Lecture 2:

Radiative forces and laser cooling

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Master ICFP, Ultracold atoms

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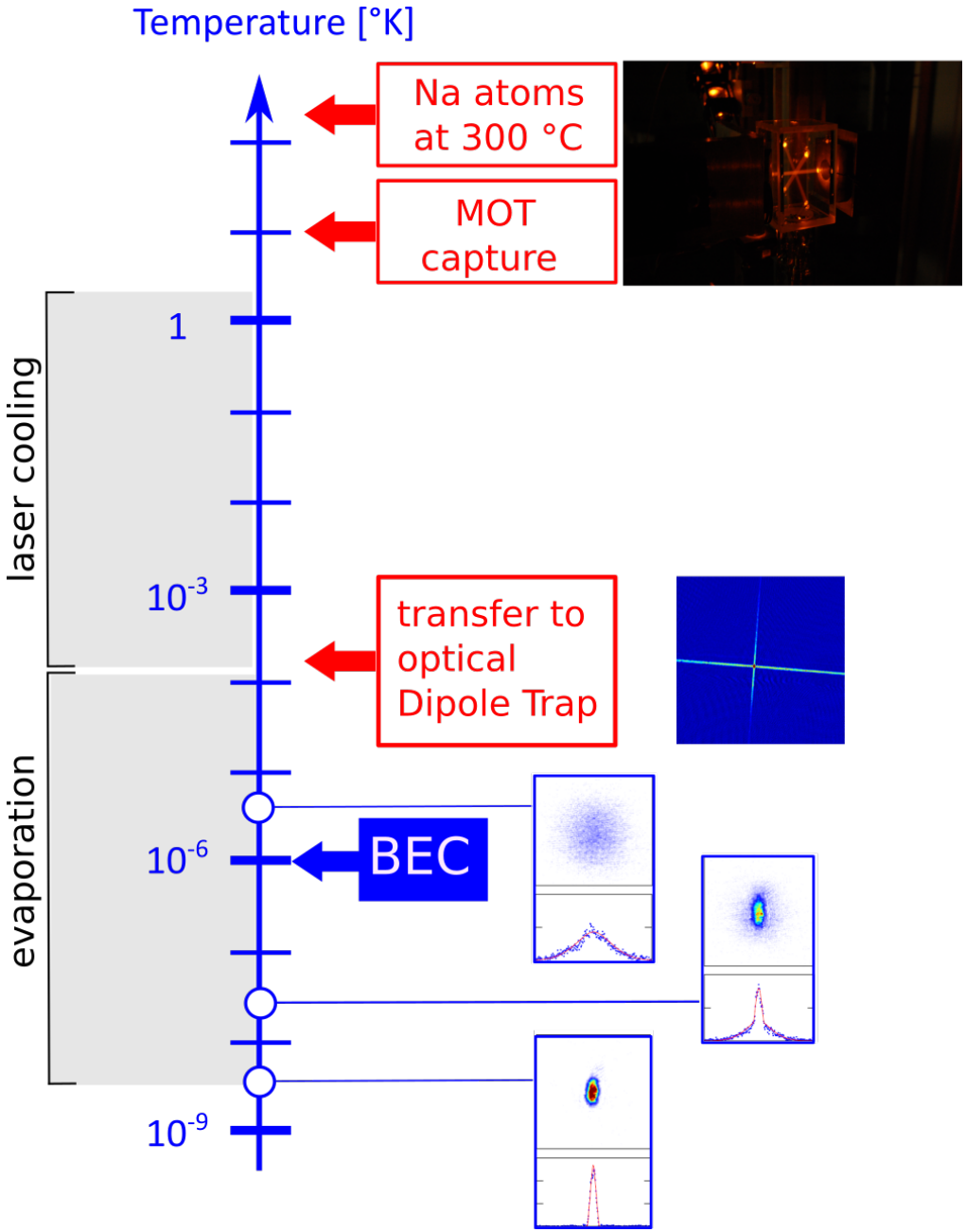
The experimental path to quantum degenerate gases

Typical experimental sequence :

- catch atoms in a magneto-optical trap
- laser cooling to $\sim 50\,\mu$K
- transfer to conservative trap (no resonant light): optical trap or magnetic trap
- evaporative cooling to BEC

Take a picture of the cloud.

Repeat

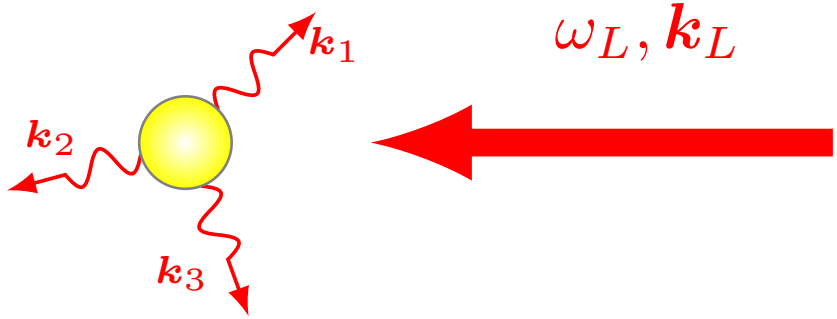


Two-level atom irradiated by a near-resonant laser:

In a period of duration Δt, the atom undergoes $N_{\rm abs}$ fluorescence cycles (absorption followed by spontaneous emission).

Atomic momentum changes by

$$\Delta \boldsymbol{p}_{\rm at} = N_{\rm abs} \hbar \boldsymbol{k}_L - \sum_i \hbar \boldsymbol{k}_i^{\rm (sp)}$$



Averaging over many cycles :

$$\overline{N_{\rm abs}} \to \Gamma_{\rm sp} \Delta t$$

$$\sum_i \overline{\hbar \boldsymbol{k}_i^{\rm (sp)}} \to 0$$

$\Gamma_{\rm sp}$: spontaneous emission rate = net absorption rate

Average force on the atom :

$$\overline{\boldsymbol{F}} = \frac{\Delta \boldsymbol{p}_{\rm at}}{\Delta t} = \Gamma_{\rm sp} \hbar \boldsymbol{k}_L$$

Very large forces! $\frac{F_{\rm max}}{M} \sim 10^5 - 10^6\, g$ for most atoms.

1. Semi-classical theory of radiative forces
   - Radiation pressure
     - Zeeman slowing of an atomic beam
   - Dipole force

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   - Sub-Doppler cooling

Equations of motion of the external atomic variables: Ehrenfest theorem

Fully quantized atom-light Hamiltonian :

$$\hat{H} = \hbar\omega_{eg}|e\rangle\langle e| + \frac{\hat{\boldsymbol{P}}^2}{2M} + \hat{H}_{\mathrm{e-m}} + \hat{V}_{\text{A-L}}(\hat{\boldsymbol{R}}) + \hat{V}_{\text{A-V}}(\hat{\boldsymbol{R}})$$

So far we have ignored the atomic center of mass position. How to treat it in a simple way ?

