Master ICFP

Ultracold Atoms

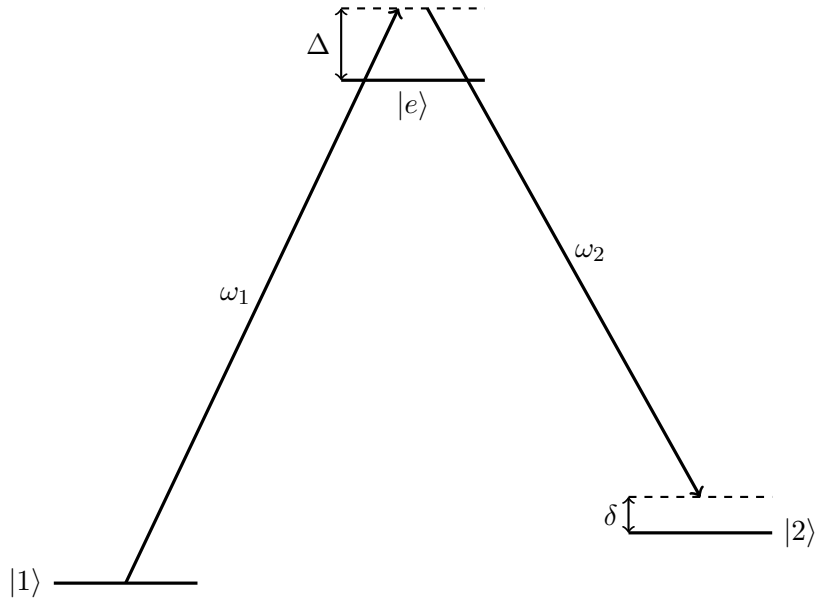
Exercise class n°7: Bragg diffraction of a Bose-Einstein condensate

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We study the diffraction of a Bose-Einstein condensate by a moving optical lattice. We show that one can coherently inject momentum to the atoms and prepare quantum superpositions with several momentum components. Finally we show an example of interferometer based on Bragg diffraction.

1 Preliminary : Two-photon Raman transition

We consider the coupling between two long-lived states $|1\rangle$, $|2\rangle$ from two-photon transitions via an excited state $|e\rangle$. The coupling is generated by two lasers, of frequencies ω_1 and ω_2. The frequency difference $\omega_1 - \omega_2$ is equal to the energy difference between states $|1\rangle$ and $|2\rangle$, plus an energy offset δ. The one-photon Rabi couplings are denoted as $\Omega_i \mathrm{e}^{-i\omega_i t}$ $(i = 1, 2)$.



1. Write down the evolution equations of a general quantum superposition in states $|1\rangle$, $|2\rangle$ and $|e\rangle$.

2. Using the rotating wave approximation, show that the system can be written, in a dressed state basis, as a time-independent Hamiltonian of matrix

$$H = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega_1 & 0 \\ \Omega_1^* & -2\Delta & \Omega_2^* \\ 0 & \Omega_2 & -2\delta \end{pmatrix}$$

in the $(|1\rangle, |e\rangle, |2\rangle)$ basis.

We write the state as

$$|\psi(t)\rangle = c_1(t)\,|1\rangle + c_2(t)\,|2\rangle + c_e(t)\,|e\rangle\,.$$

We assume that the lasers are far detuned by Δ from one-photon resonances. The state $|e\rangle$ then evolves on a time scale $\sim 1/\Delta$.

3. By averaging over this fast time scale, show that the amplitude of the excited state is given by

$$c_e = \frac{1}{2\Delta}\left(\Omega_1^* c_1 + \Omega_2^* c_2\right).$$

Note that this amplitude could also be deduced from first-order perturbation theory.

4. Deduce that the system can be described as an effective two-level system, the states $|1\rangle$ and $|2\rangle$ being coupled by a single field of effective Rabi frequency

$$\Omega_{\text{eff}} = \frac{\Omega_1 \Omega_2^*}{2\Delta}.$$

2 Bragg diffraction

We consider the Bragg diffraction of a BEC on a moving optical lattice, created by two counter-propagating laser beams of frequency difference δ.

These beams couple, via a two-photon process, two states of same internal state but different momenta. The recoil energy and velocity are $E_r = h \times 25$ kHz and $v_r = 2.9\,\text{cm}\,\text{s}^{-1}$, respectively. The optical lattice is applied after time of flight so that the effect of interactions is negligible.

We first consider the resonant case $\delta = 0$.

5. What are the momentum states coupled by Bragg diffraction? Write down the corresponding Hamiltonian.

6. Consider a transition between $|p\rangle$ and $|p + 2\hbar k\rangle$. What is the total variation of photon numbers in the laser beams ? Give a microscopic interpretation of the atomic momentum change.

7. Show that this process is forbidden for an atom initially at rest because it does not conserve the total energy.

8. Represent on a graph the atom dispersion and the relevant two-photon transitions. What value of the detuning δ allows for a resonant coupling of the $|p\rangle$ and $|p + 2\hbar k\rangle$ states?

