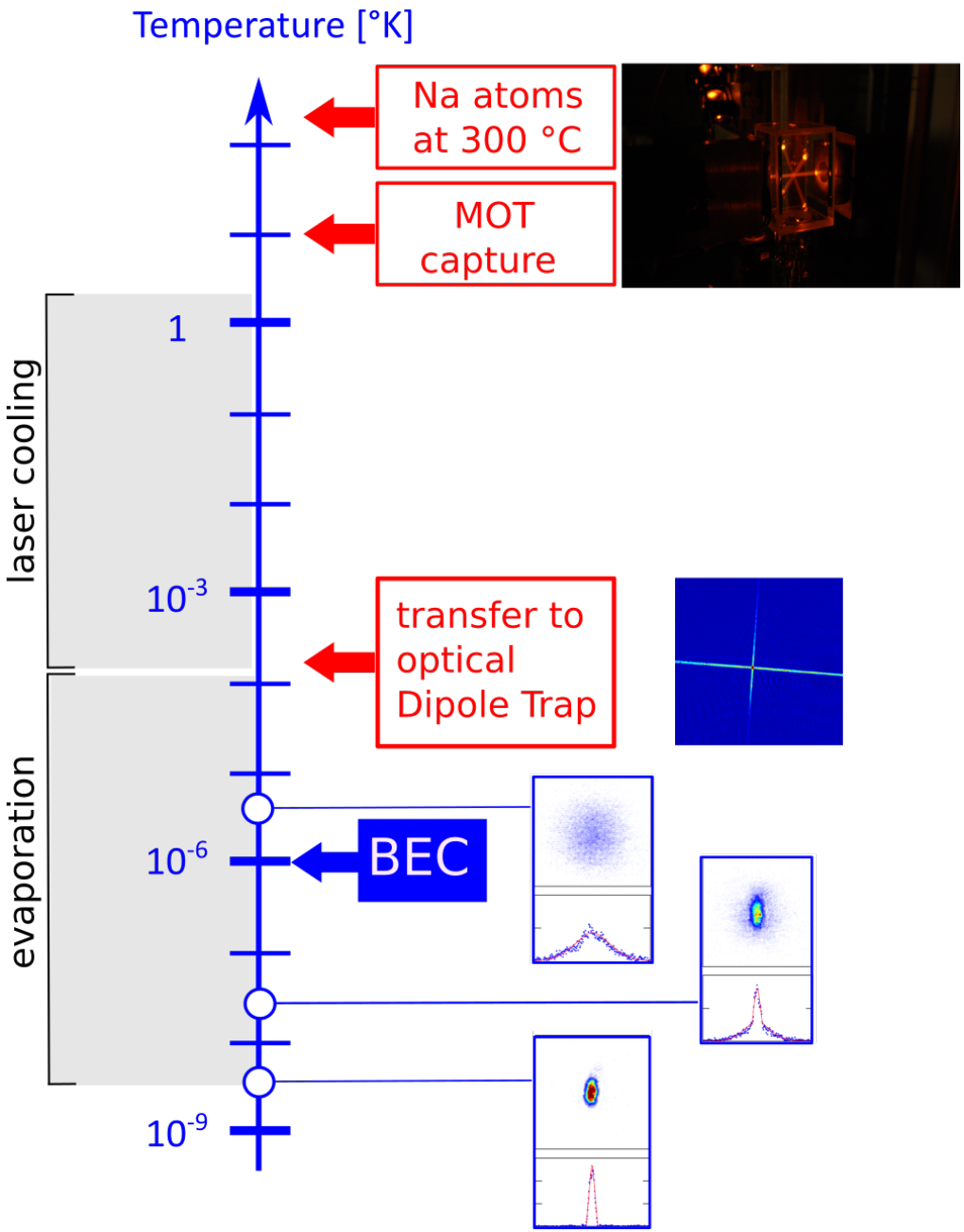
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$$\boldsymbol{F}_{\rm dip}\left(\boldsymbol{r}\right) = -u_{\rm st}\hbar\boldsymbol{\nabla}\Omega_L(\boldsymbol{r}) = -\frac{\hbar\delta_L}{2}\frac{s}{1+s}\frac{\boldsymbol{\nabla}\Omega_L}{\Omega_L}, \qquad \frac{s}{1+s} = \frac{\frac{\Omega_L^2}{2}}{\delta_L^2 + \frac{\Gamma^2}{4} + \frac{\Omega_L^2}{2}}$$

Properties of the dipole force :

- $\boldsymbol{F}_{\rm dip}$ vanishes on resonance,
- $\boldsymbol{F}_{\rm dip}$ negligible close to resonance as compared to radiation pressure $\boldsymbol{F}_{\rm rp}$,
- conservative : $\boldsymbol{F}_{\rm dip} = -\boldsymbol{\nabla}U(\boldsymbol{r})$, with the dipole potential U given by

$$U(\boldsymbol{r}) = \frac{\hbar\delta_L}{2}\ln\left[1+s(\boldsymbol{r})\right]$$

For a standing wave : $E_L = E_0\cos(\boldsymbol{k}\cdot\boldsymbol{r})$, $\varphi_L = \text{cst}$, the dipole force is given by

$$U(\boldsymbol{r}) = \frac{\hbar\delta_L}{2}\ln\left[1+s_0\cos^2(\boldsymbol{k}\cdot\boldsymbol{r})\right]$$

The potential is periodic, trapping atoms at the nodes or antinodes of the standing wave depending on the detuning (J. Dalibard's lectures).
Redistribution of photons between the $\leftarrow$ and $\rightarrow$ propagating waves by stimulated emission.

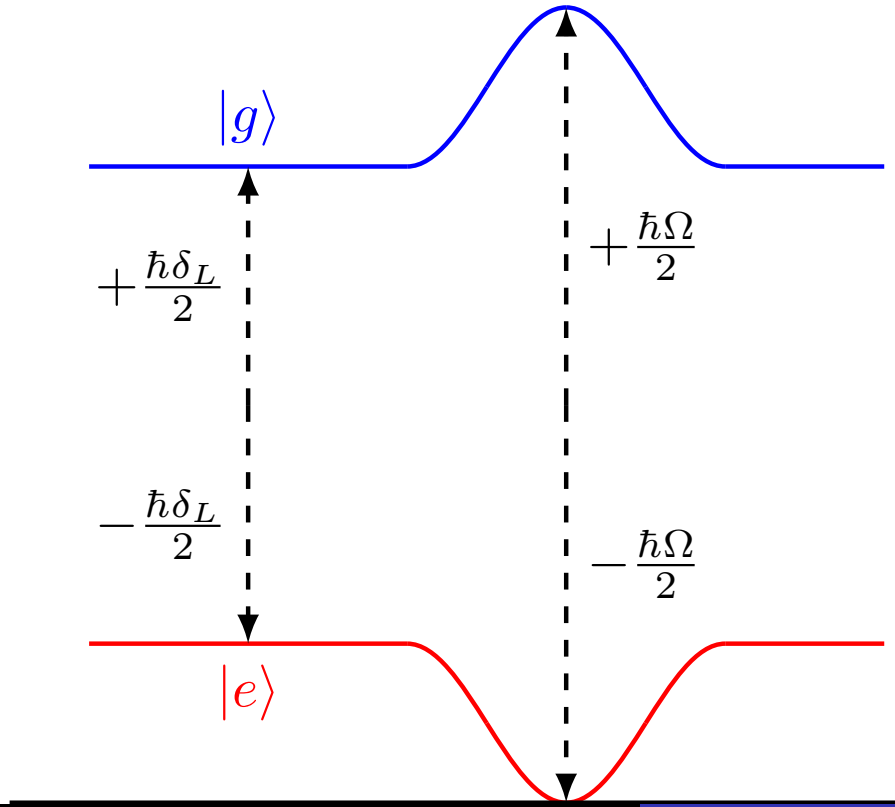
Another interpretation of the dipole force

Large detunings : $\delta_L \gg \Gamma, \Omega_L$ or equivalently $s \ll 1$

$$U(\boldsymbol{r}) \approx \frac{\hbar \delta_L s(\boldsymbol{r})}{2} = \frac{\hbar \Omega_L^2(\boldsymbol{r})}{4\delta_L}$$