Ehrenfest theorem : $i\hbar\frac{d}{dt}\langle\hat{O}\rangle = \left\langle[\hat{O},\hat{H}]\right\rangle$

$$\frac{d}{dt}\langle\hat{\boldsymbol{R}}\rangle = \frac{1}{i\hbar}\left\langle[\hat{\boldsymbol{R}},\hat{H}]\right\rangle = \frac{\langle\hat{\boldsymbol{P}}\rangle}{M}$$
$$\frac{d}{dt}\langle\hat{\boldsymbol{P}}\rangle = \frac{1}{i\hbar}\left\langle[\hat{\boldsymbol{P}},\hat{H}]\right\rangle = -\left\langle\boldsymbol{\nabla}\hat{V}_{\text{A-L}}(\hat{\boldsymbol{R}})\right\rangle - \left\langle\boldsymbol{\nabla}\hat{V}_{\text{A-V}}(\hat{\boldsymbol{R}})\right\rangle$$

In accordance with the qualitative description in the beginning, the net force from the electromagnetic vacuum vanishes,

$$\left\langle\boldsymbol{\nabla}\hat{V}_{\text{A-V}}(\hat{\boldsymbol{R}})\right\rangle = 0.$$

However, the mechanical contribution of the vacuum is NOT identically zero: only the mean force vanishes, not the fluctuations around the mean (to be discussed later).

Semi-classical approximation :

- Treat the atom as a wavepacket centered around $\langle \hat{\boldsymbol{R}} \rangle$, of extension Δx in real space ($\langle \hat{\boldsymbol{P}} \rangle, \Delta p$ in momentum space).
- replaces the full quantum dynamics of the atomic wavepacket by the classical trajectory defined by $\langle \hat{\boldsymbol{R}} \rangle, \langle \hat{\boldsymbol{P}} \rangle$:

$$\frac{d}{dt}\langle \hat{\boldsymbol{P}} \rangle = -\left\langle \boldsymbol{\nabla} \hat{V}_{\text{A-L}}(\hat{\boldsymbol{R}}) \right\rangle \approx -\boldsymbol{\nabla} V_{\text{A-L}} \left(\left\langle \hat{\boldsymbol{R}} \right\rangle \right) \equiv \boldsymbol{F}_{\text{rad}} \left(\langle \hat{\boldsymbol{R}} \rangle \right)$$

Writing the laser electric field as

$$\mathcal{E}_L(\boldsymbol{r}) = \boldsymbol{\varepsilon}_L E_L(\boldsymbol{r}) \cos[\omega_L t + \varphi_L(\boldsymbol{r})],$$

the semi-classical average radiative force is then given by

$$\boldsymbol{F}_{\text{rad}}(\boldsymbol{r}) = \cos[\omega_L t + \varphi_L(\boldsymbol{r})]\ \hbar \boldsymbol{\nabla} \Omega(\boldsymbol{r}, t) - \hbar \Omega(\boldsymbol{r}, t)\ \sin[\omega_L t + \varphi_L(\boldsymbol{r})]\ \boldsymbol{\nabla} \varphi_L(\boldsymbol{r}).$$

Here $\boldsymbol{r} = \langle \hat{\boldsymbol{R}} \rangle$, $\langle \hat{d} \rangle = \langle \hat{\boldsymbol{d}} \rangle \cdot \boldsymbol{\varepsilon}_L$, and

$$\hbar \Omega(\boldsymbol{r}, t) = \langle \hat{\boldsymbol{d}}(t) \cdot \boldsymbol{\varepsilon}_L \rangle E_L(\boldsymbol{r}).$$

Average radiative force: general expression for an atom at rest

Semi-classical radiative force

$$\boldsymbol{F}_{\rm rad}\left(\boldsymbol{r}\right) = \cos[\omega_L t + \varphi_L(\boldsymbol{r})]\ \hbar\boldsymbol{\nabla}\Omega - \hbar\Omega\ \sin[\omega_L t + \varphi_L(\boldsymbol{r})]\ \boldsymbol{\nabla}\varphi_L(\boldsymbol{r}),$$
$$\hbar\Omega(\boldsymbol{r},t) = \langle\hat{\boldsymbol{d}}\cdot\boldsymbol{\varepsilon}_L\rangle(t)E_L(\boldsymbol{r}).$$

The average dipole settles very quickly (on a time scale $\sim\Gamma^{-1}$) to the stationary solution of the optical Bloch equations,

$$\langle\hat{d}\rangle \approx \langle\hat{d}\rangle_{\rm st} = 2d\left[u_{\rm st}\cos(\omega_L t+\varphi_L) - v_{\rm st}\sin(\omega_L t+\varphi_L)\right].$$

where $u/v_{\rm st}$ depend on $\Omega_L(\boldsymbol{r})$ and $\delta_L(\boldsymbol{r})$.

Putting everything together, one gets a mean radiative force (averaged in time over an optical period)

$$\boldsymbol{F}_{\rm rad}\left(\boldsymbol{r}\right) = -du_{\rm st}\ \hbar\boldsymbol{\nabla}\Omega_L(\boldsymbol{r}) - dv_{\rm st}\ \hbar\Omega_L(\boldsymbol{r})\boldsymbol{\nabla}\varphi_L$$

with the Rabi frequency

$$\hbar\Omega_L(\boldsymbol{r}) = -dE_L(\boldsymbol{r})\ :\ \text{Rabi frequency}.$$

The two components of the mean radiative force are called

- **dipole force** $\propto u_{\rm st}$: linked to the reactive response of the atom (dispersion), and to the gradient of amplitude,
- **radiation pressure** $\propto v_{\rm st}$: linked to the dissipative response of the atom (absorption), and to the gradient of phase.

Validity of the semi-classical treatment: the broad line condition

The semi-classical approximation makes sense only if the wavepacket

- well localized in real space: $\Delta x \ll \lambda_L$,
- well localized in momentum space: $\frac{k_L \Delta p}{M} \ll \Gamma$.

  This condition states that all relevant momentum components in the wavepacket interact with the laser in the same manner before and after having absorbed a photon.

  Since $\Delta x \Delta p \geq \hbar$, this also implies

$$\Delta x \geq \frac{\hbar}{\Delta p} \gg \frac{\hbar k_L}{M\Gamma}$$

The two conditions are compatible iif the broad line condition is met:

$$E_R = \frac{\hbar^2 k_L^2}{2M} \ll \hbar\Gamma$$

Separation of time scales :

- $\omega_R = E_R/\hbar$: typical rate of evolution of external variables,
- Γ : typical rate of evolution of internal variables.

The fast internal variables adapt almost instantaneously to the slower external ones.