We now compare these results with experiments of the Boulder group.
Figure 1 shows absorption pictures of BEC with and without applying the optical lattice and after a time-of-flight.

9. Check that the observed displacement of 0.56 mm of the diffracted cloud after a 10 ms time-of-flight corresponds to a single two-photon process. Explain why only part of the cloud is diffracted. How should one choose the duration of the Bragg pulse to diffract the whole cloud ?

Figure 2 shows absorption pictures obtained with different detunings δ. The last picture corresponds to the case of a single Bragg beam.

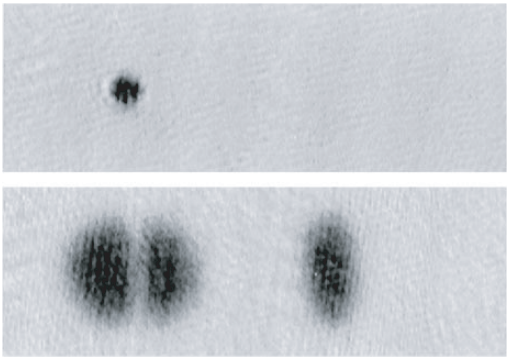


Figure 1: Optical diffraction of a Bragg diffracted condensate. Top : absorption image before the lattice is applied for 55 μs. Bottom : image after a pulse of the optical lattice and after a 10 ms time-of-flight. (*from PRL 82 871 (1999)*)

10. Explain the origin of the different diffraction positions.

11. What process can explain the last absorption picture, which exhibits atoms lying on a circle? What is the position of the center of the circle?

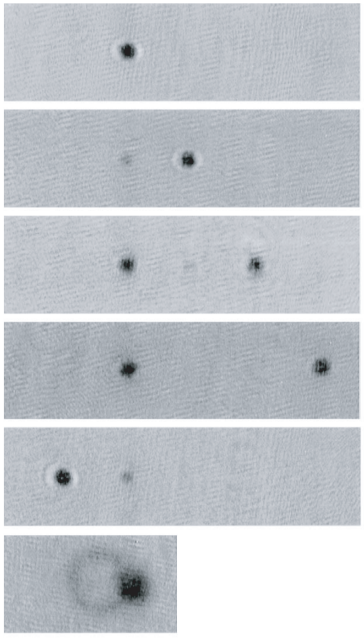


Figure 2: Absorption images for different frequency differences between the beams of the moving optical lattice. (from top to bottom : 0, 98, 200, 300, -98 kHz). The bottom image corresponds to the case with a single beam. (*from PRL 82 871 (1999)*)

3 Application to interferometry

We now briefly describe a scheme for performing interferometry experiments, based on Bragg diffraction. The principle and the results are described in figure 3.

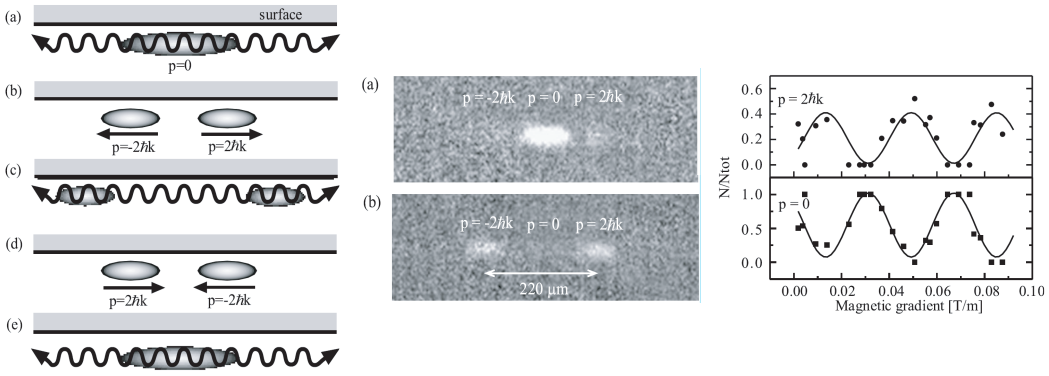


Figure 3: Left : Scheme of the Bragg interferometer. The cloud is initially spliced in two wavepackets with opposite momenta $\pm 2\hbar k$. After a given waiting time the two packets are reflected and when they overlap again they are recombined to form the interference pattern. The phase difference between the two arms is varied by changing an external magnetic field gradient. Center : Time of flight image for two different phase shifts between the two arms. Right : Variation of the population in the relevant momentum states for different gradients. (*from PRL* ***94****, 090405 (2005)*)

12. Explain how to prepare a quantum superposition of two wavepackets with same amplitude and opposite momenta.

13. Write down the expression of the phase shift created by the magnetic field gradient.

14. Give the expression of the final atomic state at the exit of the interferometer.