



Département
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Lecture 3: Atom trapping and Evaporative cooling

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MASTER ICFP, ULTRACOLD ATOMS

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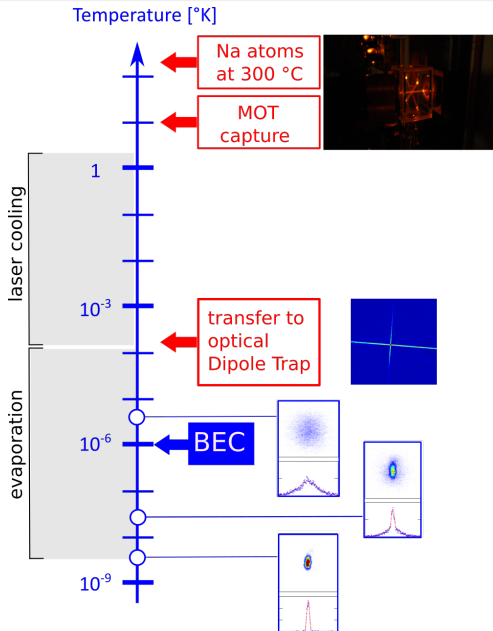
The experimental path to quantum degenerate gases

Typical experimental sequence :

- catch atoms in a **magneto-optical trap**
- **laser cooling** to $\sim 50 \mu\text{K}$
- transfer to **conservative trap** (no resonant light): optical trap or magnetic trap
- **evaporative cooling** to BEC

Take a **picture** of the cloud.

Repeat



① The magneto-optical trap

Reabsorption and collective phenomena

② Conservative traps

Optical dipole traps

Magnetic traps

③ Evaporative cooling

A short discussion of inter-atomic collisions

Kinetic description of evaporative cooling

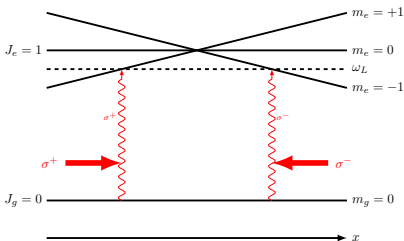
The magneto-optical trap : general principle

Atomic structure :

- $J_g = 0 \rightarrow J_e = 1$ transition
- magnetic moment μ in state e

Cooling + trapping in one dimension
by the combination of

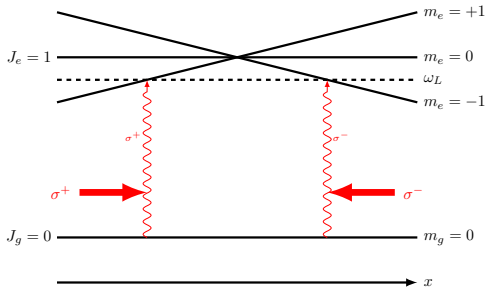
- Two counter-propagating beams
with frequency ω_L , wavevector
 $\mp k_L$, polarizations $\sigma^\pm$
- inhomogeneous magnetic field
 $B(x) = b'x$



Weak saturation $s \ll 1$: the $\sigma^\pm$ -polarized beam (with wavevector $\pm k_L$, respectively) exerts a radiation pressure force on the atom, with a detuning

$$\delta_{\pm k_L} = \delta_L \mp \left(k_L v + \frac{\mu b' x}{\hbar} \right).$$

The magneto-optical trap : general principle



The σ^+ -polarized beam with wavevector $+k_L$ drives the $|m_g = 0\rangle \rightarrow |m_e = +1\rangle$ transition. Effective detuning:

$$\delta_{+k_L} = \omega_L - k_L \cdot v - \omega_{eg} - \frac{\mu b' x}{\hbar} = \delta_L - \left(k_L v + \frac{\mu b' x}{\hbar} \right).$$

For the other beam with wavevector $-k_L$ driving the $|m_g = 0\rangle \rightarrow |m_e = -1\rangle$ transition,

$$\delta_{-k_L} = \delta_L + \left(k_L v + \frac{\mu b' x}{\hbar} \right).$$

For weak saturation $s \ll 1$, we add the forces exerted by each beam independently (as done for optical molasses),

$$F_{\text{tot}} = \frac{\Gamma \hbar k_L}{2} \frac{\Omega_L^2}{2} \left(\frac{1}{\delta_{+k_L}^2 + \frac{\Gamma^2}{4}} - \frac{1}{\delta_{-k_L}^2 + \frac{\Gamma^2}{4}} \right)$$

For $|k_L v + \mu b' x| \ll |\delta_L|$, we have

$$F_{\text{tot}} \approx -M\alpha v - M\omega_{\text{MOT}}^2 x$$

$$\alpha = \frac{\Gamma s_0 \hbar k_L^2}{M} \frac{-2\delta_L}{\delta_L^2 + \frac{\Gamma^2}{4}} \qquad \omega_{\text{MOT}}^2 = \frac{\Gamma s_0 \hbar k_L^2}{M} \frac{-2\delta_L}{\delta_L^2 + \frac{\Gamma^2}{4}} \frac{\mu b'}{\hbar k_L}$$

