Lecture 1:

Presentation – Refresher on atom-light interactions

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Master ICFP, Ultracold atoms

January 18, 2021

Lecture slides, exercise classes and other material available at

http://www.phys.ens.fr/ nascimbene/enseignement.php

References on atomic physics

- Claude Cohen-Tannoudji and David Guéry-Odelin, *Advances In Atomic Physics: An Overview*, World Scientific.
- Claude Cohen-Tannoudji, Jacques Dupont-Roc and Gilbert Grynberg, *Processus d'interaction entre photons et atomes*, CNRS Editions.

More details on laser cooling (in French): lectures notes by Jean Dalibard (http://www.phys.ens.fr/ dalibard/)

- Unpublished Master lecture notes : *Atomes ultrafroids*
- Collège de France lectures 2015: *Une brève histoire des atomes froids*
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Quantum gases:

- Wolfgang Ketterle *et al.*, arXiv:cond-mat/9904034 (1999) and arXiv:0801.2500 (2008)
- Sandro Stringari and Lev Pitaevskii, *Bose-Einstein condensation*, Oxford University Press.

A brief historical overview of AMO physics

Atomic physics, born with spectroscopy at the end of the 19th century, progressed hand-in-hand with quantum mechanics in the years 1900-1930.

Today we often speak of AMO - Atomic, Molecular and Optical - physics : *dilute* gases (as opposed to *dense* liquids and solids).

A common view in the early 1950's was that AMO physics was essentially understood, with little left to discover.

Sixty years later, this view has been proven wrong by a serie of revolutions, each leading to the next one :

- the 1960's : the laser
- the 1970's : laser spectroscopy
- the 1980's : laser cooling and trapping of atoms and ions
- the 1990's : quantum degenerate atomic gases (Bose-Einstein condensates and Fermi gases)
- the 2000's : femtosecond frequency combs

Control over the quantum state of an atom

The quantum mechanical description of an atom introduces several quantum numbers to describe its state :

- internal quantum numbers to describing the relative motion of electrons with respect to the nuclei, *e.g.* electronic angular momentum $\hat{\boldsymbol{J}}$,
- external quantum numbers, *e.g.* center of mass position $\hat{\boldsymbol{R}}$.

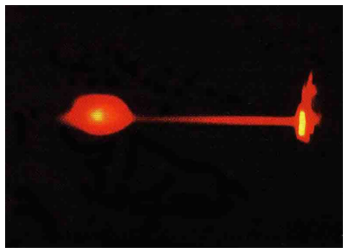
In spectroscopy, electromagnetic fields are used to probe the structure of internal states. Extensions of the same techniques developped for spectroscopy allow one to control the internal degrees of freedom coherently.

Laser cooling and trapping techniques allow one to do the same with the external degrees of freedom of the atom.

Milestones in ultracold atom physics

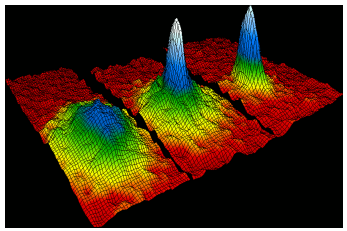
- first deflection of an atomic beam observed as early as 1933 (O. Frisch)
- revival of study of radiative forces in the lates 1970's; first proposals for laser cooling of neutral atoms (Hänsch – Dehmelt) and ions (Itano – Wineland)
  Why ? **Rise of the LASER**

Laser cooling and trapping:

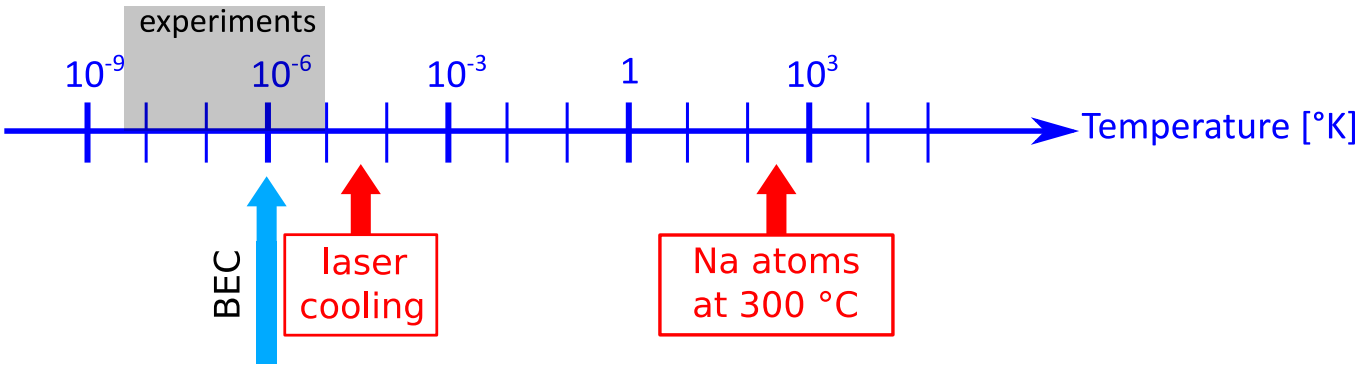


- 1980 : Slowing and bringing an atomic beam to rest
- 1985 : Optical molasses
- 1988 : magneto-optical traps , sub-Doppler cooling
- 1997 : Nobel Prize for Chu, Cohen-Tannoudji, Phillips

Quantum degenerate gases:

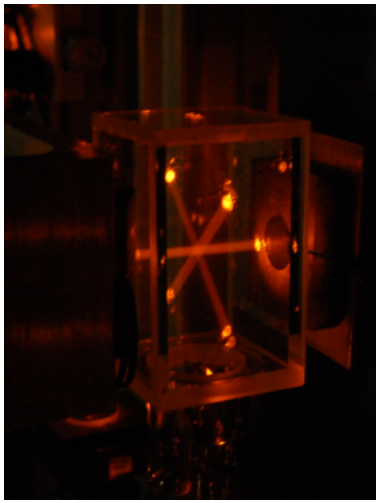


- Bose-Einstein condensation in 1995 [Cornell, Wieman, Ketterle : Nobel 2001]
- Degenerate Fermi gases in 2001 [Jin, JILA]

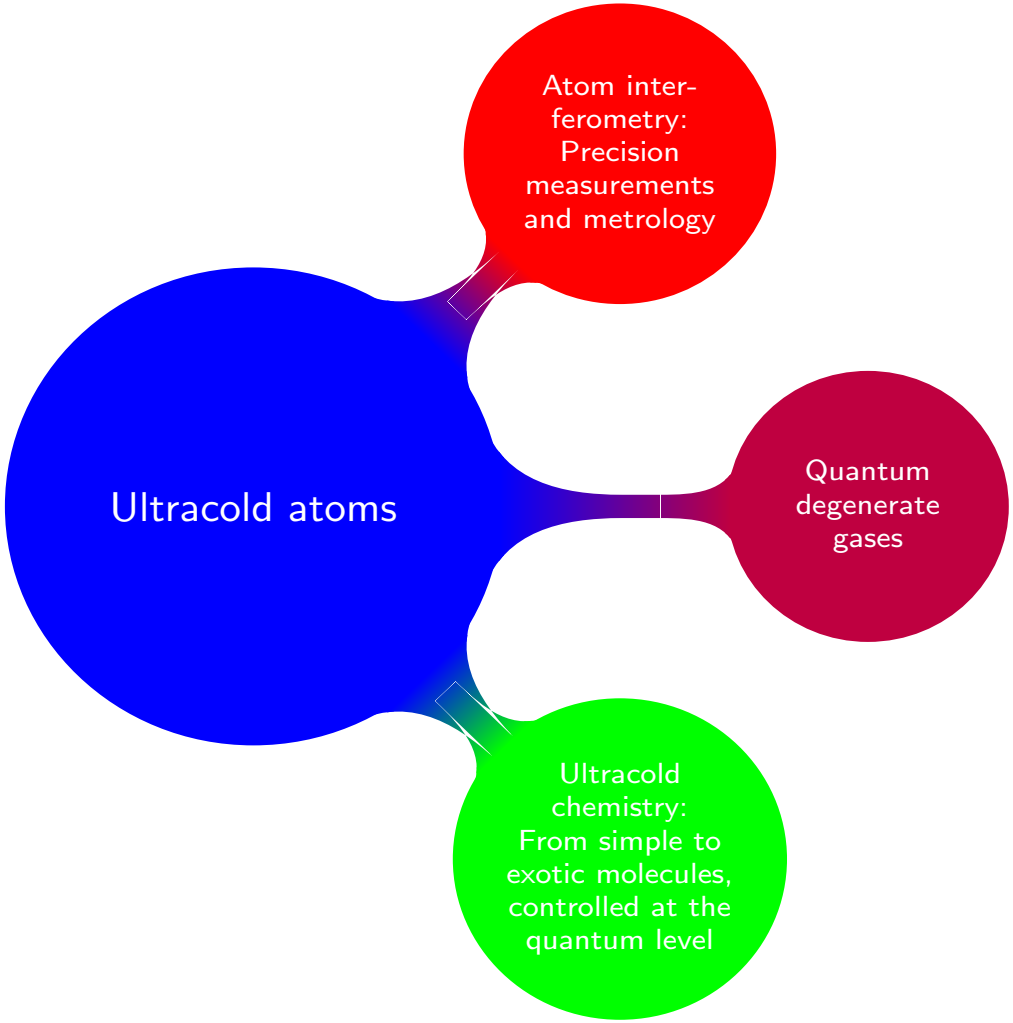


Why are ultracold atoms (and molecules) still interesting today ?

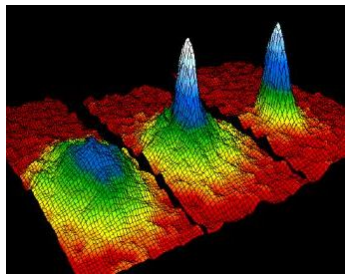
Laser cooling and trapping of neutral atoms



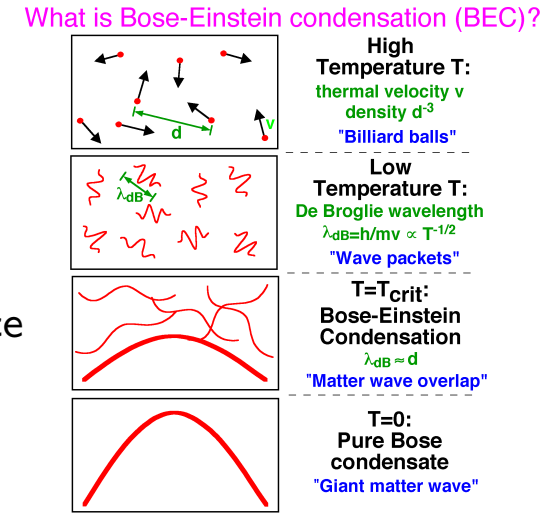
Nobel Prize 1997 :
S. Chu,
C. Cohen-Tannoudji,
W. D. Phillips



Quantum gases :
Bose-Einstein condensation (1995)
Degenerate Fermi gases (1999)



Nobel Prize 2001 :
E. Cornell,
W. Ketterle,
C. Wieman



Quantum degeneracy : phase space density $\lambda_{\mathrm{dB}} \gtrsim n^{-1/3}$

$\lambda_{\mathrm{dB}} = \sqrt{\frac{2\pi\hbar^2}{m k_B T}}$: thermal De Broglie wavelength

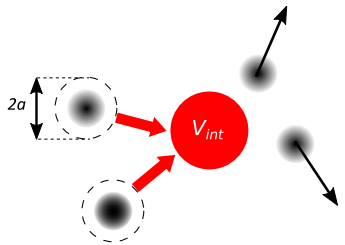
n: spatial density $\qquad$ $n^{-1/3}$: mean interparticle distance

From W. Ketterle group website, http://www.cua.mit.edu/

Interacting atoms, but dilute gas: $na^3 \ll 1$

a : scattering length for $s-$wave interactions

$8\pi a^2$: scattering cross-section (bosons)

