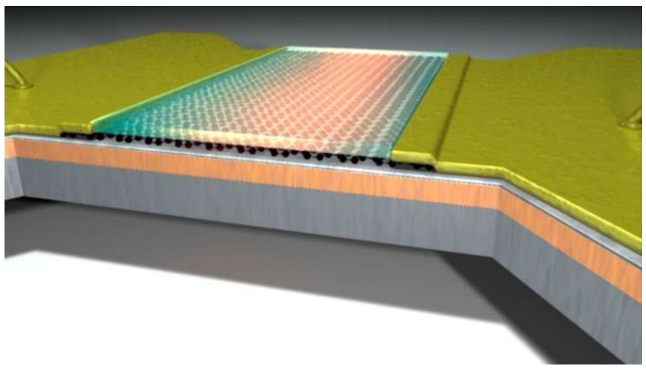
laboratoire pierre aigrain
électronique et photonique quantiques

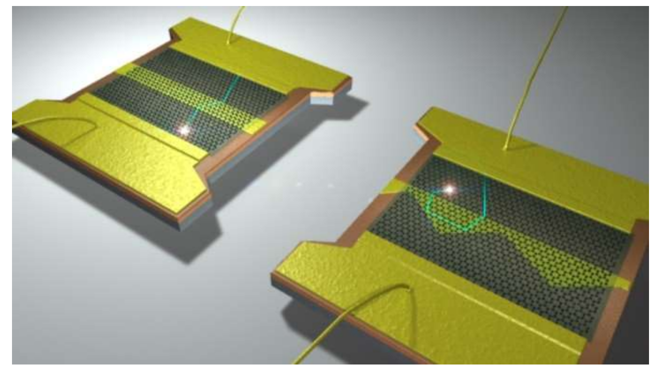
Electronic transport in graphene (today)

An introduction with focus on



hot electrons

(electron-phonon)



ballistic's

(Dirac Fermion Optics)

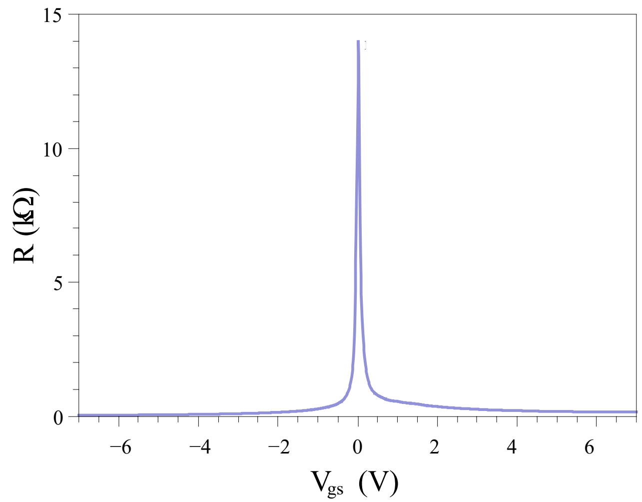
Bernard Plaçais

placais@lpa.ens.fr

GRAPHENE FLAGSHIP

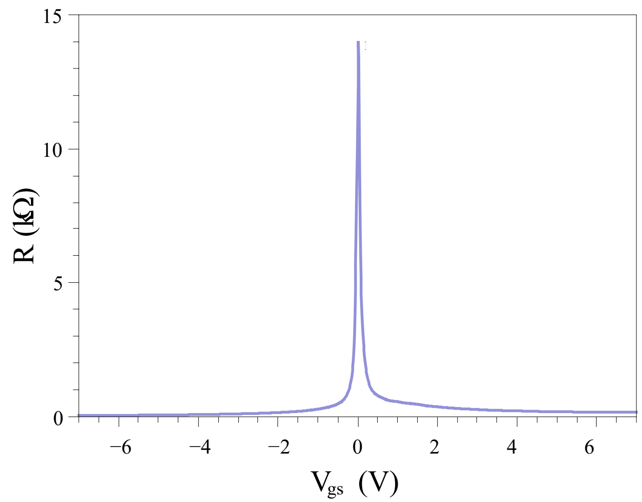
electronic transport

linear (quantum) transport

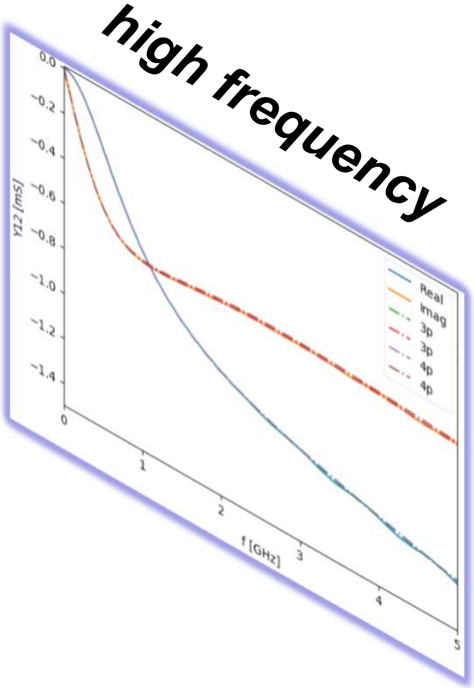


« electronic transport trilogy »

linear (quantum) transport

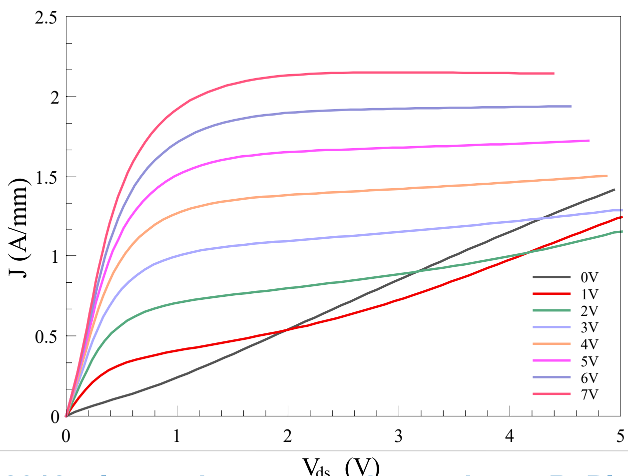


high frequency

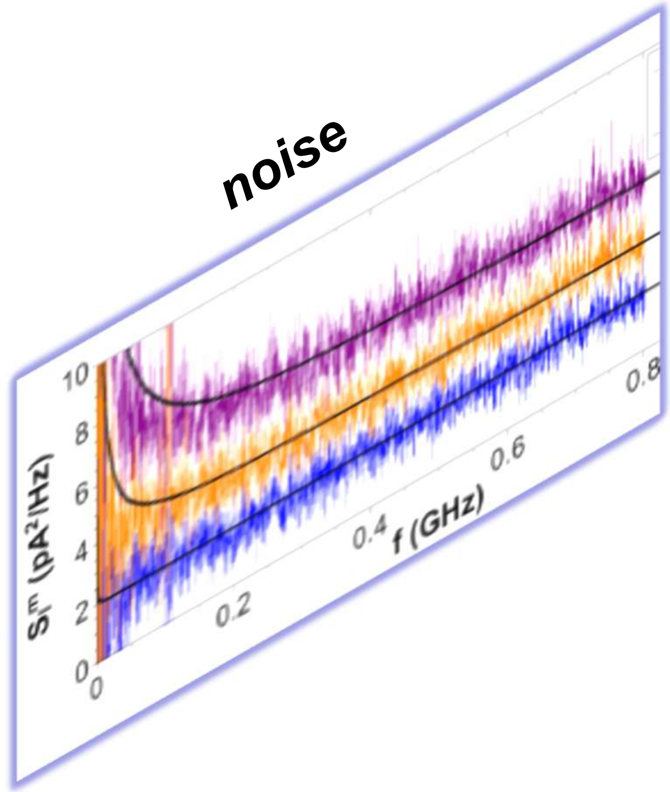


But also

non-linear

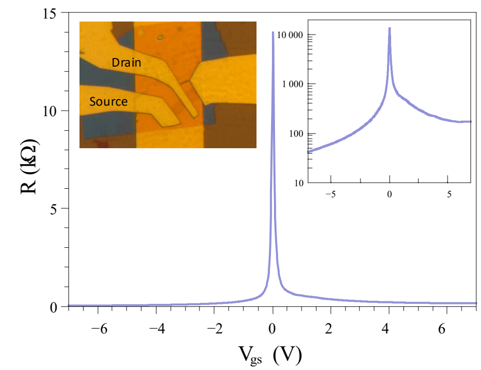


noise

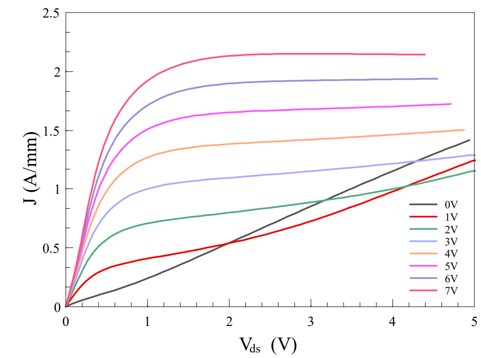


Outline

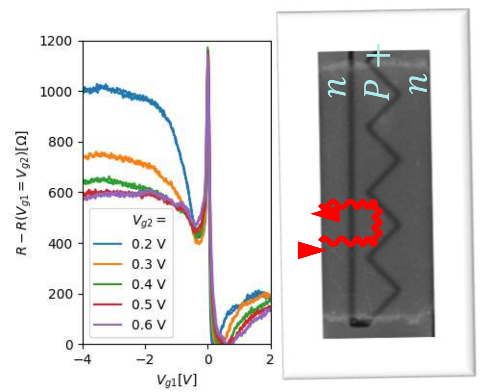
I. Low-field : from DC to high frequency
- Field-effect, density of states, conductivity,
- Scattering, mean free-path and mobility
- Quantum capacitance and Kinetic inductance



II. High-Field :
- Current saturation by optical phonon scattering
- Hot electrons effects and phonon relaxation



III. Ballistic's
- Landauer conductance and shot noise
- Klein tunneling across p-n junctions
- Dirac Fermion optics devices



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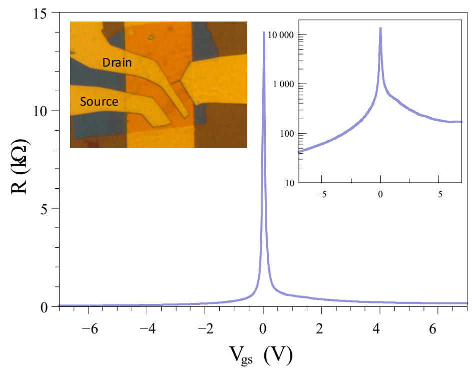

II. High-Field :
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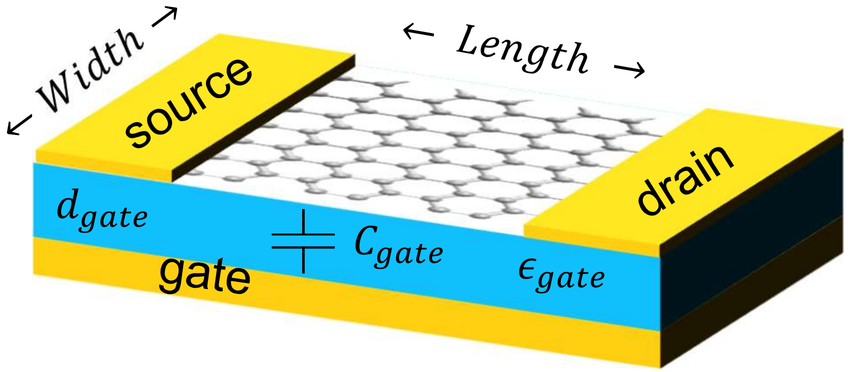
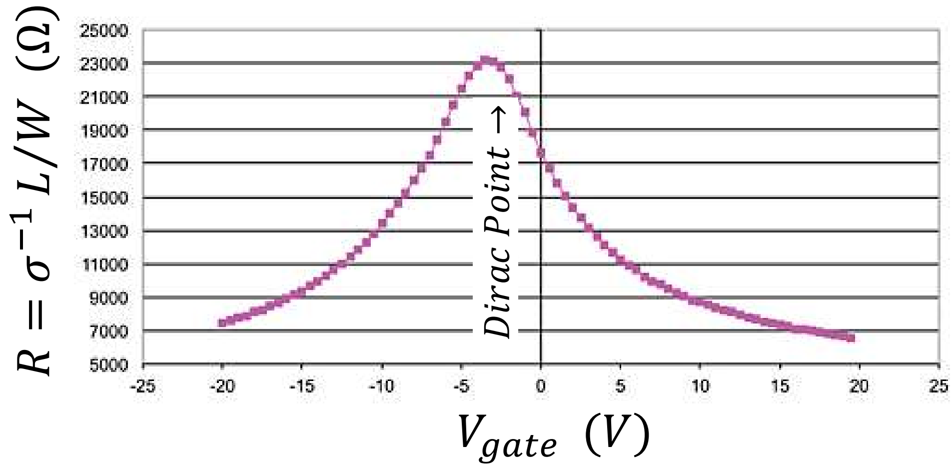
A sample with drain, source and gate





Gate capacitance:

$$ne = C_{gate}(V_{gate} - V_{DP}) \ ; \ C_{gate} = \frac{\epsilon_0 \epsilon_{gate}}{d_{gate}} \qquad 0.03 \to 3 \ mF/m^2$$

Doping range :

$$n_{max} = \frac{\epsilon_0 \epsilon_{gate}}{e} \times \frac{V_{max}}{d_{gate}} \sim 10^{13} cm^{-2} \qquad n_{12} \equiv 10^{12} cm^{-2}$$

$$n_{min} = Max\{10^{-4} n_{max}, \delta n_{puddles}\} \sim 10^{10} cm^{-2}$$

Electronic energy/momentum scales : $v_F = 10^6 m/s$

$k_F = \sqrt{\pi n}$ colors of Dirac Fermion light $\lambda_F = 10, 30, 100 \ nm$

$\varepsilon_F = \mp \hbar v_F k_F = \mp \hbar v_F \sqrt{\pi n}$ $\varepsilon_F(n_{12}) = 115 \ meV \ [-350, 350]$

Doping resolution $\delta \varepsilon_F \approx 0.35 \ meV \leq kT$

Glossary of scattering parameters

GDR-I GRAPHENE &CO GDR GRAPHENE &CO

<u>Field effect mobility</u> $(V_{drift} \equiv \mu E)$ — $\mu(10^{12}) = 0.1 \rightarrow 100\ m^2/Vs$

$J = neV_{drift} = ne\mu E_{ds} = \mu C_{gate} E_{ds}$ — $C_{gate} = 0.03 \rightarrow 3\ mF/m^2$

$\sigma = ne\mu$ — $\sigma(10^{12}) = 0.16 \rightarrow 160\ mS$

<u>Einstein relation, compressibility</u>

$\sigma = e^2\, \partial n/\partial \varepsilon_F \times D(\varepsilon_F)$

$\sigma \equiv C_Q(\varepsilon_F) \times D(\varepsilon_F)$

<u>Quantum capacitance (spectroscopy)</u>

$C_Q(\varepsilon_F) = e^2\, 2\varepsilon_F/\pi\hbar^2 v_F^2$ — $C_Q(10^{11}) = 2.75\ mF/m^2$