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For a plane wave :

$$\mathcal{E}_L(\boldsymbol{r}) = \boldsymbol{\varepsilon}_L E_L \cos[\omega_L t - \boldsymbol{k}_L \cdot \boldsymbol{r}],$$

$E_L = \text{cst}$, $\varphi_L = -\boldsymbol{k}_L \cdot \boldsymbol{r}$

The dipole force vanishes, and the radiation pressure force is

$$\boldsymbol{F}_{\text{rp}}(\boldsymbol{r}) = -v_{\text{st}} \hbar \Omega_L \boldsymbol{\nabla} \varphi_L, \qquad v_{\text{st}} = \frac{\Gamma}{2\Omega_L} \frac{s}{1+s},$$

$$= \underbrace{\frac{\Gamma s}{2(1+s)}}_{\Gamma(\rho_{ee})_{\text{st}}} \hbar \boldsymbol{k}_L = \Gamma_{\text{sp}} \hbar \boldsymbol{k}_L$$

We introduced the rate of spontaneous emission (or absorption), $\Gamma_{\text{sp}} = \Gamma(\rho_{ee})_{\text{st}}$.

Same result as the heuristic approach (beginning of the lecture) !

Properties of the radiation pressure force :

- maximum on resonance
- saturates to $F_{\text{sat}} = \frac{1}{2}\Gamma \hbar \boldsymbol{k}_L$ for high intensities
- very large ! $\frac{F_{\text{sat}}}{M} \sim 10^5 - 10^6\, g$ for most atoms.

Detuning for an atom moving at velocity $\boldsymbol{v}$:

$$\delta_L = \omega_L - \omega_{eg} - \boldsymbol{k}_L \cdot \boldsymbol{v}$$

Issues arise when working with a beam with a thermal distribution :

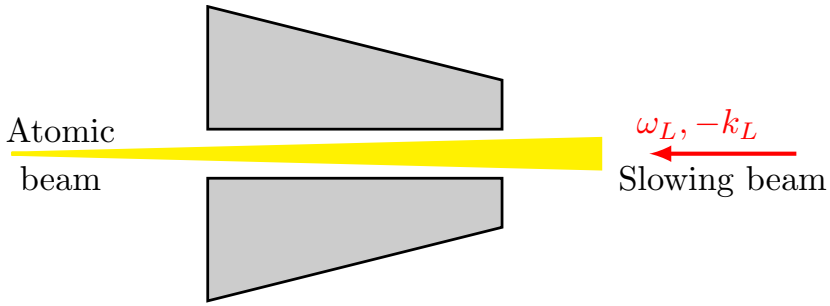
- all velocity class in a thermal beam (Boltzmann distribution) are not addressed
- if atom with an initial velocity v_0 are decelerated, they move out of resonance due to the Doppler shift.

Essentially the same problem as thermal broadening in spectroscopy.

Solution: Adjust detuning with an inhomogeneous magnetic field

$$\delta_L = \omega_L - \omega_{eg} + k_L v_z + \Delta\mu B(z)$$

Use the Zeeman effect to counter the Doppler effect.
$\Delta\mu$: difference of magnetic moments in the excited and ground state.

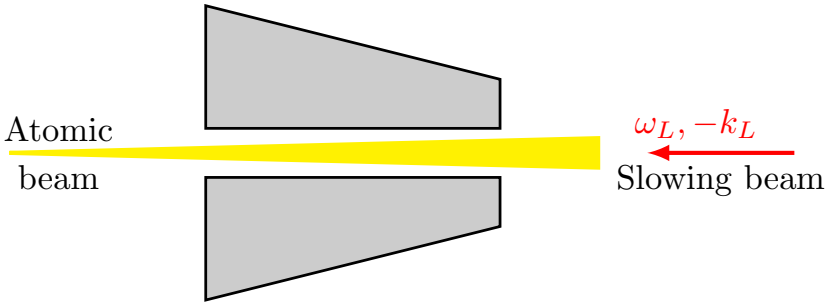


Phillips and Metcalf, Phys. Rev. Lett. 48, 596 (1982)

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Design of the magnetic field profile : Maximum force, $F_{\max} = \frac{\Gamma\hbar\boldsymbol{k}_L}{2}\frac{s_0}{1+s_0}$, with s_0 the saturation parameter for $\delta_L = 0$.

$B(z)$ such that an atom entering with a velocity v_i remains on resonance as it is decelerated to $v_f \approx 0$ with a constant force $F_{\max}$.

$$B(z) = \frac{\hbar\delta_L}{\mu} - \frac{\hbar k_L v_i}{\mu}\sqrt{1 - \frac{z}{L}}, L = \frac{v_i^2}{a_{\max}}$$

All velocities $v \leq v_i$ are also slowed to zero.

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$$\boldsymbol{F}_{\rm dip}\left(\boldsymbol{r}\right) = -u_{\rm st}\hbar\boldsymbol{\nabla}\Omega_L(\boldsymbol{r}) = -\frac{\hbar\delta_L}{2}\frac{s}{1+s}\frac{\boldsymbol{\nabla}\Omega_L}{\Omega_L}, \qquad \frac{s}{1+s} = \frac{\frac{\Omega_L^2}{2}}{\delta_L^2 + \frac{\Gamma^2}{4} + \frac{\Omega_L^2}{2}}$$

Properties of the dipole force :

- $\boldsymbol{F}_{\rm dip}$ vanishes on resonance,
- $\boldsymbol{F}_{\rm dip}$ negligible close to resonance as compared to radiation pressure $\boldsymbol{F}_{\rm rp}$,
- conservative : $\boldsymbol{F}_{\rm dip} = -\boldsymbol{\nabla}U_{\rm dip}(\boldsymbol{r})$, with the dipole potential U given by

$$U_{\rm dip}(\boldsymbol{r}) = \frac{\hbar\delta_L}{2}\ln\left[1+s(\boldsymbol{r})\right]$$

For a standing wave : $E_L = E_0\cos(\boldsymbol{k}\cdot\boldsymbol{r})$, $\varphi_L = \text{cst}$, the dipole force is given by

$$U_{\rm dip}(\boldsymbol{r}) = \frac{\hbar\delta_L}{2}\ln\left[1+s_0\cos^2(\boldsymbol{k}\cdot\boldsymbol{r})\right]$$

The potential is periodic, trapping atoms at the nodes or antinodes of the standing wave depending on the detuning (last lectures by S. Nascimbène).
Redistribution of photons between the $\leftarrow$ and $\rightarrow$ propagating waves by stimulated emission.

