Master ICFP

Ultracold Atoms

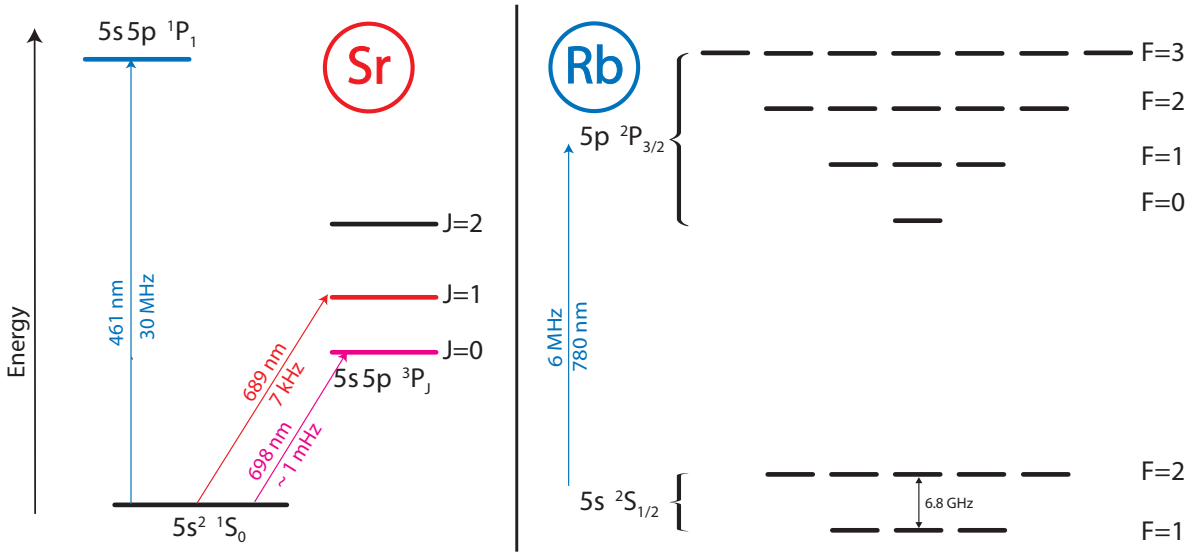
Exercise class n°1: Laser cooling and trapping

Sylvain Nascimbene

We study different examples of manipulation of atoms by light. After describing usual atomic level structures we study laser cooling and magneto-optical trapping. Finally we show that trapping atoms in an optical lattice allows to modify the fluorescence spectrum. This technique is currently used in optical atomic clocks to get sub-recoil linewidths.

1 Atomic levels and optical transitions

We show below the low-lying electronic levels of a one-electron atom (^{87}Rb) and a two-electron atom (^{84}Sr), together with the characteristics of the most relevant optical transitions.



1. Remind the electronic structure of Rb and Sr. What is the meaning of the spectroscopic notation $^{2S+1}X_J$?

2. Comment on and explain the atomic level structures of ^{87}Rb and ^{84}Sr.

3. Recall the selection rules for electric dipole one-photon transitions. Which ones are allowed among the transitions shown above ? Explain why the other transitions are weakly allowed and comment on the linewidth values.

2 Laser cooling

4. Find a numerical evaluation of the Doppler temperature for the different transitions described above.

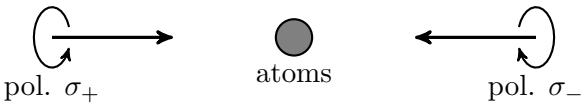
5. Explain why in experiments with Sr atoms one first makes a magneto-optical trap (MOT) on the 'blue' transition and then transfers the atoms into a MOT on the 'red' transition.

3 Magneto-Optical Trapping of Sr atoms

We study the principle of the Sr magneto-optical trap on a simplified 1D model along the z axis.

We recall the formula of the radiation pressure force for a laser beam of saturation parameter s, propagating along $\vec{u}$:

$$\vec{F}_{\text{rad}} = \frac{\hbar k \Gamma}{2} \frac{s}{1+s} \vec{u} \quad \text{with} \quad s = \frac{\Omega^2}{\delta^2 + \Gamma^2/4}.$$



6. Write down the force resulting from the superposition of two-counter propagating beams detuned from resonance by δ_0, in the absence of magnetic field. We assume that the laser intensity is not saturant. Plot the radiative force amplitude as a function of the atom velocity v.