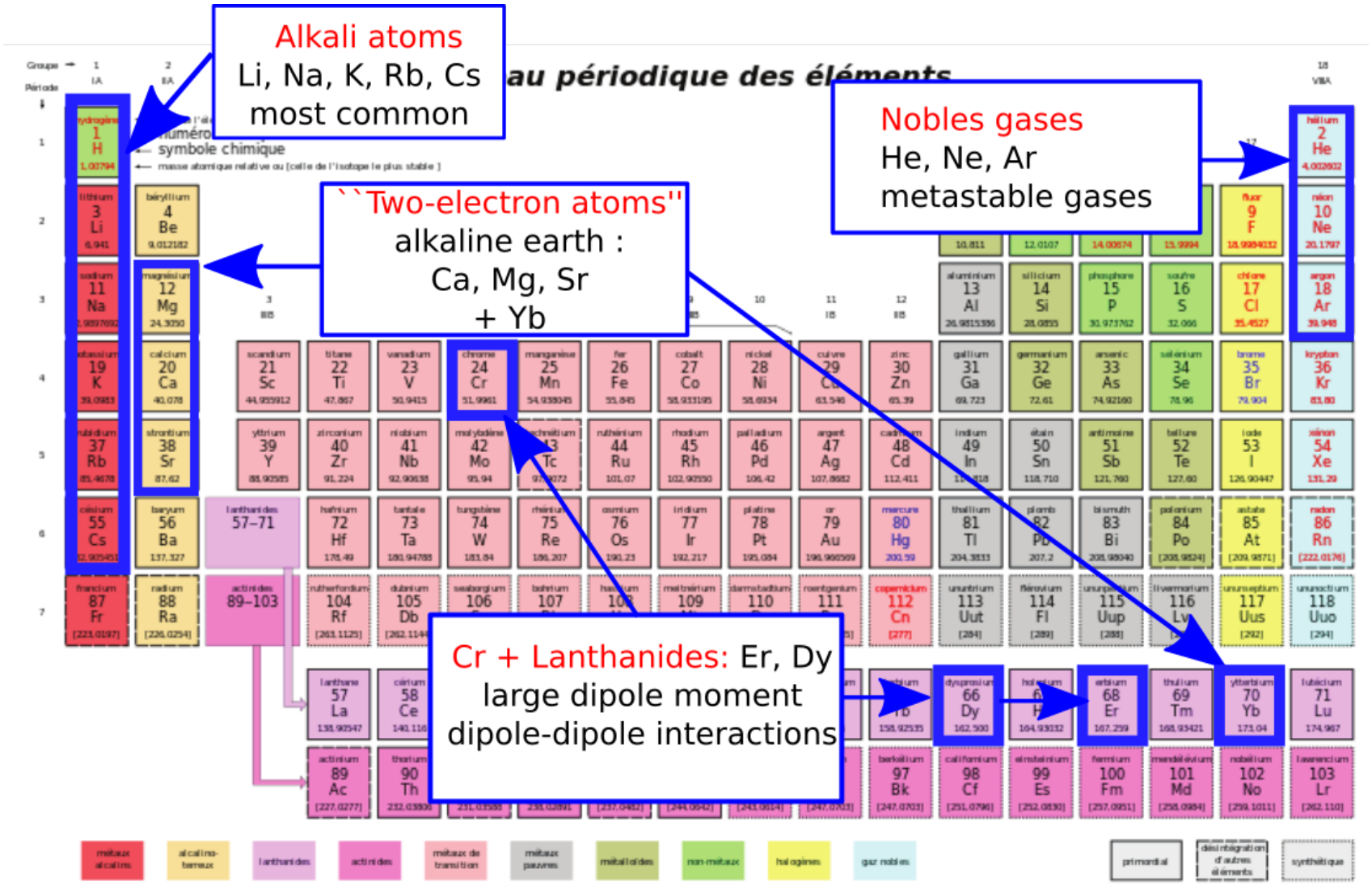


$$a \ll n^{-1/3} \ll \lambda_{\mathrm{dB}}$$

Typical values (BEC of ^{23}Na atoms) :

$$a \sim 2\,\mathrm{nm}$$
$$n^{-1/3} \sim 100\,\mathrm{nm}$$
$$\lambda_{\mathrm{dB}} \sim 1\,\mu\mathrm{m} \text{ at } T = 100\,\mathrm{nK}$$

Which atomic species ?



Outline of the course

1. Introduction and overview of experimental techniques (2 lectures),
   - *Atomic structure; Magneto-optical trap*
   - *Atomic interferometry*
2. Non-interacting quantum gases (1 lecture),
3. Atom-atom scattering in the ultracold limit (1/2 lecture),
   - *Study of a model potential; scattering length*
4. Bose-Einstein condensates (1/2 lecture),
   - *Collective modes and free expansion of a Bose-Einstein condensate*
   - *Vortices in rotating Bose-Einstein condensates*
5. Coherence of quantum gases (1 lecture),
   - *First-order correlation function of a Bose gas*
   - *Hanbury-Brown-Twiss effect with cold atoms*

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6. Ultracold fermions ; BEC-BCS crossover (2 lectures),
   - *Superfluid hydrodynamics; boson-fermion mixtures*
7. Optical lattices ; superfluid-Mott insulator transition (3 lectures).
   - *Bloch oscillations*
   - *Particle-hole excitations of a Mott insulator*

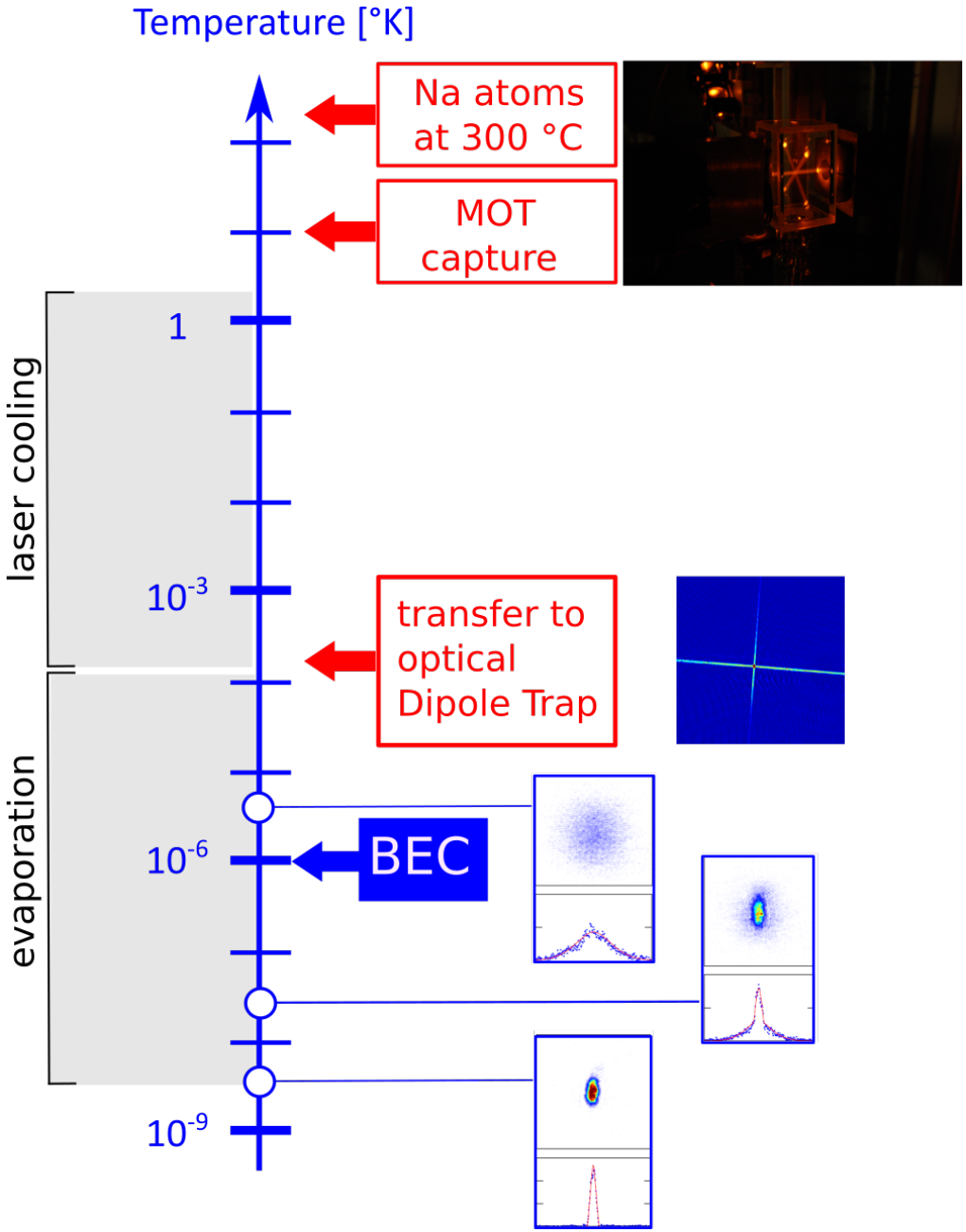
The experimental path to quantum degenerate gases

Typical experimental sequence :

- catch atoms in a magneto-optical trap
- laser cooling to $\sim 50\,\mu$K
- transfer to conservative trap (no resonant light): optical trap or magnetic trap
- evaporative cooling to BEC

Take a picture of the cloud.

Repeat



Necessity of cooling in two steps: "bright" then "dark"

Quantum gases : Phase-space density $\mathcal{D} = n\lambda_{\rm th}^3 \geq 1$

- Step 1: laser cooling in a magneto-optical trap (MOT)

Laser cooling relies on the interaction between the atoms and a near-resonant laser. Spontaneous emission of photons is at the same time

- essential to provide necessary dissipative mechanism to cool the motional degrees of freedom of the atoms,
- but also intrinsically random. Randomness $\Longrightarrow$ limit temperature !

Typical MOT:

$N \sim 10^9$ atoms, $\quad n \lesssim 10^{11}$ at/cm^3, $\quad T \sim 10\,\mu$K, $\quad \mathcal{D} \sim 10^{-6} - 10^{-5}$

To overcome the limitations of laser cooling, all experiments (with one exception) follow the same path :

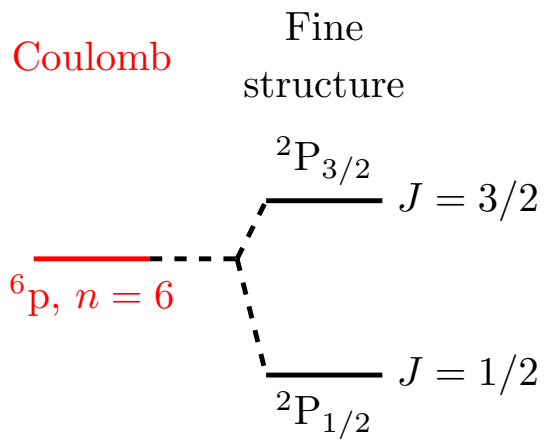
- Step 2 : evaporative cooling in a conservative trap
  - trapping "in the dark" in a *conservative trap* (optical or magnetic),
  - forced evaporative cooling to reach quantum degeneracy.

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- center-of mass variables $\hat{\boldsymbol{R}}, \hat{\boldsymbol{P}}$
- internal variables, *e.g.* electronic angular momentum $|J, m_J\rangle$

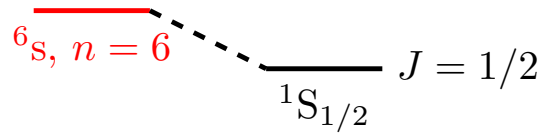
Example : ^{133}Cs

- alkali atom : one peripheral s electron
- similar structure as hydrogen or other alkalis
- fine structure: electronic angular momentum $\hat{\boldsymbol{J}} = \hat{\boldsymbol{S}} + \hat{\boldsymbol{L}}$ good quantum number



Notation:

- $g_i \equiv n, J, m_J$
- $e_j \equiv n', J', m_{J'}$



Possibly still degenerate : Zeeman degeneracy $2J + 1$ lifted by the Zeeman effect, nuclear spin I coupled to electronic degrees of freedom by the hyperfine interaction, ...