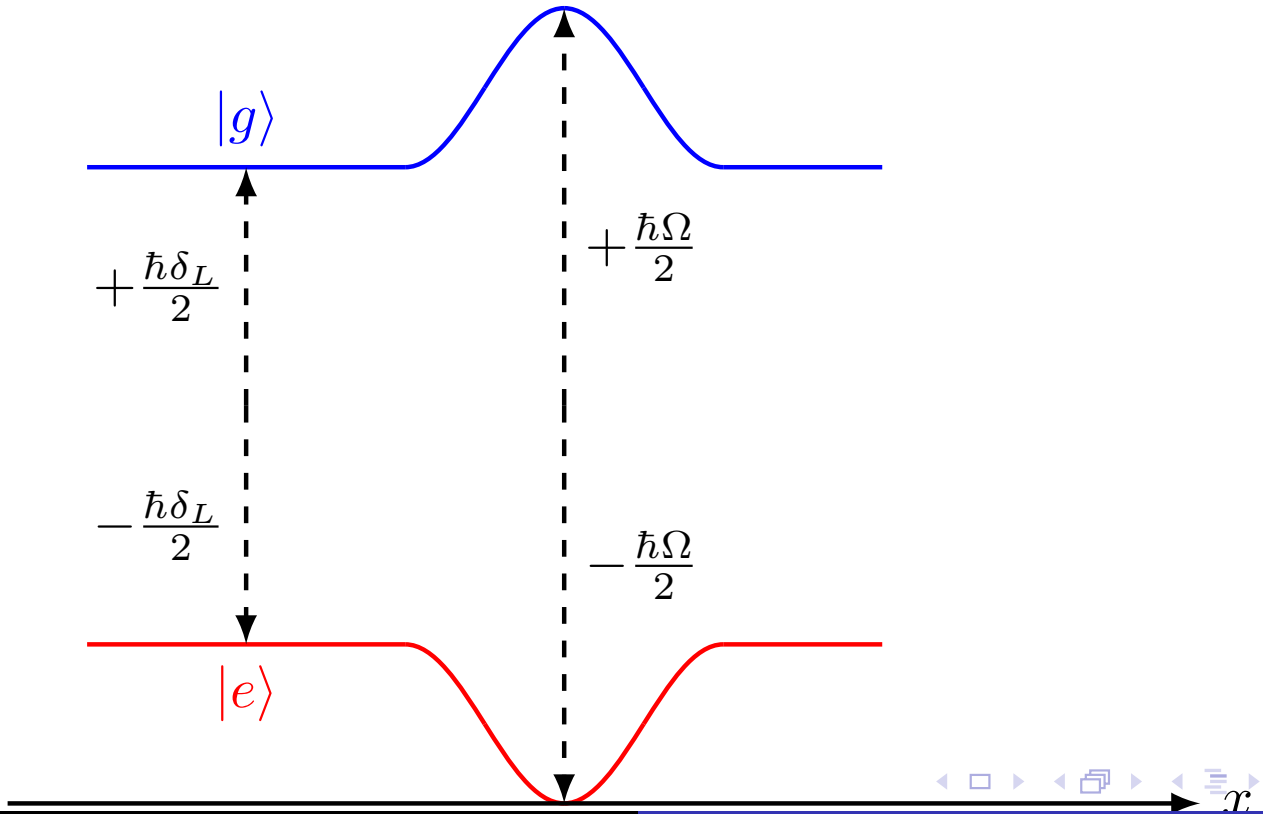
Another interpretation of the dipole force

Large detunings : $\delta_L \gg \Gamma, \Omega_L$ or equivalently $s \ll 1$

$$U_{\rm dip}(\boldsymbol{r}) \approx \frac{\hbar \delta_L s(\boldsymbol{r})}{2} = \frac{\hbar \Omega_L^2(\boldsymbol{r})}{4\delta_L}$$

Interpretation as a position-dependent light shift of the ground state atoms:

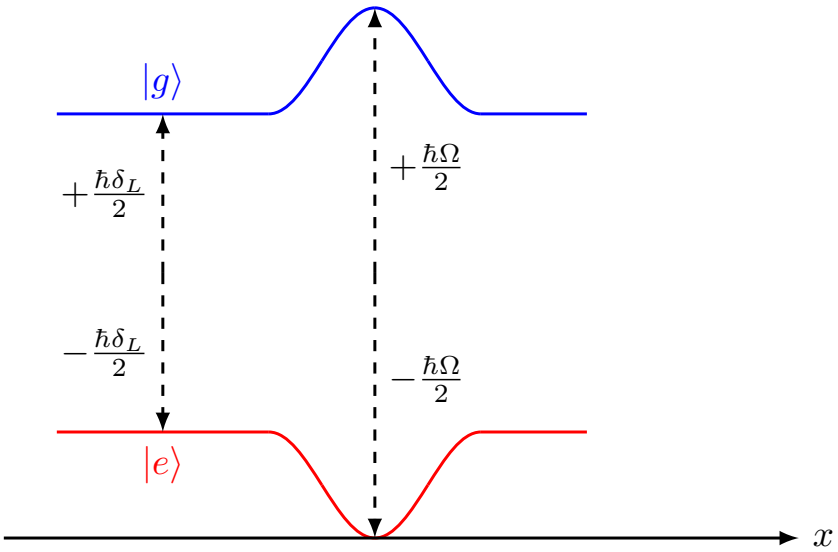
$$\Delta E_g = \frac{\hbar(\Omega - \delta_L)}{2} = \frac{\hbar}{2}\left(\sqrt{\delta_L^2 + \Omega_L^2} - \delta_L\right) \approx \frac{\hbar \Omega_L^2}{4\delta_L} \text{ if } \delta_L \gg \Omega_L \text{ for } \delta_{\rm L} > 0$$



Dipole potential for large detunings : $\delta_L \gg \Gamma, \Omega_L$ or equivalently $s \ll 1$

$$U_{\rm dip}(\boldsymbol{r}) \approx \frac{\hbar \delta_L s(\boldsymbol{r})}{2} = \frac{\hbar \Omega_L^2(\boldsymbol{r})}{4\delta_L} = \frac{\hbar \Gamma^2}{8\delta_L} \frac{I(\boldsymbol{r})}{I_{\rm sat}}$$

Interpretation as a position-dependent light shift of $|g\rangle$:



- $\delta_L > 0$: repulsive force (atoms are repelled from the intensity maxima),
- $\delta_L < 0$: attractive force (atoms are attracted to intensity maxima).

- Residual spontaneous emission :

$$\Gamma_{\rm sp} = \frac{\Gamma s}{2} = \frac{\Gamma}{\delta_L} \frac{U_{\rm dip}}{\hbar}$$

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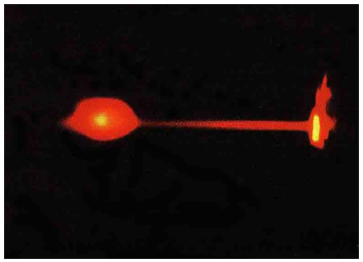
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Optical molasses

First optical molasse demonstrated in 1985 (S. Chu's group, Bell labs)

3 pairs of counter-propagating beams to provide 3D cooling

- a few thousand Sodium atoms caught for about 1 second
- cooling transition in the visible ($\lambda_L = 589\,\text{nm}$): visible by naked eye !



S. Chu *et al.*, Phys. Rev. Lett. 55, 48 (1985)

Temperature measurement by time of flight :

An atom initially at $\boldsymbol{r}_0$ with velocity $\boldsymbol{v}_0$ follows a ballistic trajectory, $\boldsymbol{r}(t) = \boldsymbol{r}_0 + \boldsymbol{v}_0 t$. At long time of flight, the density distribution reflects the initial velocity distribution.