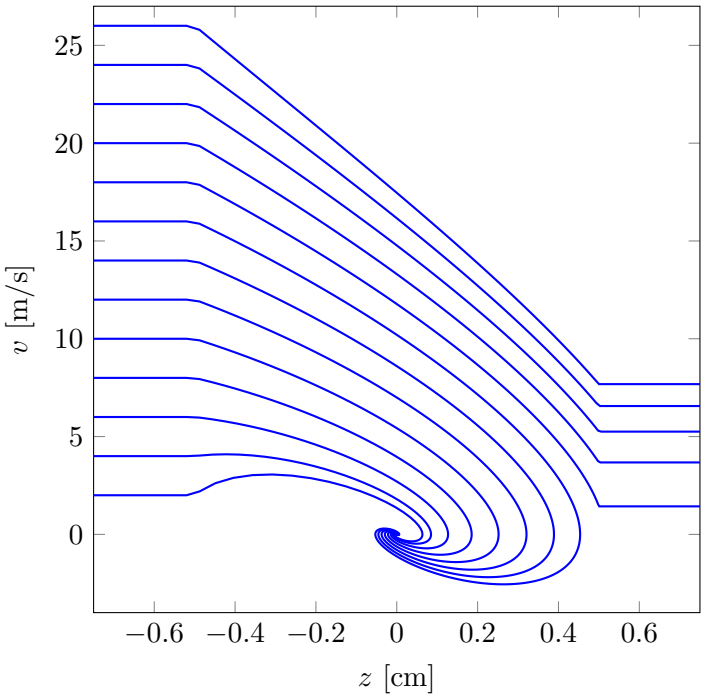
7. We now take into account the magnetic field gradient applied for magneto-optical trapping. The magnetic field $B = Gz$ results in Zeeman shifts $\Delta E = g_J m_J \mu_B B$, where g_J is the Landé factor, equal to 3/2 in our case. Plot the energy levels versus the atomic position.

8. Assuming that the two laser fields have opposite circular polarizations, write down the radiation pressure force for an atom at position z and velocity v.

9. Plot the trajectory of an atom captured in the magneto-optical trap in the (z, v) plane.

10. In order to estimate the capture velocity of the magneto-optical trap, we consider an atom slowed with the maximum acceleration $a_{\text{max}} = \hbar k \Gamma / 2m$ over the whole MOT beam size w. Give the expression of the maximum incoming velocity v_{c} of captured atoms, and calculated its value for a beam size $w = 1\,\text{cm}$.

11. We plot below several calculated trajectories for a MOT operated on the blue line, with $\delta = -3\Gamma$, $s = 0.5$, $G = 10\,\text{Gauss/cm}$. Each line corresponds to the trajectory of an atom sent from the left side with a positive velocity. Comment on this graph.

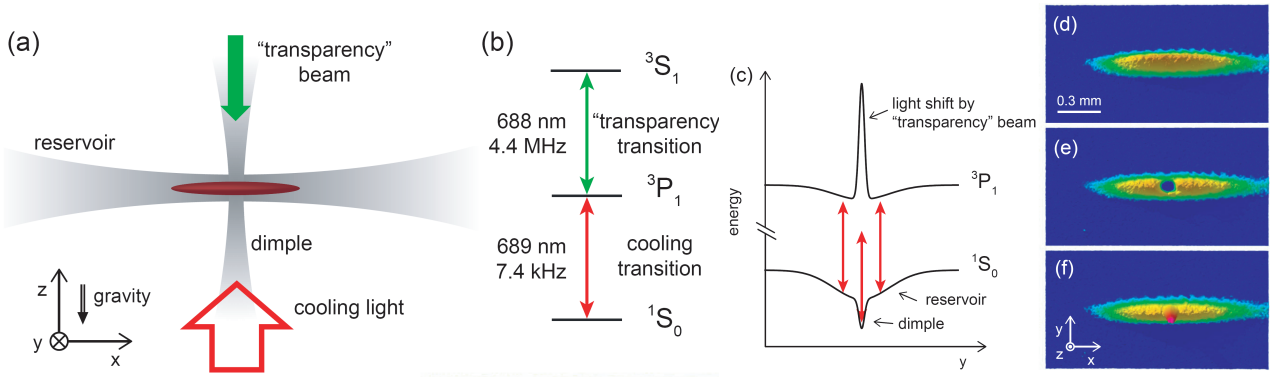


4 Optical dipole traps and lightshift

We propose to illustrate the optical dipole force with a recent experiment, which demonstrates the Bose-Einstein condensation of Sr atoms using laser cooling only (i.e. without evaporative cooling).

Even if the temperature obtained in a 'red' MOT is quite low ($\leqslant 1\,\mu$K), the density is not large enough to reach the BEC threshold. The atoms are transferred into an optical dipole trap, in which further laser cooling is operated.