- Viscous damping for $\delta_L < 0$ (red detuning), with a friction coefficient α

Doppler (and sub-Doppler) cooling exactly as in optical molasses (near the center, where the Zeeman shift is small)

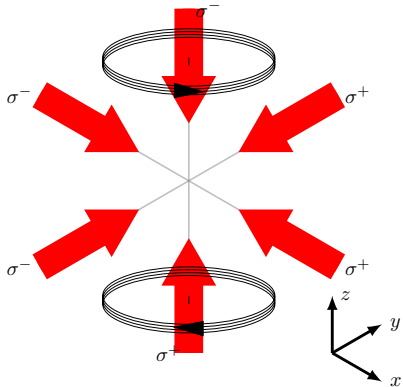
- Harmonic trapping for $\delta_L < 0$ (red detuning), with a frequency ω_{MOT}

The magneto-optical trap in 3D

MOT configuration in three dimensions :

- Three pairs of counter-propagating beams with frequency ω_L , wavevector $\mp k_L$, polarizations $\sigma^\pm$
- inhomogeneous magnetic field created by two coils with opposite current

$$\mathbf{B} \approx \begin{pmatrix} -b'x \\ -b'y \\ 2b'z \end{pmatrix}$$



Spatial distribution:

$$n(x) = n_0 e^{-\frac{x^2}{2R_{\text{MOT}}^2}}, \quad R_{\text{MOT}} = \sqrt{\frac{k_B T}{M \omega_{\text{MOT}}^2}}$$

This predicts $R \sim 50 \mu\text{m}$, much smaller than what is experimentally observed for large MOTs ($N > 10^7$ atoms).

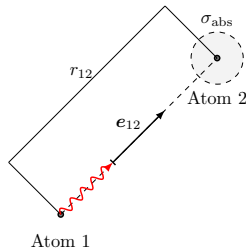
Reabsorption of spontaneously emitted photons :

A photon emitted spontaneously by atom (say 1) can be reabsorbed by another one (say 2), with probability $\frac{d\sigma_{\text{reabs}}}{d\Omega} = \frac{\sigma_{\text{reabs}}}{4\pi r_{12}^2}$ [with σ_{reabs} is the reabsorption cross-section].

Mean momentum transfer during a time $\Delta t \gg \Gamma^{-1}$:

$$\Delta \mathbf{p} = \mathbf{F}_{1 \rightarrow 2} \Delta t = \Gamma_{\text{sp}} \Delta t \frac{\sigma_{\text{abs}}}{4\pi r_{12}^2} \hbar \mathbf{k}_L \mathbf{e}_{12}$$

Long-range force mediated by spontaneous photons.



$$\mathbf{F}_{1 \rightarrow 2} = \Gamma_{\text{sp}} \hbar k_L \frac{\sigma_{\text{abs}}}{4\pi r_{12}^2} \mathbf{e}_{12}$$

Equivalent to a Coulomb-like force between two charges $q_1 = q_2 = q$, with

$$\frac{q^2}{\epsilon_0} \rightarrow \hbar \Gamma_{\text{sp}} k_L \sigma_{\text{abs}}.$$

T. Walker, D. Sesko, C. Wieman, PRL 64, 408 (1990).

A. Steane, M. Chowdhury and C. Foot, J. Opt. Soc. Am. B 9, 2142 (1992).

Electrostatic analogy in the limit of a continuous medium.

- Long-range forces also strongly affect the MOT dynamics, which is non-linear. Analogies with plasmas : self-sustained oscillations, Coulomb explosion after release [L. Pruvost *et al.*, PRA 61, 063408 (2000)] ...
- Absorption from the MOT beams also play a role: it adds another contribution to the radiation pressure with the same form as the reabsorption force but opposite in sign [J. Dalibard, Optics Comm. 68, 203 (1988)]. The overall force is normally dominated by the reabsorption part [Walker *et al.*, PRL 1990].

Magneto-optical trap for large atom numbers : equilibrium shape

Equilibrium shape of a MOT in the regime of large reabsorption:

Assume **uniform density in a sphere of radius R** . The problem is then mapped to the one of a uniformly charged sphere with charge density $\rho = qn$.

Gauss theorem:

$$\mathbf{E}(r) = \frac{\rho r}{3\epsilon_0} \mathbf{e}_r$$

Coulomb force :

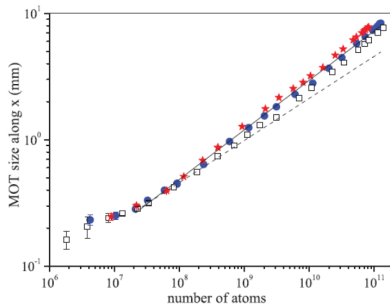
$$\mathbf{F}_{\text{Coulomb}} = \frac{q\rho r}{3\epsilon_0} \mathbf{e}_r = \frac{n\Gamma_{\text{sp}}\hbar k_L \sigma_{\text{abs}} r}{3} \mathbf{e}_r = nC r \mathbf{e}_r$$