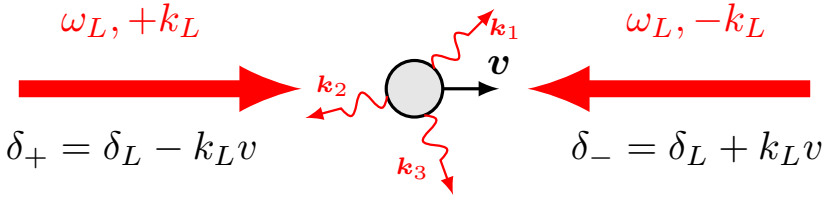
Ballistic expansion, cloud radius : $R(t) = \sqrt{\frac{k_B T}{M}} t$

Result of the measurement : $T \approx 200\,\mu\text{K}$

Laser cooling and optical molasses

Cooling configuration in one dimension :

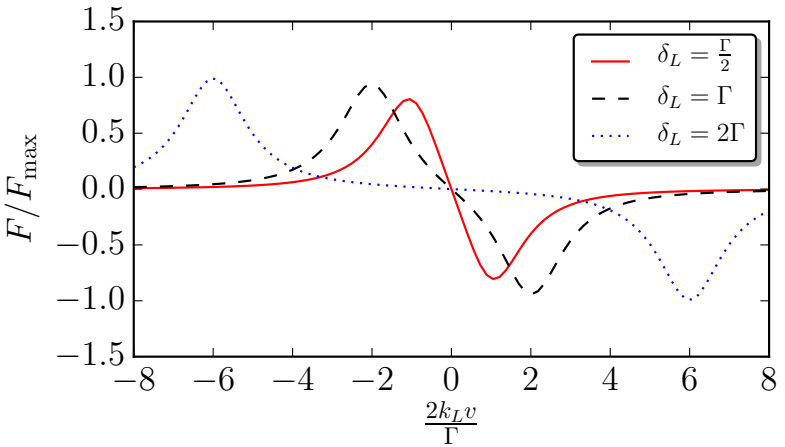
- Two counter-propagating beams (frequency ω_L, wavevector k_L)
- moving atoms with velocity $\boldsymbol{v}$



Weak saturation $s \ll 1$: $F_\pm \approx \pm \frac{\Gamma \hbar k_L}{2} s_\pm = \pm \frac{\Gamma \hbar k_L}{4} \frac{\Omega_L^2}{\delta_\pm^2 + \frac{\Gamma^2}{4}}$

In this limit (linear response regime), one can add independently the forces exerted by each beam to obtain a total *average* force

$$F_{\text{tot}} = F_+ + F_- = \frac{\Gamma \hbar k_L}{2} \frac{\Omega_L^2}{2} \left(\frac{1}{\delta_+^2 + \frac{\Gamma^2}{4}} - \frac{1}{\delta_-^2 + \frac{\Gamma^2}{4}} \right)$$



Maximum for $k_L v = |\delta_L|$:

$$F_{\max} = \frac{\Gamma \hbar k_L}{2} \frac{I}{I_{\text{sat}}}$$

For low velocities $k_L v \ll \Gamma$, we have

$$F_{\text{tot}} = -M \alpha v, \qquad \alpha = -\frac{\Gamma \hbar k_L^2}{M} \frac{2 s \delta_L}{\delta_L^2 + \frac{\Gamma^2}{4}}$$

Viscous damping for $\delta_L < 0$ (red detuning), with a friction coefficient α (units : T^{-1})

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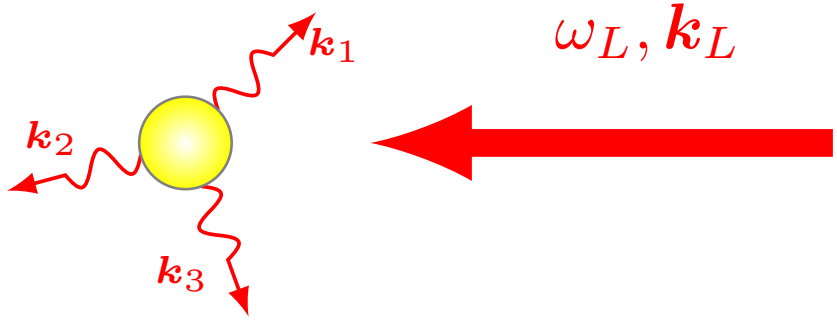
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So far we have computed the *average* radiative force $F_{\rm rad}$ exerted by the laser on the atom. Fluctuations around these average are also present (due to the stochastic character of the atom-light interaction).

Recall our basic picture :

Atomic momentum changes by

$$\Delta \boldsymbol{p}_{\rm at} = N_{\rm abs} \hbar \boldsymbol{k}_L - \sum_i \hbar \boldsymbol{k}_i^{(\rm sp)}$$



The radiation pressure force fluctuates around its average $\boldsymbol{F}_{\rm rp}$ because of two factors :

- random momentum kicks due to the random direction of emission,
- random number of fluorescence cycles during Δt.

Brownian particle submitted to a deterministic force $F_{\rm rad}$ and to a fluctuating force $\delta F(t)$.

We assume that the fluctuating part has a short memory time, *i.e.* is characterized by a correlation function

$$\langle \delta F(t')\delta F(t'')\rangle = 2D_p\delta(t'-t'').$$

The equation of motion of the Brownian particle is

$$M\ddot{x} = F_{\rm rad} + \delta F(t) \implies v(t) = v(0) + \frac{1}{M}\int_0^t dt' \left(F_{\rm rad}(t') + \frac{1}{M}\delta F(t')\right)$$

Averaging over many realizations of the fluctuating force ($\langle \delta F(t)\rangle = 0$):

$$\langle v\rangle = v(0) + \frac{1}{M}\int_0^t dt', F_{\rm rad}(t')$$

$$\Delta v^2 = \langle v^2\rangle - \langle v\rangle^2 = \frac{1}{M^2}\int_0^t dt' \int_0^t dt'' \langle \delta F(t')\delta F(t'')\rangle$$

This gives

$$\Delta p^2 = M^2\Delta v^2 = 2D_p t$$

Fluctuations of the force lead to a brownian motion in momentum space characterized by a diffusion coefficient D_p.