Modes of the electomagnetic field: plane waves indexed by $\mu = (\boldsymbol{k}_\mu, \boldsymbol{\epsilon}_\mu)$

- momentum $\hbar \boldsymbol{k}_\mu$
- polarisation vector $\boldsymbol{\epsilon}_\mu \perp \boldsymbol{k}_\mu$
- energy $\hbar\omega_\mu = \hbar c |\boldsymbol{k}_\mu|$

"Quantization box" of volume L^3 with periodic boundary conditions

Quantized electric field $\hat{\boldsymbol{E}}$:

$$\hat{\boldsymbol{E}} = \sum_\mu E_\mu \, e^{i\boldsymbol{k}_\mu \cdot \boldsymbol{r}} \, \hat{a}_\mu \, \boldsymbol{\epsilon}_\mu \; + \; \text{h.c.} \qquad \text{with} \; E_\mu = i\sqrt{\frac{\hbar\omega_\mu}{2\epsilon_0 L^3}}$$

$\hat{a}_\mu$: annihilation operator of a photon in mode μ

Laser field in quantum optics :

coherent state of the e-m field in the laser mode $L \equiv \omega_L, \boldsymbol{k}_L, \boldsymbol{\epsilon}_L$

$$|\alpha_L\rangle = \sum_{n=0}^{\infty} \frac{\alpha_L^n}{\sqrt{n!}} |n : \boldsymbol{k}_L, \boldsymbol{\varepsilon}_L\rangle, \quad \text{with } \hat{a}_L|\alpha_L\rangle = \alpha_L|\alpha_L\rangle$$

$\langle\alpha_L|\hat{a}_L^\dagger \hat{a}_L|\alpha_L\rangle = |\alpha_L|^2 = N_L$, Number of photons in the laser mode

N.B.: in quantum electrodynamics, coherent states are naturally produced by classical radiating sources.

Unitary transformation of the atom + field Hamiltonian leads to

$$\hat{\boldsymbol{E}}(\boldsymbol{r}) \longrightarrow \underbrace{\boldsymbol{\mathcal{E}}_L(\boldsymbol{r},t)}_{\text{Classical laser field}} + \underbrace{\hat{\boldsymbol{E}}_{\mathrm{V}}(\boldsymbol{r})}_{\text{Quantized vacuum field}}$$

Classical driving field:

$$\boldsymbol{\mathcal{E}}_L(\boldsymbol{r},t) = \frac{E_{\mu=L}(\boldsymbol{r})}{L^{3/2}} \boldsymbol{\varepsilon}_{\mu=L} \left(\alpha_L e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega_L t)} + \text{h.c.}\right) = E_L \boldsymbol{\varepsilon}_L \cos\left[\omega_L t + \varphi_L(\boldsymbol{r})\right]$$

We have chosen the laser mode as a plane wave in the previous discussion.

Other modes of the e-m field, *e.g.* standing or Gaussian waves, are often relevant as well.

We thus write the laser field in the most general form:

$$\boldsymbol{\mathcal{E}}_L(\boldsymbol{r},t) = E_L(\boldsymbol{r})\boldsymbol{\varepsilon}_L \cos\left[\omega_L t + \varphi_L(\boldsymbol{r})\right]$$

	$\varphi_L(\boldsymbol{r})$	$E_L(\boldsymbol{r})$
Plane wave	$\boldsymbol{k}_L \cdot \boldsymbol{r}$	uniform
Standing wave	uniform	$\propto \cos(\boldsymbol{k}_L \cdot \boldsymbol{r})$

Photon flux through the boundary of the quantization box: $\frac{N_L}{L^3} \times c\,dt\,L^2$.

Energy flux per unit area: $\delta E = \frac{N_L \hbar \omega_L}{L^3}\, c\,dt = c\,w\,dt$.

Density of e-m energy per unit volume: $w = \frac{N_L \hbar \omega_L}{L^3} = \frac{\epsilon_0}{2}|\boldsymbol{E}_L|^2$

Optical intensity = energy flux per unit surface [units W/m^2]:

$$I = cw = \frac{\epsilon_0 c}{2}|\boldsymbol{E}_L|^2.$$

Operational quantity to measure the laser "strength".

Typical values vary strongly in ultracold atoms experiments, depending on the application:

- $\sim$mW/cm^2 (near-resonant lasers: cooling, ...),
- $\sim \mu$W/cm^2 (near-resonant lasers: imaging),
- ~ 10W/cm^2 (far off-resonant lasers for optical trapping).

Atom-light interaction: Dipolar electric approximation

Dipolar electric Hamiltonian :

$$\hat{V}_{A-\mathrm{em}} = -\hat{\boldsymbol{d}} \cdot \hat{\boldsymbol{E}}(\hat{\boldsymbol{R}})$$

- $\hat{\boldsymbol{d}}$: electric dipole of the atom (purely atomic, acts on internal variables).
- $\hat{\boldsymbol{R}}$: center of mass of the atom (purely atomic, acts on external variables).
- $\hat{\boldsymbol{E}}(\hat{\boldsymbol{R}})$: mixed operator, acts on electromagnetic and atomic external variables.

Remarks:

- Coulomb gauge of quantum electrodynamics.
- Approximation valid if atomic dimensions ($\sim$ Å) $\ll$ wavelength (UV, visible, IR, ...): the e-m field is approximately uniform over the extent of the electronic cloud.
- Neglect higher-order terms in the multipolar expansion (magnetic dipole, electric quadrupole, ...) smaller by a factor at least $\sim \alpha_{\mathrm{FS}}^2$, with $\alpha_{\mathrm{FS}} \approx 1/137$ the fine structure constant.

Electric dipole operator in the atomic basis:

$$\hat{\boldsymbol{d}} = \sum_{g_i, e_j} \boldsymbol{d}_{ij} |e_j\rangle\langle g_i| + \mathrm{h.c.}, \qquad \boldsymbol{d}_{ij} = \langle e_j | \hat{\boldsymbol{d}} | g_i \rangle$$

$\{|g_i\rangle\}_i, \{|e_j\rangle\}_j$ denotes the ground state and excited states manifolds (internal variables).

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$$\hat{V}_{A-\mathrm{em}} = -\sum_{\mu} E_{\mu} \quad \left(\hat{\boldsymbol{d}} \cdot \boldsymbol{\epsilon_{\mu}}\right) \quad \frac{e^{i\boldsymbol{k}_{\mu}\cdot\boldsymbol{r}}}{L^{3/2}} \quad \hat{a}_{\mu} + \text{ h.c.}$$

Matrix elements for the transition $|\boldsymbol{k}_i, g_i; 1:\mu\rangle \longrightarrow |\boldsymbol{k}_f, e_j; \varnothing\rangle$:

$$\langle \boldsymbol{k}_f, e_j; \varnothing | \hat{V}_{A-\mathrm{em}} | \boldsymbol{k}_i, g_i; 1:\mu\rangle = -\frac{E_{\mu}}{L^{3/2}} \underbrace{\langle \boldsymbol{k}_f | e^{i\boldsymbol{k}_{\mu}\cdot\boldsymbol{r}} | \boldsymbol{k}_i\rangle}_{(1)} \underbrace{\langle e_j | \left(\hat{\boldsymbol{d}} \cdot \boldsymbol{\epsilon_{\mu}}\right) | g_i\rangle}_{(2)} \langle \varnothing | \hat{a}_{\mu} | 1:\mu\rangle$$

1. $= \int d^3\boldsymbol{r}\, e^{i(\boldsymbol{k}_i + \boldsymbol{k}_{\mu} - \boldsymbol{k}_f)\cdot\boldsymbol{r}} \propto \delta(\boldsymbol{k}_i + \boldsymbol{k}_{\mu} - \boldsymbol{k}_f)$: conservation of momentum
2. $\langle J', m_{J'} | \left(\hat{\boldsymbol{d}} \cdot \boldsymbol{\epsilon_{\mu}}\right) | J, m_J\rangle$: conservation of angular momentum

Absorption of a photon $(\boldsymbol{k}_\mu, \hbar\omega_\mu)$: atomic state changes from $|g, \boldsymbol{k}_i\rangle$ to $|e, \boldsymbol{k}_f\rangle$

Conservation of momentum : $\boldsymbol{k}_f = \boldsymbol{k}_i + \boldsymbol{k}_\mu$

Conservation of energy : $\omega_\mu = \omega_{eg} + \frac{\hbar k_\mu^2}{2M} + \frac{\hbar}{M}\boldsymbol{k}_\mu \cdot \boldsymbol{k}_i$

$E_R = \hbar\omega_R = \frac{\hbar^2 k_\mu^2}{2M}$: recoil energy/frequency, $\frac{\hbar}{M}\boldsymbol{k}_\mu \cdot \boldsymbol{k}_i = \boldsymbol{k}_\mu \cdot \boldsymbol{v}_i$: Doppler shift

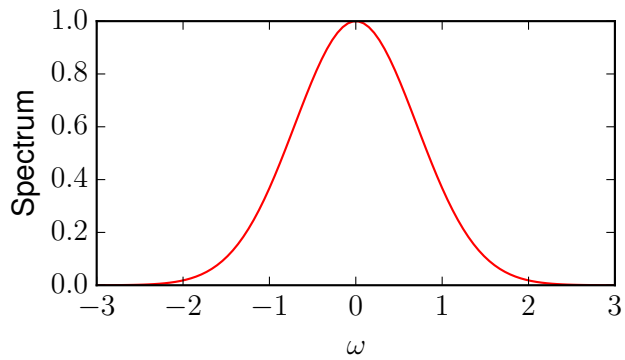
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Doppler broadening of spectral lines:

Gas in thermal equilibrium : $P(\boldsymbol{v}) \propto e^{-\frac{M\boldsymbol{v}^2}{2k_B T}}$

$\Longrightarrow$ Gaussian absorption spectrum

spectral width $\Delta\omega = q\sqrt{\frac{8k_B T}{\pi M}}$



Emission of a photon $(\boldsymbol{k}_\mu, \hbar\omega_\mu)$: $|e, \boldsymbol{k}_i\rangle \rightarrow |e, \boldsymbol{k}_f\rangle$

$\boldsymbol{k}_f = \boldsymbol{k}_i - \boldsymbol{k}_\mu$

$\omega = \omega_{eg} - \frac{\hbar k_\mu^2}{2M} + \frac{\hbar}{M}\boldsymbol{k}_\mu \cdot \boldsymbol{k}_i$

Same Doppler shift but opposite recoil shift

Laser polarisation $\boldsymbol{\varepsilon}$: $\boldsymbol{\varepsilon} \cdot \boldsymbol{k}_L = 0$

Decomposition in the standard basis $\boldsymbol{\varepsilon} = \sum_{q=0,\pm1} \varepsilon_q \boldsymbol{e}_q$

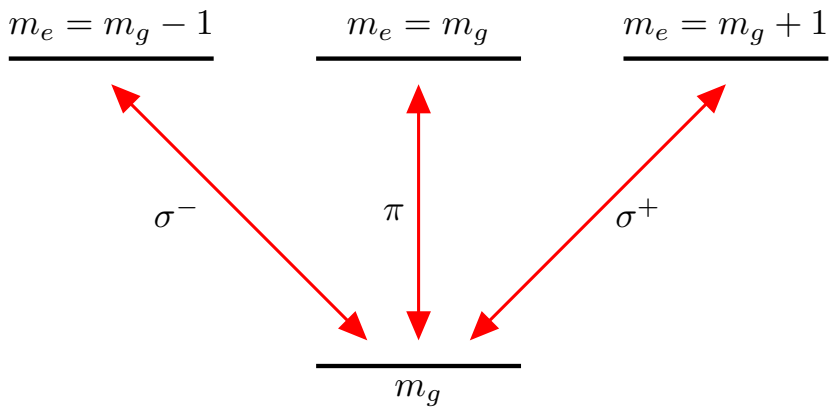
$$\boldsymbol{e}_+ = \frac{-1}{\sqrt{2}} (\boldsymbol{e}_x + i\boldsymbol{e}_y), \qquad \boldsymbol{e}_- = \frac{1}{\sqrt{2}i} (\boldsymbol{e}_x - i\boldsymbol{e}_y), \qquad , \boldsymbol{e}_0 = \boldsymbol{e}_z$$

$$\sigma_\pm\text{-polarized light: } \boldsymbol{\varepsilon} = \boldsymbol{e}_\pm, \qquad \pi\text{-polarized light: } \boldsymbol{\varepsilon} = \boldsymbol{e}_z$$

N.B. in atomic physics one usually chooses z along the direction of a static magnetic field (if any), not necessarily along the laser propagation direction $\boldsymbol{k}_L$.