Equilibrium, neglecting the viscous force :

$$\mathbf{F}_{\text{Coulomb}} + \mathbf{F}_{\text{MOT}} = 0$$
$$\Rightarrow n = \frac{m\omega_{\text{MOT}}^2}{C} = \frac{2|\delta_L|\Gamma}{\delta_L^2 + \frac{\Gamma^2}{4}} \frac{\mu b'}{\hbar \Gamma \sigma_{\text{abs}}}$$

Typical values :

$$n \sim 10^{10} \text{ at/cm}^3, R \sim \text{mm}$$



A. Camarra, R. Kaiser, G. Labeyrie, PRA 90, 063404 (2014)

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③ Evaporative cooling

A short discussion of inter-atomic collisions

Kinetic description of evaporative cooling

$$\mathbf{F}_{\text{dip}}(\mathbf{r}) = -u_{\text{st}} \hbar \nabla \Omega_L(\mathbf{r}) = -\frac{\hbar \delta_L}{2} \frac{s}{1+s} \frac{\nabla \Omega_L}{\Omega_L}, \quad \frac{s}{1+s} = \frac{\frac{\Omega_L^2}{2}}{\delta_L^2 + \frac{\Gamma^2}{4} + \frac{\Omega_L^2}{2}}$$

Properties of the dipole force :

- $\mathbf{F}_{\text{dip}}$ vanishes on resonance,
- $\mathbf{F}_{\text{dip}}$ negligible close to resonance as compared to radiation pressure $\mathbf{F}_{\text{rp}}$,
- conservative : $\mathbf{F}_{\text{dip}} = -\nabla U(\mathbf{r})$, with the **dipole potential** U given by

$$U(\mathbf{r}) = \frac{\hbar \delta_L}{2} \ln [1 + s(\mathbf{r})]$$

For a standing wave : $E_L = E_0 \cos(\mathbf{k} \cdot \mathbf{r})$, $\varphi_L = \text{cst}$, the dipole force is given by

$$U(\mathbf{r}) = \frac{\hbar \delta_L}{2} \ln [1 + s_0 \cos^2(\mathbf{k} \cdot \mathbf{r})]$$

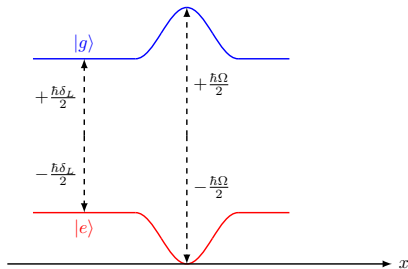
The potential is periodic, trapping atoms at the nodes or antinodes of the standing wave depending on the detuning (basis for optical lattices, covered in the last part of the course).

Redistribution of photons between the $\leftarrow$ and $\rightarrow$ propagating waves by stimulated emission.

Dipole potential for large detunings : $\delta_L \gg \Gamma, \Omega_L$ or equivalently $s \ll 1$

$$U(\mathbf{r}) \approx \frac{\hbar \delta_L s(\mathbf{r})}{2} = \frac{\hbar \Omega_L^2(\mathbf{r})}{4\delta_L} = \frac{\hbar \Gamma^2}{8\delta_L} \frac{I(\mathbf{r})}{I_{\text{sat}}}$$

Interpretation as a position-dependent light shift of $|g\rangle$:



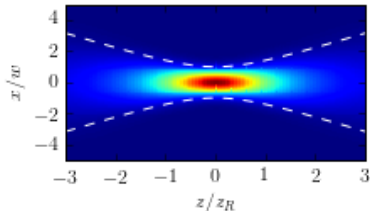
- $\delta_L > 0$: repulsive force (atoms are repelled from the intensity maxima),
- $\delta_L < 0$: attractive force (atoms are attracted to intensity maxima).

- Residual spontaneous emission :

$$\Gamma_{\text{sp}} = \frac{\Gamma s}{2} = \frac{\Gamma}{\delta_L} \frac{U}{\hbar}$$

Actual laser beams have a Gaussian transverse profile. This creates a dipole potential energy of the form

$$U(\mathbf{r}) = -U_0 \frac{e^{-2\frac{x^2+y^2}{w^2}}}{1 + \left(\frac{z}{z_R}\right)^2}$$



Potential map

Harmonic trap near the bottom of the potential (particles with energy $\ll U_0$):

$$U(\mathbf{r}) = -U_0 + \sum_{x_\alpha=x,y,z} \frac{1}{2} m \omega_\alpha^2 x_\alpha^2$$