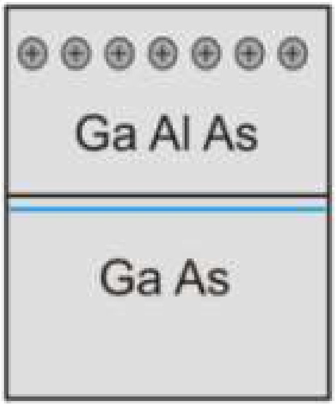
<u>Diffusion constant (spectroscopy)</u>

$D(\varepsilon_F) = \sigma/C_Q$ — $D(10^{12}) = 0.5 \rightarrow 5\ m^2/s$

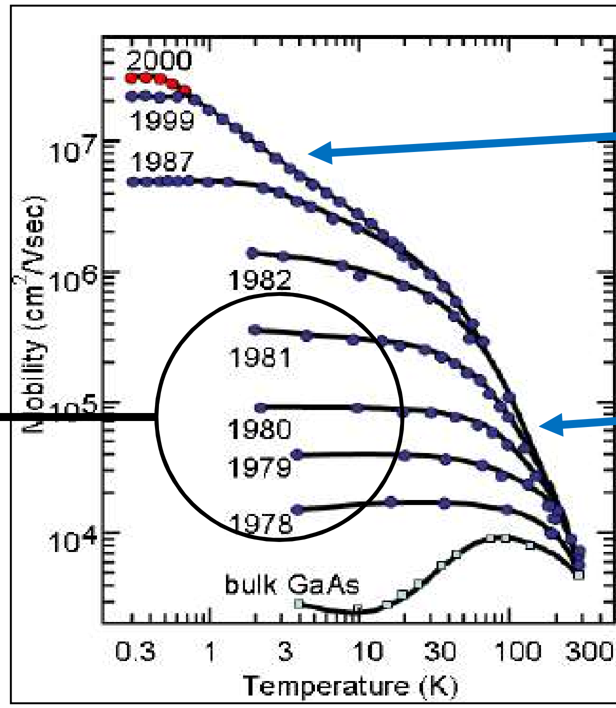
<u>Mean-free-path</u>

$l_{mfp} \equiv V_F \tau_{mfp} = \mu \frac{\varepsilon_F}{eV_F}$; — $l_{mfp}(10^{12}) = 0.01 \rightarrow 10\ \mu m$

Temperature dependence of mobility



$n \sim 10^{11}\ cm^{-2}$



Acoustic phonons

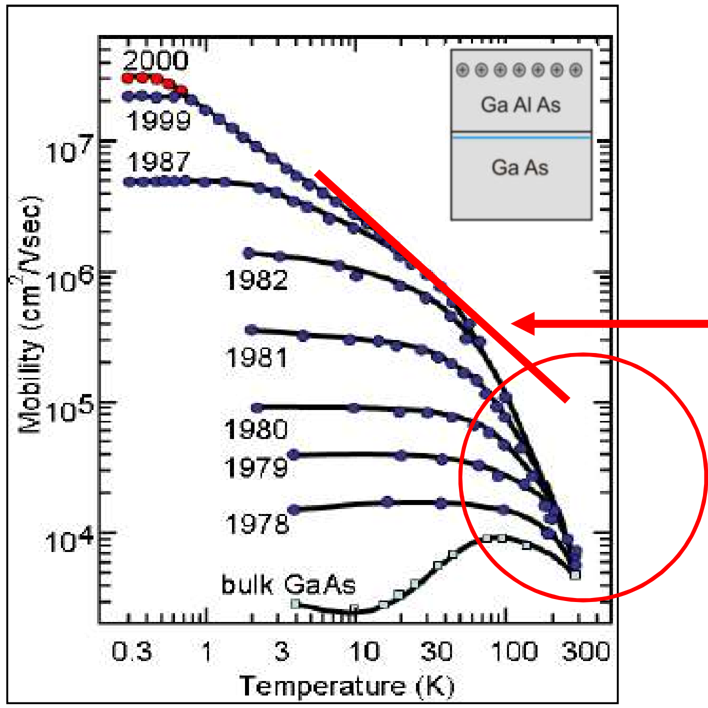
Impurity plateau

Optical phonons

Progress in mobility have been slow !

L. Pjeiffer et al. PRL2003

Intrinsic graphene mobility

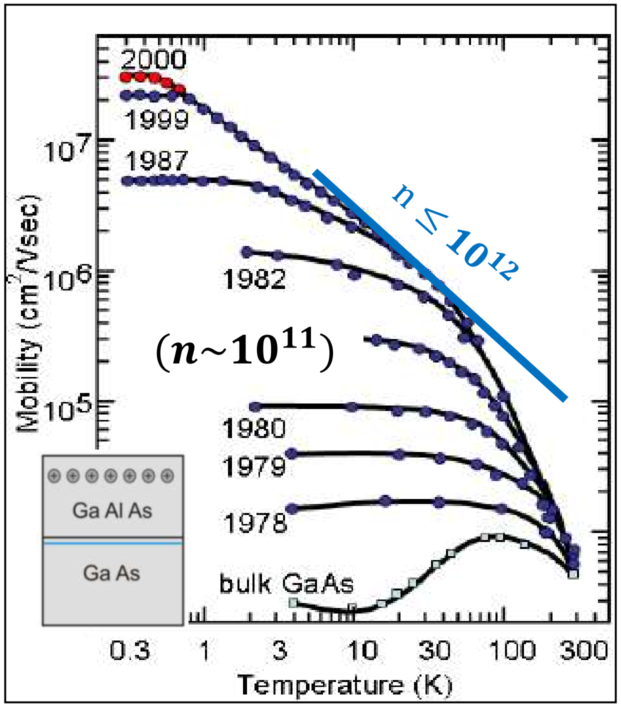


Acoustic phonons limit in graphene ($n \sim 10^{12}\ cm^{-2}$)

optical phonons elusive below 2000K

Graphene mobility outperforms GaAs 2DEGs ar 300K !!!

Graphene mobility pride



Graphene mobility does outperforms GaAs 2DEGs !

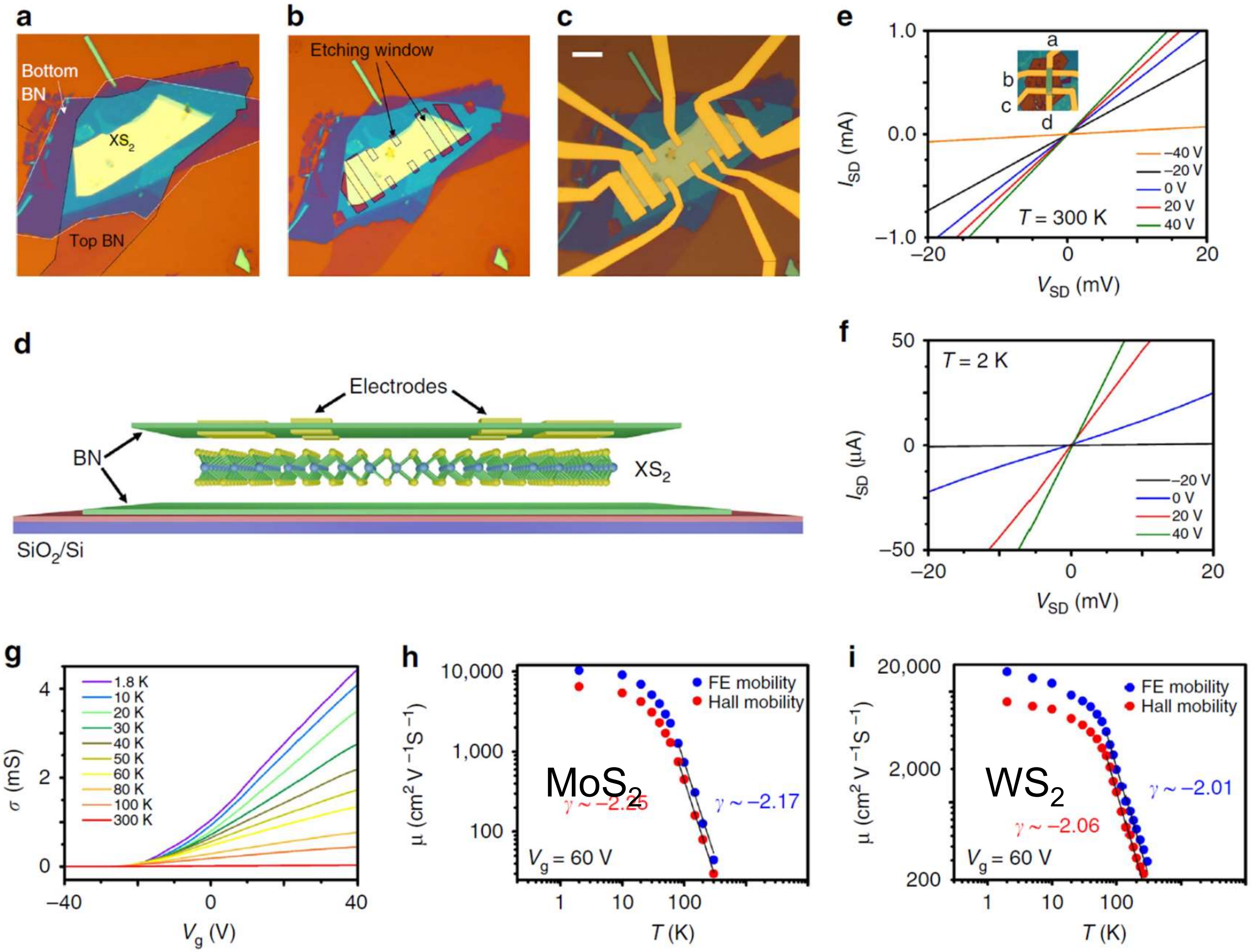
"Who wants to win MILLIONS ?"

(The true reason why we express mobility in cm^2/Vs)

(2012) ………… **(2018)**

μ_{12} (cm^2/Vs)	**100**	**1000**	**10 000**	**100 000**	**1 000 000**
quality	bad	fair	good	very good	excellent
examples		G/SiO2	G/Al2O3 G/BN	BN/G/BN 300K	BN/G/BN 3K
applications	Optics (joke !)	Sensors	High-field & Opto	High frequency	Ballistics

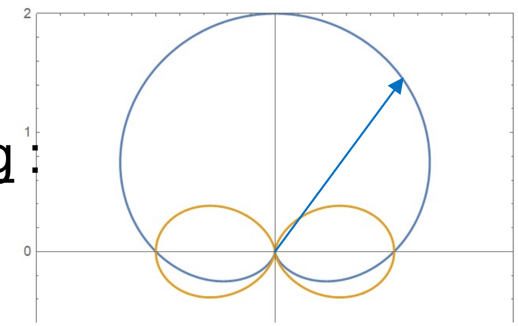
………. &Co : TMD mobility not as large



Z. Wu, …. N. Wang, Nat Comm (2016) : Even–odd layer-dependent magnetotransport of high-mobility Q-valley electrons in transition metal disulfides ;
Morva et al., PRL 2017 : WSe2, SLG/BLG, …/…

impurity scattering is fascinating ....

Resistance is backscattering $\{k_F, \sigma\} \rightarrow \{-k_F, -\sigma\}$



<u>Pseudospin momentum locking suppresses backscattering</u> :

$$\frac{1}{\tau_{tr.}} = \int (1-\cos\theta)(1+\cos\theta)V_q(\theta)d\theta \propto V_q(\pi/2)$$

Scattering pocket shrinks and Dirac Fermion have to take the U-turn !

<u>Spinor scattering have more (tunable) chanels</u> :

$$H_K = \hbar v_F \sigma \cdot q \quad + \underbrace{V(q)\hat{I}}_{\text{scalar}} \quad + \underbrace{\alpha\sigma\cdot U}_{\text{gauge-field}} \quad + \underbrace{\delta m^* \sigma_z}_{\text{Dirac-mass}}$$

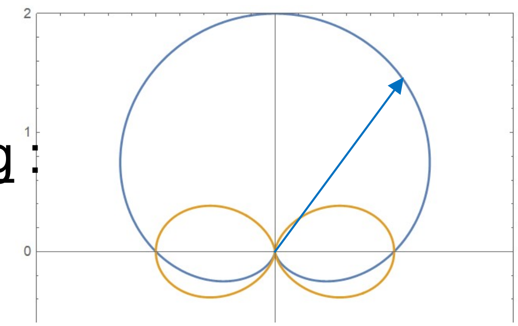
$$\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

mechanisms	*scattering time*	*conductivity*
local impurity	$\tau \sim 1/k_F$	$\sigma \sim Const$
local impurity	$\tau \sim \ln k_F/k_F$	$\sigma \sim \ln n_c$
random Dirac-mass	$\tau \sim Const$	$\sigma \sim \sqrt{n_c}$
charged impurity	$\tau \sim k_F$	$\sigma \sim n_c$
resonnant scattering	$\tau \sim k_F \ln^2(k_F)$	$\sigma \sim n_c \ln^2 n_c$
ripples	$\tau \sim k_F^{(2H-1)}$	$\sigma \sim n_c^H$

Graphene : E. Pallecchi et al., Phys. Rev. B (2011), H. Graef et al., in preparation

……….. but today less and less relevant

Resistance is backscattering $\{k_F, \sigma\} \rightarrow \{-k_F, -\sigma\}$



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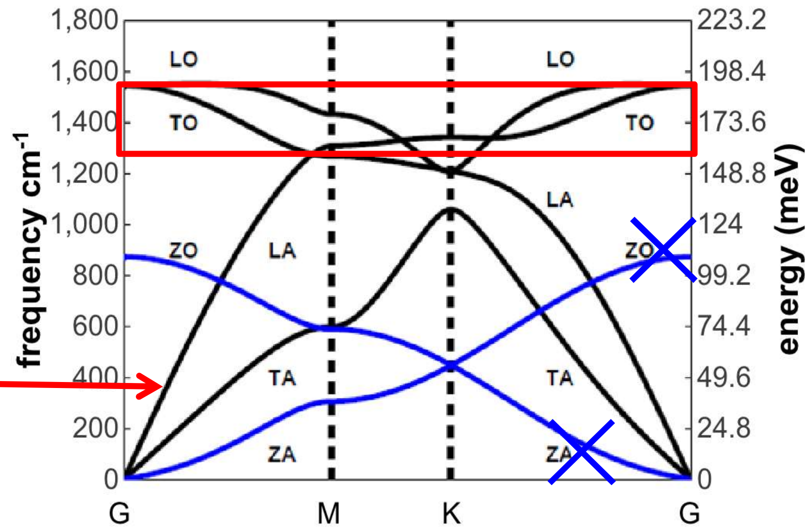
Cannot avoid phonons

large AC-phonons velocity

$s \approx 2\ 10^4\ m/s$



OP-energy band

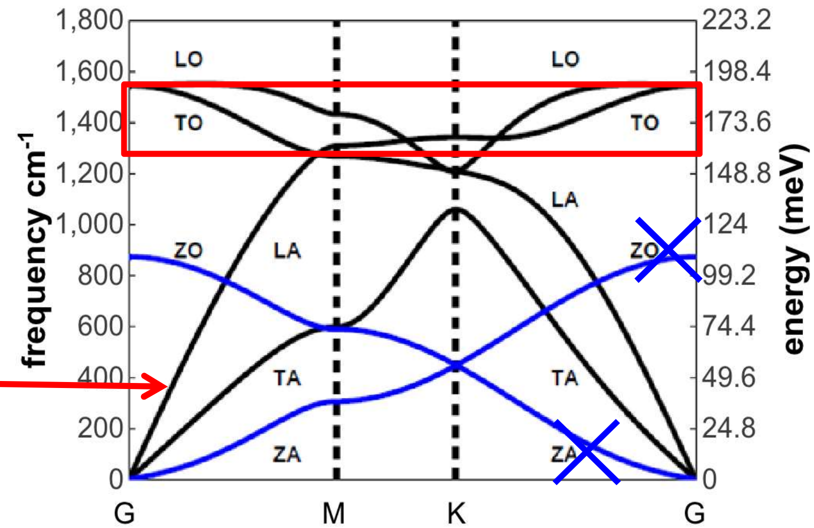
$\hbar\Omega_{OP} \approx 170 - 200\ meV$

$(\hbar\Omega_{GaAS} \sim 30\ meV)$

activated OP scattering

magic sp2 bonding !