Interpretation as a position-dependent light shift of the ground state atoms:
For $\delta_L > 0$:

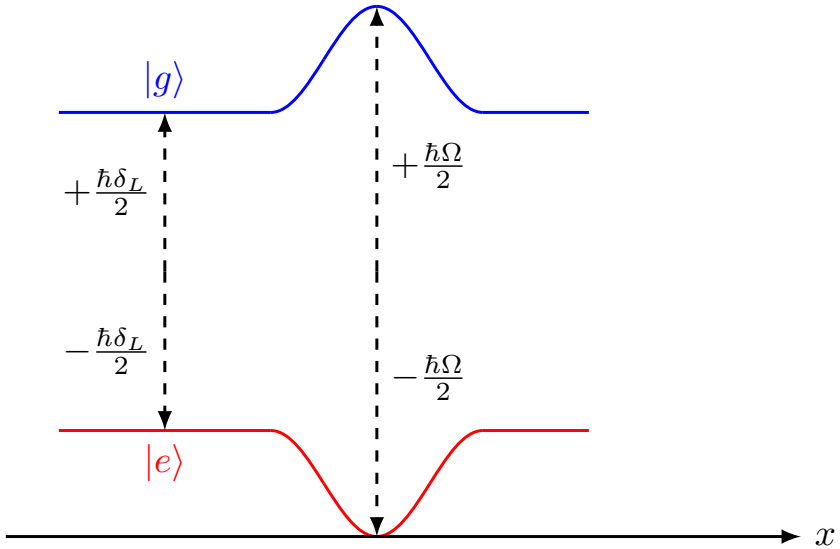
$$\Delta E_g = \frac{\hbar(\Omega - \delta_L)}{2} = \frac{\hbar}{2}\left(\sqrt{\delta_L^2 + \Omega_L^2} - \delta_L\right) \approx \frac{\hbar \Omega_L^2}{4\delta_L} \text{ if } \delta_L \gg \Omega_L$$



Dipole potential for large detunings : $\delta_L \gg \Gamma, \Omega_L$ or equivalently $s \ll 1$

$$U(\boldsymbol{r}) \approx \frac{\hbar \delta_L s(\boldsymbol{r})}{2} = \frac{\hbar \Omega_L^2(\boldsymbol{r})}{4\delta_L} = \frac{\hbar \Gamma^2}{8\delta_L} \frac{I(\boldsymbol{r})}{I_{\rm sat}}$$

Interpretation as a position-dependent light shift of $|g\rangle$:



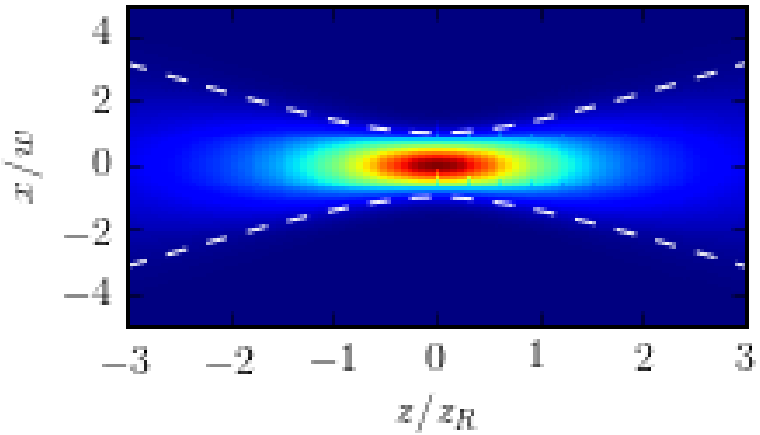
- $\delta_L > 0$: repulsive force (atoms are repelled from the intensity maxima),
- $\delta_L < 0$: attractive force (atoms are attracted to intensity maxima).

- Residual spontaneous emission :

$$\Gamma_{\rm sp} = \frac{\Gamma s}{2} = \frac{\Gamma}{\delta_L} \frac{U}{\hbar}$$

Actual laser beams have a Gaussian transverse profile. This creates a dipole potential energy of the form

$$U(\boldsymbol{r}) = -U_0 \frac{e^{-2\frac{x^2+y^2}{w^2}}}{1+\left(\frac{z}{z_R}\right)^2}$$



Harmonic trap near the bottom of the potential (particles with energy $\ll U_0$):

$$U(\boldsymbol{r}) = -U_0 + \sum_{x_\alpha = x,y,z} \frac{1}{2} m\omega_\alpha^2 x_\alpha^2$$

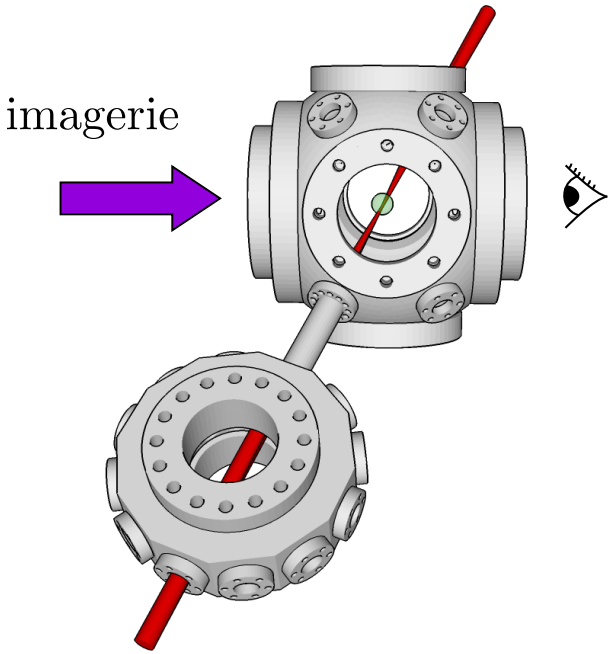
$$\omega_x = \omega_y = \sqrt{\frac{4U_0}{Mw^2}}, \qquad \omega_z = \sqrt{\frac{2U_0}{Mz_R^2}} = \frac{w}{\sqrt{2}z_R}\omega_x$$

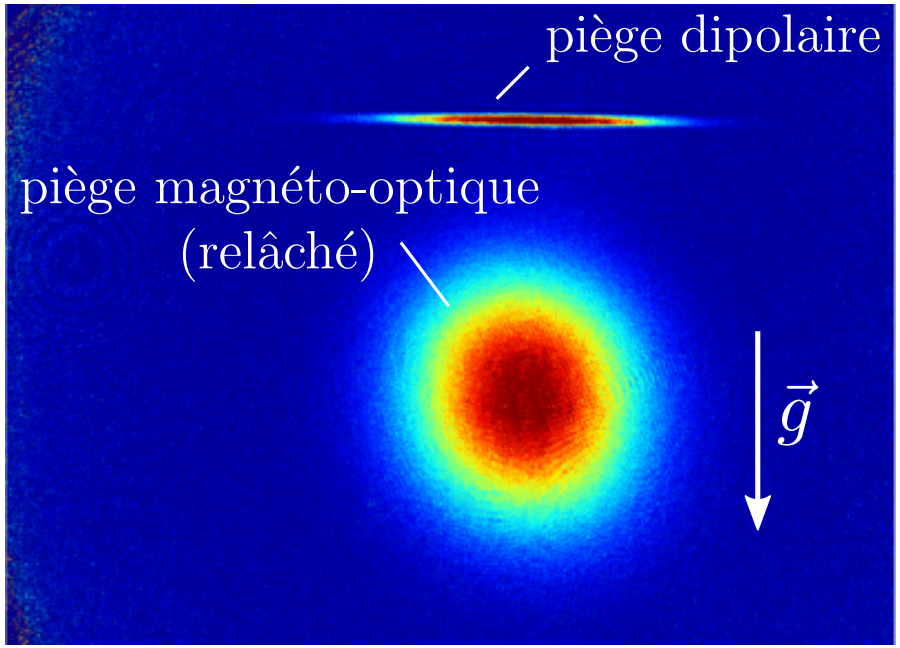
Typical example for Ytterbium atoms :