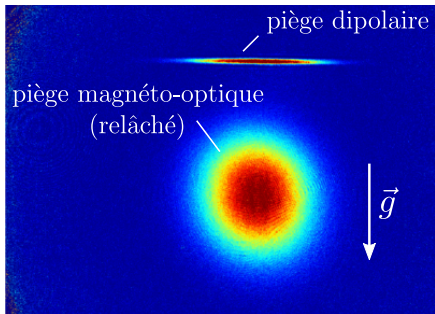
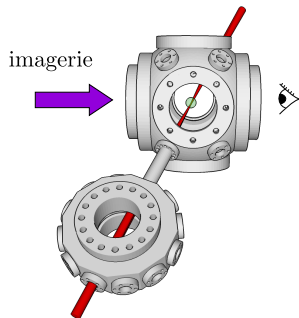
$$\omega_x = \omega_y = \sqrt{\frac{4U_0}{Mw^2}}, \quad \omega_z = \sqrt{\frac{2U_0}{Mz_R^2}} = \frac{w}{\sqrt{2}z_R} \omega_x$$

Typical example for Ytterbium atoms :

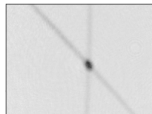
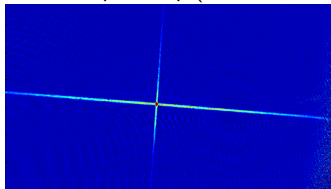
- $\lambda_{eg} = 399 \text{ nm}$
- $\lambda_L = 532 \text{ nm}$
- $\delta_L = -2 \cdot 10^{14} \text{ Hz}$
- $P = 10 \text{ W}$, $w = 50 \mu\text{m}$
- $s \approx 10^{-6}$
- $I = 2 \text{ MW/m}^2$ at focus
- $U_0 \approx k_B \times 700 \mu\text{K}$
- $\omega_x/(2\pi) \approx 1.2 \text{ kHz}$
- $\omega_z/(2\pi) \approx 3 \text{ Hz}$

Loading a dipole trap from a MOT

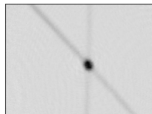
Single beam dipole trap (side view) :



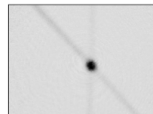
Crossed dipole trap (viewed from above):



$t=100$ ms



$t=300$ ms



$t=600$ ms

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Kinetic description of evaporative cooling

Atoms coming close together interact (short-range interaction potential). The result of the collision process allows us to distinguish two types of collisions : elastic or inelastic.

Elastic collisions :

- Redistribution of momenta on a “collision sphere”, conserving total momentum and energy.

$$\begin{aligned} \mathbf{p}_1 + \mathbf{p}_2 &\longrightarrow \mathbf{p}'_1 + \mathbf{p}'_2 \\ \frac{1}{2M}(\mathbf{p}_1^2 + \mathbf{p}_2^2) &\longrightarrow \frac{1}{2M}(\mathbf{p}'_1{}^2 + \mathbf{p}'_2{}^2) \end{aligned}$$

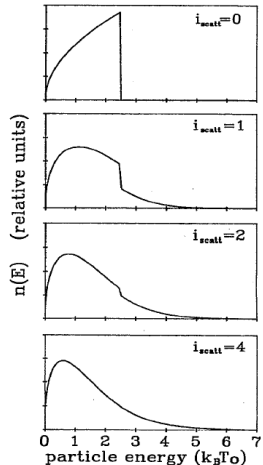
- Thermalization of trapped atoms, governed by the rate of elastic collisions :

$$\gamma_{\text{coll}} = n\sigma_{\text{coll}}v_{\text{th}}$$

- n : spatial density
- σ_{coll} : collisional cross-section
- $v_{\text{th}} = \sqrt{\frac{8k_B T}{\pi M}}$: mean thermal velocity.

How long does it take to thermalize ?

- Numerical simulation of the Boltzmann equation (assuming ergodicity) [Snoke and Wolf, PRB 39, 4030 (1989).].
- A few collisions are enough to relax to the quasi-equilibrium distribution.
- Thermalization time = a few collision times $\gamma_{\text{coll}}^{-1}$.



Atoms coming close together interact (short-range interaction potential). The result of the collision process allows us to distinguish two types of collisions : elastic and inelastic.

Inelastic collisions : *reactive* collisions changing the atom internal state

- collision with hot atoms from the background gas (with energy ~ 300 K)
- *reactive* collisions changing the internal state of the atoms
 - two-body collision with a change in internal state

$$\begin{aligned}\mathbf{p}_1 + \mathbf{p}_2 &\longrightarrow \mathbf{p}'_1 + \mathbf{p}'_2 \\ \frac{1}{2M}(\mathbf{p}_1^2 + \mathbf{p}_2^2) &\longrightarrow \frac{1}{2M}(\mathbf{p}'_1{}^2 + \mathbf{p}'_2{}^2) + \Delta E_{\text{internal}}\end{aligned}$$

- three-body recombination to a molecular state (binding energy $E_m < 0$)

$$\begin{aligned}\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 &\longrightarrow \mathbf{p}'_3 + \mathbf{p}_m \\ \frac{1}{2M}(\mathbf{p}_1^2 + \mathbf{p}_2^2 + \mathbf{p}_3^2) &\longrightarrow \frac{\mathbf{p}'_3{}^2}{2M} + \frac{\mathbf{p}_m^2}{2(2M)} - E_m\end{aligned}$$

Inelastic collision processes typically produce atoms with energy *much* higher than the trap depth.