Selection rules for allowed transitions :

$$\Delta J = 0, \pm 1$$
$$\Delta m_J = 0, \pm 1$$



Computation of matrix elements of d

Express the dipole operator in the standard basis as for the polarization $\boldsymbol{\varepsilon}$,

$$\hat{\boldsymbol{d}} = \sum_{q=0,\pm1} \hat{d}_q \boldsymbol{e}_q, \qquad \hat{d}_q = \sum_{g_i,e_j} \langle e_j|\hat{d}_q|g_i\rangle |e_j\rangle\langle g_i| + \text{h.c.}, \qquad \hat{\boldsymbol{d}}\cdot\boldsymbol{\varepsilon} = \sum_{q=0,\pm1} \hat{d}_q \varepsilon_q$$

Wigner-Eckart theorem:

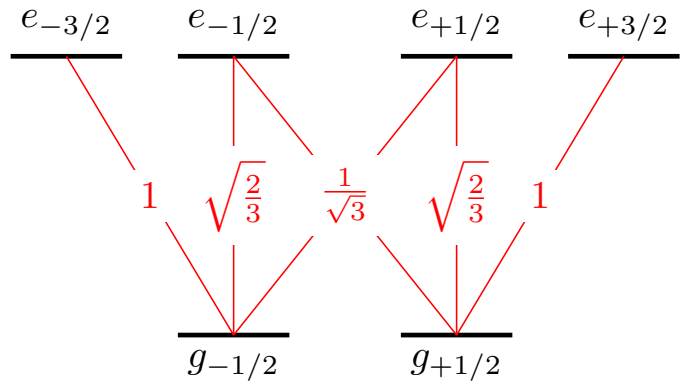
$$\langle J', m_{J'}|\hat{d}_q|J, m_J\rangle = \underbrace{d_{JJ'}}_{\text{reduced dipole}} \times \underbrace{\langle J,', m_{J'}|J, m_J; 1, q\rangle}_{\text{Clebsch-Gordan coefficient}}$$

$d_{JJ'}$ is a reduced dipole matrix element characterizing the $J \to J'$ transition.

The Clebsch-Gordan coefficients determine the selection rules.

Example: $J = 1/2 \to J' = 3/2$ transition

m_J	$m_{J'}$	q	$\langle e_j\|\hat{d}_q\|g_i\rangle$
$\frac{1}{2}$	$\frac{3}{2}$	$+1$	d
$\frac{1}{2}$	$\frac{1}{2}$	0	$\sqrt{\frac{2}{3}}d$
$+\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{\sqrt{3}}d$



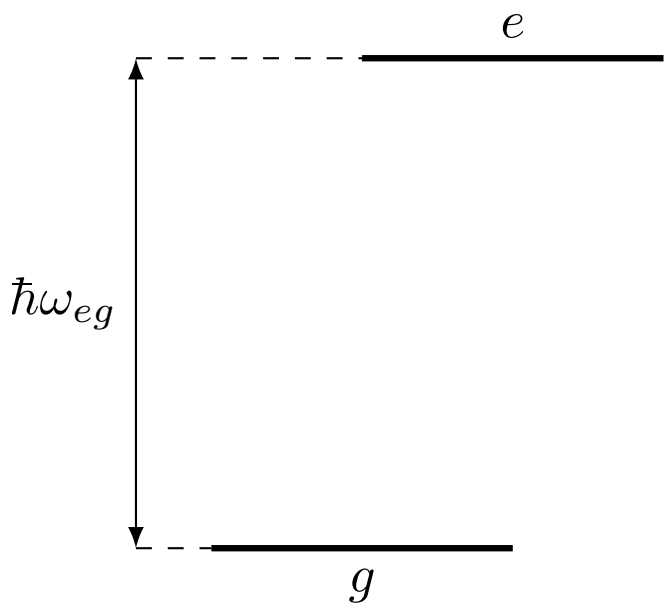
$$\sum \left|\langle J, m_J; 1, q|J,', m_{J'}\rangle\right|^2 = 1$$

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Atom in free space:

$$H_{\mathrm{at}} = \frac{\hat{\boldsymbol{P}}^2}{2M} + \hbar\omega_{eg}|e\rangle\langle e|$$

- center-of-mass momentum/position $\hat{P}/\hat{R}$
- two internal states g/e



Dipole operator :

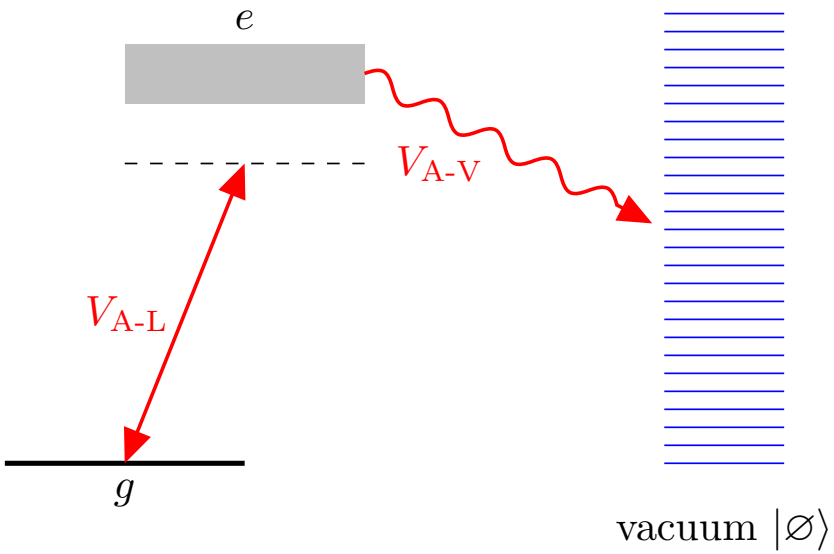
$$\hat{\boldsymbol{d}} = \boldsymbol{d}\hat{D}^+ + \boldsymbol{d}^*\hat{D}^-$$

$$\hat{D}^+ = |e\rangle\langle g| = \left(\hat{D}^-\right)^\dagger$$

- Bohr frequency $\omega_{eg} = \frac{E_e - E_g}{\hbar}$

The standard model of quantum optics: two-level atom interacting with the e-m field

Atom+field Hamiltonian : $H_{\text{atom+field}} = \underbrace{H_{\text{at}} + V_{\text{A-L}}}_{\text{atomic}} + \underbrace{H_{\text{field}}}_{\text{e-m}} + \underbrace{V_{\text{A-V}}}_{\text{atomic + e-m}}$



Rotating wave approximation :

$$V_{\text{A-L}} \underset{\text{RWA}}{\approx} \frac{\hbar\Omega_L}{2} e^{-i(\omega_L t+\varphi_L)}|e\rangle\langle g| + \text{h.c.} \qquad \hbar\Omega_L = -\boldsymbol{d}\cdot\boldsymbol{\varepsilon}_L E_L$$

$$V_{\text{A-V}} \underset{\text{RWA}}{\approx} \left(\sum_\mu V_\mu \hat{a}_\mu e^{i(\boldsymbol{k}_\mu\cdot\hat{\boldsymbol{R}}-\omega_\mu t)}\right)|e\rangle\langle g| + \text{h.c.} \qquad V_\mu = -\frac{\boldsymbol{d}\cdot\boldsymbol{\varepsilon}_\mu E_\mu}{2L^{3/2}}$$

Master equation for the atomic density operator

Atom and field are entangled : the proper quantum mechanical labeling of a state is $|\phi\rangle_{\text{atom}} \otimes |\alpha\rangle_L \otimes |\eta\rangle_{m\neq L}$. We need a purely atomic description that erases all information about the electromagnetic field, except for its effect on the atomic variables.

Density matrix formalism : $\hat{\sigma}$ describes the state of the composite atom+field system

- $\text{Tr}(\hat{\sigma}) = 1$
- $\text{Tr}(\hat{\sigma}^2) \leq 1$
- $\langle \hat{O} \rangle = \text{Tr}(\hat{\sigma}\hat{O})$
- $i\hbar \frac{d}{dt}\hat{\sigma} = [\hat{H}_{\text{atom+field}}, \hat{\sigma}]$

Reduced atomic density matrix :

$$\hat{\rho} = \text{Tr}_{\text{field}}\left[\hat{\sigma}\right] = \begin{pmatrix} \hat{\rho}_{gg} & \hat{\rho}_{ge} \\ \hat{\rho}_{eg} & \hat{\rho}_{ee} \end{pmatrix} : \text{ purely atomic operator}$$

N.B. $\hat{\rho}_{\alpha\beta} = \langle\alpha|\hat{\rho}|\beta\rangle$ is still an operator in general with respect to the external degrees of freedom of the atom.

$$i\hbar \frac{d}{dt}\hat{\rho} = [\hat{H}_{\text{at}} + \hat{V}_{A-L}, \hat{\rho}] + \text{Tr}_{\text{field}}\left[\hat{V}_{A-V}, \hat{\sigma}_{\text{atom+field}}\right]$$

$\hat{V}_{A-L}$ corresponds to coherent driving by the classical laser field
$\boldsymbol{\mathcal{E}}_L = \boldsymbol{\epsilon}_L E_L \cos(\omega_L t + \varphi_L)$.

$\hat{V}_{A-V}$ corresponds to the coupling to the electromagnetic field vacuum (spontaneous emission): Dissipation.

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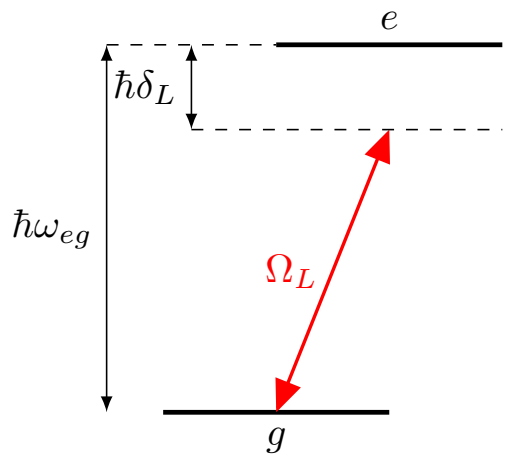
We assume for the rest of the lecture that the atom is at rest, and that its external degrees of freedom can be ignored (this will be covered in the next lecture). The term $V_{\text{A-L}}$ gives a contribution :

$$i\hbar\frac{d}{dt}\hat{\rho} = [\hat{H}_{\text{at}} + \hat{V}_{A-L}, \hat{\rho}]$$

$$\left(\frac{d}{dt}\rho_{ee}\right)_{\text{las.}} = -i\frac{\Omega_L}{2}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$

$$\left(\frac{d}{dt}\rho_{gg}\right)_{\text{las.}} = i\frac{\Omega_L}{2}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$

$$\left(\frac{d}{dt}\tilde{\rho}_{eg}\right)_{\text{las.}} = i\delta_L\tilde{\rho}_{eg} + i\frac{\Omega_L}{2}\left(\rho_{ee} - \rho_{gg}\right)$$



with $\delta_L = \omega_L - \omega_{eg}$ and $\tilde{\rho}_{ge} = e^{-i(\omega_L t+\varphi_L)}\rho_{ge}$, $\tilde{\rho}_{eg} = e^{i(\omega_L t+\varphi_L)}\rho_{eg}$.