Phonon resistivity



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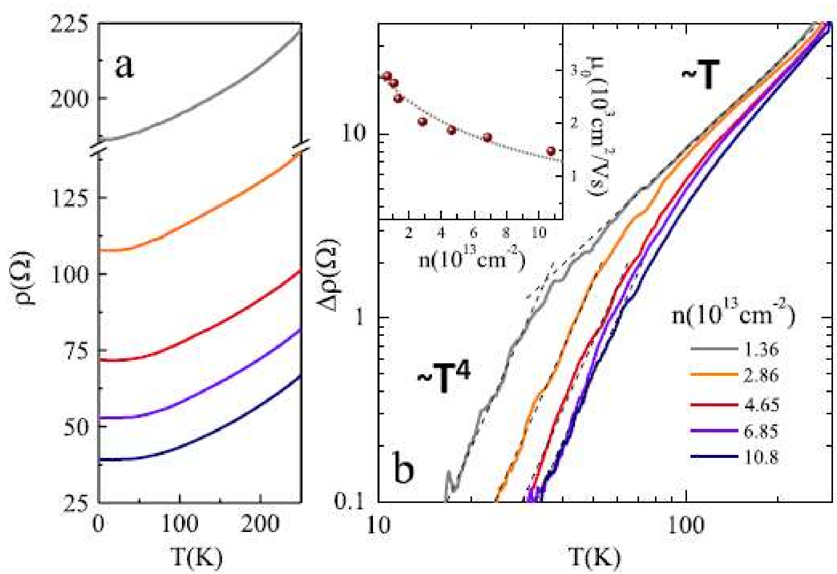


Efetov-Kim, PRL2010

$\rho = \rho_{imp} + \rho_{ph}(\mathrm{T})$

Doping dependent residual resistivity

Doping independent phonon resistivity

$\Delta\rho(T) = \rho_{ph}(\mathrm{T})$

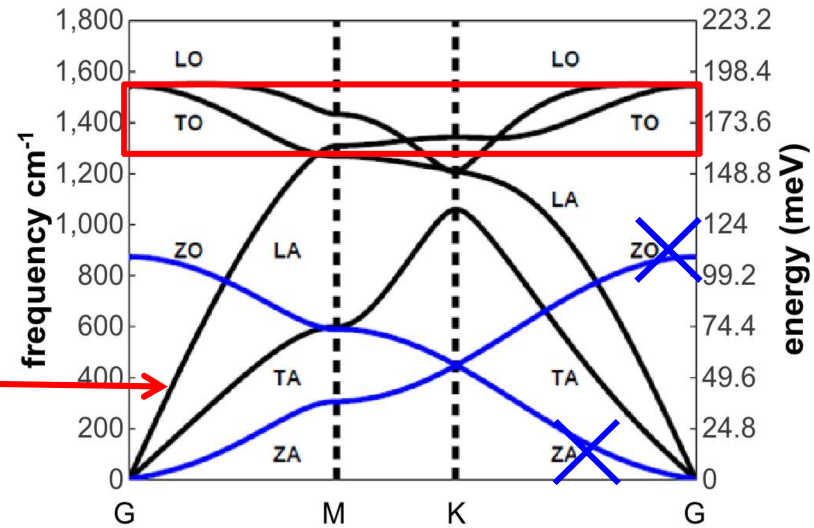
2D Bloch-Gruneisen T^4 regime

T-linear at high temperature

Phonon resistivity theory

large AC-phonons velocity

$s \approx 2\ 10^4\ m/s$



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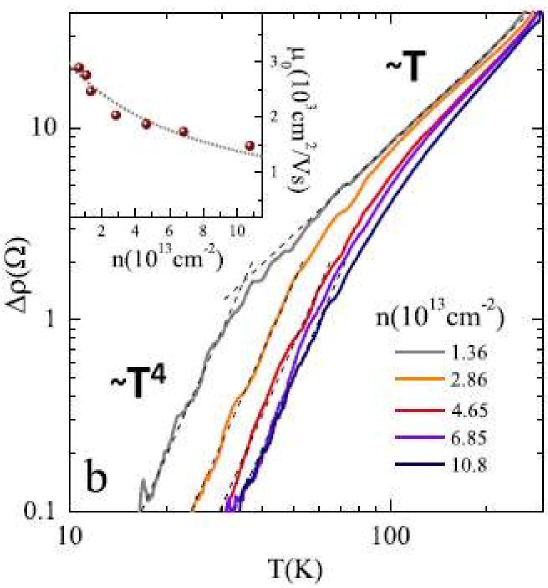


Efetov-Kim, PRL2010

Deformation potential coupling $D \sim 15eV$

Very small phonon resitivity

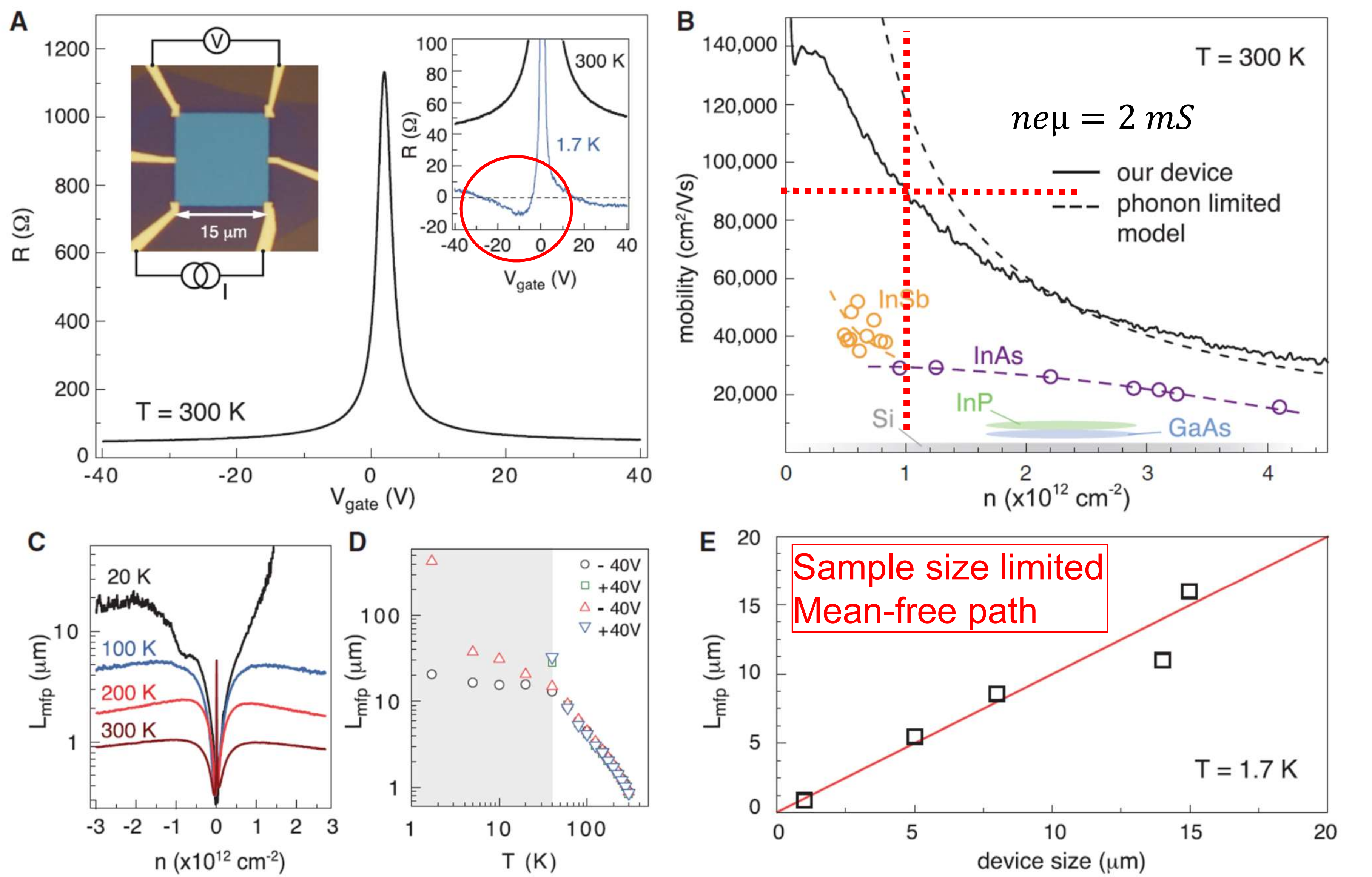
$$\rho_{ph}(n,T) = \frac{\pi}{4\hbar e^2 V_F{}^2} \frac{D^2 kT}{\rho_m s^2} \approx 0.1 \times T\ \ \Omega\ \ !!!!!!$$

Room temperature :

$\sigma_{ph}(n) \approx Cte$; $\mu_{ph}(n) \propto 1/n$; $l_{ph}(n) \propto 1/\sqrt{n}$

Hwang et al., PRB 2008

Today : hBN encapsulated graphene



Exfoliated : L. Wang et al., Science 342, 614 (2013) ; CVD+pick-up: L. Banszerus et al., Science Adv. 2015

High mobility needed for high frequency

<u>Diffusion constant</u>

$D(\varepsilon_F) = V_F{}^2 \tau_{mfp}/2$

$\mu(10^{12}) = 0.1 \rightarrow 10\ m^2/Vs$

$D(10^{12}) = 0.5 \rightarrow 5\ m^2/s$

$\tau = \frac{L^2}{D} \sim ps\ \ with\ L = 1\mu m$

High mobility needed for high bias (see Lecture-II)

High mobility needed for ballistic's (see Lecture-III)

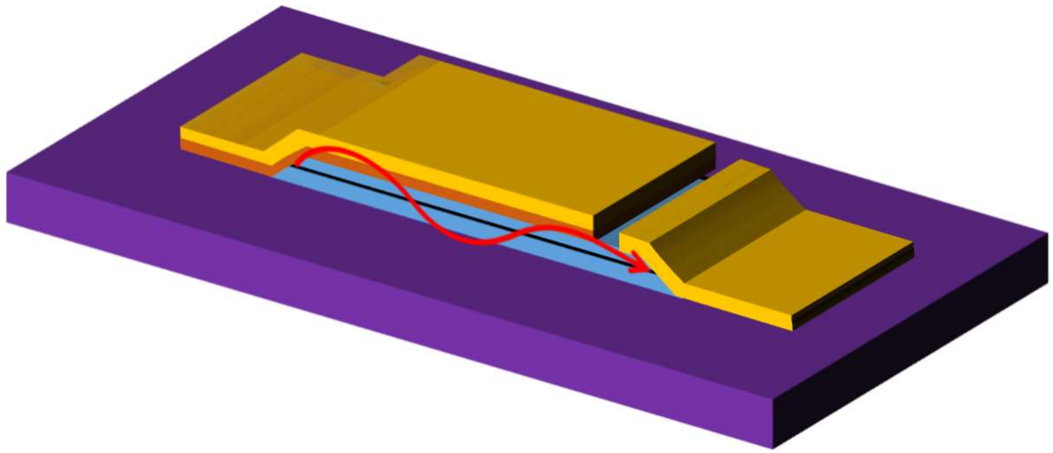
<u>Mean-free-path</u>

$l_{mfp}(10^{12}) = 1 \rightarrow 10\ \mu m$

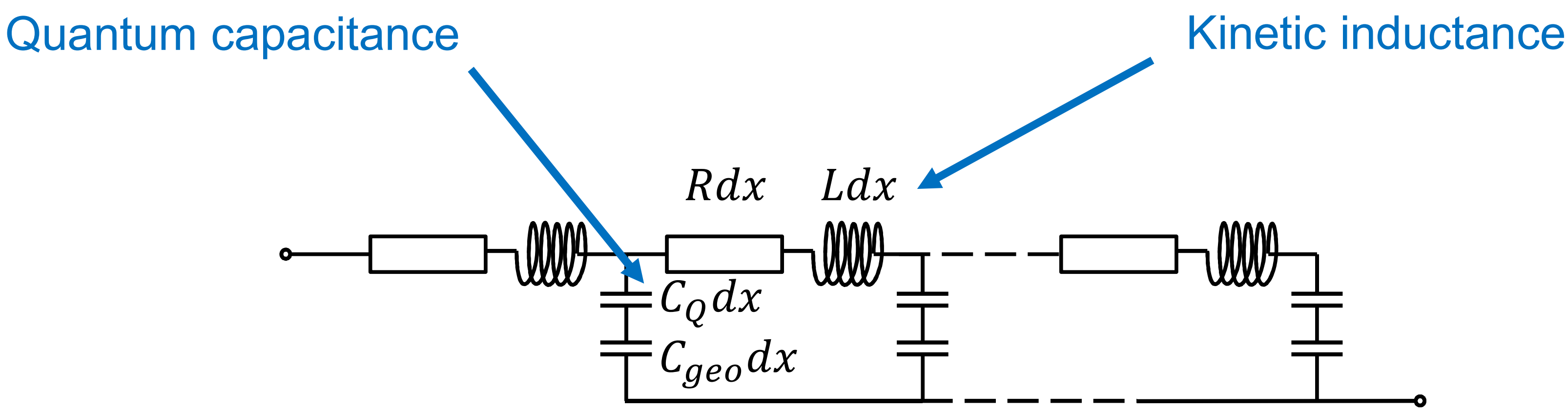
High mobility for quantum Hall effect (Lecture IV)
High mobility for electron quantum optics (Lecture V) ……

