Master ICFP

Ultracold Atoms

Exercise class n°3: Thermal fraction of a Bose-Einstein condensate

Sylvain Nascimbene

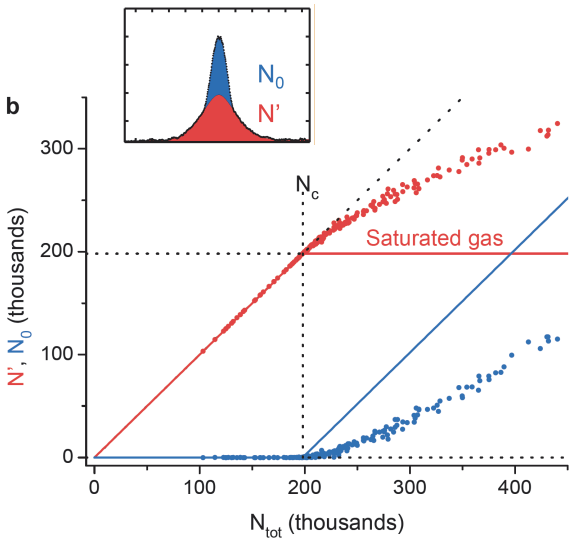
We discuss the notion of condensed fraction of a 3D BEC at finite temperature. We first calculate the condensed fraction for an ideal Bose gas, and then show that interactions play a significant role in the description of typical experimental systems.

Ideal Bose Gas

We first consider an ideal gas of bosons of mass m at a finite temperature T. We want to estimate the atom number in the thermal component N' and the atom number in the condensate N_0 as a function of the total atom number $N = N_0 + N'$. We use the semi-classical approximation for the calculation of densities.

1. Calculate the atomic density n of a uniform gas as a function of $\beta = 1/k_B T$ and the chemical potential μ. We will use the polylogarithm function: $g_{3/2}(z) = \sum_{\ell=1}^{\infty} \frac{z^\ell}{\ell^{3/2}}$.
2. Use the local density approximation to find the expression of the density $n(r)$ in a harmonic trap of angular frequency ω as a function of the chemical potential μ_0 at the trap bottom.
3. Give the expression of the critical atom number as a function of temperature. We will use the Rieman function $\zeta(3) = \sum_{\ell=1}^{\infty} \frac{1}{l^3} \simeq 1.20$.
4. What is the value of the critical temperature ?
5. How does the condensed fraction vary with temperature ?
6. Plot the curves of N' and N_0 as a function of N.

We report below experimental results obtained for a ^{39}K cloud at constant temperature (Phys. Rev. Lett. 106, 230401 (2011)).



We observe a non saturation of excited atoms in the experiment. To understand this behavior, we refine our model including atomic interactions at the mean-field level. The coupling constant associated with interactions is denoted $g = 4\pi\hbar^2 a/m$.

Spatial profile of the BEC

We assume that the BEC density profile can be described in the Thomas-Fermi limit.

7. What is the spatial profile of the BEC density $n_0(r)$?

8. Calculate the chemical potential of the cloud μ_0.

Non-saturation of the thermal fraction

We also take into account interactions in the thermal component, via a Hartree-Fock mean-field energy shift acting as an effective potential

$$V_{\text{eff}}(r) = V(r) + 2g[n_0(r) + n'(r)],$$

where $V(r)$ is the external trapping potential and $n_0(r)$ and $n'(r)$ are respectively the thermal and condensed densities. As the atomic density is usually quite low in the thermal component, we neglect in the following the mean-field shift due to thermal atoms.

9. Calculate and plot the effective potential, assuming the condensed part in the Thomas-Fermi regime.

10. Show that, to lowest order in $\mu_0/(k_B T)$,

$$\frac{N'}{N_{\text{id}}^c} = 1 + \frac{\zeta(2)}{\zeta(3)}\frac{\mu_0}{k_B T},$$

where N_{id}^c is the critical atom number for an ideal gas.
We give

$$\int_0^x d\epsilon\,\sqrt{\epsilon}\,e^{\epsilon - x} + \int_x^\infty d\epsilon\,\sqrt{\epsilon}\,e^{x-\epsilon} \approx \frac{\sqrt{\pi}}{2}(1+x) \qquad \text{for} \quad x \ll 1.$$

11. Comment on the experimental results plotted below, measured at constant temperature.