- $\lambda_{eg} = 399\,nm$
- $\lambda_L = 532\,nm$
- $\delta_L = -2 \cdot 10^{14}$ Hz
- $P = 10$ W, $w = 50\,\mu$m
- $s \approx 10^{-6}$
- $I = 2\,\mathrm{MW/m^2}$ at focus
- $U_0 \approx k_B \times 700\,\mu$K
- $\omega_x/(2\pi) \approx 1.2$ kHz
- $\omega_z/(2\pi) \approx 3$ Hz

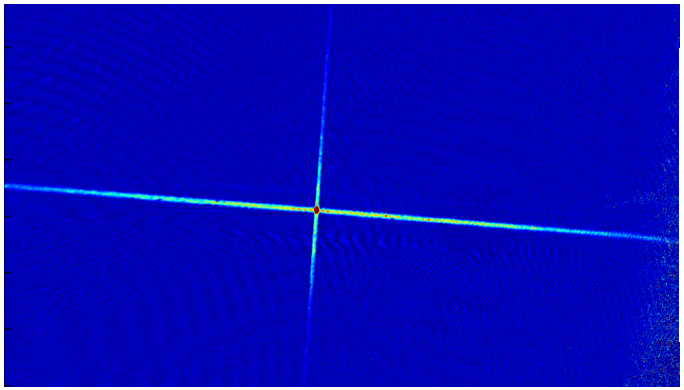
Loading a dipole trap

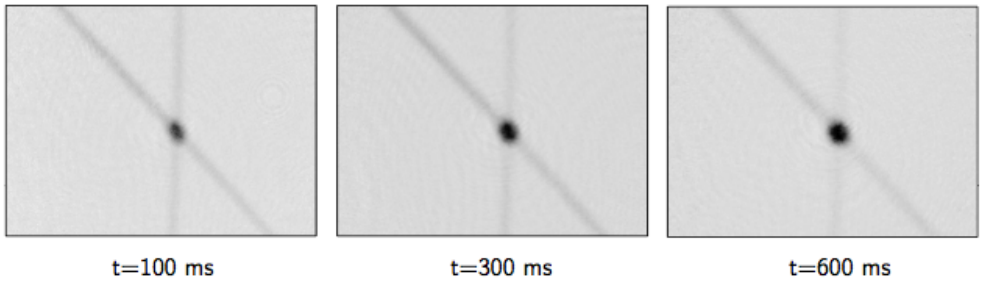
Single beam dipole trap (side view) :





Crossed dipole trap (viewed from above):





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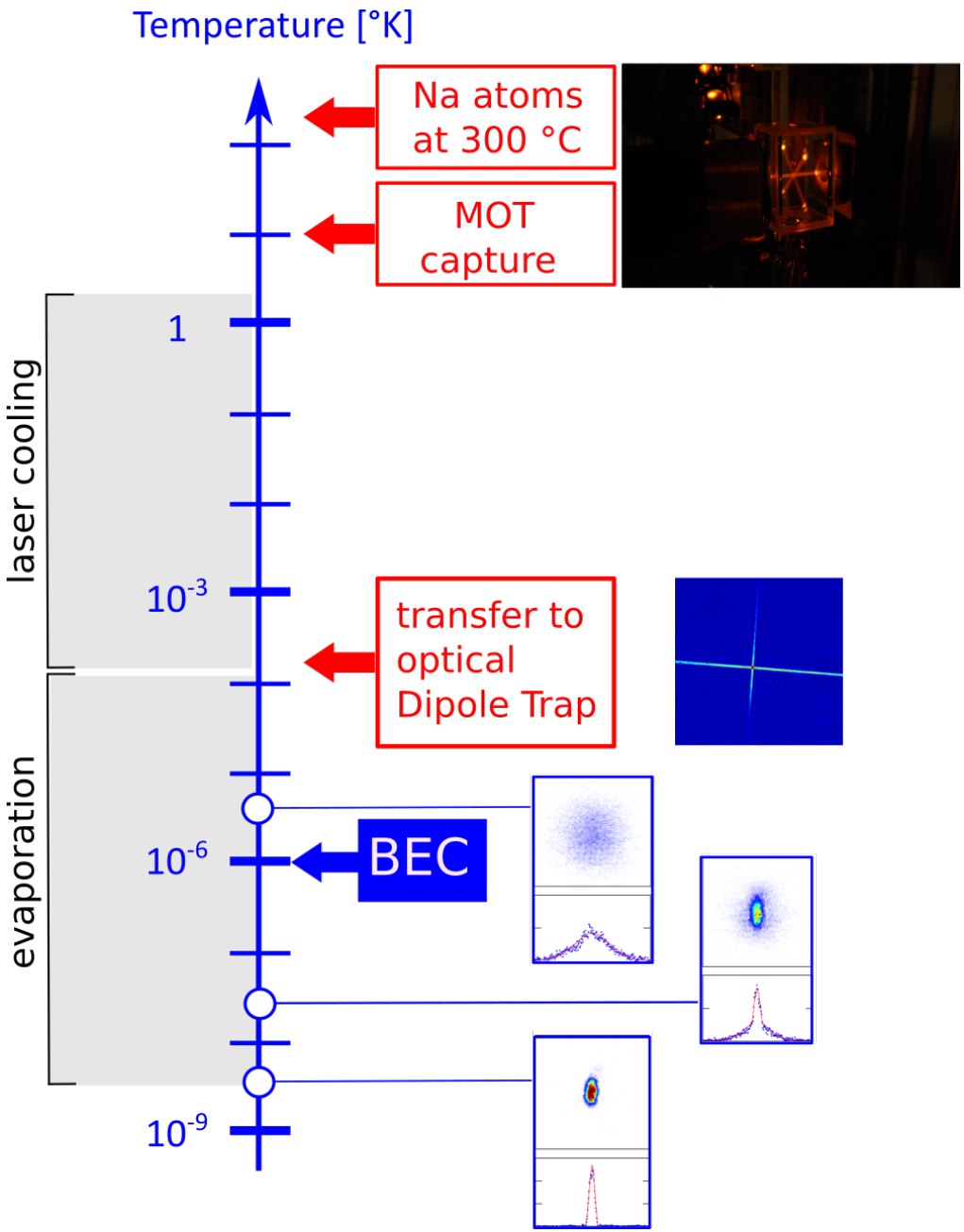
The experimental path to quantum degerate gases

Typical experimental sequence :