HF : capacitance and inductance

Wavelength on the order of sample length

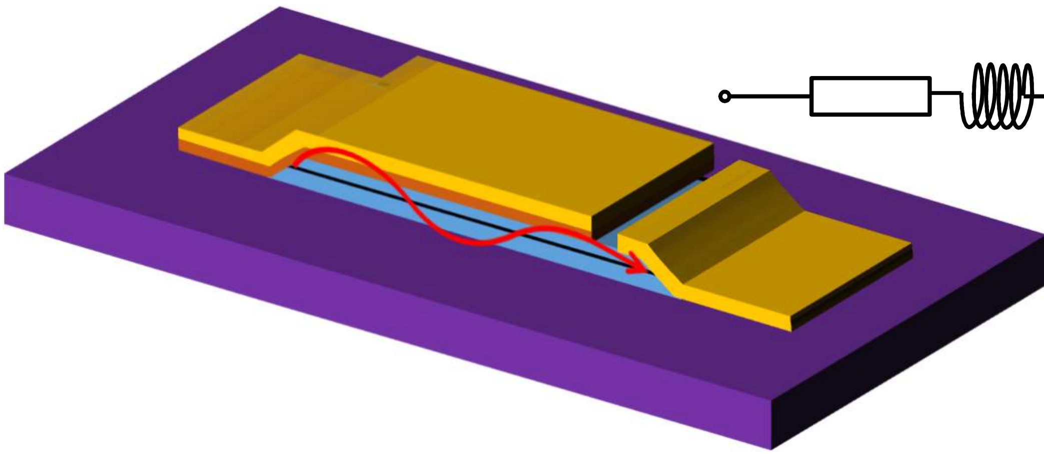


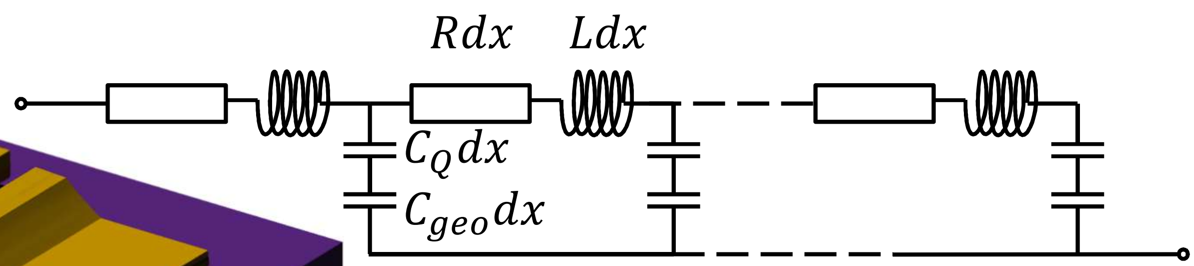
described as a propagation line with a lumped element description



High-frequency : graphene (new) wave

Finite compressibility renormalizes the gate capacitance

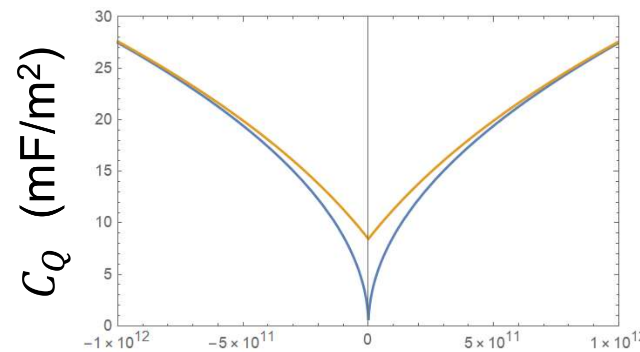




Quantum capacitance :

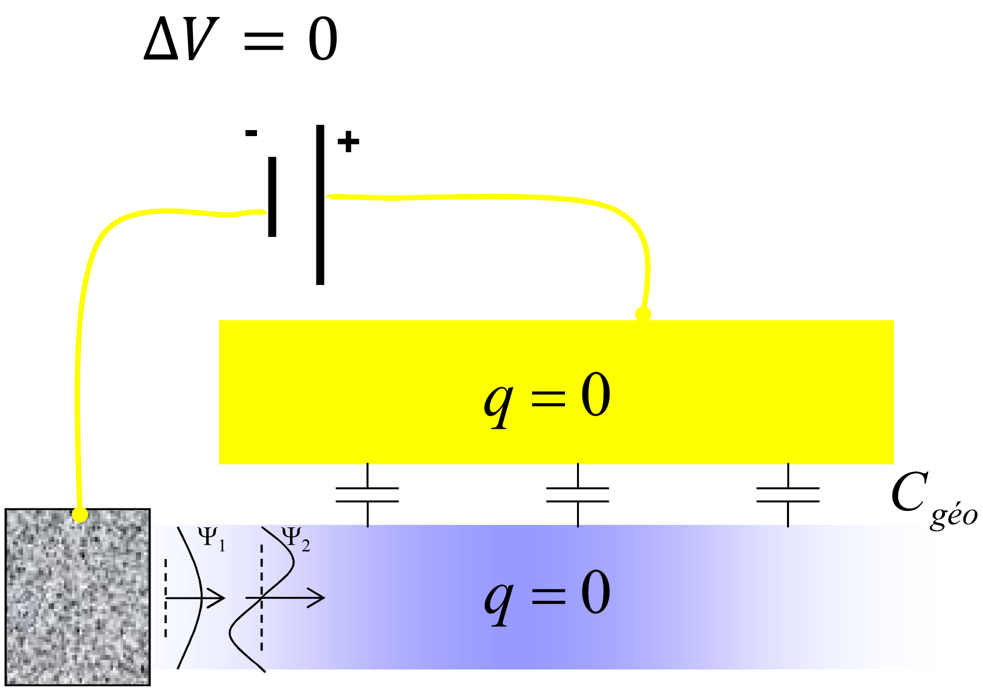
$$C_Q = e^2\, \partial n/\partial \varepsilon_F \rightarrow e^2 N(\varepsilon_F) = \frac{2e^2 \varepsilon_F}{\pi\, \hbar^2 {V_F}^2}$$

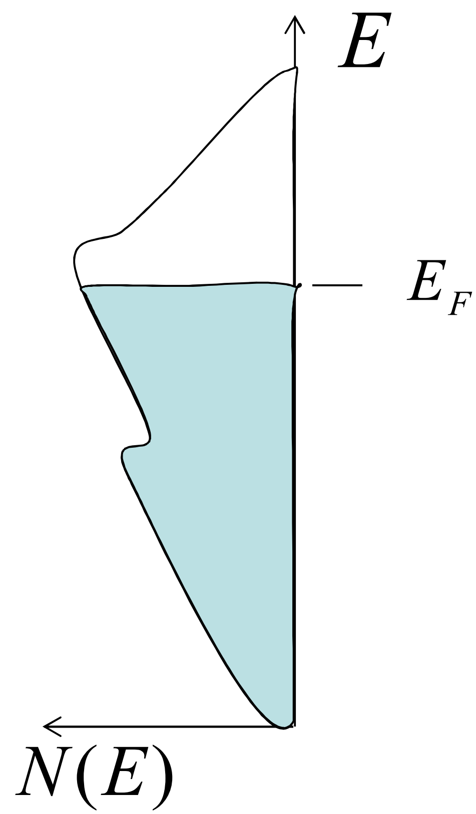
$$C_Q(T) = \frac{2e^2 k_B T}{\pi\, \hbar^2 {V_F}^2} \times ln\left[2 + 2\cosh\left(\frac{\varepsilon_F}{k_B T}\right)\right]$$



$C_{geo} \sim 3\ mF/m^2$

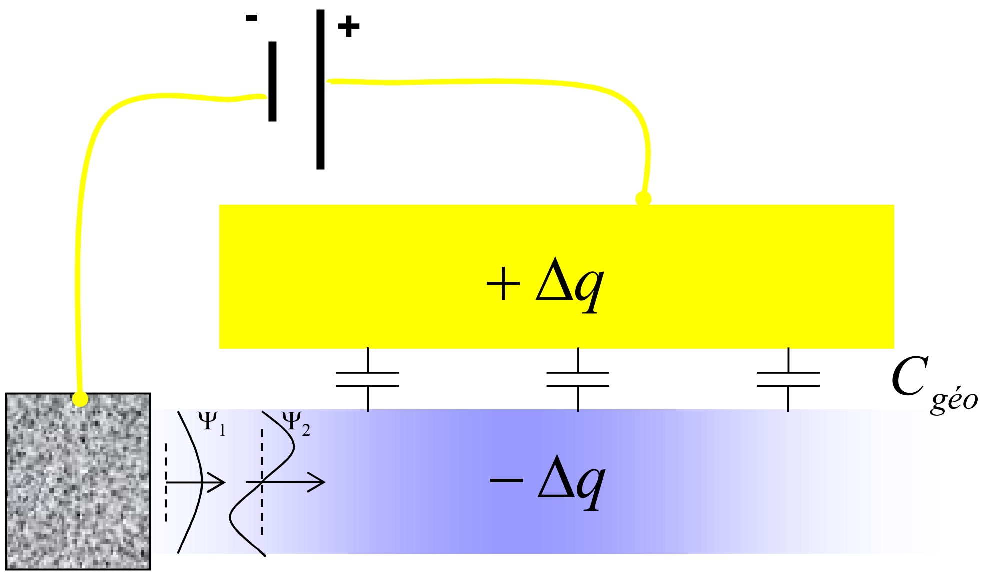
Q-capacitance or electronic compressibility

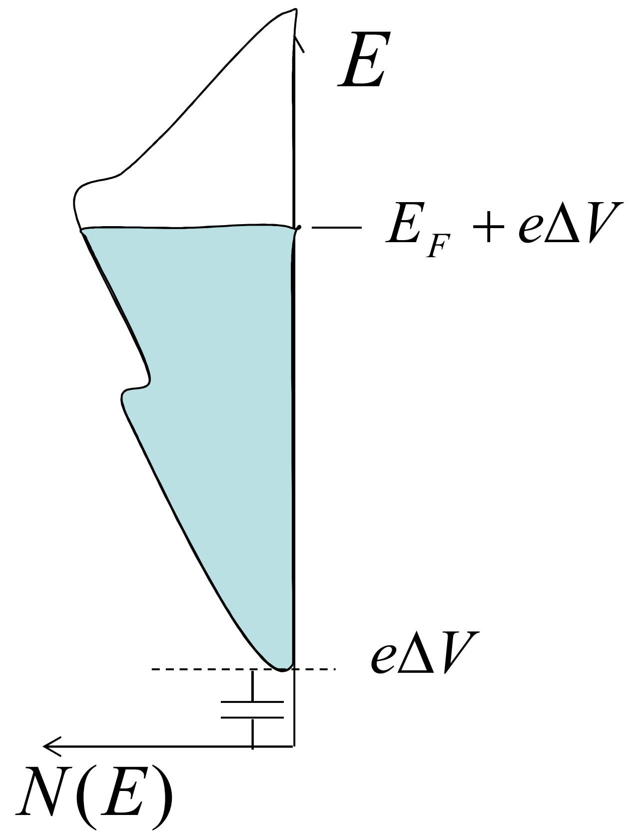




Q-capacitance or electronic compressibility

$$\Delta V = -\Delta\mu_F / e$$





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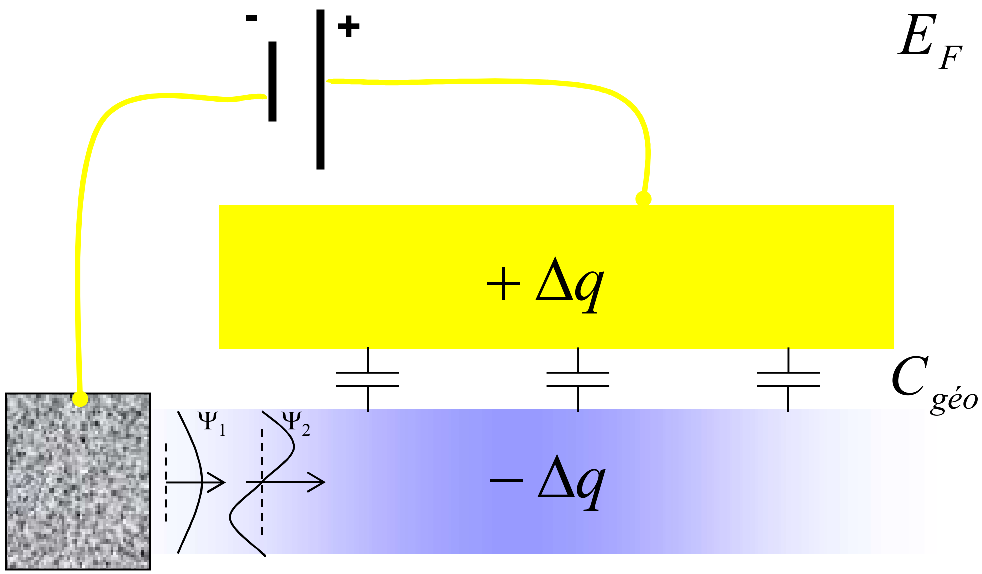
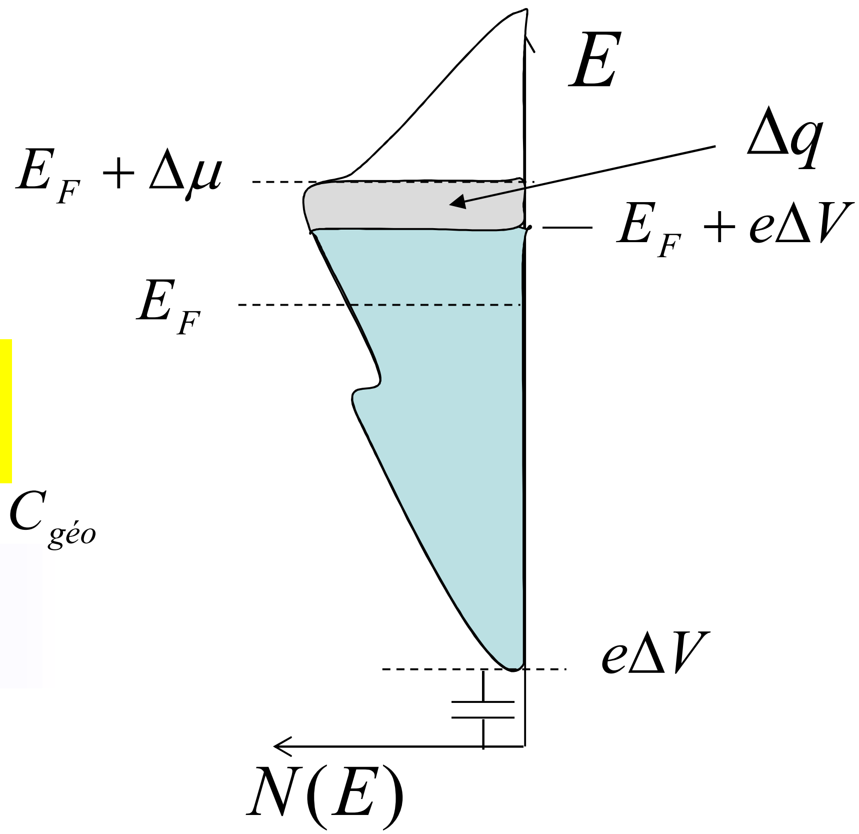
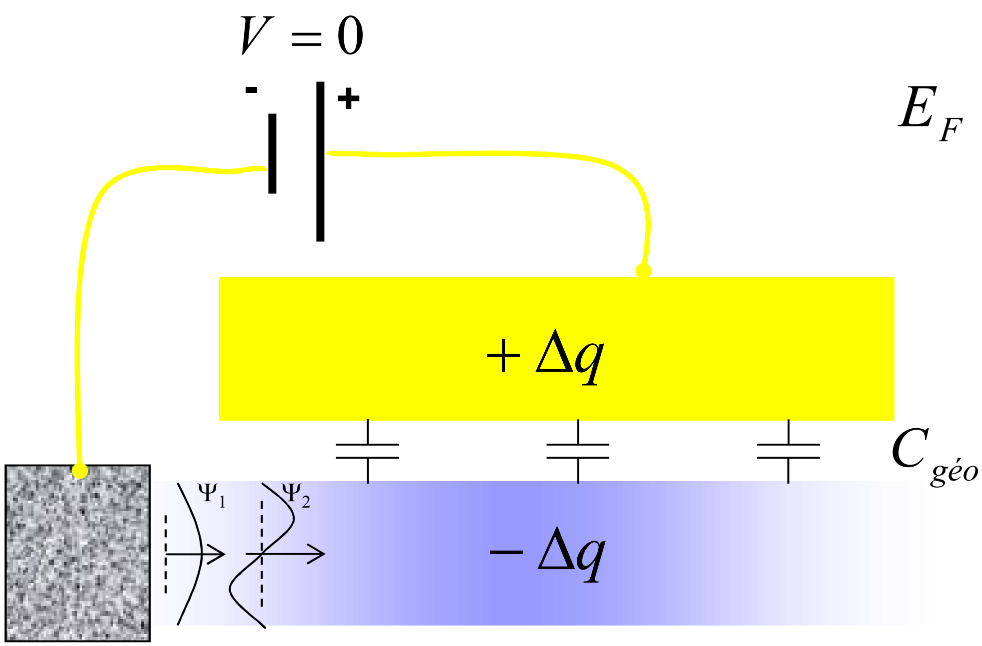
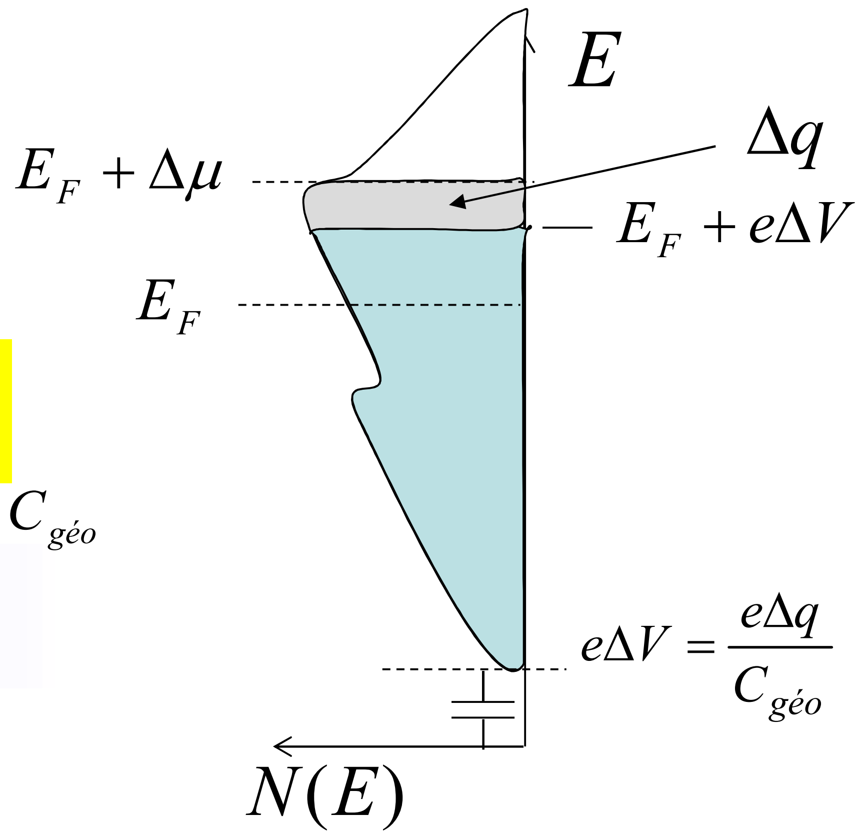