Trap losses described by a rate equation:

$$\dot{n} = - \underbrace{\gamma_1 n}_{\text{background}} - \underbrace{\gamma_2 n^2}_{\text{two-body}} - \underbrace{\gamma_3 n^3}_{\text{three-body}}$$

n : spatial density

γ_m : rate for m -body process

$\gamma_{1,2,3} \ll \gamma_{\text{coll}}$ to have (approximately) ergodic behavior.

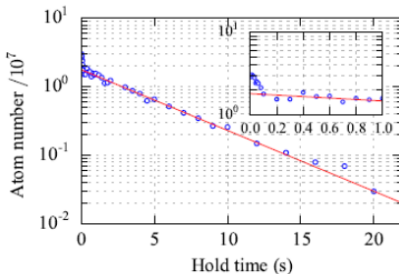


Figure from Matthias Scholl's PhD thesis

Ultracold atoms can be considered in equilibrium only in a *kinetic sense*: The true thermodynamic equilibrium state of the atomic bodies are all solid at sub-K temperatures.

In this sense atomic gases are only metastable systems from a thermodynamic point of view.

Consider a gas of atoms held in a trap (potential $U(\mathbf{r})$) of *finite depth* (say U_0). Atoms with energy lower than U_0 stay trapped, atoms with higher energy escape.

After a few collisions times, the gas equilibrates (in a kinetic sense) to a **truncated Maxwell-Boltzmann distribution**. The phase-space density $f(\mathbf{r}, \mathbf{p})$ is given by

$$f(\mathbf{r}, \mathbf{p}) = \frac{1}{Z} e^{-\frac{\mathbf{p}^2}{2Mk_B T} - \frac{U(\mathbf{r})}{k_B T}} \Theta \left(U_0 - \frac{\mathbf{p}^2}{2M} - U(\mathbf{r}) \right).$$

Θ is the Heaviside step function.

Verified in experiments and in numerical solutions of the Boltzmann equation [e.g. Luiten et al., PRA 53, 381 (1996)].

Ergodicity: f depends only on the mechanical energy of the atoms $\frac{\mathbf{p}^2}{2M} + U(\mathbf{r})$.

For a harmonic trap $U(\mathbf{r}) = \frac{1}{2} M \omega^2 \mathbf{r}^2$ (approximation valid for energies $\ll U_0$),

$$\frac{1}{Z} = N \left(\frac{\hbar}{M R_{\text{th}} v_{\text{th}}} \right)^3 = n(0) \lambda_{\text{th}}^3 \propto \frac{N \omega^3}{T^3}, \quad R_{\text{th}} = \sqrt{\frac{k_B T}{M \omega^2}}, \quad v_{\text{th}} = \sqrt{\frac{k_B T}{M}}$$

Principle of evaporative cooling

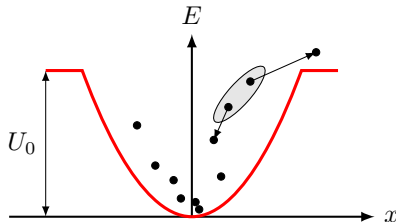
Atoms trapped in a potential of depth U_0 , undergoing elastic collisions :

- two atoms with energy close to U_0 collide
- result: one “cold” atom and a “hot” one with energy $> U_0$
- rethermalization of the $N - 1$ atoms remaining in the trap results in a lower mean energy per atom.

Evaporation parameter:

$$\eta = \frac{U_0}{k_B T} \sim 8 - 11$$

Evaporated atoms have an energy $\sim \eta k_B T$: evaporation proceeds from the *tails of the distribution*.



Rate of evaporation:

- f_η : fraction of atoms with energy $> U_0 = \eta k_B T$. For a Boltzmann gas, $f_\eta \propto \sqrt{\eta} e^{-\eta}$.
- atoms with energy $\approx \eta k_B T$ have a velocity $\propto \sqrt{\eta}$

$$\gamma_{\text{ev}} \sim \gamma_{\text{coll}} \eta e^{-\eta} \ll \gamma_{\text{coll}}$$

More details in: W. Ketterle and N. J. Van Druten, Evaporative cooling of trapped atoms, available from the Ketterle group website.

Simple estimate of the efficiency of evaporative cooling

Step-like model of evaporation: each step $N \rightarrow N - dN, T \rightarrow T - dT$

We write the mean energy lost by the evaporated atoms as $-d\bar{E} = (\eta + \kappa)k_B T$, with

$$\eta = \frac{U_0}{k_B T} \sim 10$$

and $\kappa \sim 1$ the “excess energy” with respect to the trap depth.

Mean energy before the collision : $\bar{E} = 3Nk_B T$.

Mean energy after evaporation:

$$\bar{E} - d\bar{E} = \bar{E} - (\eta + \kappa)k_B T dN$$

Mean energy after thermalization:

$$\bar{E} - d\bar{E} = 3(N - dN)k_B(T - dT) \approx 3Nk_B T - 3k_B T dN - 3Nk_B dT$$

We get

$$(\eta + \kappa - 3) \frac{dN}{N} = 3 \frac{dT}{T} \quad \Rightarrow \quad \left(\frac{T_f}{T_i} \right) = \left(\frac{N_f}{N_i} \right)^\alpha, \quad \alpha = \frac{\eta + \kappa}{3} - 1 \sim 2 - 3$$

Losing a factor 10 on N cools the sample by a factor $\sim 100 - 1000$!