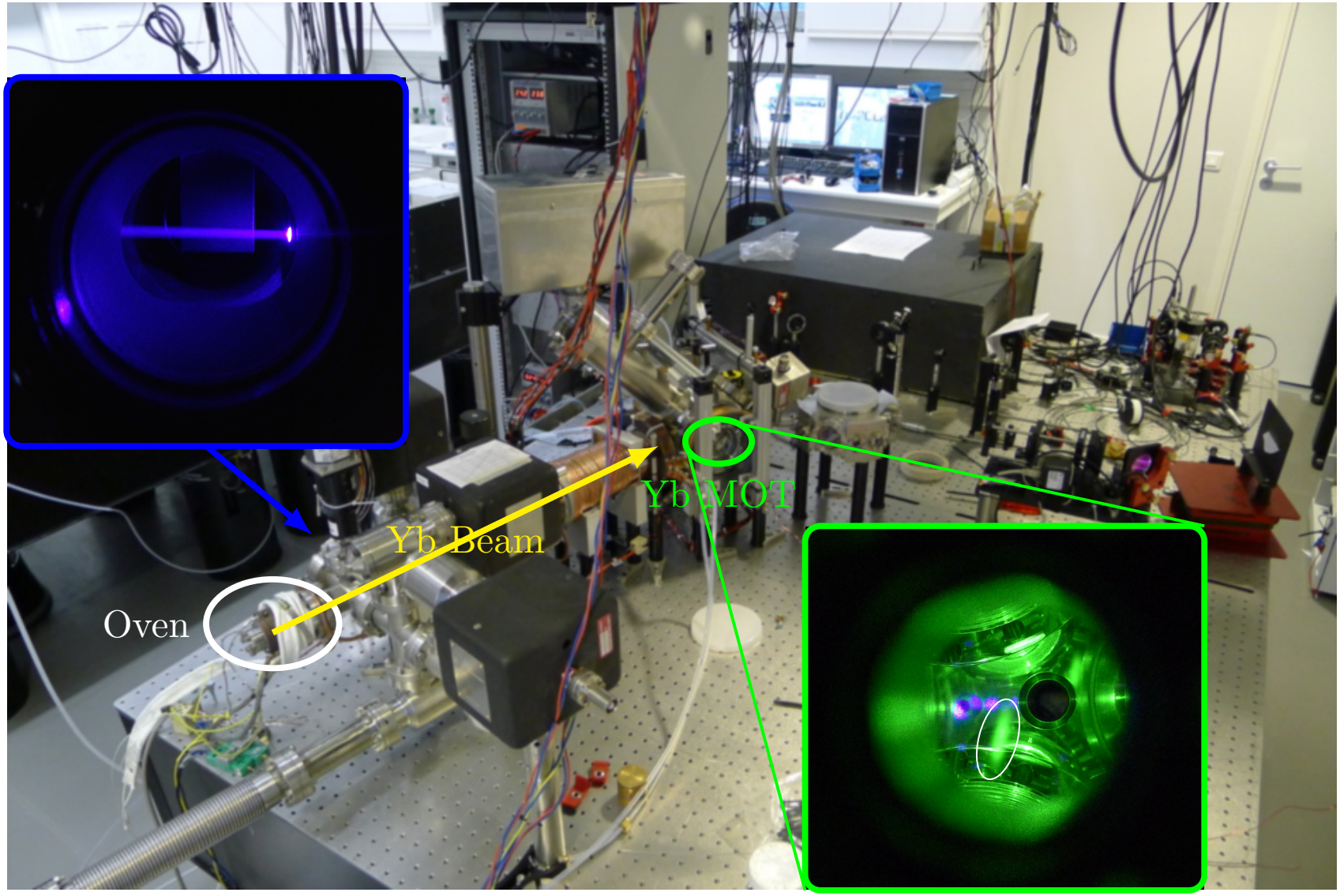
- catch atoms in a magneto-optical trap
- laser cooling to $\sim 50\,\mu$K
- transfer to conservative trap (no resonant light): optical trap or magnetic trap
- evaporative cooling to BEC

Take a picture of the cloud

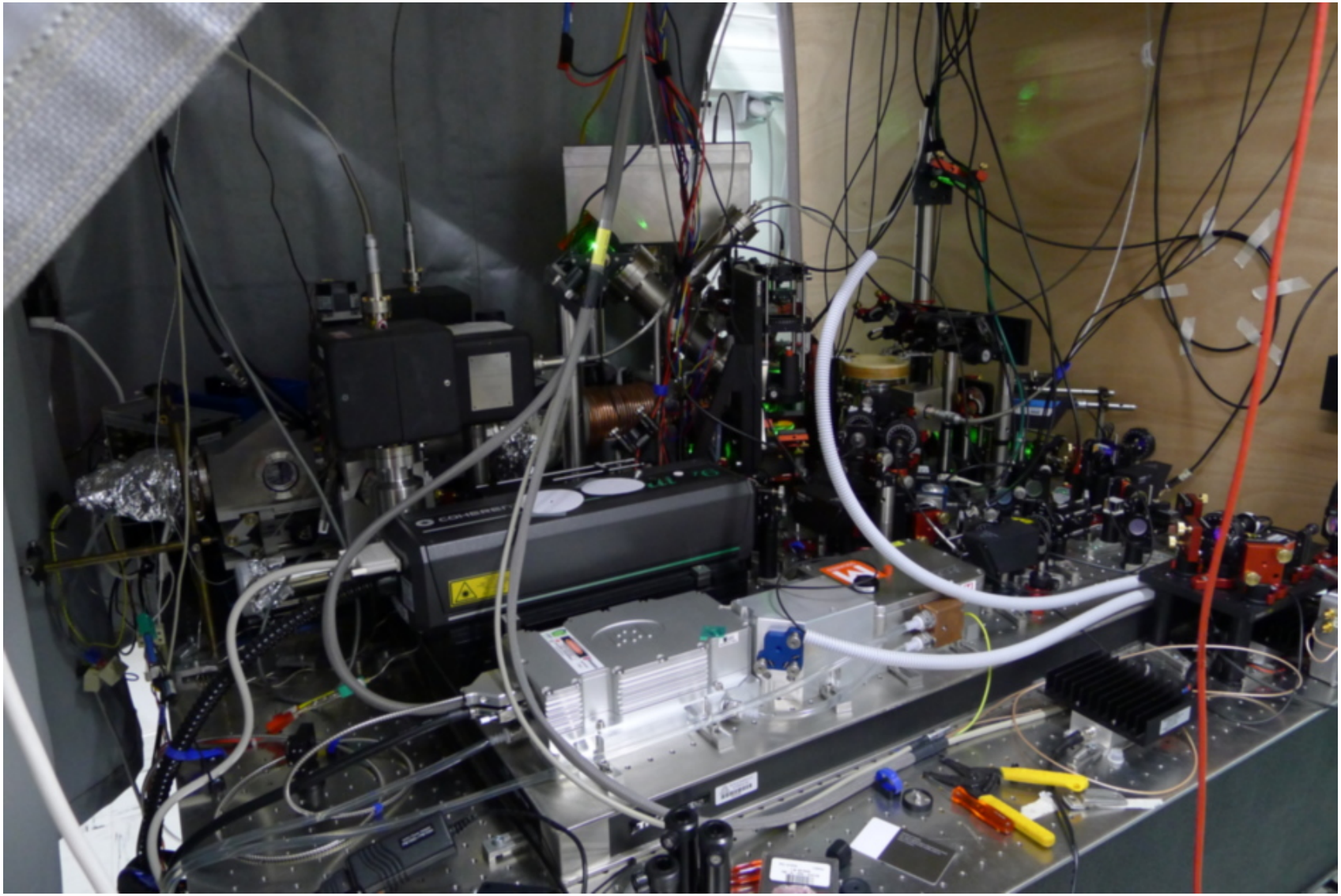
Repeat



Ytterbium BEC experiment at LKB



Ytterbium BEC experiment at LKB



Atoms coming close together interact (short-range interaction potential). The result of the collision process allows us to distinguish two types of collisions : elastic or inelastic.

Elastic collisions :

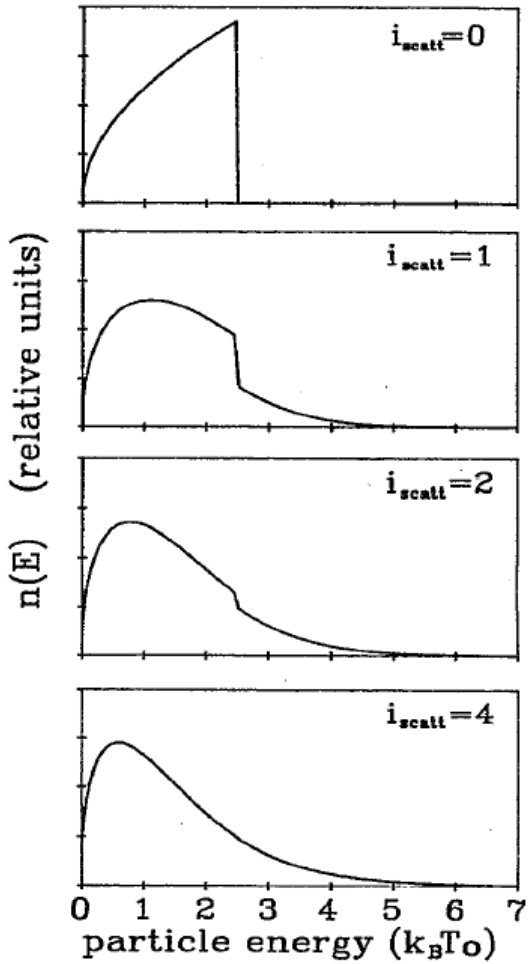
- Redistribution of momenta on a "collision sphere", conserving total momentum and energy.
- Thermalization of trapped atoms, governed by the rate of elastic collisions :

$$\gamma_{\mathrm{coll}} = n\sigma_{\mathrm{coll}} v_{\mathrm{th}}$$

  - n: spatial density
  - σ_{coll}: collision cross-section
  - $v_{\mathrm{th}} = \sqrt{\frac{8k_B T}{\pi M}}$: mean thermal velocity.