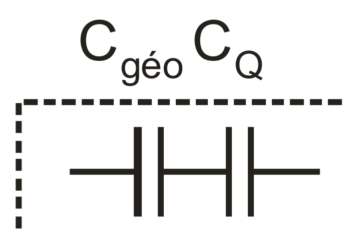
Quantum capacitance





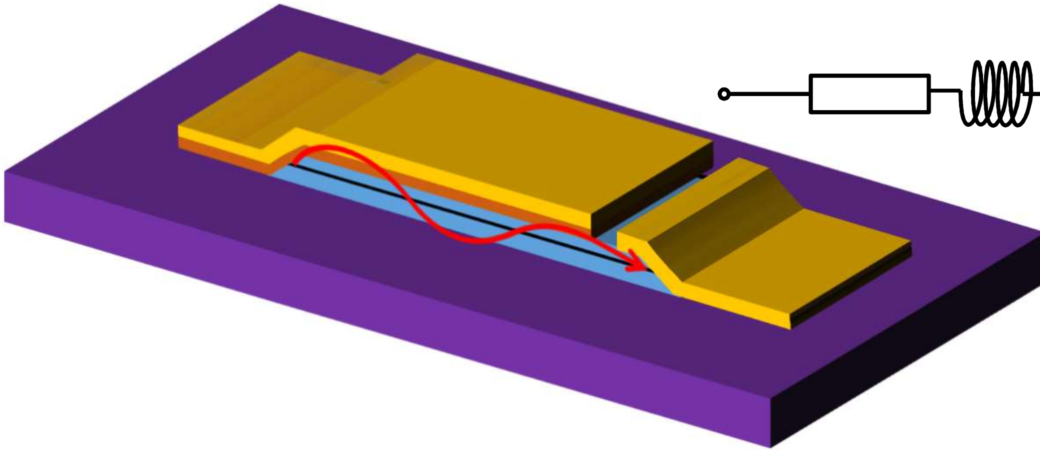
$$\frac{1}{C_\mu} = \frac{1}{C_{geo}} + \frac{1}{C_Q}$$

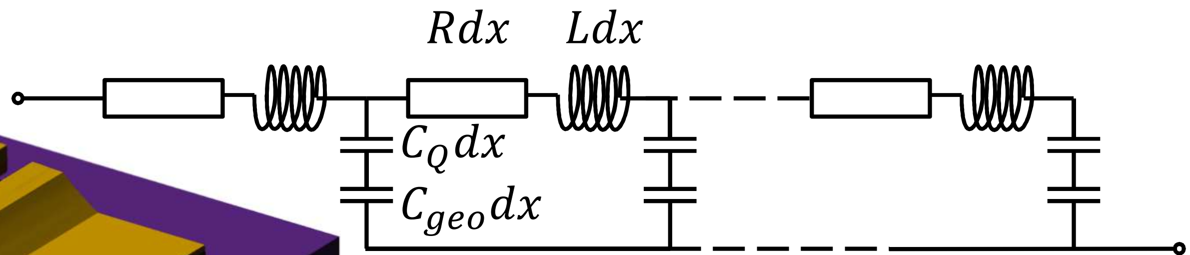
$$\Delta\mu_F = e\Delta V + \frac{\Delta q}{e}\frac{1}{N(E)} = e\Delta q\left(\frac{1}{C_{geo}} + \frac{1}{C_Q}\right)$$



High-frequency : graphene (new) wave

Graphene as a propagation line, a lumped element description with :



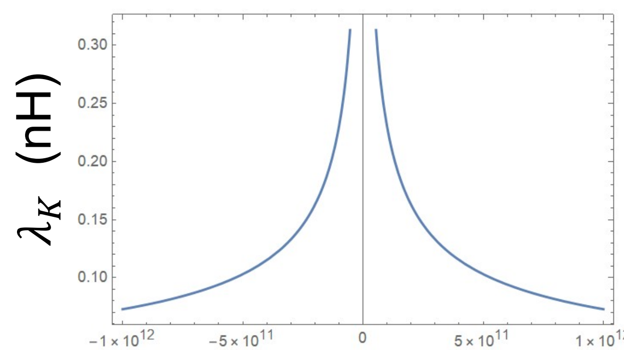


<u>Kinetic inductance</u> :

$$\rho_{Drude}(\omega) = \frac{1 + i\omega\tau_{scatt.}}{ne\mu} = \frac{1}{ne\mu} + i\omega\lambda_K$$

$$dU = \frac{1}{2}\, nmv_F^2 \equiv \frac{1}{2}\lambda_K J^2$$

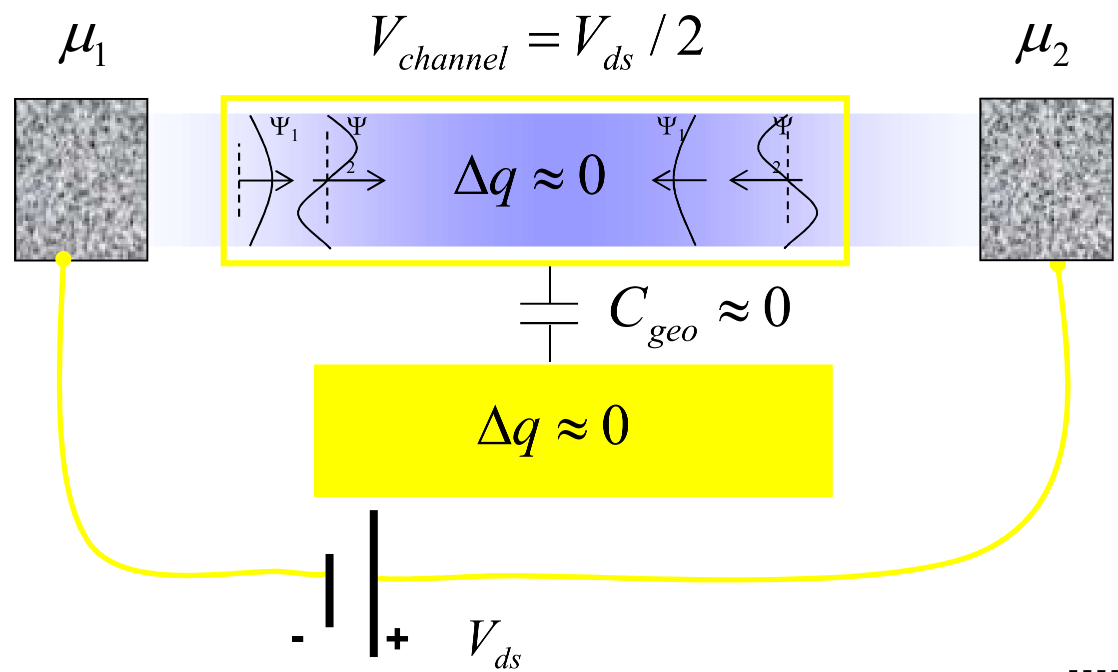
$$\lambda_K(\varepsilon_F) = \frac{\pi\,\hbar^2}{\varepsilon_F e^2} \quad \left\{\equiv \frac{m}{ne^2}\right\}$$

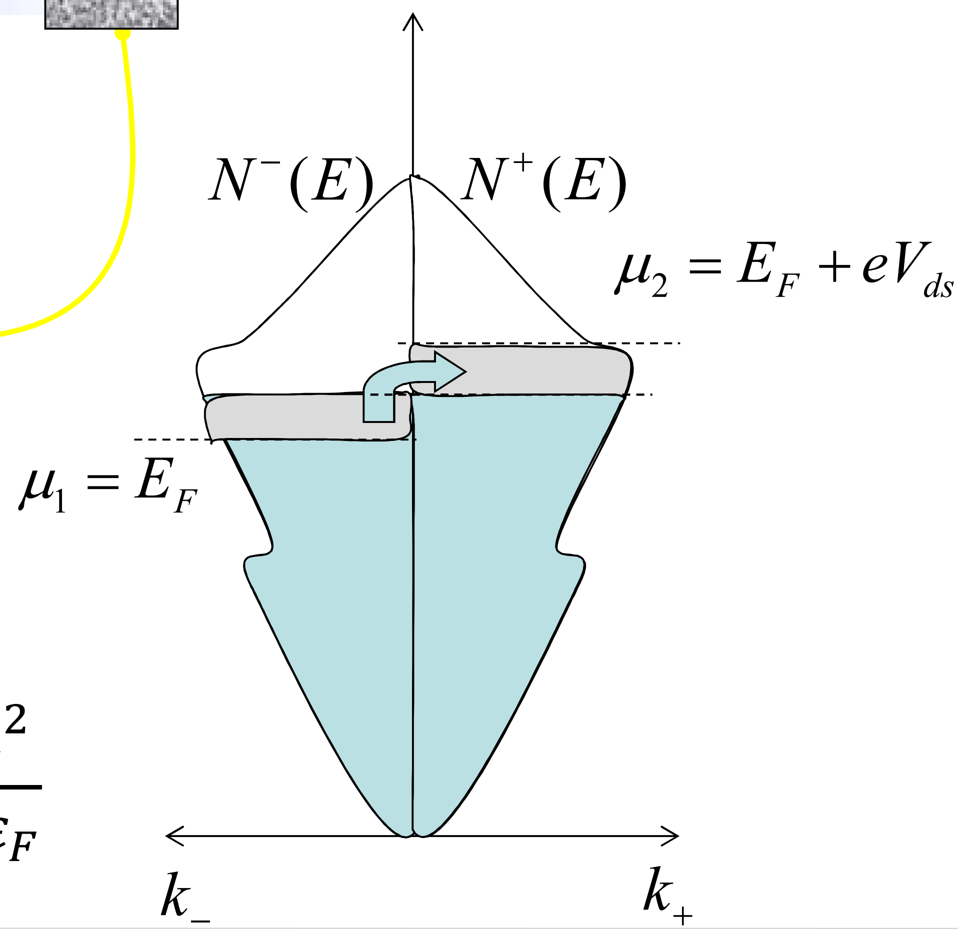


$$L_{geo} \sim \mu_0 W \sim 1\ pH$$

H. Yoon et al., Nature Nanotech 2014 : Measurement of collective dynamical mass of Dirac fermions in graphene

Kinetic inductance, a collective mass effect





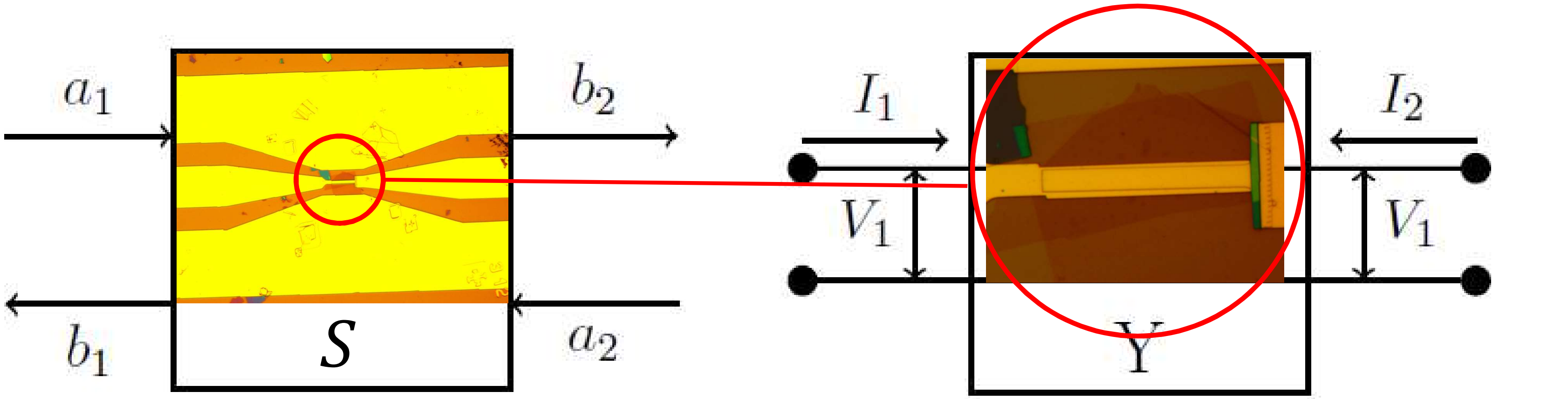
$$\Delta U = \left(N^+(\varepsilon_F) \times \frac{eV}{2}\right) \times \frac{eV}{2} = \frac{1}{2} L_K I^2$$

$$V = I / G_{Landauer} \quad \Rightarrow \quad \lambda_K = \frac{\pi \hbar^2}{e^2 \varepsilon_F}$$

