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# Supporting Online Material for

## An On-Demand Coherent Single-Electron Source

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**This PDF file includes:**

SOM Text
Figs. S1 and S2
References

## Modeling of the single electron source

To compare experiments and theory, we use the simplified 1D model of a Quantum Dot coupled to the 2D lead by a QPC transmitting one channel. When considering the linear response of the current to a small amplitude high frequency voltage applied the top gate, the system is a quantum R-C circuit. It was studied and described in ref.[1]. When a square wave voltage of high amplitude is applied, the system becomes a Single Electron Source and a source of quantized ac current.

From ref.[2], it is found that the Coulomb blockade effect can be neglected in a first approximation. To model the dot spectrum we use a constant energy level spacing $\Delta$ which is a good approximation in the edge state regime. Denoting $r$ and $t$ the amplitude reflection and transmission coefficients of the QPC ($r^2 = 1 - D$, $t = \sqrt{D}$), we first calculate the scattering amplitude of the RC circuit at energy $\epsilon$:

$$s(\epsilon) = r - t^2 e^{i\varphi} \sum_{n=0}^{\infty} (re^{i\varphi})^n = \frac{r - e^{i\varphi}}{1 - re^{i\varphi}}$$

where $\varphi = 2\pi\epsilon/\Delta$ is the phase accumulated for a single turn in the quantum dot. Then, the dot density of states is given by:

$$N(\epsilon) = \frac{1}{2i\pi} s^{+} \frac{\partial s}{\partial \varepsilon} = \frac{1}{\Delta} \frac{1 - r^2}{1 - 2r\cos 2\pi \frac{\varepsilon}{\Delta} + r^2}$$

To model the transmission $D$ dependence on the voltage $V_G$ applied on the QPC gate, we use a Fermi-Dirac like function [3]:

$$D(V_G) \quad = \quad \frac{1}{1 + e^{-\frac{V_G - V_0}{\Delta V}}}$$

Screening effects may lead to slight changes of the above formula, but practically we have not seen any obvious modifications. $V_G$ also controls dot potential by capacitive coupling which is modeled here by a linear shift of the dot energy spectrum. This model was successfully used to describe the linear response of quantum RC circuits [1] using the appropriate expressions of the admittance in the case $eV_{exc} << \hbar\omega$ [4]. Figure 1 shows that it accounts very well for phase resolved measurements of the linear-regime admittance. Here the model is compared with data taken on the sample used in the present paper. The agreement is very good using a unique set of parameters ($\Delta$, $V_0$, $\Delta V$ and geometric capacitance) for a wide frequency range (0.18-1.5 GHz). From this comparison, we thus know accurately the transmission of the QPC as a function of $V_G$ ($V_0 = -896mV$ and $\Delta V = 2.9\ mV$) as well as the charging energy of the dot ($\sim 2.5K$) and the geometrical capacitance ($C \geq 3fF$, which corresponds to 0.6 K).

In the present paper, the same model is used to describe the non linear response of our circuit, using equation (1) below. As discussed in the main text (see fig.4), full agreement with experiment is achieved taking as only parameters the one extracted from the linear response. In figure 2, we present more experimental data supporting this quantitative agreement for the full range of dot potential.

## Calculation of the charge injection:

We calculate the current in the circuit in response to a high-amplitude periodic square excitation ($eV_{exc} >> \hbar\omega$ where $2V_{exc}$ is the peak to peak amplitude). Our calculations are a direct extension of the work of Büttiker et al.[4], which was devoted to the case of a small