Fluctuations of the radiation pressure :

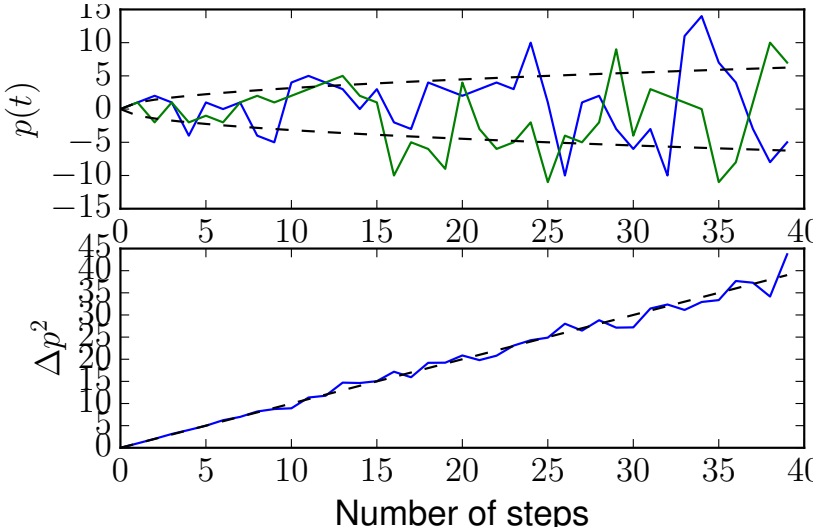
Random momentum kicks for each spontaneous emission.

Photons spontaneously emitted at a rate $\Gamma_{\rm sp} = \Gamma(\rho_{ee})_{\rm st}$.

Random walk of step size $\pm\hbar k_L$ in momentum space:

$$\left\langle \frac{\Delta p^2}{(\hbar k_L)^2} \right\rangle = N_{\rm step}$$

On average: $N_{\rm step} = \Gamma_{\rm sp}\Delta t$ steps per time interval $\Delta t \gg \Gamma^{-1}$



$$\Delta p^2 = (\hbar k_L)^2 \Gamma_{\rm sp} \Delta t = 2 \underbrace{\frac{\Gamma(\hbar k_L)^2}{4} \frac{s}{1+s}}_{\equiv (D_p)_{\rm rec}} \Delta t$$

Fluctuations of the radiation pressure :

Random walk of step size $\pm\hbar k_L$ in momentum space, with (on average) $\Gamma_{\rm sp}\Delta t$ steps per time interval $\Delta t \gg \Gamma^{-1}$

$$(D_p)_{\rm rec} = \frac{\Gamma(\hbar k_L)^2}{4}\frac{s}{1+s}$$

Fluctuations of the number of fluorescence cycles per time interval :

Fluctuations in the number of absorbed photons during Δt with poisson statistics, $\Delta N^2_{\rm abs} = \overline{N}_{\rm abs} = \Gamma(\rho_{ee})_{\rm st}\Delta t$.

$\Delta p^2 = \Delta N^2_{\rm abs}(\hbar k_L)^2 = 2\,(D_p)_{\rm abs}\,\Delta t$, where

$$(D_p)_{\rm abs} = \frac{\Gamma(\hbar k_L)^2(\rho_{ee})_{\rm st}}{2} = \frac{\Gamma(\hbar k_L)^2}{4}\frac{s}{1+s}$$

NB: The probability distribution of the incident photons is Poissonian. However there are corrections in resonance fluorescence, expressed by the so-called *Mandel Q-factor* such that $\Delta N_{\rm abs} = \overline{N}_{\rm abs}(1+Q)$. For resonant fluorescence, $Q < 0$: after absorbing a photon, an atom cannot absorb another one for a time $\sim \Gamma^{-1}$ (*i.e.* until the first photon has been reemitted). We forget about this correction for the following.

Fluctuations of the radiation pressure :

Random walk of step size $\pm\hbar k_L$ in momentum space, with (on average) $\Gamma_{\rm sp}\Delta t$ steps per time interval $\Delta t \gg \Gamma^{-1}$

$$(D_p)_{\rm rec} = \frac{\Gamma(\hbar k_L)^2}{4}\frac{s}{1+s}$$

Fluctuations of the number of fluorescence cycles per time interval :

Fluctuations in the number of absorbed photons during Δt.

$$(D_p)_{\rm abs} = (Dp)_{\rm rec} = \frac{\Gamma(\hbar k_L)^2}{4}\frac{s}{1+s}$$

Momentum diffusion coefficient:

$$D_p = (D_p)_{\rm rec} + (D_p)_{\rm abs} = \frac{\Gamma(\hbar k_L)^2}{2}\frac{s}{1+s}$$

Optical molasses steady-state results from a balance between the cooling provided by the mean friction force (*not* deterministic!) and the heating ($\equiv$ momentum diffusion) associated with the force fluctuations.

Langevin equation :

$$\dot{v} = -\alpha v + \frac{\delta F(t)}{M} \implies v(t) = v(0)e^{-\alpha t} + \frac{1}{M}\int_0^t \delta F(t')e^{-\alpha(t-t')}dt'$$

Average over different realizations of the fluctuating force :

$$\langle v \rangle = v(0)e^{-\alpha t}$$

$$\Delta v^2 = \langle v^2 \rangle - v(0)^2 e^{-2\alpha t} = \frac{1}{M^2}\int_0^t dt' \int_0^t dt'' \langle \delta F(t')\delta F(t'')\rangle e^{-\alpha(2t-t'-t'')}$$

Short memory : $\langle \delta F(t')\delta F(t'')\rangle = 2D_p\delta(t'-t'')$

$$\Delta v^2 = \frac{D_p}{M^2\alpha}\left(1-\exp^{-2\alpha t}\right) = \begin{cases} \frac{2D_p t}{M^2}, & t \ll \alpha^{-1} \\ \\ \frac{D_p}{M^2\alpha}, & t \gg \alpha^{-1} \end{cases}$$

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We define a temperature (in the long times limit) by

$$\frac{k_B T}{2} = \frac{1}{2} M \langle \Delta v^2 \rangle \implies k_B T = \frac{D_p}{M\alpha} \quad \text{Einstein's relation}$$

$$\frac{D_p}{M} = \frac{\Gamma s (\hbar k_L)^2}{2M}$$

$$s \ll 1$$

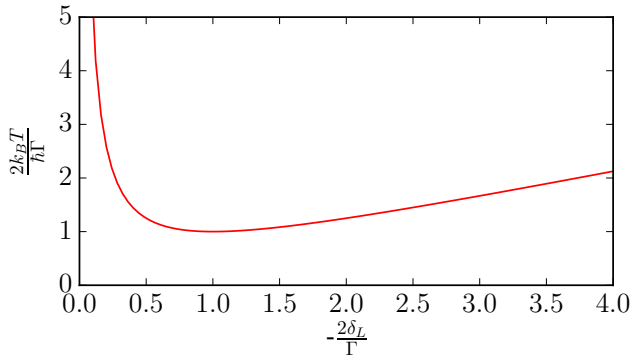
$$\alpha = \frac{\Gamma s (\hbar k_L)^2}{M} \frac{2|\delta_L|}{\delta_L^2 + \frac{\Gamma^2}{4}}$$

$$\delta_L < 0$$

Doppler temperature :

$$k_B T_{\mathrm{D}} = \frac{\hbar\Gamma}{4}\left(\frac{2|\delta_L|}{\Gamma} + \frac{\Gamma}{2|\delta_L|}\right)$$

Minimum for $\delta_L = -\frac{\Gamma}{2}$, $k_B T_{\mathrm{D,min}} = \frac{\hbar\Gamma}{2}$.