For a **fixed trap frequency** ω and a gas in quasi-equilibrium, the spatial density n , the phase space density $\mathcal{D}$, elastic collision rate γ_{coll} scale as :

$$n = \frac{N}{(2\pi)^{3/2} R_{\text{th}}^3} \propto \frac{N\omega^3}{T^{3/2}} \sim N^{1-\frac{3\alpha}{2}}$$

$$\mathcal{D} = n\lambda_{\text{th}}^3 \propto \frac{N\omega^3}{T^3} \sim N^{1-3\alpha}$$

$$\gamma_{\text{coll}} = n\sigma v_{\text{th}} \propto \frac{N\omega^3}{T} \sim N^{1-\alpha}$$

$\alpha > 1/3$: increase of the phase space density of the sample.

$\alpha > 1$: increase of the collision rate.

$$\begin{aligned}n &\propto \frac{N\omega^3}{T^{3/2}} \sim N^{1-\frac{3\alpha}{2}} \\ \mathcal{D} &\propto \frac{N\omega^3}{T^3} \sim N^{1-3\alpha} \\ \gamma_{\text{coll}} &\propto \frac{N\omega^3}{T} \sim N^{1-\alpha}\end{aligned}$$

Practical problem : reach $\mathcal{D} \sim 1$ before loss processes set in.

Rate of evaporation $\propto \gamma_{\text{coll}}\eta e^{-\eta}$ must be much faster than the lifetime of the sample.

Forced evaporation : decrease trap depth as the gas cools down to keep η roughly constant ($\sim 8 - 10$). Scaling laws are $T \propto U_0$ and

$$\begin{aligned}n &\propto \frac{N\omega^3}{T^{3/2}} \sim U_0^{\frac{2-3\alpha}{2\alpha}} \\ \mathcal{D} &\propto \frac{N\omega^3}{T^3} \sim U_0^{\frac{1-3\alpha}{\alpha}} \\ \gamma_{\text{coll}} &\propto \frac{N\omega^3}{T} \sim U_0^{\frac{1-\alpha}{\alpha}}\end{aligned}$$

Best scenario is to reach the runaway regime: evaporation at ever increasing γ_{coll} ($\alpha > 1$). Feasible in magnetic traps (see Appendix).

Detailed optimization can be complicated : it must also account for background

The trap depth U_0 is varied by reducing the laser power, since $U_0 \propto P$.

However, the **trap frequencies also decrease**, as $\omega \propto \sqrt{P} \propto \sqrt{U_0}$.

The spatial density n , the phase space density $\mathcal{D}$, elastic collision rate γ_{coll} now scale as

$$\begin{aligned}n &\propto \frac{N\omega^3}{T^{3/2}} \sim U_0^{\frac{1}{\alpha}} \\ \mathcal{D} &\propto \frac{N\omega^3}{T^3} \sim U_0^{\frac{1}{\alpha} - \frac{3}{2}} \\ \gamma_{\text{coll}} &\propto n \frac{N\omega^3}{T} \propto U_0^{\frac{1}{\alpha} + \frac{1}{2}}\end{aligned}$$

Therefore, **in optical traps, the collision rate never increases and runaway evaporation does not happen.**

Fortunately, the collisions rates after loading from a MOT are high (a few thousands per second, typically). The rule of the game is then to optimize the initial conditions and the “trajectory” $U_0(t)$ such that one reaches $\mathcal{D} \sim 1$ before the collision rate has dropped too much and evaporation stops.

Example of evaporative cooling “trajectory”

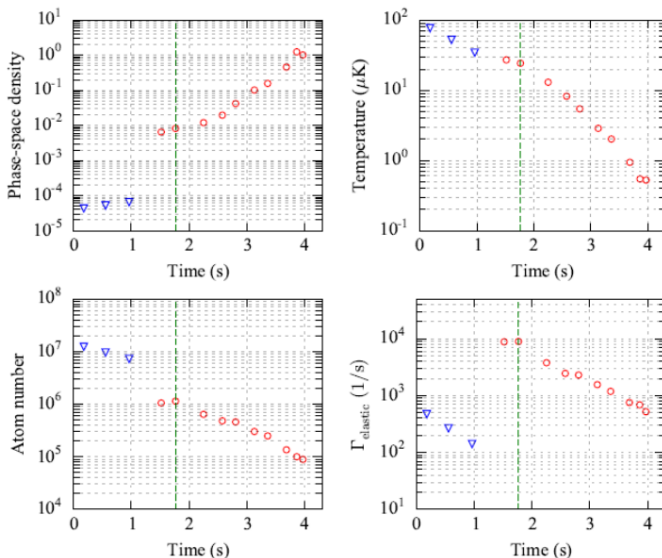


Figure from Matthias Scholl's PhD thesis

Blue triangles: single beam DT; red circles : crossed dipole trap

Complement on magnetic traps

Magnetic traps

Zeeman Hamiltonian in a non-uniform magnetic field $\mathbf{B}(\mathbf{r})$:

$$U_{\text{mag}} = -\mathbf{m} \cdot \mathbf{B}(\mathbf{r})$$

Assuming that the internal states can follow adiabatically the change of $\mathbf{B}$ as the atom move in space,

$$\langle F, m_F | U_{\text{mag}} | F, m_F \rangle = g_F m_F |\mathbf{B}(\mathbf{r})|$$