Coherent evolution leading linear superpositions of $|g\rangle$ and $|e\rangle$ (cf. Rabi oscillations)

Light shift of the bare atomic levels:

$$E_{\pm} = \pm\sqrt{\Omega_L^2 + \delta_L^2}$$

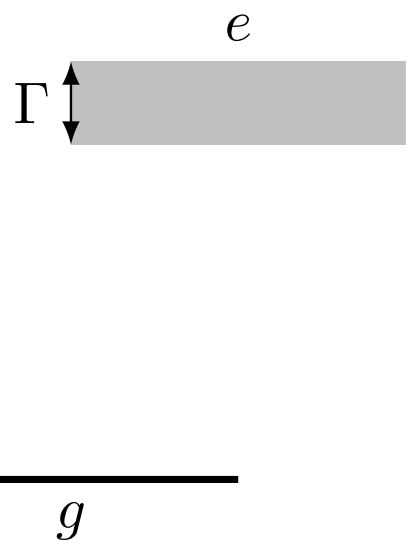
Shift proportional for small Ω_L to the light intensity : $\Delta E_g = -\Delta E_e = -\frac{\Omega_L^2}{4\delta_L}$

The term $V_{\text{A-V}}$ gives a contribution :

$$i\hbar \frac{d}{dt}\hat{\rho} = \text{Tr}_{\text{field}}\left[\hat{V}_{A-V}, \hat{\sigma}_{\text{atom+field}}\right]$$

Result of the calculation :

$$\left(\frac{d}{dt}\rho_{ee}\right)_{\text{s.e.}} = -\Gamma\rho_{ee}$$
$$\left(\frac{d}{dt}\rho_{gg}\right)_{\text{s.e.}} = \Gamma\rho_{ee}$$
$$\left(\frac{d}{dt}\rho_{eg}\right)_{\text{s.e.}} = -\frac{\Gamma}{2}\rho_{eg}$$



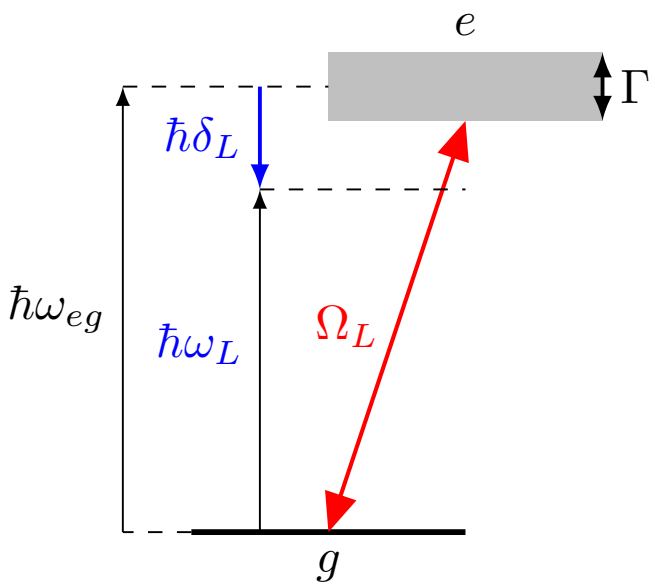
$\Gamma = \frac{\omega_{eg}^3 d^2}{6\pi\epsilon_0\hbar c^2}$ is the spontaneous emission rate.

Assumptions :

- $\hat{\sigma} \approx \hat{\rho} \otimes \prod_\mu |\varnothing\rangle_\mu \ {}_\mu\langle\varnothing|$
- correlation time of the reservoir : ω_{ge}^{-1} very short compared to the time scales $(\Omega_R, \Gamma, \delta)$ over which the atomic variables evolve

Optical Bloch equations :

$$\frac{d}{dt}\rho_{ee} = -\Gamma\rho_{ee} - \frac{\Omega_L}{2i}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$
$$\frac{d}{dt}\rho_{gg} = \Gamma\rho_{ee} + \frac{\Omega_L}{2i}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$
$$\frac{d}{dt}\tilde{\rho}_{eg} = \left(i\delta_L - \frac{\Gamma}{2}\right)\tilde{\rho}_{eg} + i\frac{\Omega_L}{2}\left(\rho_{ee} - \rho_{gg}\right)$$



We defined: $\tilde{\rho}_{ge} = e^{-i(\omega_L t+\varphi_L)}\rho_{ge}$, $\tilde{\rho}_{eg} = e^{i(\omega_L t+\varphi_L)}\rho_{eg}$.

Laser detuning: $\delta_L = \omega_L - \omega_{eg}$

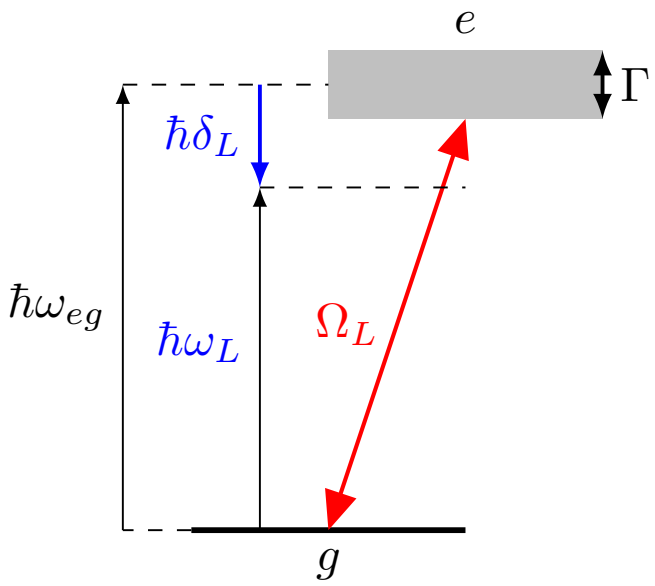
Natural linewidth of the transition: $\Gamma = \frac{\omega_{eg}^3 d^2}{6\pi\epsilon_0\hbar c^2}$

Laser electric field: $\boldsymbol{\mathcal{E}}_L = \boldsymbol{\epsilon}_L E_L \cos(\omega_L t + \varphi_L)$

Rabi frequency: $\boldsymbol{\Omega}_L = \langle e|\hat{\boldsymbol{d}}\cdot\boldsymbol{\epsilon}_L|g\rangle E_L = dE_L$

Optical Bloch equations :

$$\frac{d}{dt}\rho_{ee} = -\Gamma\rho_{ee} - \frac{\Omega_L}{2i}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$
$$\frac{d}{dt}\rho_{gg} = \Gamma\rho_{ee} + \frac{\Omega_L}{2i}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$
$$\frac{d}{dt}\tilde{\rho}_{eg} = \left(i\delta_L - \frac{\Gamma}{2}\right)\tilde{\rho}_{eg} + i\frac{\Omega_L}{2}\left(\rho_{ee} - \rho_{gg}\right)$$



Define:

$$u = \frac{1}{2}\left(\tilde{\rho}_{ge} + \tilde{\rho}_{eg}\right),$$
$$v = \frac{1}{2i}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$

$$\dot{u} = -\frac{\Gamma}{2}u + \delta_L v,$$
$$\dot{v} = -\frac{\Gamma}{2}v - \delta_L u - \Omega_L\frac{\rho_{ee} - \rho_{gg}}{2}$$

Interpretation of u and v :

$$\langle\hat{d}\rangle = 2d\left[u\cos(\omega_L t + \varphi_L) - v\sin(\omega_L t + \varphi_L)\right]$$

- u : component of the average dipole in-phase with the field
- v : component of the average dipole out-of-phase with the field

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Steady-state solutions of the OBE

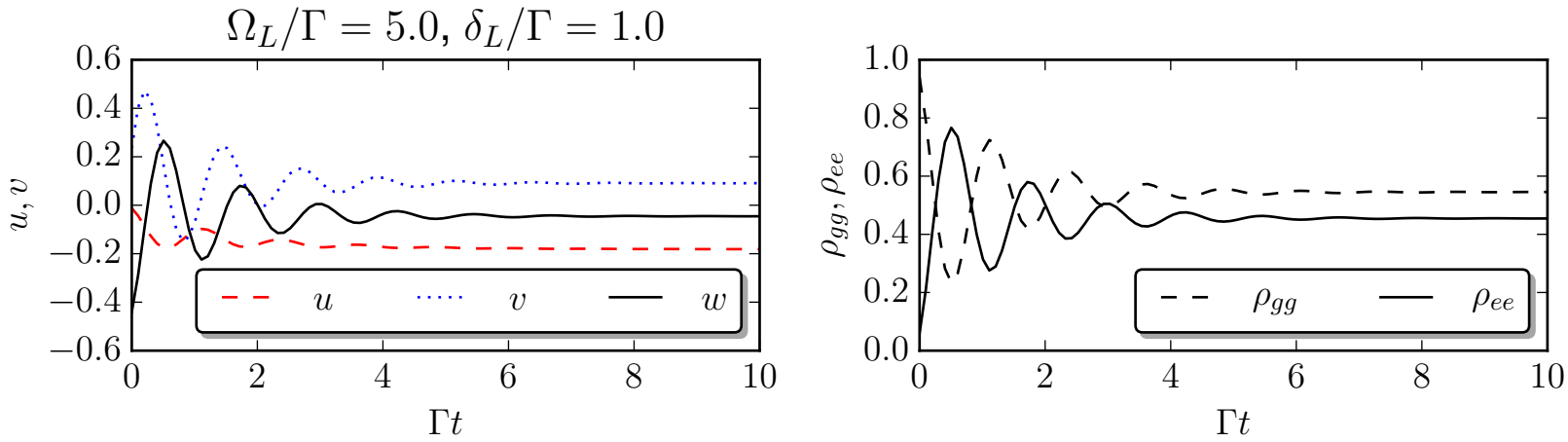
Stationary solutions :

$$u_{\mathrm{st}} = \frac{\delta_L}{\Omega_L}\frac{s}{1+s}$$
$$v_{\mathrm{st}} = \frac{\Gamma}{2\Omega_L}\frac{s}{1+s}$$
$$\rho_{ee,\mathrm{st}} = \frac{1}{2}\frac{s}{1+s}$$

Saturation parameter :

$$s = \frac{\frac{\Omega_L^2}{2}}{\delta_L^2 + \frac{\Gamma^2}{4}}$$

Numerical integration with $\rho_{ee}(0) = 0$, $u(0) = v(0) = 0$:



$$w = (\rho_{ee} - \rho_{gg})/2$$

Damped Rabi oscillations, damping to steady state after a few Γ^{-1}

Low and high intensity limits

Stationary solutions :

$$u_{\rm st} = \frac{\delta_L}{\Omega_L}\frac{s}{1+s}$$
$$v_{\rm st} = \frac{\Gamma}{2\Omega_L}\frac{s}{1+s}$$
$$\rho_{ee,\rm st} = \frac{1}{2}\frac{s}{1+s}$$

Saturation parameter :

$$s = \frac{\frac{\Omega_L^2}{2}}{\delta_L^2 + \frac{\Gamma^2}{4}}$$

High intensity $s \gg 1$: $\Omega_L^2 \gg \delta_L^2 + \frac{\Gamma^2}{4}$

$$u_{\rm st} \approx \frac{\delta_L}{\Omega_L} \to 0$$
$$v_{\rm st} \approx \frac{\Gamma}{2\Omega_L} \to 0$$
$$\rho_{ee,\rm st} \approx \frac{1}{2}$$

saturation of the near-resonant optical response of the atom

Low intensity $s \ll 1$: $\Omega_L^2 \ll \delta_L^2 + \frac{\Gamma^2}{4}$

$$u_{\rm st} \approx \frac{\delta_L}{\Omega_L} s$$
$$v_{\rm st} \approx \frac{\Gamma}{2\Omega_L} s$$
$$\rho_{ee,\rm st} \approx \frac{s}{2}$$

Linear response of a driven electric dipole ($u, v \propto |\boldsymbol{\mathcal{E}}_L|$, $s \propto |\boldsymbol{\mathcal{E}}_L|^2$)

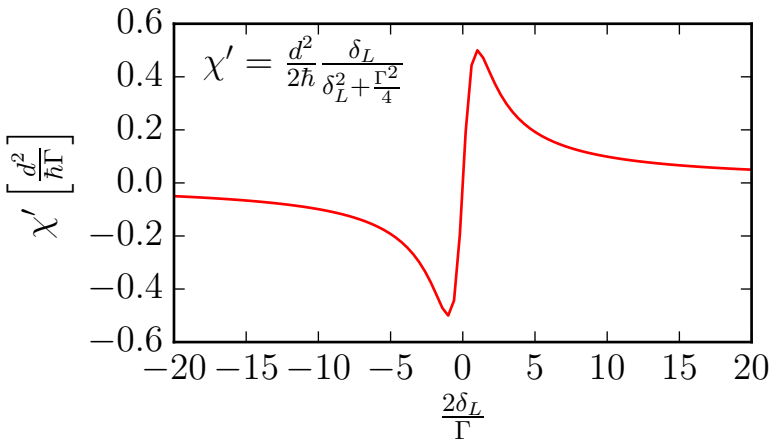
Linear response to a harmonic perturbation: $\hat{H} = \hat{H}_0 - \hat{d}E_L \cos(\omega_L t + \phi_L)$