NB: More sophisticated approaches (Fokker-Planck equation for the atomic distribution) show that the velocity distribution is Gaussian, as expected for a Maxwell-Boltzmann distribution.

Results from S. Chu's group:

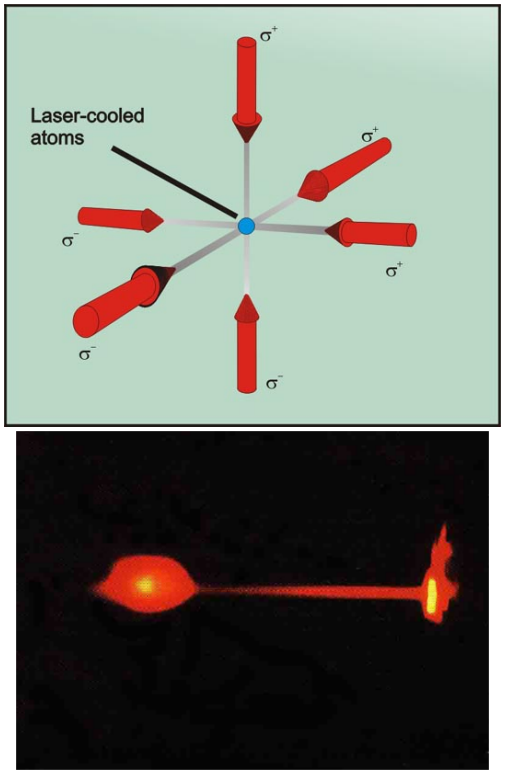
Sodium atom : $\lambda_L = 589\,\text{nm}$,
$\Gamma = 2\pi \times 10\,\text{MHz}$

$\frac{E_r}{h} \approx 25\,\text{KHz}$: broad line condition fulfilled

- A few thousand Sodium atoms caught for about 1 second:

  $D_x = \frac{k_B T_D}{M\alpha} \approx 2\,\text{mm}^2/\text{s}$ $(\frac{I}{I_{\text{sat}}} = 0.1)$

  Beam size $w \sim 1\,\text{cm} \implies$ Catching time $\sim \frac{w}{\sqrt{D_x}} \sim 6\,\text{s}$



S. Chu *et al.*, Phys. Rev. Lett. 55, 48 (1985)

$\alpha^{-1} \approx 30\,\mu\text{s}$ for $\delta_L = -\Gamma/2$, $I = 0.1 I_{\text{sat}}$

- Experimental temperature: $T \approx 200\,\mu\text{K}$

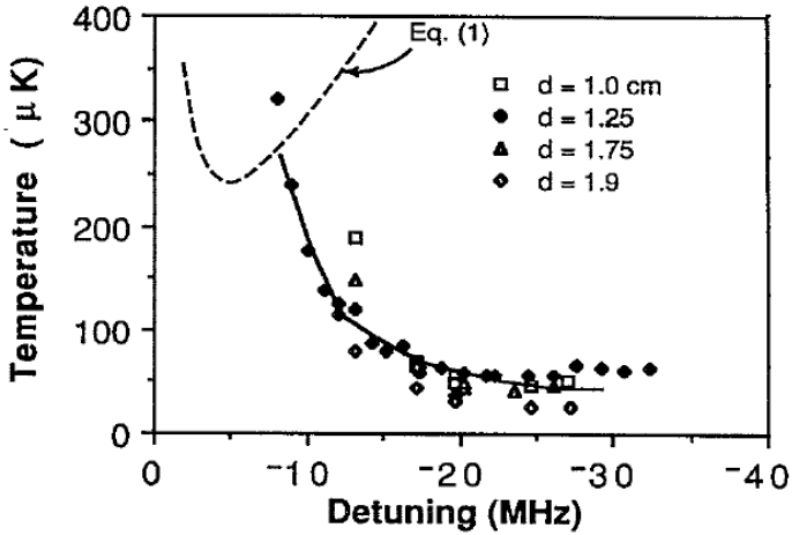
  Doppler theory : $T_D \approx 200\,\mu\text{K}$

1. Semi-classical theory of radiative forces
   - Radiation pressure
     - Zeeman slowing of an atomic beam
   - Dipole force

2. Laser cooling
   - Optical molasses
   - Friction force
   - Fluctuations of radiative forces and momentum diffusion
   - Doppler cooling
   - Sub-Doppler cooling

Experimental results for Sodium atoms

A couple of years after the first molasses, more careful measurements (Phillips' group at NIST) show that atoms in optical molasses are much colder than expected from Doppler theory !



P. Lett *et al.*, PRL (1998)

Dashed line : Doppler theory

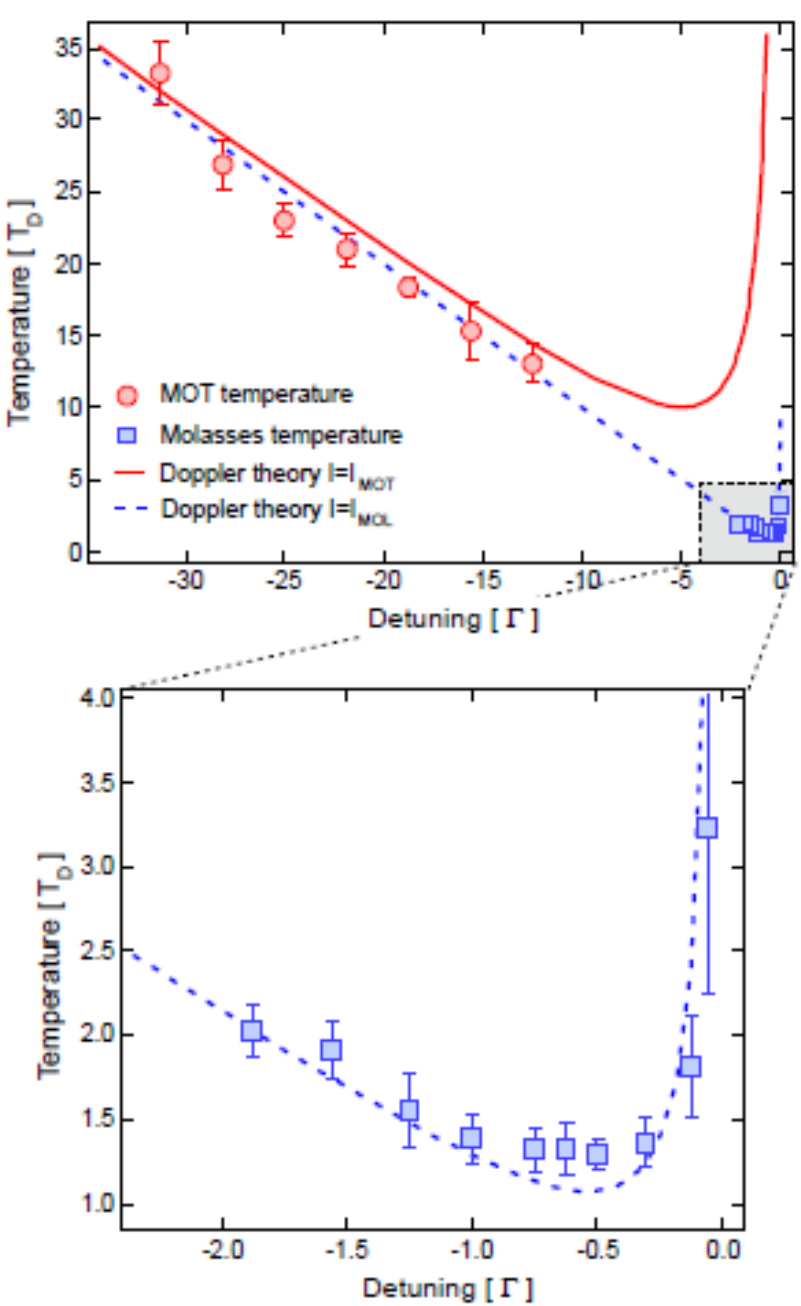
Measurements done on Sodium atoms
$\Gamma = 2\pi \times 10\,\mathrm{MHz}$