High frequency techniques

Whenever wave length is smaller than setup size

Scattering of μ-waves extract RF admittance Y



$$Y_{11} = Y_0 \frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$$

$$Y_{12} = Y_0 \frac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$$

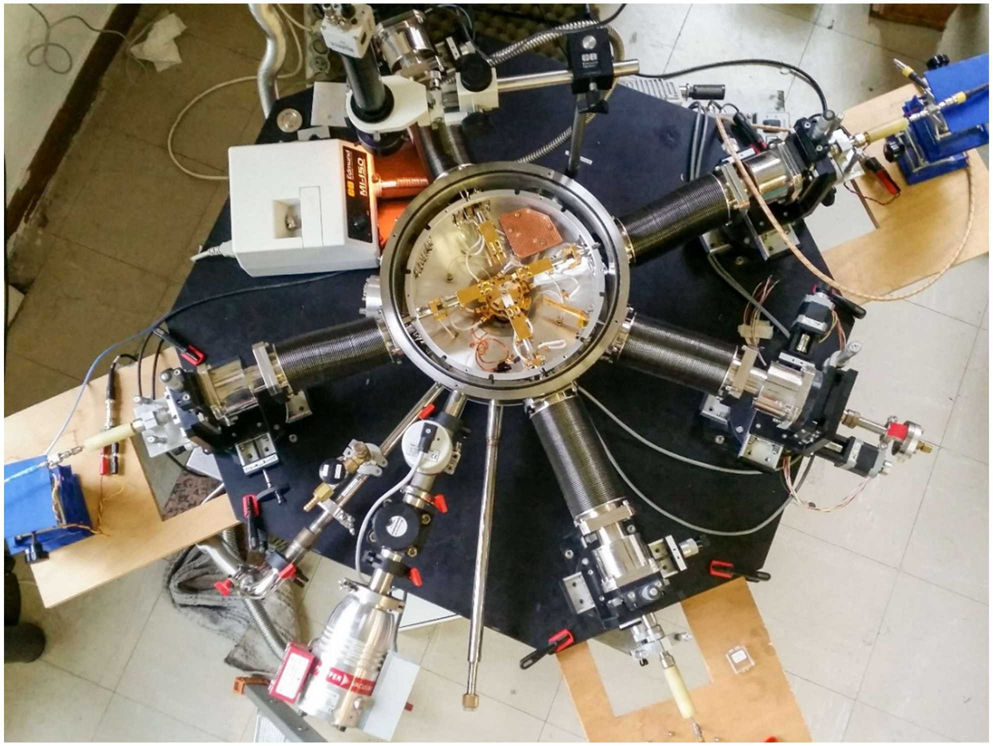
$$Y_{21} = Y_0 \frac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$$

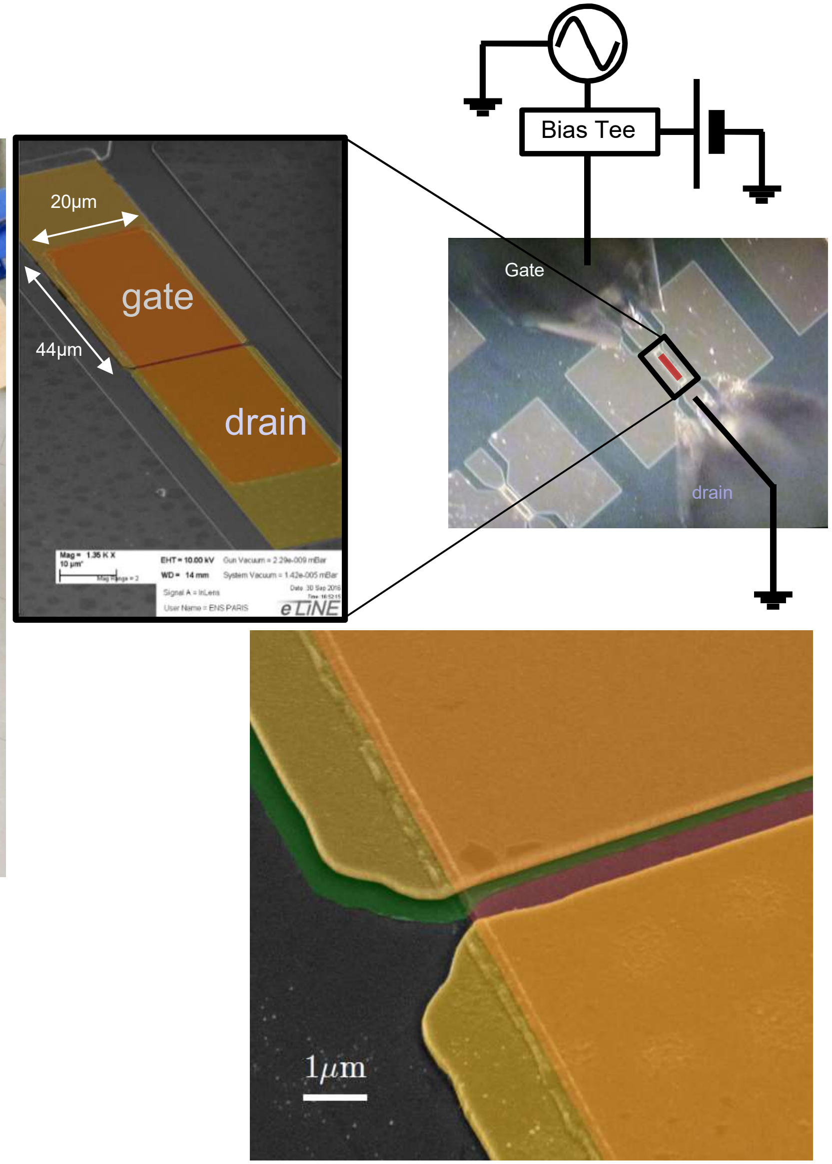
$$Y_{22} = Y_0 \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$$

D.M. Pozar, Microwave engineering, Wiley, 3rd edition (2005)

Please dont repeat that it is easy, otherwise everyone will do it ....

High frequency set-up

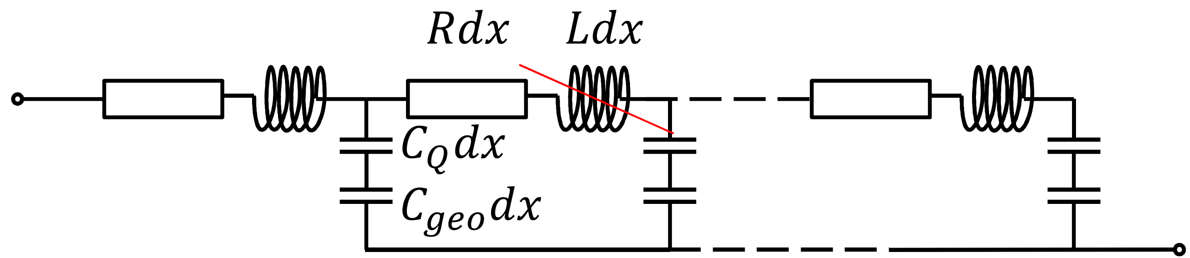


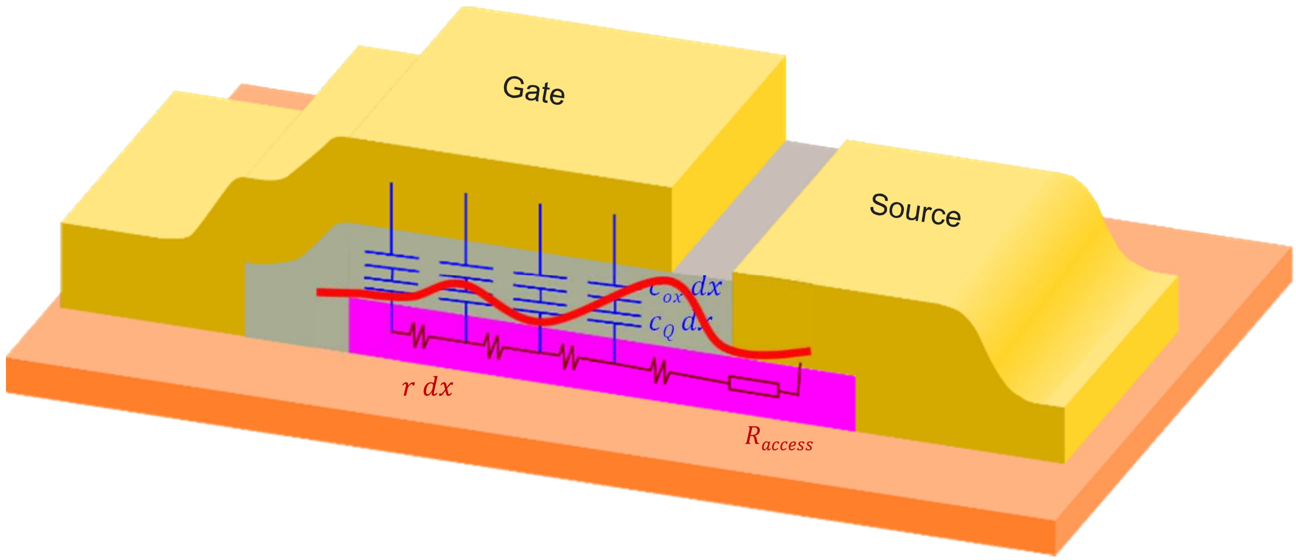


Cryogenic RF probe station:

T = 10 – 400 K
f = DC – 40 GHz
In-situ calibration

Diffusive evanescent wave $R \gg L\omega$

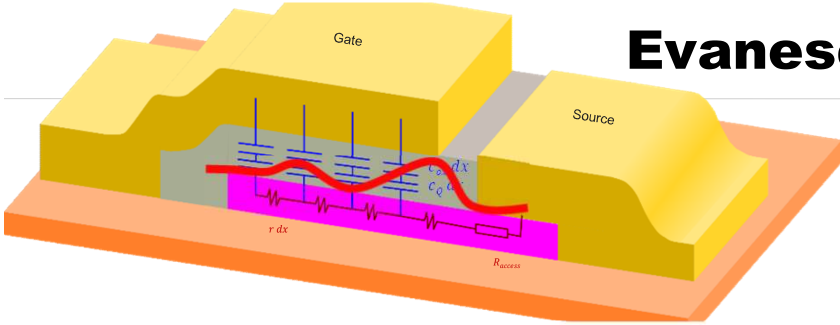




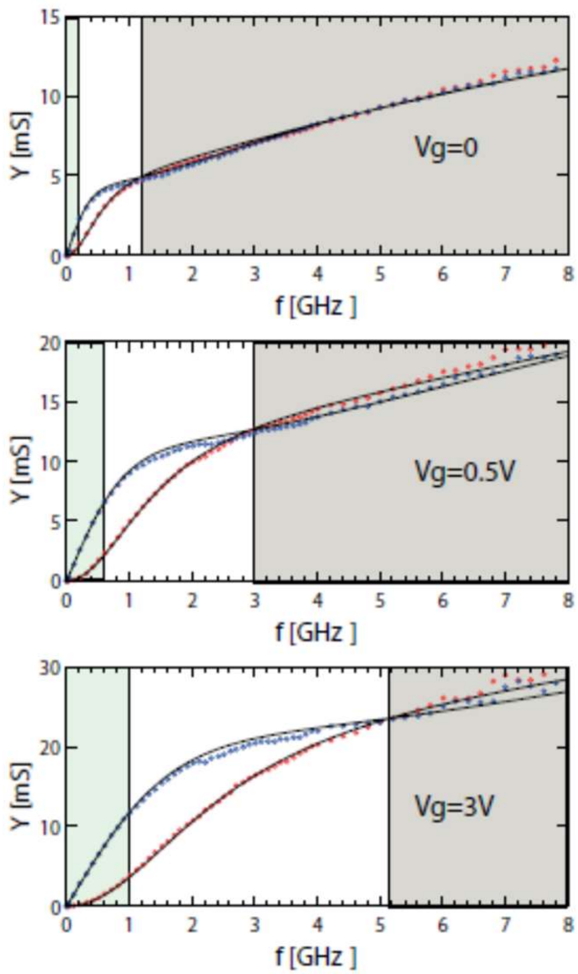
$$Y = j\omega C L W \times \frac{\tanh\left(L\sqrt{jC\omega/\sigma}\right)}{L\sqrt{jC\omega/\sigma}}$$

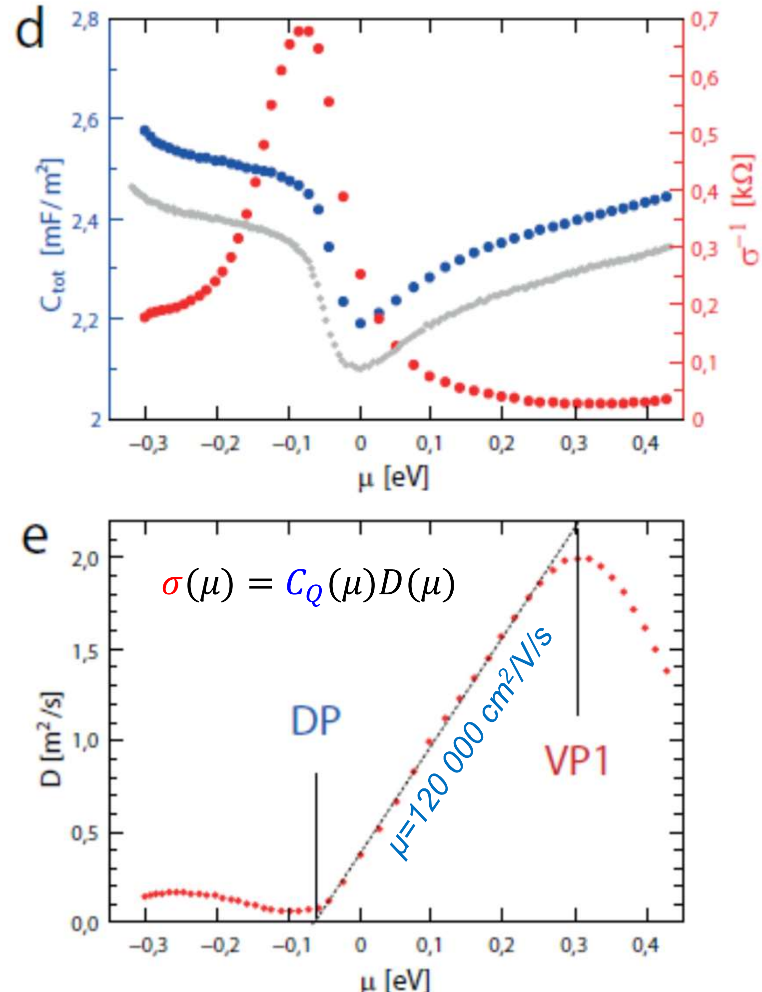
Topological Insulators : A. Inhoher et al., Phys. Rev. B (2017), Phys. Rev. Applied (2018)