How long does it take to thermalize ?

- Left: Numerical simulation of the Boltzmann equation.
- A few collisions are enough to relax to the quasi-equilibrium distribution.
- Thermalization time = a few collision times $\gamma_{\mathrm{coll}}^{-1}$.



Snoke and Wolf, PRB 39, 4030 (1989).

Quasi-equilibrium in a trap

Ultracold atoms can be considered in equilbrium only in a *kinetic sense*. The true thermodynamic equilibrium state of the atomic bodies are all solid at sub-K temperatures. In this sense atomic gases are only metastable systems (with lifetimes of seconds or even minutes).

Consider a gas of atoms held in a trap (potential $U(\boldsymbol{r})$) of *finite depth* (say U_0). Atoms with energy lower than U_0 stay trapped, atoms with higher energy escape.

After a few collisions times, the gas equilibrates (in a kinetic sense) to a Maxwell-Boltzmann distribution, with phase-space density given by

$$f(\boldsymbol{r},\boldsymbol{p}) = \frac{1}{Z} e^{-\frac{\boldsymbol{p}^2}{2Mk_BT} - \frac{U(\boldsymbol{r})}{k_BT}} \Theta\left(\frac{\boldsymbol{p}^2}{2M} + U(\boldsymbol{r}) - U_0\right).$$

Verified in experiments and in numerical solutions of the Boltzmann equation [Luiten et al., PRA 53, 381 (1996)].

Ergodicity: f depends only on the energy.

For a harmonic trap $U(r) = \frac{1}{2}M\omega^2\boldsymbol{r}^2$ (approximation valid for energies $\ll U_0$),

$$\frac{1}{Z} = \frac{N(2\pi\hbar)^3}{(2\pi)^3 R_{\rm th}^3 (Mv_{\rm th})^3} = n(0)\lambda_{\rm th}^3 \propto \frac{N\omega^3}{T^3}, \qquad R_{\rm th} = \sqrt{\frac{k_BT}{M\omega^2}}, v_{\rm th} = \sqrt{\frac{k_BT}{M}}$$

Elastic and inelastic collisions

Atoms coming close together interact (short-range interaction potential). The result of the collision process allows us to distinguish two types of collisions : elastic and inelastic.

Inelastic collisions :

- collision with hot atoms from the background gas (with energy $\sim 300\,\mathrm{K}$)
- two-body collision between cold atoms with a change in internal state
- three-body recombination of cold atoms to a molecular state (binding energy $E_m < 0$)

All these processes typically produces atoms with energy higher than the trap depth, and lead to trap losses, described by a rate equation of the form

$$\dot{n} = -\underbrace{\gamma_1 n}_{\text{background}} - \underbrace{\gamma_2 n^2}_{\text{two-body}} - \underbrace{\gamma_3 n^3}_{\text{three-body}}$$

$\gamma_{1,2,3} \ll \gamma_{\mathrm{coll}}$ to have (approximately) ergodic behavior.

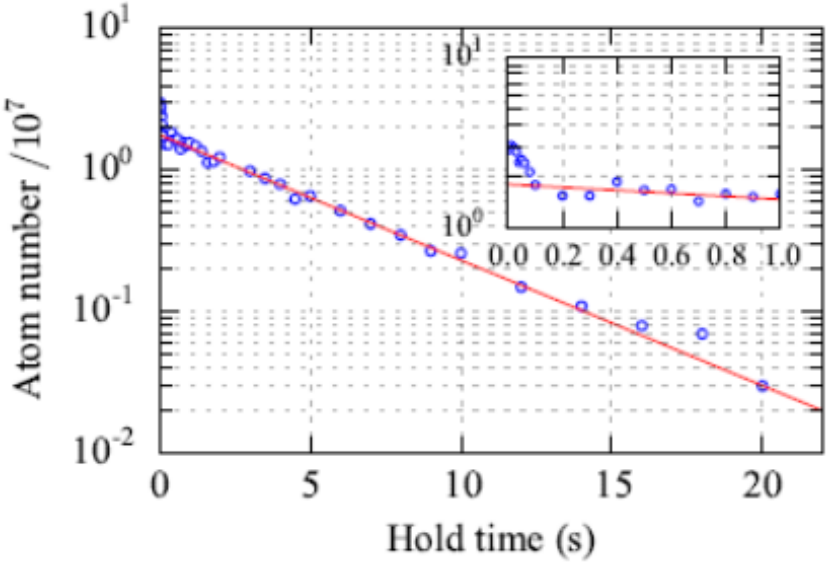


Figure from Matthias Scholl's PhD thesis

Principle of evaporative cooling

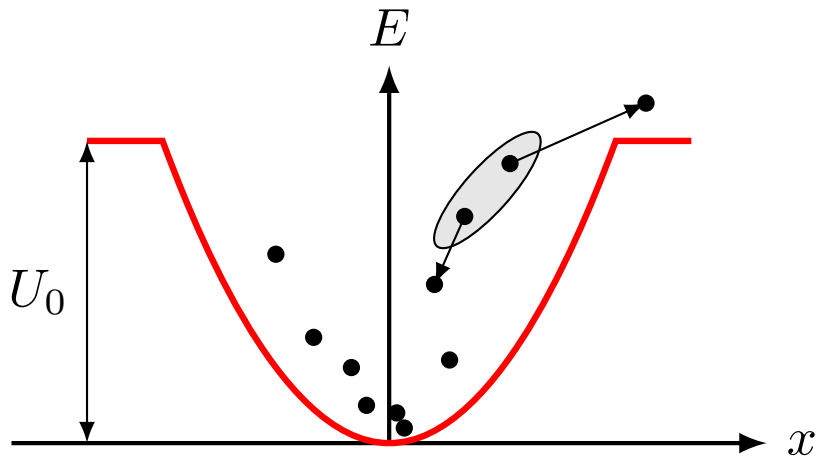
Atoms trapped in a potential of depth U_0, undergoing collisions :

- two atoms with energy close to U_0 collide
- result: one "cold" atom and a "hot" one with energy $> U_0$
- rethermalization of the $N-1$ atoms remaining in the trap results in a lower mean energy per atom.

Evaporation parameter:

$$\eta = \frac{U_0}{k_B T} \sim 8 - 11$$

Evaporated atoms have an energy $\sim \eta k_B T$: evaporation proceeds from the *tails of the distribution*.



Rate of evaporation:

- f_η: fraction of atoms with energy $> U_0 = \eta k_B T$. For a Boltzmann gas, $f_\eta \propto \sqrt{\eta} e^{-\eta}$.
- atoms with energy $\approx \eta k_B T$ have a velocity $\propto \sqrt{\eta}$

$$\gamma_{\mathrm{ev}} \sim \gamma_{\mathrm{coll}} \eta e^{-\eta} \ll \gamma_{\mathrm{coll}}$$

More details in: W. Ketterle and N. J. Van Druten, Evaporative cooling of trapped atoms, available from the Ketterle group website.

Simple estimate of the efficiency of evaporative cooling

Step-like model of evaporation: each step $N \to N - dN, T \to T - dT$

We write the mean energy lost by the evaporated atoms as $-d\overline{E} = (\eta + \kappa)k_B T$, with

$$\eta = \frac{U_0}{k_B T} \sim 8 - 11$$

and $\kappa \sim 1$ the excess "energy" with respect to the trap depth.

Mean energy before the collision : $\overline{E} = 3Nk_B T$.

Mean energy after evaporation and thermalization:

$$\begin{aligned} \overline{E} - d\overline{E} &= \overline{E} - (\eta + \kappa)k_B T dN \\ &= 3(N - dN)k_B(T - dT) \approx 3Nk_B T - 3k_B T dN - 3Nk_B dT \end{aligned}$$

We get

$$(\eta + \kappa - 3)dN = 3dT \implies \left(\frac{T_f}{T_i}\right) = \left(\frac{N_f}{N_i}\right)^{\alpha}, \qquad \alpha = \frac{\eta + \kappa}{3} - 1 \sim 2 - 3$$

Losing a factor 10 on N cools the sample by a factor $\sim 100 - 1000$!