- wrong minimal temperature
- wrong dependence with detuning

Doppler theory works fine for atoms with a simple internal structure ($J = 0 \to J = 1$).

It does not work for alkali atoms because of additional effects linked with their more complex internal structure.

Experimental results for (metastable) Helium atoms



- Experiment with ^{4}He* at IO-GS
  R. Chang *et al.*, PRA 90 (6), 063407 (2014)
- $J = 1 \rightarrow J' = 2$ transition
- Solid curve: Doppler theory for the MOT ($I_{MOT} = 100\, I_{\mathrm{sat}}$)

  Dashed curve: Doppler theory for the molasse ($I_{mol} = 0.1\, I_{\mathrm{sat}}$)
- ^{4}He* appears as a special case where sub-Doppler cooling, which could in principle be present, is in practice irrelevant.
- Similar measurements for Hg in McFerran *et al.*, Optics letters 35 (18), 3078-3080 (2010).

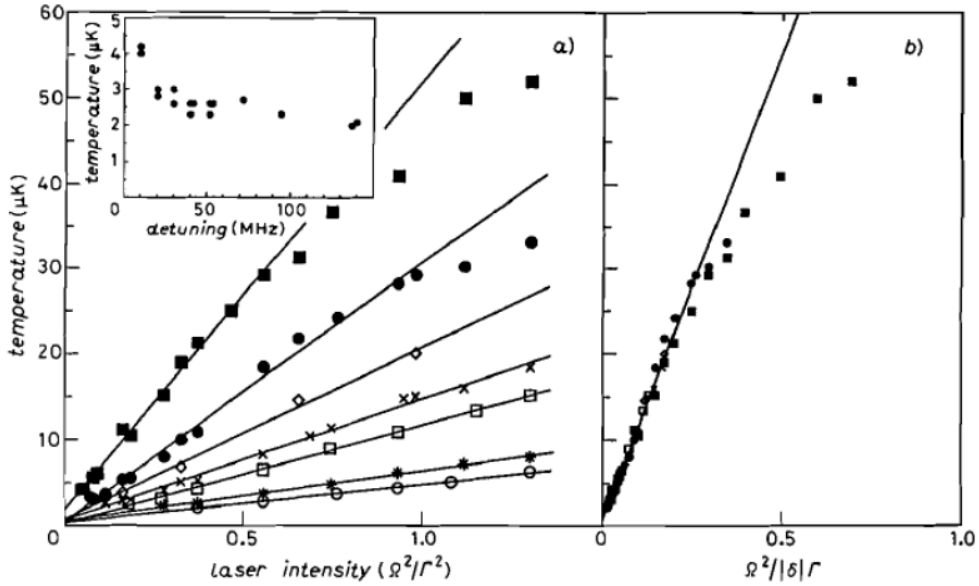
  Here sub-Doppler cooling is simply non-existent (no internal structure).

Temperature:

$$T_{\mathrm{SD}} \geq \# E_R,$$

$\#$ is a pure number, frequently a few tens ($k_B T \sim 3\,\mu$K for the above experiment).

$$T_{\mathrm{SD}} \propto \frac{\hbar \Omega_L^2}{|\delta_L|},$$

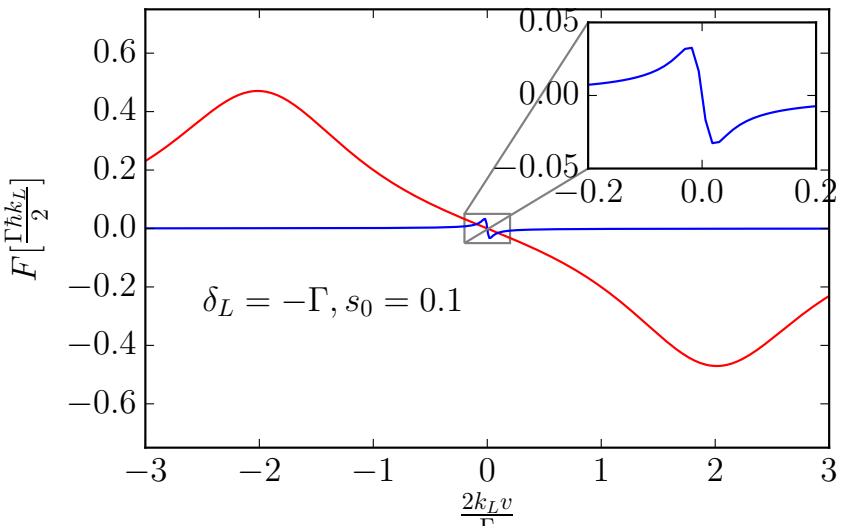


[Salomon *et al.*, EuroPhysics Letters 1990]

For Cesium atoms : $T_D \approx 130\,\mu$K, $E_R/k_B \approx 100\,$nK, $T_{\mathrm{SD}} \approx 6\,\mu$K for $\delta_L = \Gamma$, $s_0 = 0.1$

Explained by the interplay between the polarization of light and the internal structure of the atoms.

Conclusion on laser cooling



Laser cooling : two successive mechanisms leading to ultralow temperatures

- Doppler cooling : catches atoms around $v \sim \frac{\Gamma}{2k_L}$, cools atoms down to T_{D},
- Polarization gradient cooling : leads to sub-Doppler temperatures (several recoil temperatures in practice), active around $v \sim \frac{\Gamma s_0}{2k_L}$.

Only Doppler cooling for atoms lacking an internal structure (alkaline-earth atoms, like Sr, Hg, Ca; or Yb).

No trapping in optical molasses: atoms slowly diffuse away from the trapping region (intersection volume of all beams in a real, 3D configuration)