Evanescent wave spectrum



$$Y = j\omega C L W \times \frac{\tanh\left(L\sqrt{jC\omega/\sigma}\right)}{L\sqrt{jC\omega/\sigma}}$$



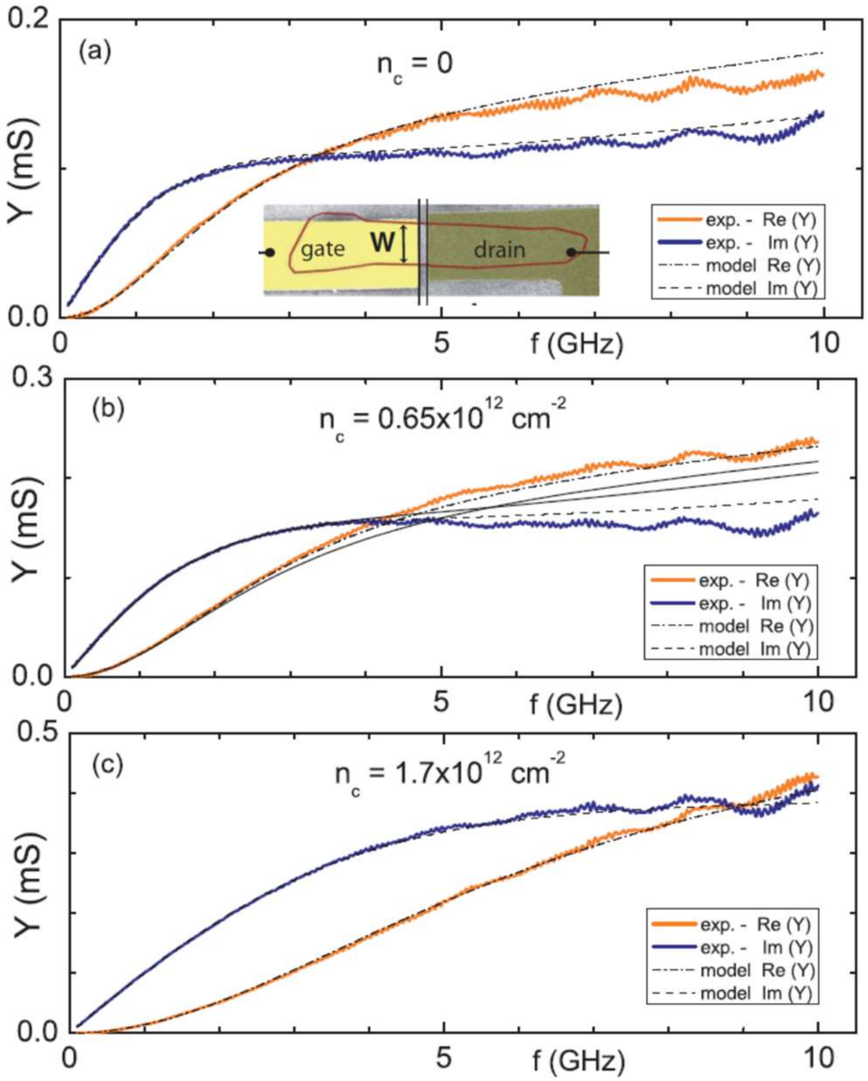


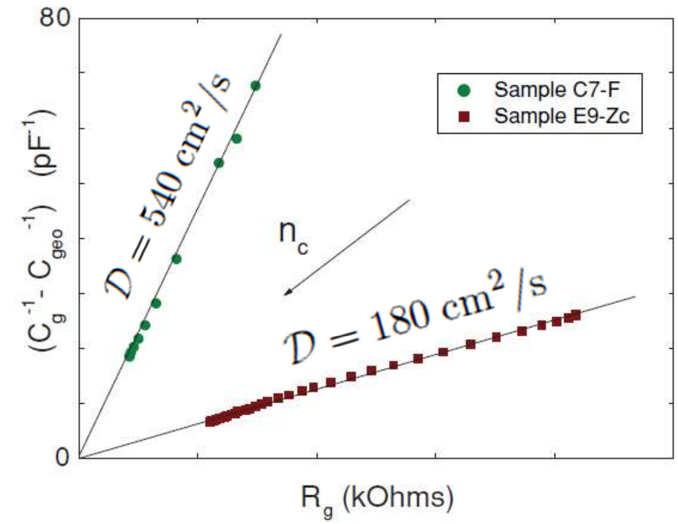
HgTe : A. Inhoher et al., Phys. Rev. B (2017), Phys. Rev. Applied (2018)

Ex.: mass disorder in graphene

Einstein : $\sigma(\varepsilon_F) = C_F(\varepsilon_F) D(\varepsilon_F)$

Diffusion constant is energy indep.





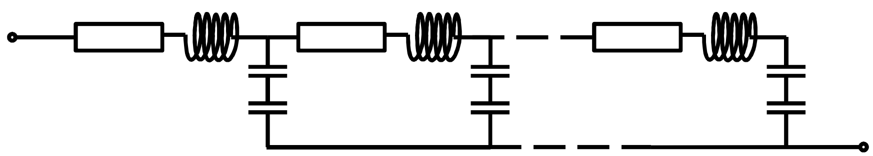
mechanisms	*scattering time*	*conductivity*
local impurity	$\tau \sim 1/k_F$	$\sigma \sim Const$
local impurity	$\tau \sim \ln k_F / k_F$	$\sigma \sim \ln n_c$
random Dirac-mass	$\tau \sim Const$	$\sigma \sim \sqrt{n_c}$
charged impurity	$\tau \sim k_F$	$\sigma \sim n_c$
resonnant scattering	$\tau \sim k_F \ln^2(k_F)$	$\sigma \sim n_c \ln^2 n_c$
ripples	$\tau \sim k_F^{(2H-1)}$	$\sigma \sim n_c^H$
acoustic phonons	$\tau \sim k_F^2$	$\sigma \sim n_c^{3/2}$

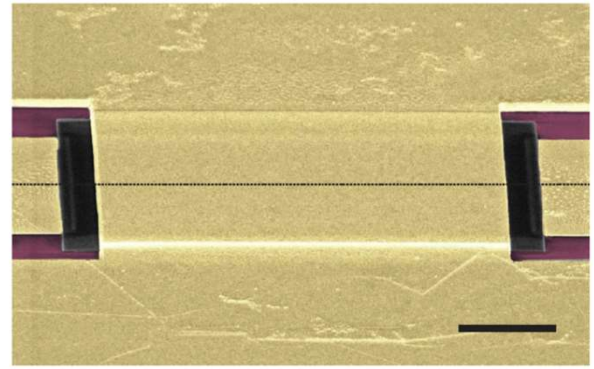
Exp.: E. Pallecchi et al., PRB 2011;

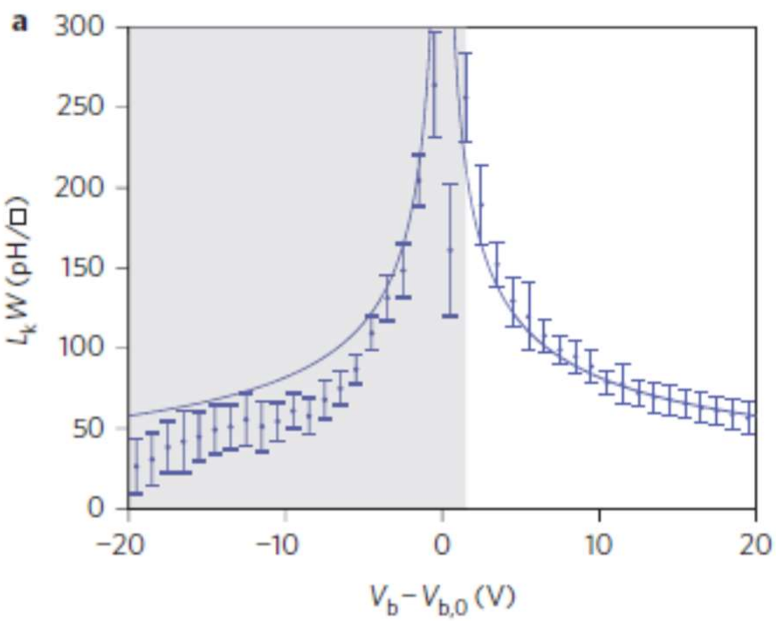
Theory : K. Ziegler et al., PRL 2006

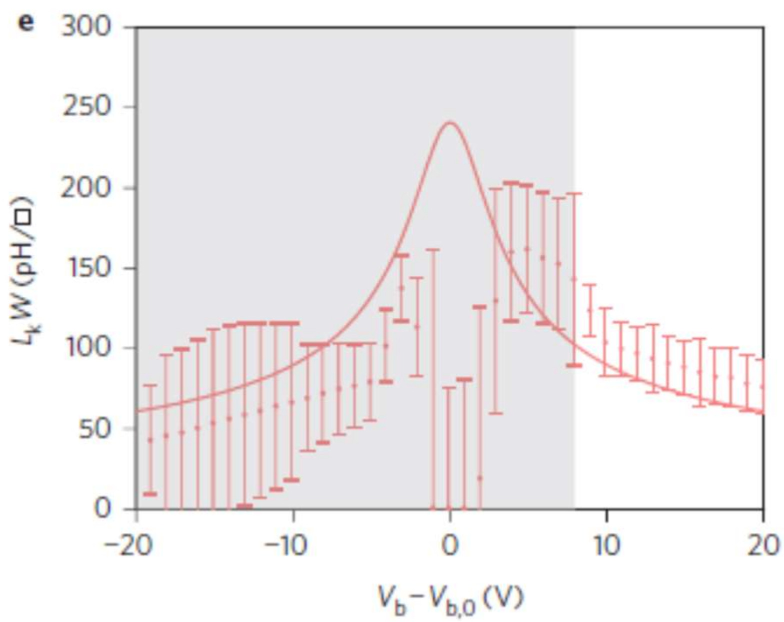
Kinetic inductance

Kinetic inductance: $\lambda_K = L_K W = \frac{\pi \hbar^2}{\varepsilon_F e^2} \equiv \frac{m}{ne^2}$







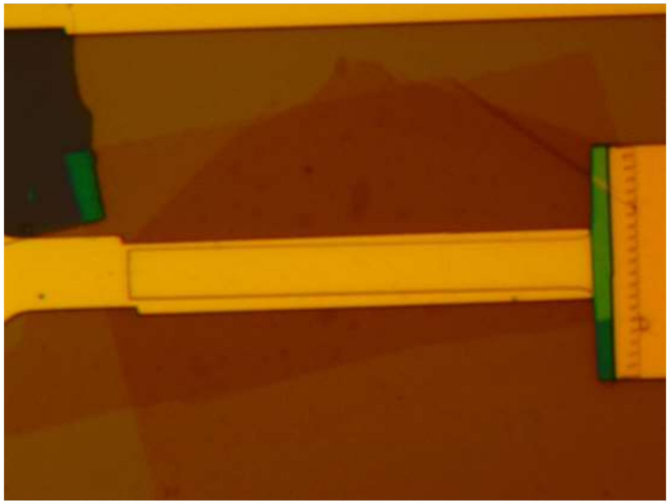


H. Yoon et al., Nature Nanotech 2014 : Measurement of collective dynamical mass of Dirac fermions in graphene

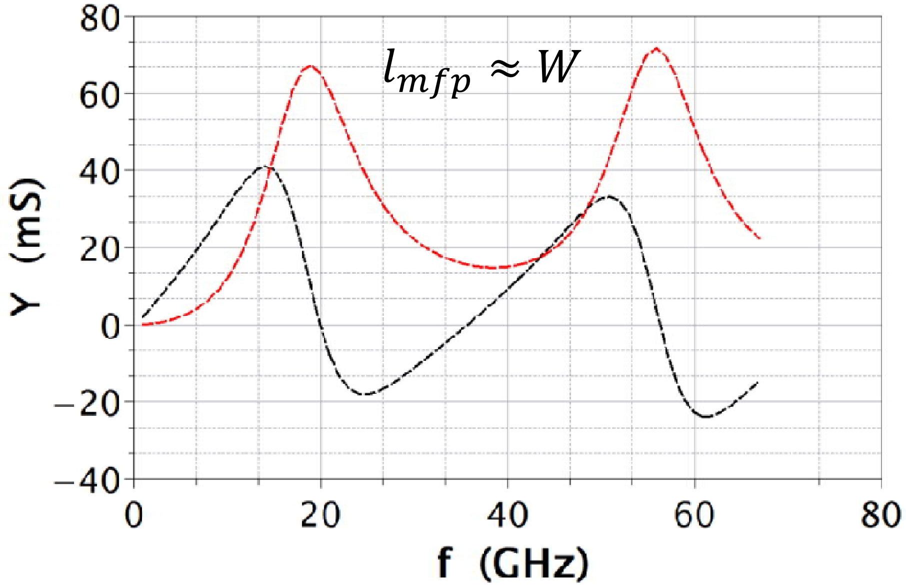
Quater wave plasma resonators

Poster David Mele

$L \times W \approx 32 \times 8\ \mu m$



$$Y = j\omega CLW \times \frac{\tanh\left(L\sqrt{jC\omega(\rho + j\omega\lambda_K)}\right)}{L\sqrt{jC\omega(\rho + j\omega\lambda_K)}}$$

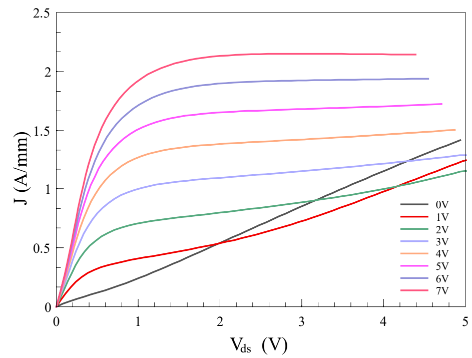


- Plasma waves at GHz frequency
- Plasma waves in doping modulated graphene
- 600 GHz high resolution RADARs : detect cables
- Fun !