Maxwell equations tell us that $|\mathbf{B}|$ can only have a minimum.

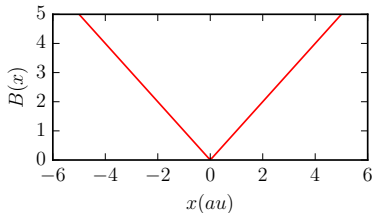
If $g_F m_F > 0$ (low-field seeking states), the potential energy is minimal where $|\mathbf{B}|$ is: this results in atom trapping near the minimum. Atoms in high field seeking states with $g_F m_F < 0$ are expelled from the potential.

NB : The state $|F, m_F\rangle$ is defined in a local reference frame, where the local quantization axis is given by the direction of $\mathbf{B}$.

Quadrupole trap :

field created by two coils with opposite current

$$\mathbf{B} \approx \begin{pmatrix} -b'x \\ -b'y \\ 2b'z \end{pmatrix},$$
$$|\mathbf{B}| = b' \sqrt{x^2 + y^2 + 4z^2}$$



Quantum adiabatic theorem : Slowly evolving quantum system, with Hamiltonian $\hat{H}(t)$

Instantaneous eigenbasis of $\hat{H}$: $\hat{H}(t)|\phi_n(t)\rangle = \varepsilon_n(t)|\phi_n(t)\rangle$.

Time-dependent wave function in the $\{|\phi_n(t)\rangle\}$ basis:

$$|\Psi(t)\rangle = \sum_n a_n(t) e^{-\frac{i}{\hbar} \int_0^t \varepsilon_n(t') dt'} |\phi_n(t)\rangle,$$

From Schrödinger equation, one gets $[\omega_{mn} = \varepsilon_m - \varepsilon_n]$:

$$\dot{a}_n = -\langle \phi_n | \dot{\phi}_n \rangle a_n(t) - \sum_{m \neq n} e^{-\frac{i}{\hbar} \int_0^t \omega_{mn}(t') dt'} \langle \phi_n | \dot{\phi}_m \rangle a_m(t),$$

- **Berry phase** : $\langle \phi_n | \dot{\phi}_n \rangle = -i\gamma_B$ is a pure phase. Wavefunction unchanged up to a phase evolution after a cyclic change.
- **The adiabatic theorem** : for *arbitrarily slow* evolution starting from a particular state n_0 ($a_n(0) = \delta_{n,n_0}$), and in the *absence of level crossings*, $a_n(t) \rightarrow \delta_{n,n_0}$ (up to a global phase).

Adiabatic approximation for slow evolutions and initial condition $a_n(0) = \delta_{n,n_0}$:

$$a_n(t) \rightarrow e^{i\gamma_B(t)} \delta_{n,n_0}$$

Validity criterion :

$$(\varepsilon_m - \varepsilon_n)^2$$

Adiabaticity criterion for a time-dependent field $\mathbf{B}(t)$:

$$\left| \langle \phi_n | \dot{H} | \phi_m \rangle \right| \sim \mu \dot{B} \ll \frac{(\varepsilon_m - \varepsilon_n)^2}{\hbar} \sim \frac{(\mu B)^2}{\hbar} = \hbar \omega_{\text{Larmor}}^2, \text{ or } \frac{\dot{\omega}_{\text{Larmor}}}{\omega_{\text{Larmor}}^2} \ll 1$$

Here $\omega_{\text{Larmor}} = \mu B / \hbar$ is the Larmor frequency quantifying the angular velocity of the spin precession around the magnetic field axis.

Adiabaticity criterion for an atom, described classically, moving in a static field $\mathbf{B}(\mathbf{r})$:

$$H = -\mathbf{m} \cdot \mathbf{B}[\mathbf{r}(t)] \implies \langle \downarrow | \dot{\mathbf{B}} | \uparrow \rangle = \cdot \langle \downarrow | (\mathbf{v} \cdot \nabla) \mathbf{B} | \uparrow \rangle \ll \frac{\mu |\mathbf{B}(\mathbf{r})|^2}{\hbar}$$

- spin-flip transitions to untrapped states near the minimum of $|\mathbf{B}|$
- result in atom losses,
- mostly affect the fast atoms.

Majorana losses is a major issue in a quadrupole trap since $\min(|\mathbf{B}|) = 0$.