For a fixed trap frequency ω, the spatial density n, the phase space density $\mathcal{D}$, elastic collision rate $\gamma_{\rm coll}$ scale as :

$$n = \frac{N}{(2\pi)^{3/2} R_{\rm th}^3} \propto \frac{N\omega^3}{T^{3/2}} \sim N^{1-\frac{3\alpha}{2}}$$

$$\mathcal{D} = n\lambda_{\rm th}^3 \propto \frac{N\omega^3}{T^3} \sim N^{1-3\alpha}$$

$$\gamma_{\rm coll} = n\sigma v_{\rm th} \propto \frac{N\omega^3}{T} \sim N^{1-\alpha}$$

$\alpha > 1/3$ results in an *increase of the phase space density of the sample*.
$\alpha > 1$ results in an *increase of the collision rate*.

Practical problem : reach $\mathcal{D} \sim 1$ before loss processes set in.

Rate of evaporation $\propto \gamma_{\rm coll} \eta e^{-\eta}$ must be much faster than the lifetime of the sample.

Forced evaporation : decrease trap depth as the gas cools down to keep η roughly constant ($\sim 8 - 10$).

Runaway eaporation: evaporation at ever increasing $\gamma_{\rm coll}$ ($\alpha > 1$). Feasible in magnetic traps (see Appendix).

The trap depth U_0 is varied by reducing the laser power, since $U_0 \propto P$.
However, the trap frequencies also decrease, as $\omega \propto \sqrt{P}$.

The spatial density n, the phase space density $\mathcal{D}$, elastic collision rate γ_{coll} scale as :

$$n \propto \frac{N\omega^3}{T^{3/2}} \sim U_0^{\frac{1}{\alpha}}$$

$$\mathcal{D} = n\lambda_{\mathrm{th}}^3 \sim U_0^{\frac{1}{\alpha}-\frac{3}{2}}$$

$$\gamma_{\mathrm{coll}} = n\sigma v_{\mathrm{th}} \propto U_0^{\frac{1}{\alpha}+\frac{1}{2}}$$

Therefore, in optical traps, the collision rate never increases and runaway evaporation does not happen. Fortunately, the collisions rates after loading from a MOT are high (a few thousands per second, typically).

The rule of the game is then to optimize the initial conditions and the "trajectory" $U_0(t)$ such that one reaches $\mathcal{D} \sim 1$ before the collision rate has dropped too much and evaporation stops.

Example of evaporative cooling "trajectory"

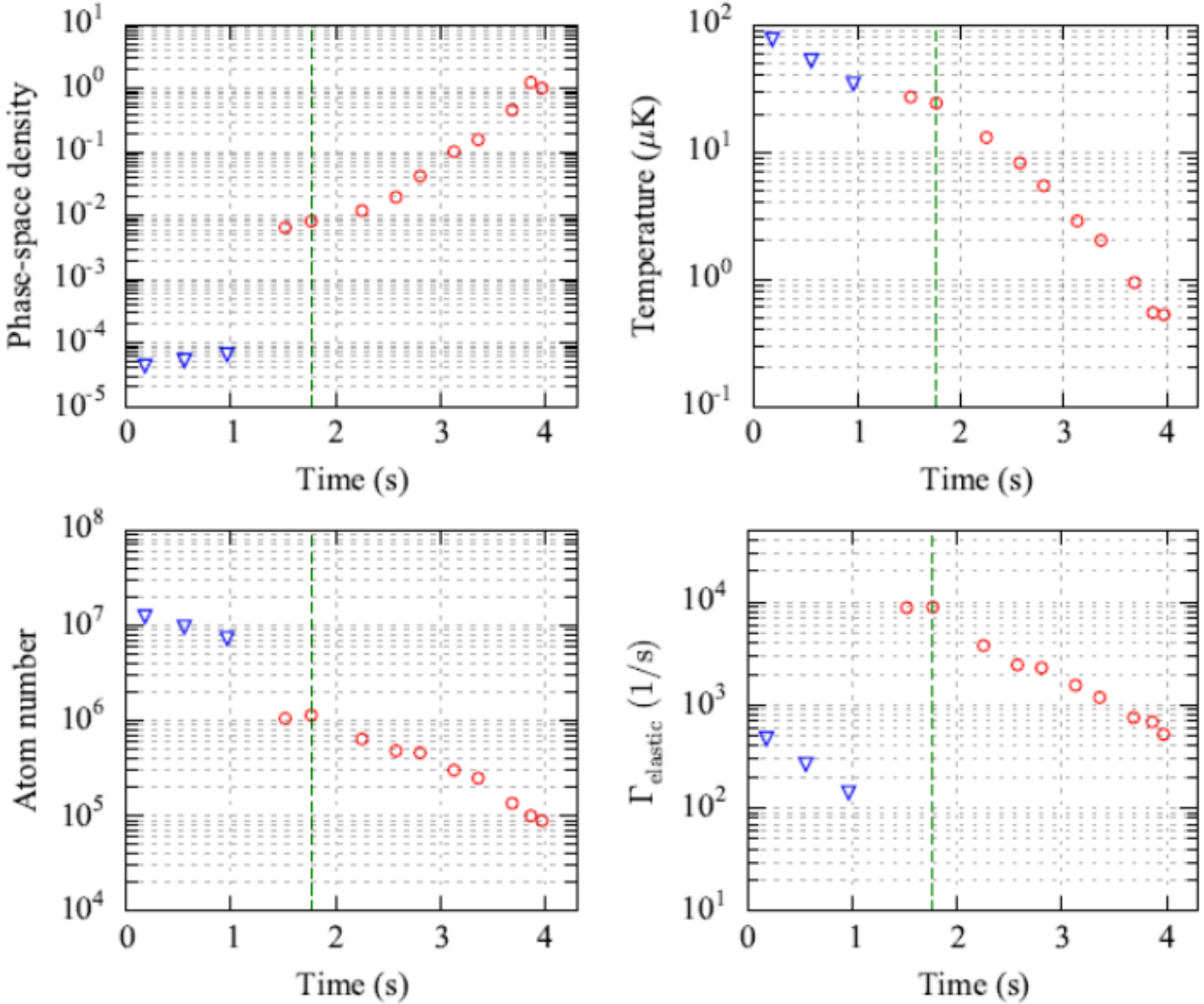


Figure from Matthias Scholl's PhD thesis

Blue triangles: single beam DT; red circles : crossed dipole trap

Complement: magnetic traps

Zeeman Hamiltonian in a non-uniform magnetic field $\boldsymbol{B}(\boldsymbol{r})$:

$$U_{\mathrm{mag}} = -\boldsymbol{m} \cdot \boldsymbol{B}(\boldsymbol{r})$$

Assuming that the internal states can follow adiabatically the change of $\boldsymbol{B}$ as the atom move in space,

$$\langle F, m_F | U_{\mathrm{mag}} | F, m_F \rangle = g_F m_F |\boldsymbol{B}(\boldsymbol{r})|$$

Maxwell equations tell us that $|B|$ can only have a minimum.

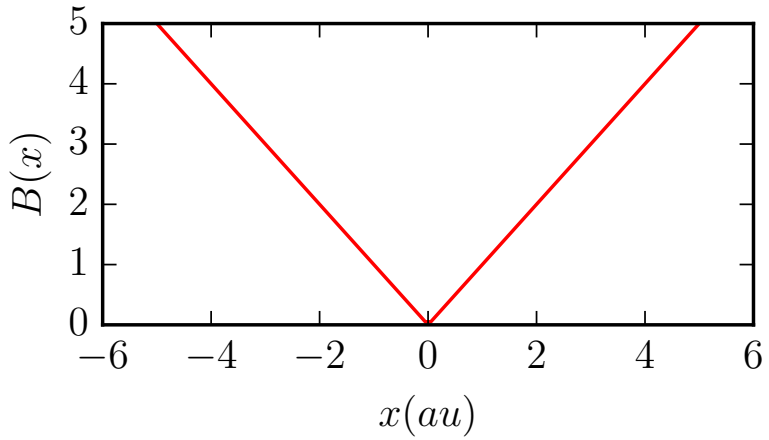
If $g_F m_F > 0$ (low-field seeking states), the potential energy is minimal where $|\boldsymbol{B}|$ is: this results in atom trapping near the minimum. Atoms in high field seeking states with $g_F m_F < 0$ are expelled from the potential.