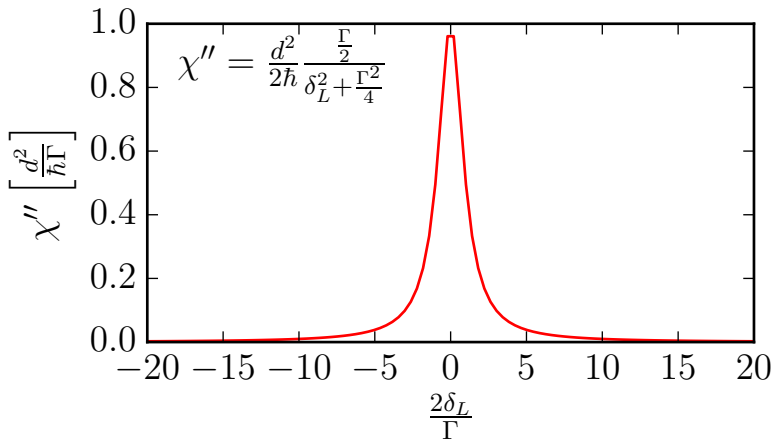
Complex susceptibility : $\chi = \chi' + i\chi''$

$$\begin{aligned}\langle \hat{d} \rangle &= \left(\chi E_L e^{i(\omega_L t + \phi_L)} + \chi^* E_L e^{-i(\omega_L t + \phi_L)} \right) \\ &= \underbrace{(\chi + \chi^*)}_{2\chi'} E_L \cos(\omega_L t + \phi_L) - \underbrace{(\chi - \chi^*)}_{2\chi''} E_L \sin(\omega_L t + \phi_L) \\ &= 2d \left[u \cos(\omega_L t + \phi_L) - v \sin(\omega_L t + \phi_L) \right]\end{aligned}$$

In the limit where $s \ll 1$, we have

$$\chi' = \frac{d u_{\mathrm{st}}}{E_L} = \frac{d^2}{\hbar} \frac{\delta_L s}{\Omega_L^2} = \frac{d^2}{2\hbar} \frac{\delta_L}{\delta_L^2 + \frac{\Gamma^2}{4}}, \qquad \chi'' = \frac{d v_{\mathrm{st}}}{E_L} \approx \frac{d^2}{\hbar} \frac{\Gamma s}{2\Omega_L^2} = \frac{d^2}{2\hbar} \frac{\frac{\Gamma}{2}}{\delta_L^2 + \frac{\Gamma^2}{4}}$$





Aside: optical intensity and saturation intensity

Optical intensity :

$$I = \frac{\text{Optical power}}{\text{Area}} = \frac{\epsilon_0 c}{2} E_L^2,$$

$$\Omega_L^2 = \frac{2d^2 I}{\hbar^2 \epsilon_0 c}$$

Saturation intensity : I_{sat} defined such that $s = 1$ on resonance ($\delta_L = 0$)

$$I_{\text{sat}} = \frac{\hbar \Gamma c k_L^3}{12\pi} \sim \text{mW/cm}^2$$

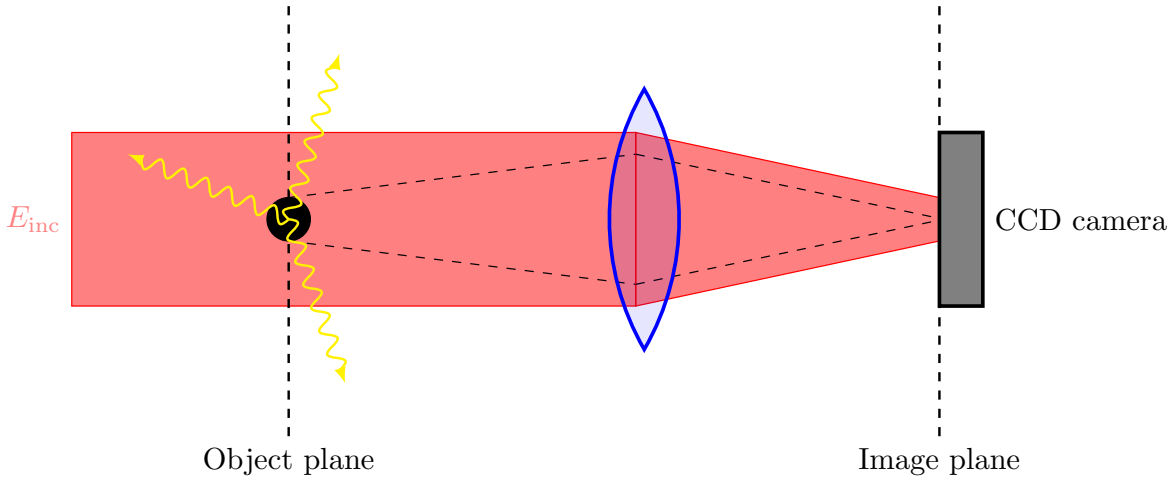
$$\Omega_L^2 = \frac{\Gamma^2}{2} \frac{I}{I_{\text{sat}}}$$

$$s = \frac{\frac{\Omega_L^2}{2}}{\delta_L^2 + \frac{\Gamma^2}{4}} = \frac{\frac{I}{I_{\text{sat}}}}{\left(\frac{2\delta_L}{\Gamma}\right)^2 + 1},$$

$$\frac{s}{1+s} = \frac{\frac{I}{I_{\text{sat}}}}{\left(\frac{2\delta_L}{\Gamma}\right)^2 + 1 + \frac{I}{I_{\text{sat}}}}$$

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Basic setup :



Camera placed in an image plane optically conjugate to the "plane of the atoms"
What are the properties of the transmitted light ?

What informations on the trapped atoms are recorded on the camera ?

Maxwell told us that an electric field propagating in a dielectric (non-magnetic) medium obeys the wave equation

$$\boldsymbol{\nabla}^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \boldsymbol{P}}{\partial t^2}$$

where $\boldsymbol{P} = \epsilon_0 \chi \boldsymbol{E}$ is the electric polarization density and $\chi_m = \chi'_m + i\chi''_m$ the electric susceptibility of the medium.

We write $\mathcal{E}_L(\boldsymbol{r}, t) = \frac{1}{2} E_L(\boldsymbol{r}, t) e^{i\varphi_L(\boldsymbol{r},t)} e^{i(k_L z - \omega_L t)} + \text{c.c.}$.

Assuming that the amplitude E_L and phase φ_L vary slowly on the scale of the laser wavelength, one can make a slowly-varying enveloppe (or paraxial) approximation:

$$\frac{dE_L}{dz} = -\frac{k_L \chi''_m}{2} E_L, \ : \text{Beer-Lambert law}$$

$$\frac{d\varphi_L}{dz} = \frac{k_L \chi'_m}{2} \varphi_L \ : \text{wave refraction}$$

If we *assume* that all atoms behave as independent dipoles with an average dipole moment $\langle \hat{\boldsymbol{d}} \rangle$, then the electric polarization density is simply

$$\boldsymbol{P} = \epsilon_0 \chi \boldsymbol{E} = n_{\mathrm{at}} \langle \hat{\boldsymbol{d}} \rangle$$

with $n_{\mathrm{at}}(\boldsymbol{r})$ the atomic density. The medium susceptibility is simply $\chi_m = n_{\mathrm{at}} \chi$ with χ the single-atom susceptibility obtained earlier.

NB : the atomic density must also vary smoothly *along* z on the scale λ_L.

The propagation equation for the amplitude and phase become

$$\frac{dE_L}{dz} = -\frac{k_L n_{\mathrm{at}} \chi'}{2} E_L, \ : \text{Beer-Lambert law}$$

$$\frac{d\varphi_L}{dz} = \frac{k_L n_{\mathrm{at}} \chi''}{2} \varphi_L \ : \text{wave refraction}$$

We rewrite the equation for $\mathcal{E}$ in terms of the laser intensity $I = \frac{\epsilon_0 c}{2}|\mathcal{E}|^2$,

$$\frac{dI}{dz} = -\kappa I(z) : \text{Beer-Lambert law}, \qquad \kappa = \frac{k_L n_{\text{at}} \chi''}{2} = \frac{3\lambda_L^2}{2\pi} \frac{n_{\text{at}}}{\left(\frac{2\delta_L}{\Gamma}\right)^2 + 1}$$

Interpretation in terms of scattered photons :

$d(\text{Photon flux}) = -n_{\text{at}}\, \sigma\, dz \times (\text{Photon flux})$, with a scattering cross-section

$$\sigma = \frac{\sigma_0}{\left(\frac{2\delta_L}{\Gamma}\right)^2 + 1}, \quad \sigma_0 = \frac{3\lambda_L^2}{2\pi}, \quad \kappa = \sigma n_{\text{at}}$$

Maximum on resonance ($\sigma = \sigma_0$ for $\delta_L \approx 0$).

The intensity on the camera (assuming that the focal depth of the imaging system is $\gg$ cloud size) gives a magnified version of the transmitted intensity,

$$I_t(x,y) = I_{\text{inc}}(x,y) e^{-\sigma \int n_{\text{at}}(x,y,z) dz}$$

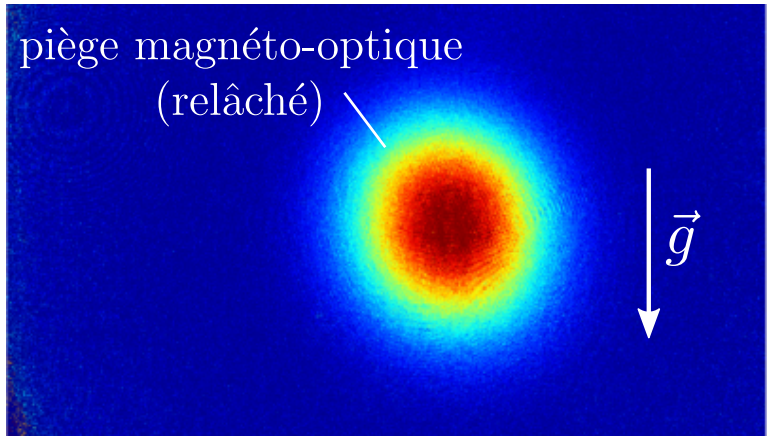
One calls $\tilde{n}(x,y) = \int n_{\text{at}}(x,y,z)dz$ the *column density* and $OD(x,y) = \sigma \tilde{n}(x,y)$ the *optical depth*.

Absorption signal :

$$\tilde{n}(x,y) = \frac{1}{\sigma} \ln \left(\frac{I_{\text{inc}}(x,y)}{I_t(x,y)} \right)$$

Absorption imaging of a MOT

Absorption image of a MOT of Yb atoms after a time of flight of $t \sim 10\,\text{ms}$, during which atom are released from the trap and expand freely.



Why do we need to take absorption pictures after a time of flight ?

- Often the optical depth is too large: $OD \sim 5 - 10$ for a MOT, $OD \sim 100$ or more for a BEC. Images are "pitch black" and dominated by technical noise.
- To avoid photon reabsorption and multiple scattering, which makes the above description invalid and the interpretation of images difficult.

In-situ images also possible, though easier with dispersive techniques than with absorption imaging.

Time-of-flight (t.o.f.) experiment :

- suddenly switch off the trap potential at $t = 0$,
- let the cloud expand for a time t.

The spatial distribution for long times is proportional to the initial *momentum distribution* $\mathcal{P}_0$ evaluated at $\boldsymbol{p} = \frac{M\boldsymbol{r}}{t}$:

$$\boxed{n_{\rm at}(\boldsymbol{r}, t) \longrightarrow_{t\to\infty} \left(\frac{M}{t}\right)^3 \mathcal{P}_0\left(\boldsymbol{p} = \frac{M\boldsymbol{r}}{t}\right)}$$

This results holds for a classical or a quantum gase, indistinctively, *provided one can neglect the role of interactions during the expansion*.

For non-degenerate gases, temperature can be inferred from the cloud size after t.o.f.

For a Boltzmann gas,

$$\frac{\langle p_x^2 \rangle}{2M} = \frac{k_B T}{2}$$

and suitable generalizations for Bose or Fermi gases.

Proof for a classical gas-I

A classical gas is described by its phase-space density $f(\mathbf{r}, \boldsymbol{p})$, such that

$$\mathcal{P}(\boldsymbol{p}) = \int d^3\boldsymbol{r}\, f(\mathbf{r}, \boldsymbol{p}) \ : \text{momentum distribution}$$

$$n_{\mathrm{at}}(\boldsymbol{r}) = \int d^3\boldsymbol{p}\, f(\mathbf{r}, \boldsymbol{p}) \ : \text{spatial distribution.}$$

Liouville's theorem: A phase-space element of volume $d^3\boldsymbol{r}_0 d^3\boldsymbol{p}_0$ centered on $(\boldsymbol{r}_0, \boldsymbol{p}_0)$ is conserved along the classical phase-space trajectory evolving from that point to $(\boldsymbol{r}(t), \boldsymbol{p}(t))$.
The phase space distribution function then obeys

$$f\Big(\boldsymbol{r}(t), \boldsymbol{p}(t), t\Big) = f_0(\boldsymbol{r}_0, \boldsymbol{p}_0),$$

with f_0 the initial distribution.