Two far-detuned lasers are required to reach degeneracy (see figure below). First, a transparency laser ($\lambda_T = 688\,$nm) is used to create a region in space where atoms do not see the light to avoid the associated losses (light assisted collisions, reabsorption of photons). Second, the atoms are trapped in a small size dipole trap (the so-called 'dimple' trap) at the focus of an other laser ($\lambda_D = 1065\,$nm) to enhance the density. Typical absorption images are shown in the right panels: without the transparency beam and dimple (d), with only the transparency beam (e), with both beams (f).



From S. Stellmer et al, Phys. Rev. Lett. ***110****, 263003 (2013).*

We recall the formula of the dipole force:

$$\vec{F}_{\text{dip}} = -\frac{\hbar\delta}{2}\frac{\vec{\nabla}s}{1+s}$$

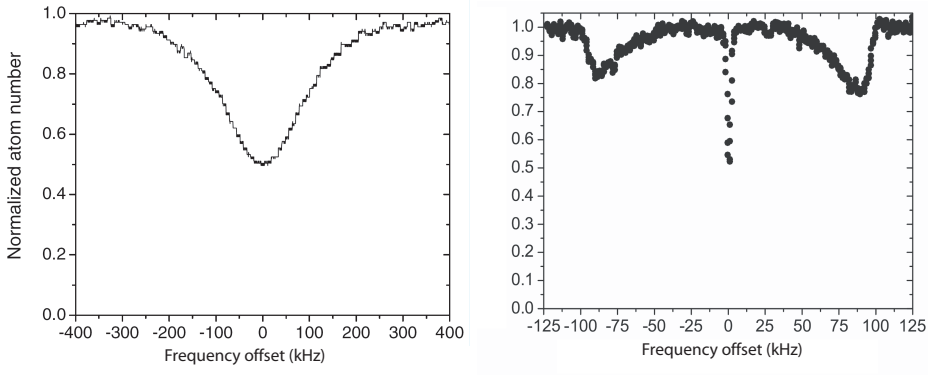
12. Write down the expression of the dipole potential created by a beam of Rabi frequency Ω and detuning δ. Simplify the expression for $\delta \gg \Omega, \Gamma$.

13. Using the level scheme of figure (b) explain how this scheme works. What is the sign of the detuning of the transparency beam with respect to the transparency transition ?

5 Optical clocks

Optical clocks are systems designed to create a frequency standard in the optical domain. They are based on the extremely narrow optical transitions such as the 698 nm transition of Sr. In a MOT the atom temperature limits the linewidth because of the Doppler effect. We describe how this limit can be overcome by trapping the atoms in a deep optical lattice.

These considerations are illustrated in the figure below, that represents resonance measurements for atomic samples in a magneto-optical trap (left graph) and in a deep optical lattice (right graph). These measurements were performed around the ultra-narrow transition of Yb in the NIST group (PRL 95, 083003 (2005) and PRL 96, 083002 (2006)).



14. Calculate the expected Doppler width for $T = 50\,\mu$K ($M = 174\,\mathrm{g}\cdot\mathrm{mol}^{-1}$, $\lambda = 578\,$nm).

We now describe the case of atoms trapped in an optical lattice. We suppose that the center of mass motion of an atom and its electronic state are decoupled. We assume that the center of mass motion is described by a harmonic oscillator of frequency Ω.

15. The optical lattice is created by the interference of two counterpropagating beams. What is the spatial variation of the laser intensity ? What is the potential profile ? We call U_0 the trap depth.

16. Show that the oscillation frequency at the bottom of each lattice well reads

$$\omega = 2\sqrt{U_0 E_r}/\hbar, \quad E_r = \frac{\hbar^2 k^2}{2m}.$$

17. Show on a graph the different possible transitions $(g, n) \to (e, n')$ between the two electronic states taking into account the motional states (quantum number n).

18. Compare the frequency difference between all the transitions with the recoil frequency in the Lamb-Dicke limit, for which the width of the ground state wavefunction of the harmonic oscillator is much smaller than $\lambda/2\pi$.

19. We remind that the light-atom interaction can be described by the electric dipole Hamiltonian $\hat{H} = -\hat{\mathbf{d}}\cdot\mathbf{E}$. Write the matrix elements in the harmonic oscillator basis, and show that recoil-free resonant transitions are expected to have the largest amplitude.

20. Explain why we expect the occurence of sideband transitions, shifted from resonance by $\pm\Omega$.

21. Compare this description with the experimental results. What is the oscillation frequency in the lattice? Compare it with the recoil frequency.

22. The optical lattice not only shifts the energy of the atoms in the electronic ground state, but also in the excited state. How should we choose the wavelength of the laser making the optical lattice in order to make a precise clock?