NB : The state $|F, m_F\rangle$ is defined in a *local* reference frame, where the local quantization axis is given by the direction of $\boldsymbol{B}$.

Quadrupole trap :
field created by two coils with opposite current

$$\boldsymbol{B} \approx \begin{pmatrix} -b'x \\ -b'y \\ 2b'z \end{pmatrix},$$

$$|\boldsymbol{B}| = b' \sqrt{x^2 + y^2 + 4z^2}$$



Adiabatic theorem

Quantum adiabatic theorem : Slowly evolving quantum system, with Hamiltonian $\hat{H}(t)$

Instantaneous eigenbasis of $\hat{H}$: $\hat{H}(t)|\phi_n(t)\rangle = \varepsilon_n(t)|\phi_n(t)\rangle$.

Time-dependent wave function in the $\{|\phi_n(t)\rangle\}$ basis:

$$|\Psi(t)\rangle = \sum_n a_n(t) e^{-\frac{i}{\hbar}\int_0^t \varepsilon_n(t')dt'} |\phi_n(t)\rangle,$$

From Schrödinger equation, one gets $[\omega_{mn} = \varepsilon_m - \varepsilon_n]$:

$$\dot{a}_n = -\langle\phi_n|\dot{\phi}_n\rangle a_n(t) - \sum_{m\neq n} e^{-\frac{i}{\hbar}\int_0^t \omega_{mn}(t')dt'} \langle\phi_n|\dot{\phi}_m\rangle a_m(t),$$

- Berry phase : $\langle\phi_n|\dot{\phi}_n\rangle = -i\gamma_B$ is a pure phase. Wavefunction unchanged up to a phase evolution after a cyclic change.
- The adiabatic theorem : for *arbitrarily slow* evolution starting from a particular state n_0 ($a_n(0) = \delta_{n,n_0}$), and in the *absence of level crossings*, $a_n(t) \to \delta_{n,n_0}$ (up to a global phase).

Adiabatic approximation for slow evolutions and initial condition $a_n(0) = \delta_{n,n_0}$:

$$a_n(t) \to e^{i\gamma_B(t)}\delta_{n,n_0}$$

Validity criterion :

$$\left(\quad\right)^2$$

Adiabaticity criterion for a time-dependent field $\boldsymbol{B}(t)$:

$$\left|\langle\phi_n|\dot{H}|\phi_m\rangle\right| \sim \mu\dot{B} \ll \frac{\left(\varepsilon_m - \varepsilon_n\right)^2}{\hbar} \sim \frac{(\mu B)^2}{\hbar} = \hbar\omega_{\mathrm{Larmor}}^2, \text{ or } \frac{\dot{\omega}_{\mathrm{Larmor}}}{\omega_{\mathrm{Larmor}}^2} \ll 1$$

Here $\omega_{\mathrm{Larmor}} = \mu B/\hbar$ is the Larmor frequency quantifying the angular velocity of the spin precession around the magnetic field axis.

Adiabaticity criterion for an atom, described classically, moving in a static field $\boldsymbol{B}(\boldsymbol{r})$:

$$H = -\boldsymbol{m}\cdot B[\boldsymbol{r}(t)] \implies \langle\downarrow|\dot{\boldsymbol{B}}|\uparrow\rangle = \cdot\langle\downarrow|\left(\boldsymbol{v}\cdot\boldsymbol{\nabla}\right)\boldsymbol{B}|\uparrow\rangle \ll \frac{\mu|\boldsymbol{B}(\boldsymbol{r})|^2}{\hbar}$$

- spin-flip transitions to untrapped states near the minimum of $|B|$
- result in atom losses,
- mostly affect the fast atoms.

Majorana losses is a major issue in a quadrupole trap since $\min(|\boldsymbol{B}|) = 0$.

Biased quadrupole trap (Ioffe trap) :

$$\boldsymbol{B} \approx \begin{pmatrix} -b'x \\ -b'y \\ B_0 + 2b'z \end{pmatrix}$$

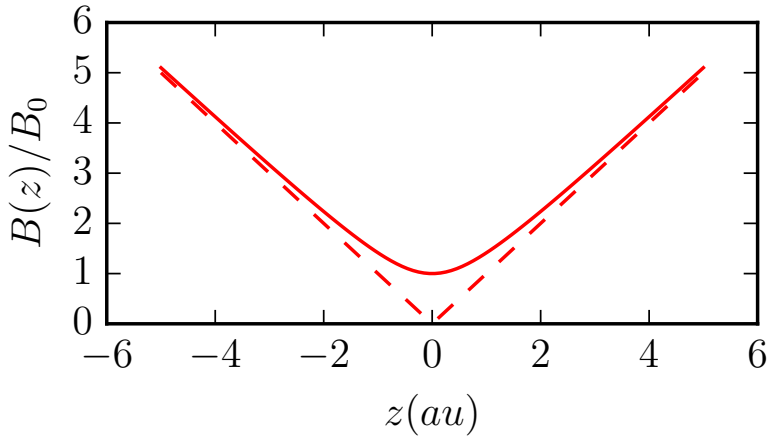
$$|\boldsymbol{B}| = \sqrt{b'^2(x^2+y^2) + (B_0 + 2b'z)^2}$$

$$\approx B_0 + b'^2(x^2+y^2) + 2b'z + \frac{2b'^2}{B_0} z^2$$

Harmonic trap with frequency $\omega_x, \omega_y \gg \omega_z$

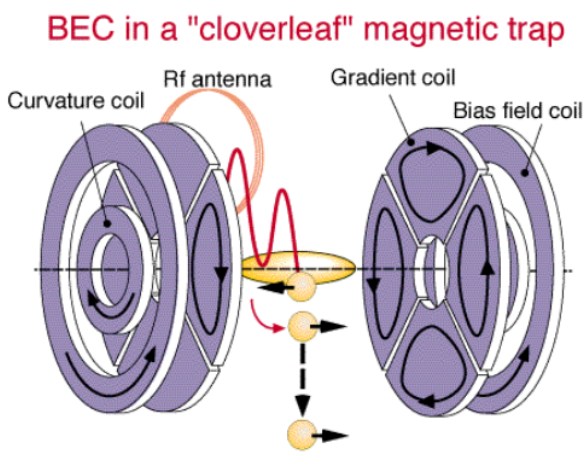
Typical values :

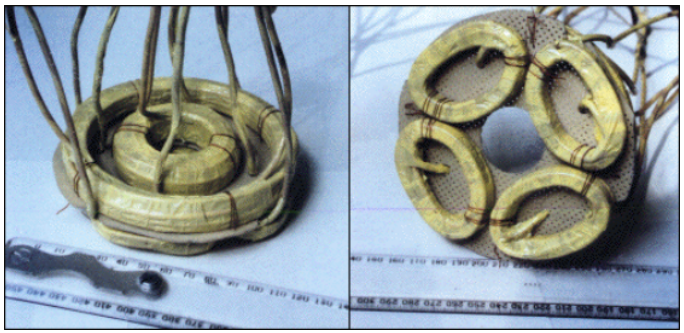
- $B_0 \sim 1 - 10\,\mathrm{G}$ [1 G=10^{-4}T],
- $b' \sim 100\,\mathrm{G/cm}$,
- $\omega_x = \omega_y \approx 2\pi \times 300\,\mathrm{Hz}$,
  $\omega_z \approx 2\pi \times 30\,\mathrm{Hz}$.