In a t.o.f. experiment, the initially confined particles are suddenly released at $t = 0$ and undergo ballistic flight for $t > 0$. The classical trajectory originating from $(\boldsymbol{r}_0, \boldsymbol{p}_0)$ is simply

$$\Big(\boldsymbol{r}(t), \boldsymbol{p}(t)\Big) = \Big(\boldsymbol{r}_0 + \frac{\boldsymbol{p}_0}{M} t, \boldsymbol{p}_0\Big)\,.$$

The spatial density after t.o.f. reads

$$n_{\mathrm{at}}(\boldsymbol{r}, t) = \int d^3\boldsymbol{p}\, f(\mathbf{r}, \boldsymbol{p}, t) = \int d^3\boldsymbol{p}_0\, f_0\left(\mathbf{r} - \frac{\boldsymbol{p}_0 t}{M}, \boldsymbol{p}_0\right)$$

The spatial density reads

$$n_{\rm at}(\boldsymbol{r},t)=\int d^3\boldsymbol{p}_0\, f_0\left(\mathbf{r}-\frac{\boldsymbol{p}_0 t}{M},\boldsymbol{p}_0\right)=\int d^3\boldsymbol{r}_0'\int d^3\boldsymbol{p}_0\, f_0(\mathbf{r}_0',\boldsymbol{p}_0)\delta\left(\boldsymbol{r}-\frac{\boldsymbol{p}_0}{M}t-\boldsymbol{r}_0'\right)$$

For sufficiently long times, the terms $\propto t$ in the δ function dominate and

$$\begin{aligned} n_{\rm at}(\boldsymbol{r},t) &\approx \int d^3\boldsymbol{r}_0'\int d^3\boldsymbol{p}_0\, f_0(\mathbf{r}_0',\boldsymbol{p}_0)\delta\left(-\boldsymbol{r}-\frac{\boldsymbol{p}_0}{M}t\right) \\ &\approx \left(\frac{M}{t}\right)^3\int d^3\boldsymbol{r}_0\, f_0\left(\mathbf{r}_0,\frac{M\mathbf{r}}{t}\right)=\left(\frac{M}{t}\right)^3\mathcal{P}_0\left(\frac{M\mathbf{r}}{t}\right). \end{aligned}$$

Validity :

Let Δr_0 and Δp_0 be the extent of f_0 in position and momentum, respectively.

The asymptotic regime is reached when

$$\Delta p_0 t/M \gg \Delta r_0.$$

Dispersive imaging :

The phase φ_L of the transmitted wave also carries information on $\tilde{n}$:

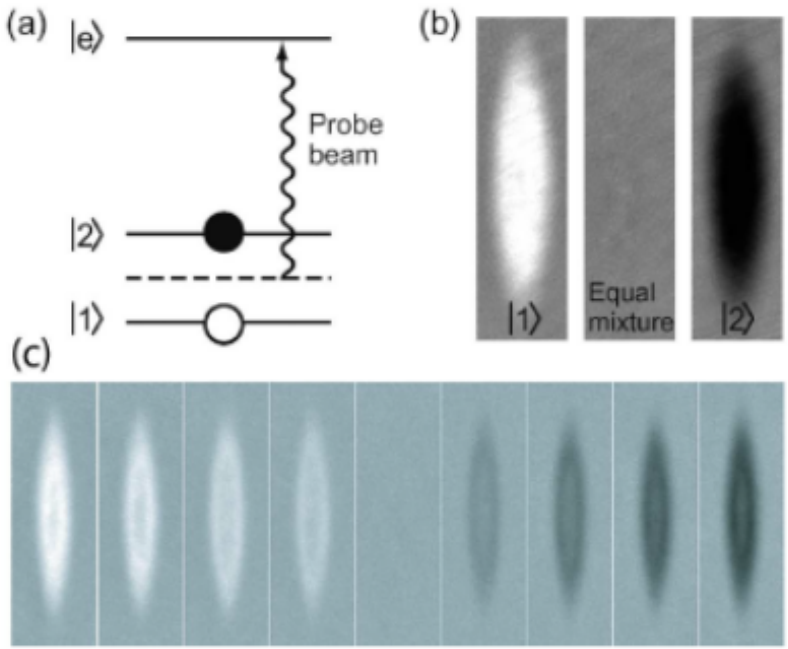
$$\frac{d\varphi_L}{dz} = (n-1)k_L = \frac{1}{2}\sigma n_{\mathrm{at}} \left(\frac{2\delta_L}{\Gamma}\right)$$

with $n = 1 + \chi'$ the optical index.

By interfering the transmitted wave with the incident wave, this information can be retrieved (*e.g.* in *phase-contrast imaging*). For details see W. Ketterle's review papers.

Usually operated off-resonance ($\delta_L \gg \Gamma$) to minimize absorption and multiple scattering.

Bypass the limitations of absorption imaging for in-situ density measurements.



Phase-contrast images of a two-component Fermi gas

- a: level scheme
- b: $\phi_L > 0$ (< 0) for state 1 (2),
- c: different mixtures with $N_2 - N_1$ increasing from left to right.

6. Complement 1: Saturated absorption spectroscopy

7. Complement 2: A minimal introduction to the dressed atom picture

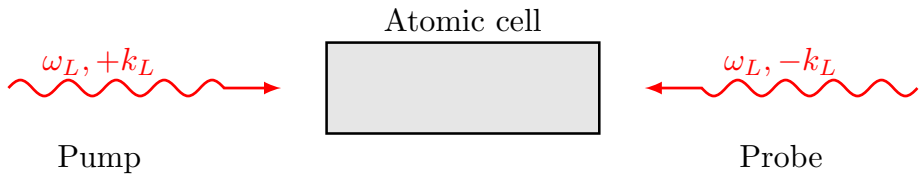
Absorption of a weak probe beam by an atomic vapour at ambiant temperature, in presence of a strong pump beam

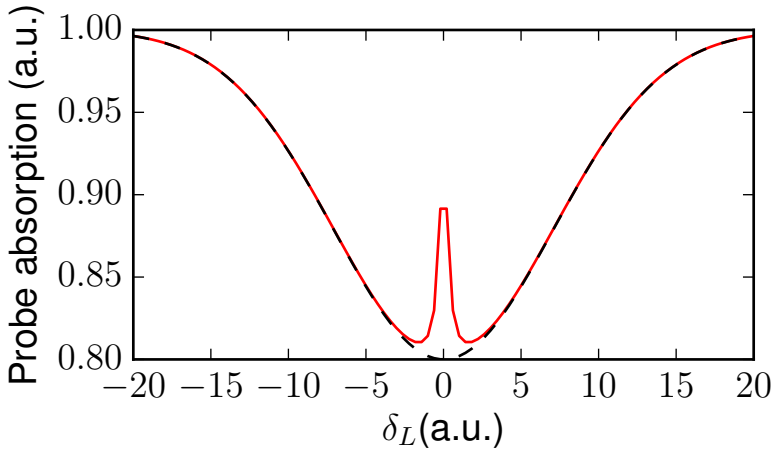
Atomic velocities v distributed according to Maxwell-Boltzmann statistics : Doppler broadening

Counter-propagating configuration :

Pump resonance: $\omega_L = \omega_{eg} - k_L v$

Probe resonance: $\omega_L = \omega_{eg} + k_L v$



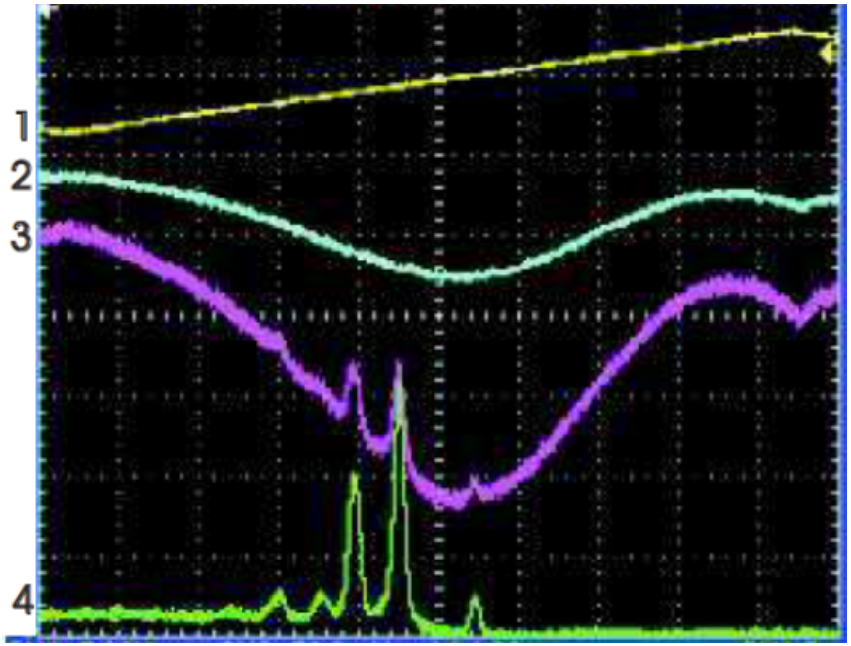


- if $|\omega_L - \omega_{eg}| \gg \Gamma$:
  - pump resonant for $v \approx -\frac{\delta_L}{k_L}$
  - probe resonant for $v \approx \frac{\delta_L}{k_L}$
- if $|\omega_L - \omega_{eg}| \lesssim \Gamma$:
  - pump and probe simultaneously resonant for $v \approx 0$
  - saturation by the pump suppresses probe absorption

Transitions $^5S_{1/2}$, $F = 2 \rightarrow {}^5P_{3/2}$, $F' = 1, 2, 3$ in ^{87}Rb

1. 1: laser frequency scan
2. 2: absorption, probe alone
3. 3: absorption, pump + probe
4. 4: difference 2-3

Figures from T. Riegler and T. Volz, Max Planck Institut for Quantum Optics, Munich



1. Non-linear features of the resonant atomic response allows one to beat the Doppler limit.
2. Enabled by the discovery of lasers (and their availability at many different wavelengths).
3. Saturated absorption spectroscopy is widely used in experiments to lock laser frequencies to atomic resonances.

Crossover resonances in saturated absorption spectroscopy

One expect 3 transitions: $^5S_{1/2}$, $F = 2 \rightarrow {}^5P_{3/2}$, $F' = 1, 2, 3$, and therefore 3 spectral absorption lines.
There are 6 in the experiment ?