Biased quadrupole trap (loffe trap) :

$$\mathbf{B} \approx \begin{pmatrix} -b'x \\ -b'y \\ B_0 + 2b'z \end{pmatrix}$$

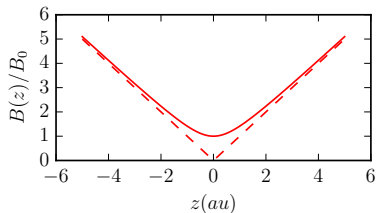
$$|\mathbf{B}| = \sqrt{b'^2(x^2 + y^2) + (B_0 + 2b'z)^2}$$

$$\approx B_0 + b'^2(x^2 + y^2) + 2b'z + \frac{2b'^2}{B_0}z^2$$

Harmonic trap with frequency $\omega_x, \omega_y \gg \omega_z$

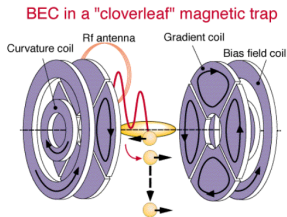
Typical values :

- $B_0 \sim 1 - 10 \text{ G}$ [$1 \text{ G} = 10^{-4} \text{ T}$],
- $b' \sim 100 \text{ G/cm}$,
- $\omega_x = \omega_y \approx 2\pi \times 300 \text{ Hz}$,
 $\omega_z \approx 2\pi \times 30 \text{ Hz}$.

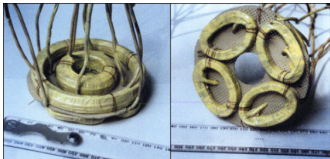


Gallery of magnetic traps

"Cloverleaf" trap :



MIT, March '96 [M.-O. Mewes et al., PRL 77, 416 (1996)]



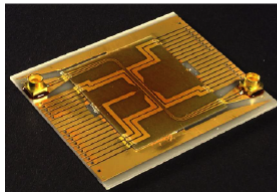
Ketterle group, MIT

<http://cua.mit.edu/>

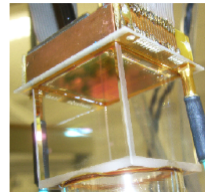
- Macroscopic trap :
 $l \sim 1 - 10 \text{ cm}$,
 $I \sim 100 - 1000 \text{ A}$
- kW of electrical power to dissipate (water cooling)
- switch-off not trivial/ power

Atom chips:

a



b



Reichel group, LKB

<http://www.lkb.ens.fr/~Atom-Chips-Group->

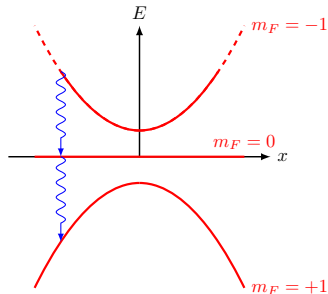
- "Microscopic" trap : $l \sim 0.1 - 1 \text{ mm}$, $I \sim 1 \text{ A}$
- little power dissipated, compact
- chip problematic for some applications
- usually modest vacuum level

Many variants for both designs (practically one per experiment ...)

Evaporative cooling in magnetic traps: radio-frequency evaporation

Forced evaporation: $U_0(t)$ decreases in time (in a controlled way). Optimizing the evaporation trajectory is done by controlling the atom lost by the evaporation process *via* the trap depth.

In magnetic traps, this is achieved by radio-frequency evaporation : a radiofrequency field is applied, inducing transitions from a trapped state to a non-trapped state for a given energy $\hbar\omega_{\text{rf}} = U_0$.

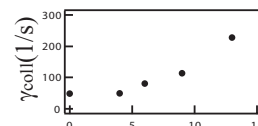
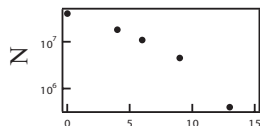
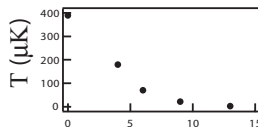
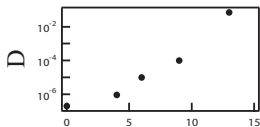


Usually $\eta = U_0/k_B T \approx \text{cst}$. Then, the scaling with trap depth is $T \sim U_0$, $N \sim U_0^{\frac{1}{\alpha}}$, and

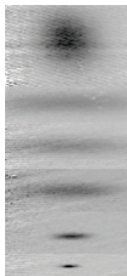
$$n \sim U_0^{\frac{1}{\alpha} - \frac{3}{2}}, \quad \mathcal{D} \sim U_0^{\frac{1}{\alpha} - 3}, \quad \gamma_{\text{coll}} \sim U_0^{\frac{1}{\alpha} - 1}$$

Optimization can be done (usually experimentally) to reach the runaway regime.

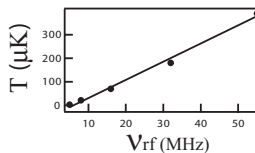
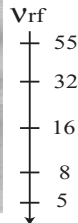
Illustration of evaporation in magnetic traps



Durée d'évaporation (s)



piège magnétique
avant compression



F. Gerbier, PhD thesis