Gallery of magnetic traps

"Cloverleaf" trap :

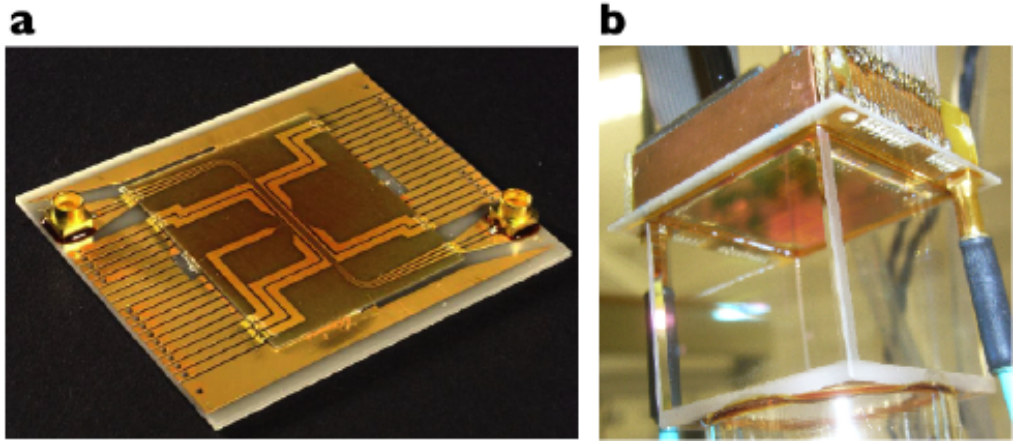




Ketterle group, MIT
http://cua.mit.edu/

- Macroscopic trap :
  $l \sim 1 - 10\,\text{cm}$,
  $I \sim 100 - 1000\,\text{A}$
- kW of electrical power to dissipate (water cooling)

Atom chips:



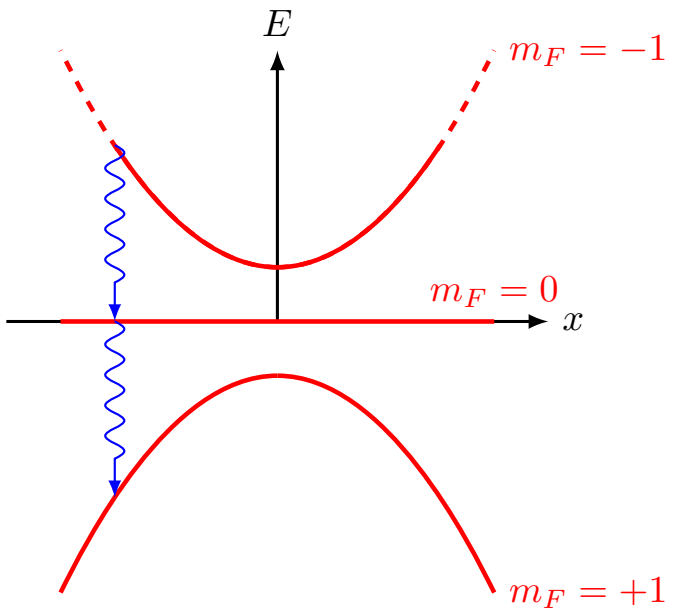
Reichel group, LKB
http://www.lkb.ens.fr/-Atom-Chips-Group-

- "Microscopic" trap : $l \sim 0.1 - 1\,\text{mm}$, $I \sim 1\,\text{A}$
- little power dissipated, compact
- chip problematic for some applications
- usually modest vacuum level

Many variants for both designs (practically one per experiment ...)

Forced evaporation: $U_0(t)$ decreases in time (in a controlled way). Optimizing the evaporation trajectory is done by controlling the atom lost by the evaporation process *via* the trap depth.

In magnetic traps, this is achieved by radio-frequency evaporation : a radiofrequency field is applied, inducing transitions from a trapped state to a non-trapped state for a given energy $\hbar\omega_{\rm rf} = U_0$.

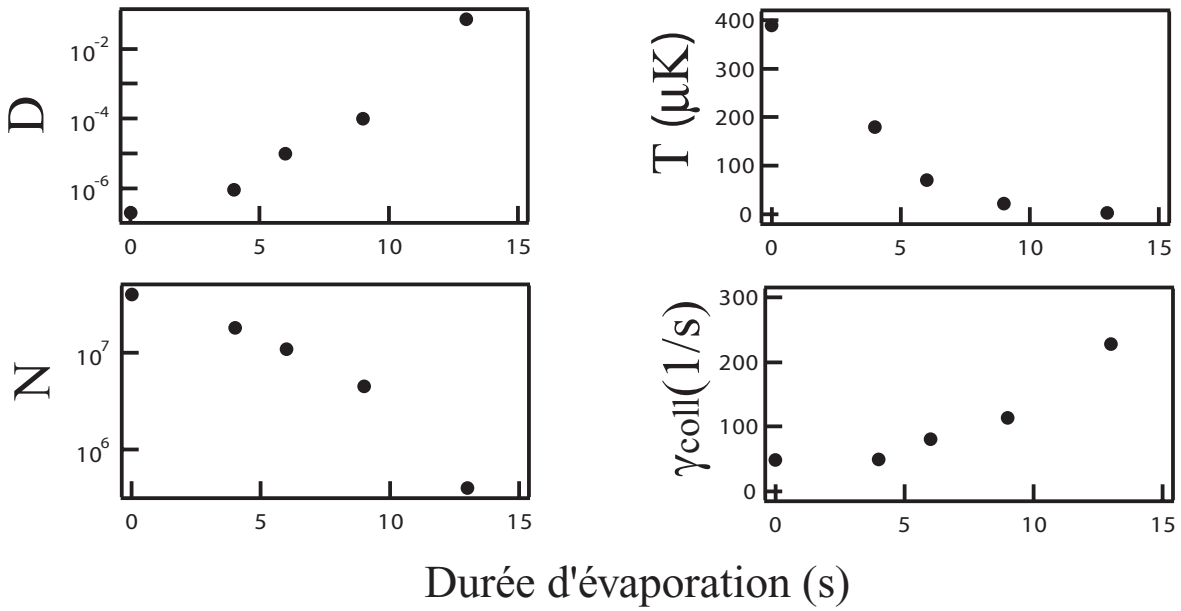


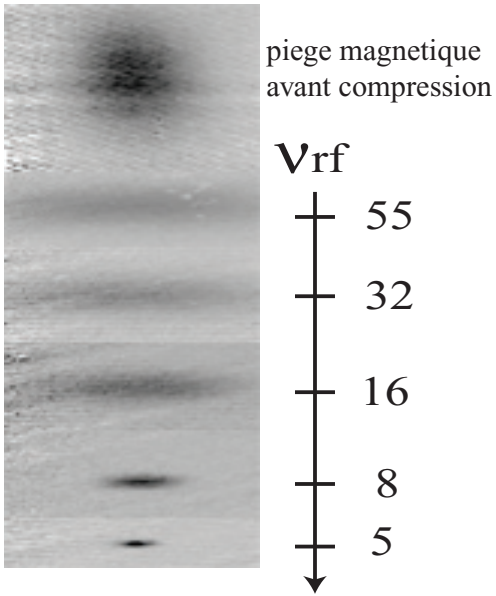
Usually $\eta = U_0/k_B T \approx$cst. Then, the scaling with trap depth is $T \sim U_0$, $N \sim U_0^{\frac{1}{\alpha}}$, and

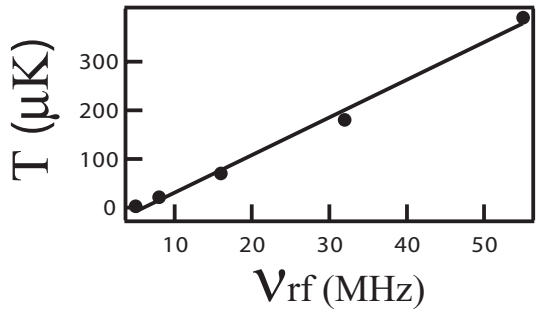
$$n \sim U_0^{\frac{1}{\alpha}-\frac{3}{2}}, \quad \mathcal{D} \sim U_0^{\frac{1}{\alpha}-3} \quad \gamma_{\rm coll} \sim U_0^{\frac{1}{\alpha}-1}$$

Optimization can be done (usually experimentally) to reach the runaway regime.

Illustration of evaporation in magnetic traps







F. Gerbier, PhD thesis