Additional crossover resonances :

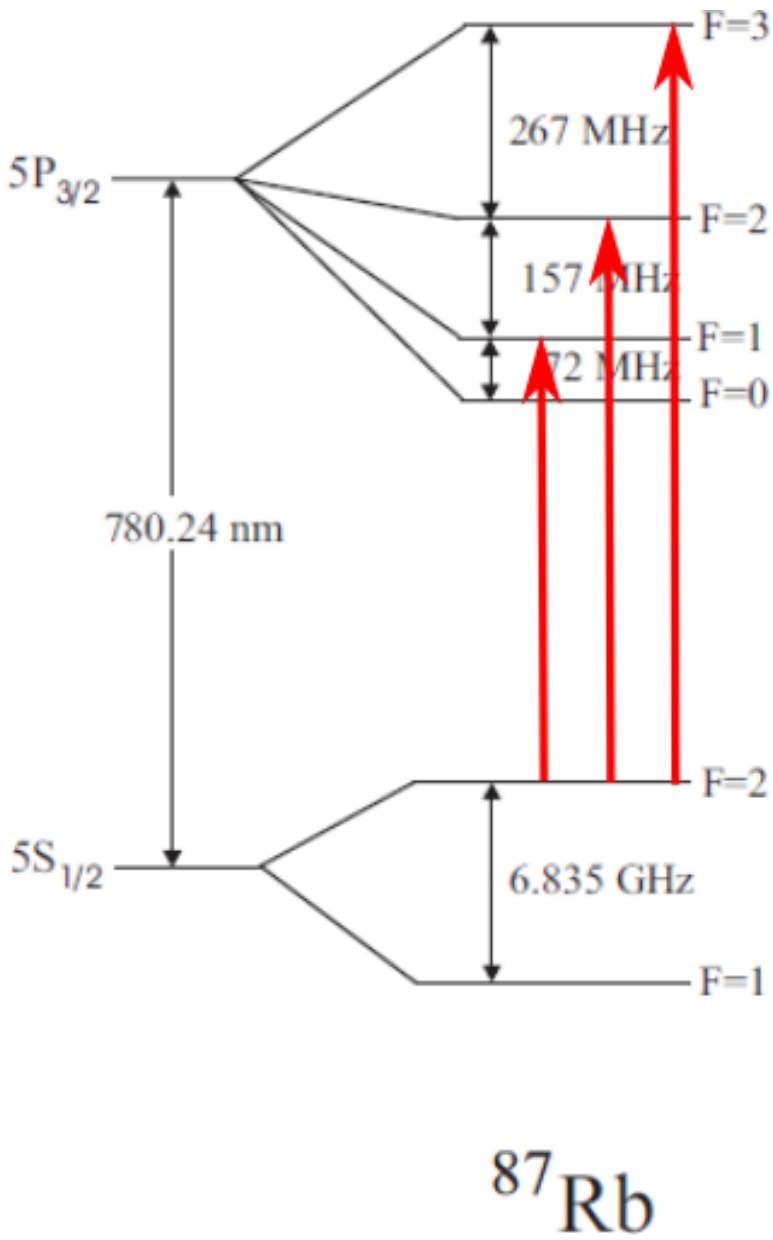
- one ground state g, 2 excited states $e_{1/2}$
- crossover when pump resonant with $g \rightarrow e_1$ and probe resonant with $g \rightarrow e_2$ (or the opposite):

$$\omega_L = \omega_{e_1 g} - k_L v$$
$$\omega_L = \omega_{e_2 g} + k_L v$$

- Both lasers simultaneously resonant for atoms in the same velocity class when

$$\omega_L = \frac{\omega_{e_1 g} + \omega_{e_2 g}}{2}$$

- useful in practice to lock (intense lines) and to measure hyperfine splittings



6. Complement 1: Saturated absorption spectroscopy

7. Complement 2: A minimal introduction to the dressed atom picture

Quantum treatment of the laser mode :

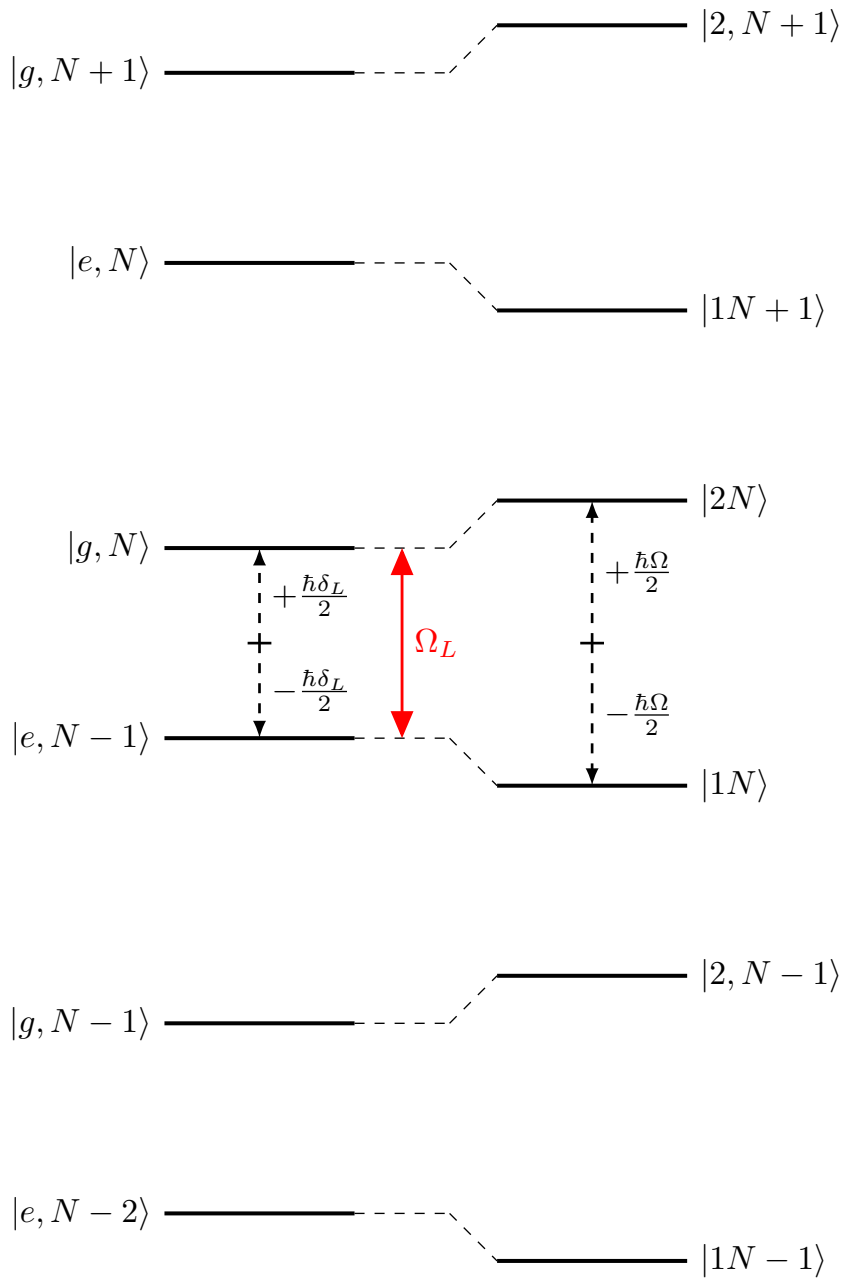
The laser mode is defined by a (fictitious or real) cavity of volume L^3, and assumed to be in a coherent state, with $N_L = \langle \hat{a}_L^\dagger \hat{a}_L \rangle \gg 1$.

Hilbert space : spanned by states of the form $|a; N\rangle$ where $a = g, e$ labels the atom internal state and N the number of photons in the laser mode.

The energy levels of the $\{\text{atom} + \text{laser}\}$ system group into multiplicities $\mathcal{E}_N$, well-separated from each other.

The new eigenstates $|1 - N\rangle$ and $|2 - N\rangle$ are called *dressed states*, and are both superpositions of the bare states $|g; N\rangle$ and $|e, N - 1\rangle$.

$$\left.\begin{matrix} E_{2,N} \\ E_{1,N} \end{matrix}\right\} = \frac{E_e - E_g - \hbar\omega_L}{2} + N\hbar\Omega_L \left\{\begin{matrix} +\frac{\hbar\Omega}{2} \\ -\frac{\hbar\Omega}{2} \end{matrix}\right.$$

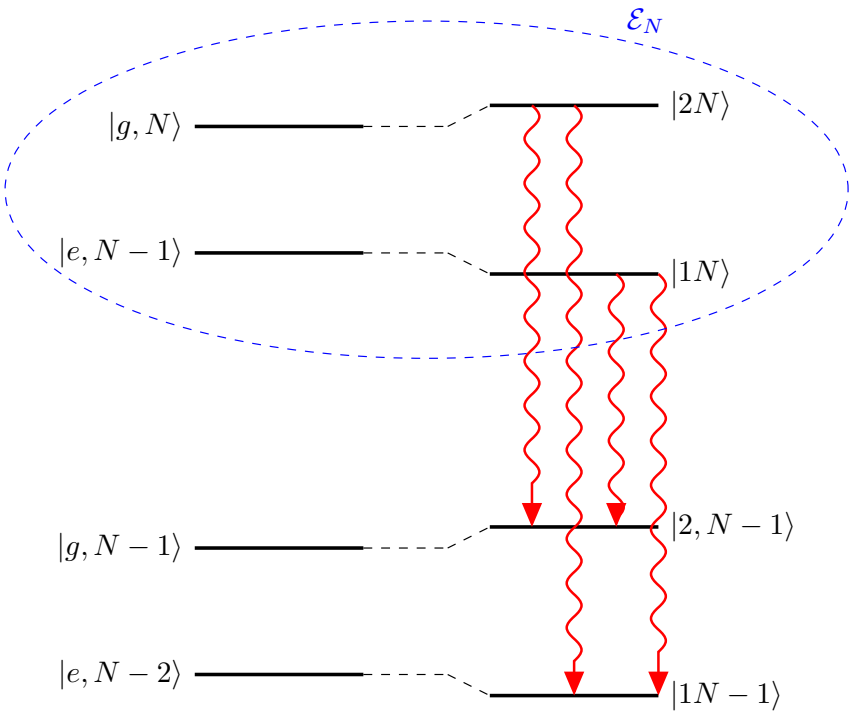


Spontaneous emission :

Spontaneous emission populates photonic modes outside of the $\{\text{atom} + \text{laser}\}$ system

Transition $\mathcal{E}_N \to \mathcal{E}_{N-1}$ after each spontaneous emission event

Radiative cascade of the dressed atom

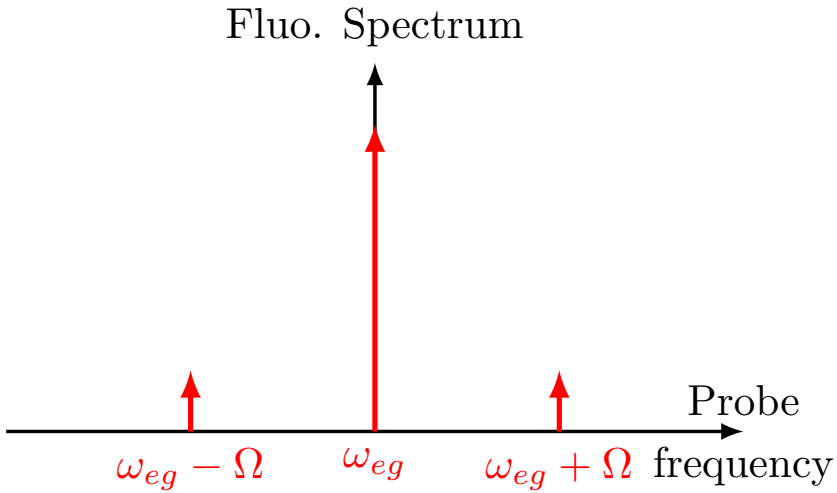


Mollow triplet :

Possible Bohr frequencies :

- $E_{2,N} - E_{2,N-1} = \hbar\omega_L$
- $E_{2,N} - E_{1,N-1} = \hbar(\omega_L + \Omega)$
- $E_{1,N} - E_{2,N-1} = \hbar(\omega_L - \Omega)$
- $E_{1,N} - E_{1,N-1} = \hbar\omega_L$

Triplet of peaks in the fluorescence spectrum



Radiative cascade of the dressed atom:

The radiative decay $\mathcal{E}_N \to \mathcal{E}_{N-1}$ from spontaneous emission can be described by an effective Hamiltonian

$$\hat{H}_{\text{eff}} = \begin{pmatrix} \frac{\delta_L}{2} & \frac{\Omega_L}{2} \\ \frac{\Omega_L}{2} & -\frac{\delta_L + i\Gamma}{2} \end{pmatrix}$$

Eigenvalues for $\delta_L = 0$:

$$E_\pm = \frac{-i\Gamma}{4} \pm \frac{1}{2}\sqrt{\Omega_L^2 - \frac{\Gamma^2}{4}} = \begin{cases} = -i\frac{\Gamma}{4}\left(1 \mp \sqrt{1 - \left(\frac{2\Omega_L}{\Gamma}\right)^2}\right) & \text{if } \Omega_L < \frac{\Gamma}{2} \\ = \frac{1}{2}\sqrt{\Omega_L^2 - \frac{\Gamma^2}{4}} - i\frac{\Gamma}{4} & \text{if } \Omega_L \geq \frac{\Gamma}{2} \end{cases}$$

With this simple approach, we understand (qualitatively)

- the Mollow triplet in the fluorescence spectrum for high intensities,
- the occurrence of damping with a rate $\sim \Gamma$,
- the change from underdamped to overdamped dynamics when Ω_L is reduced below a value $\frac{\Gamma}{2}$.

Advantage of the dressed atom approach : perhaps less convenient for computations than the optical Bloch equations, but provides clear and (usually) simple physical pictures to interpret the results (end results are of course equivalents in the limit $N_L \to \infty$).