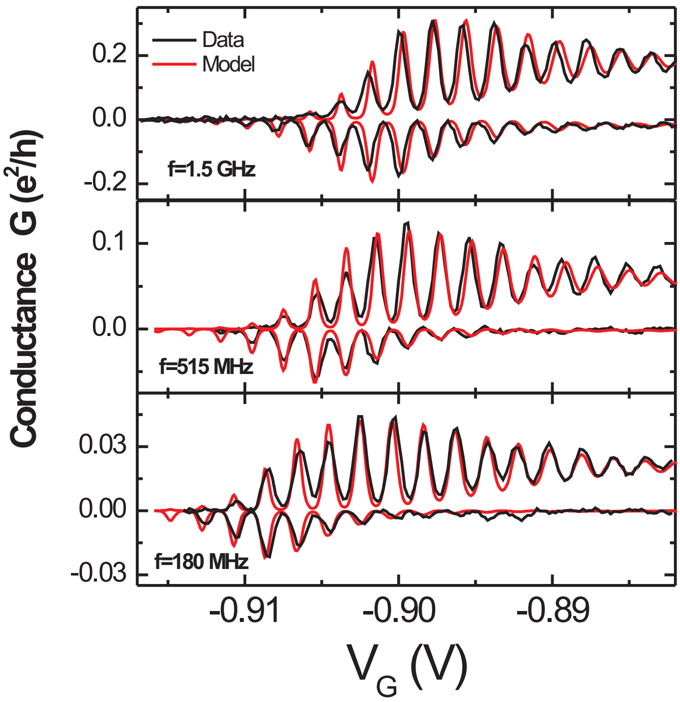


Figure 1: Complex conductance $G$ of our sample as a function of the gate voltage $V_G$ at $T = 200$ mK for f = 0.18, 0.515 and 1.5 GHz : Experimental results (black lines) and simulations (red lines). For clarity, we have plotted $Im(G) > 0$ and $-Re(G) < 0$.

amplitude sine excitation ($eV_{exc} << \hbar\omega$). In our calculations [5], the effect of the geometrical dot capacitance is neglected ($e^2/C << \Delta$) which is justified by the experimental values ($C \approx 4e^2/\Delta$). We calculate the low frequency expansion of the $(2k+1)^{th}$ harmonic of the current:

$$I_{(2k+1)\omega} = \frac{i2V_{exc}}{\pi(2k+1)} \int d\epsilon \Big[ -i(2k+1)\omega e^2 N(\epsilon) + \frac{h}{2e^2}[e^2 N(\epsilon)(2k+1)\omega]^2 \Big] \frac{f(\epsilon - 2eV_{exc}) - f(\epsilon)}{2eV_{exc}} \quad (1)$$

where $i2V_{exc}/\pi(2k+1)$ is the $(2k+1)^{th}$ harmonic of the excitation voltage and $N(\epsilon)$ is the dot density of states. The above equation shows that, in the non-linear regime, our circuit is still equivalent to an RC circuit with $V_{exc}$-dependent capacitance and resistance given by:

$$\begin{aligned} \widetilde{C_q} &= e^2 \int d\epsilon N(\epsilon) \frac{f(\epsilon - 2eV_{exc}) - f(\epsilon)}{2eV_{exc}} \\ \widetilde{R_q} &= \frac{h}{2e^2} \frac{\int d\epsilon N(\epsilon)^2 \frac{f(\epsilon - 2eV_{exc}) - f(\epsilon)}{2eV_{exc}}}{[\int d\epsilon N(\epsilon) \frac{f(\epsilon - 2eV_{exc}) - f(\epsilon)}{2eV_{exc}}]^2}. \end{aligned}$$

In the time domain, this correspond to an exponentially decaying current :

$$\begin{aligned} I(t) &= \frac{q}{\tau} e^{-t/\tau} \\ q &= \widetilde{C_q} \times 2V_{exc} \\ \tau &= \widetilde{R_q}\widetilde{C_q} \end{aligned}$$

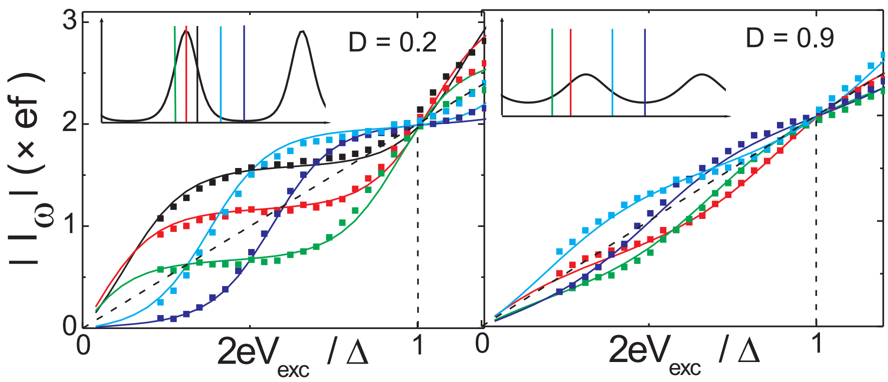


Figure 2: $|I_\omega|$ as a function of $2eV_{exc}/\Delta$ for different dot potentials at $D \approx 0.2$ (left) and $D \approx 0.9$ (right). Points correspond to experimental values and lines to theoretical predictions. Insets: schematic representation of the dot density of states $N(\epsilon)$

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