Outline

I. Low-field : from DC to high frequency
- Field-effect, density of states, conductivity,
- Scattering, mean free-path and mobility
- Quantum capacitance and kinetic inductance

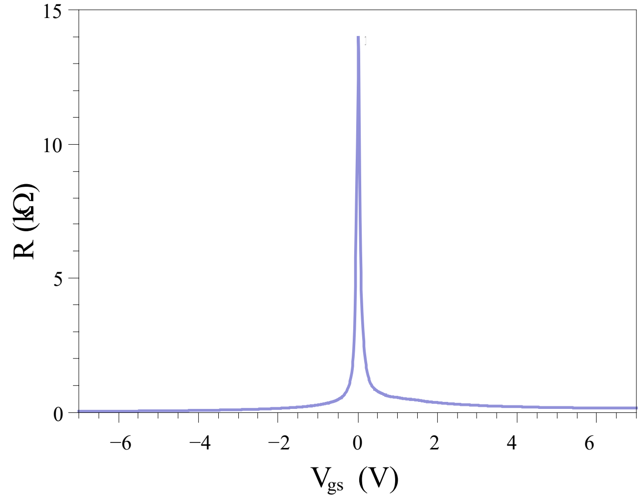
II. High-Field transport
- Motivation : Field effect transistors
- Current saturation by optical phonon scattering
- Hot electrons effects and phonon relaxation



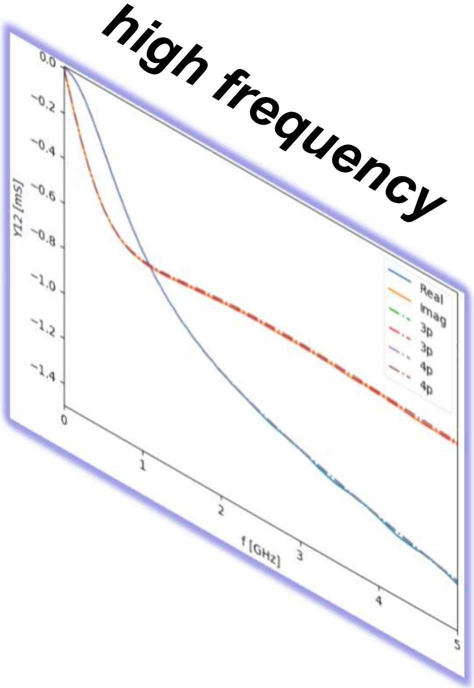
III. Ballistic's
- Landauer conductance and shot noise
- Klein tunneling across p-n junctions
- Dirac Fermion optics devices

Mobility bashing

linear (quantum) transport

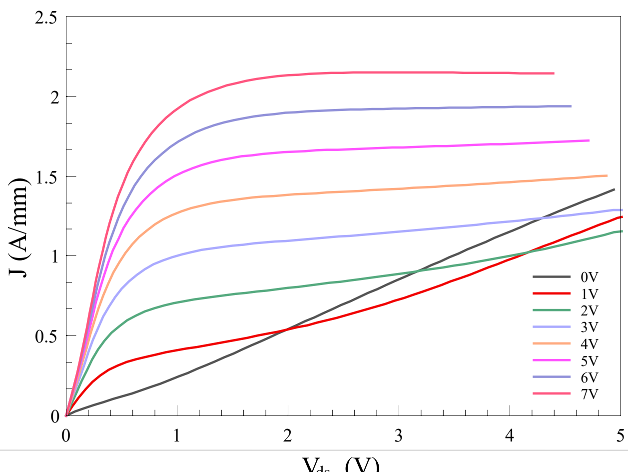


high frequency

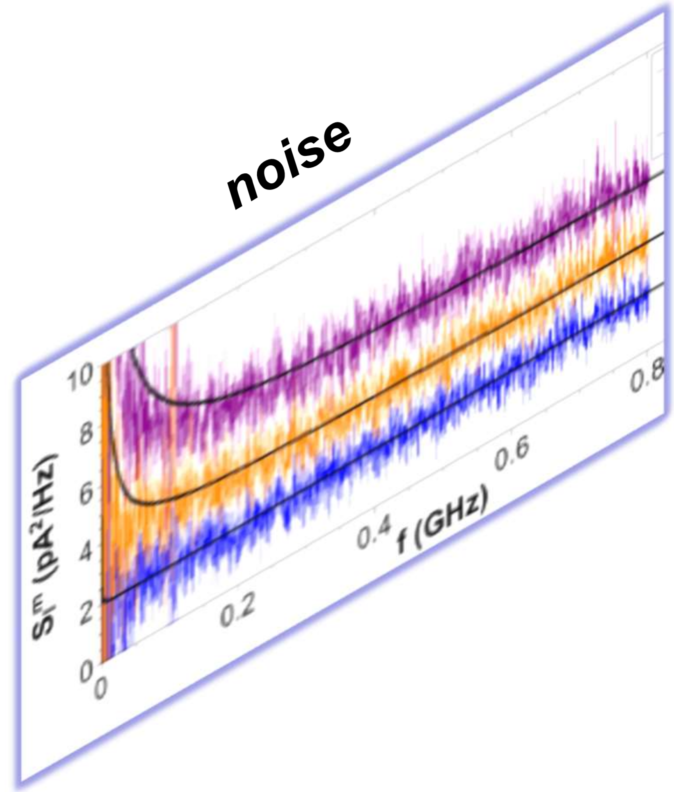


But also

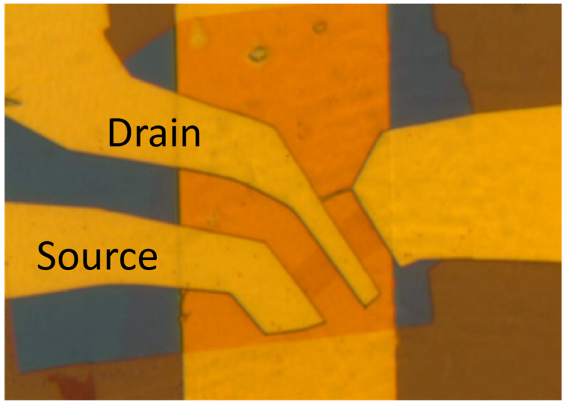
non-linear

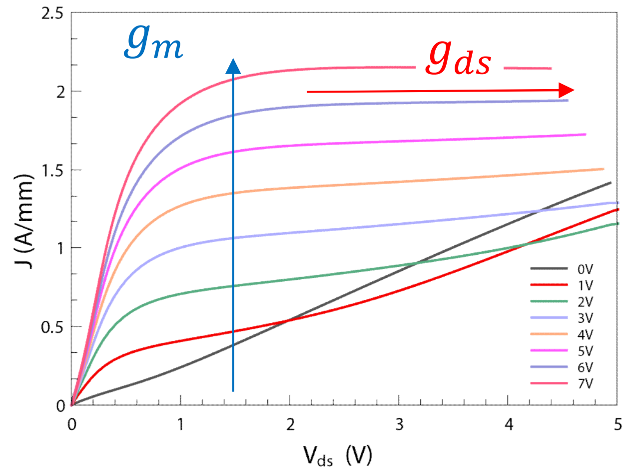


noise



A low tech field effect transistors

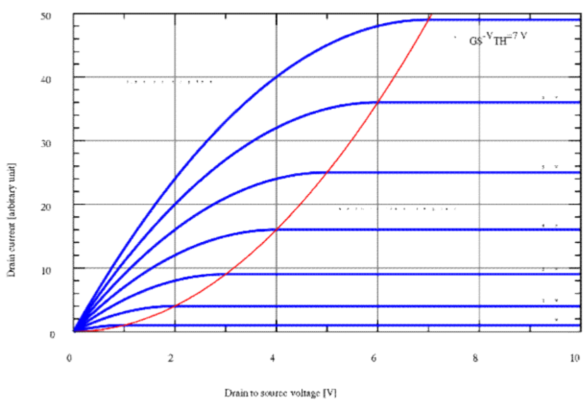


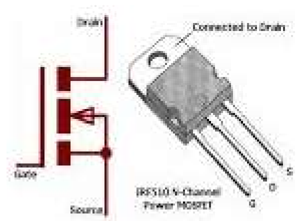


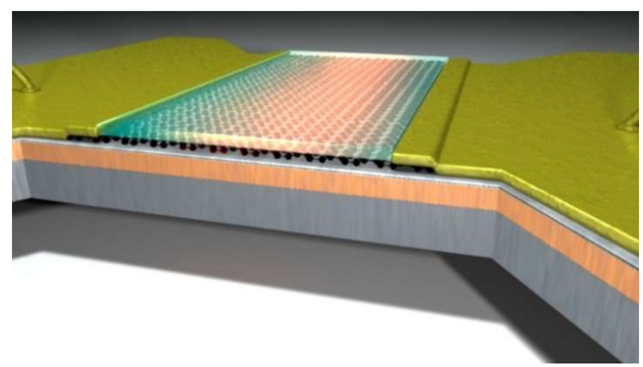
<u>Current saturation is needed for voltage gain (and cut-off frequencies)</u>

$$Gain = \partial V_{ds}/\partial V_{gs} = (\partial I_{ds}/\partial V_{gs})/(\partial I_{ds}/\partial V_{ds}) = g_m/g_{ds} \approx 10$$

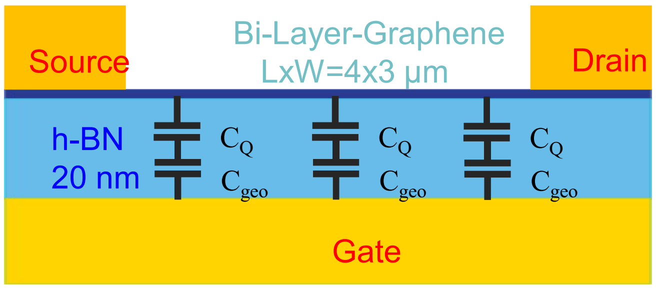
High bias $\Longrightarrow$ Large Joule power $\Longrightarrow$ hot electrons



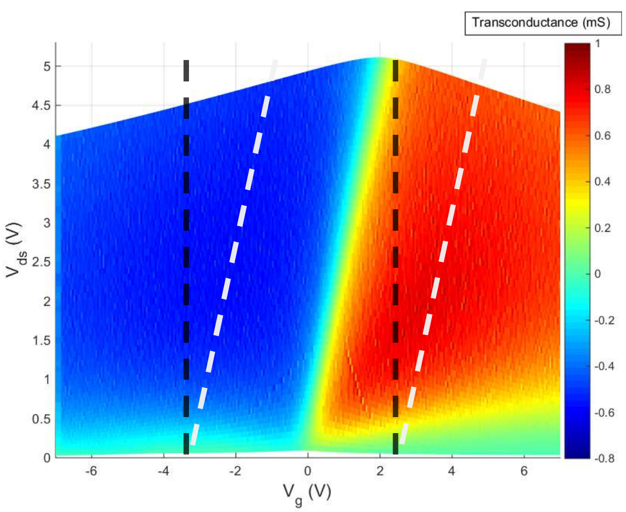




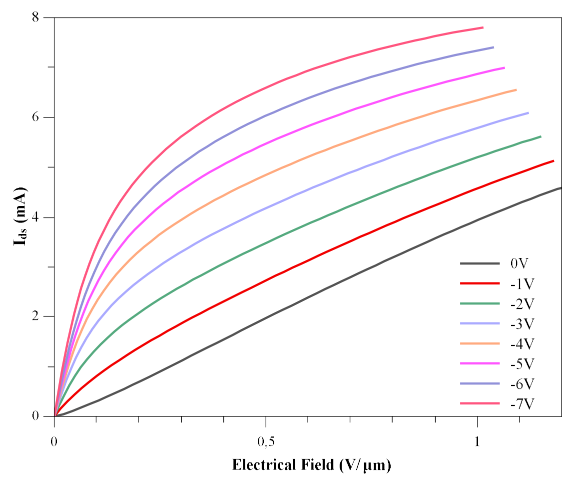
Saturation depends on polarity (drain gating)

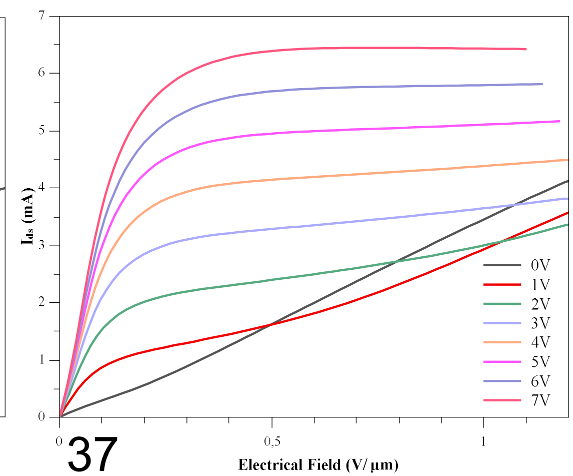


Transconductance



Constant gate voltage





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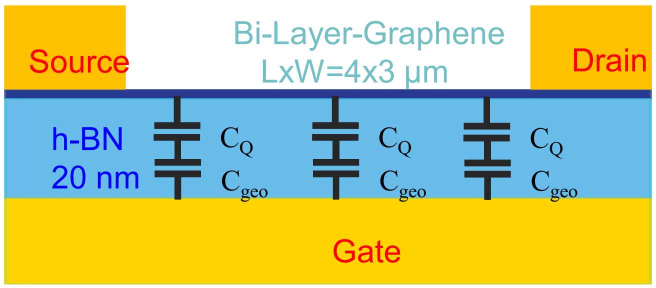
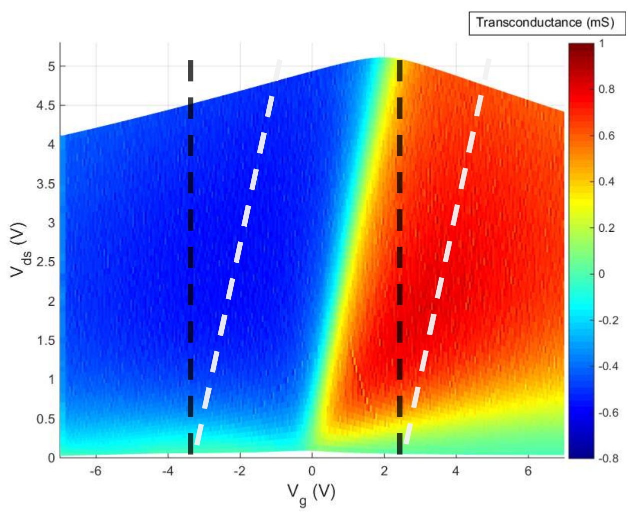
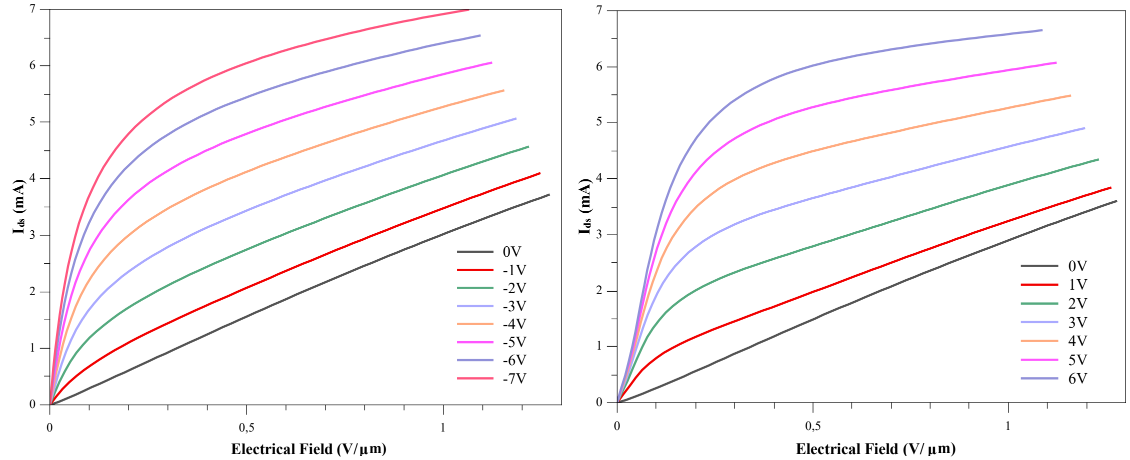
Saturation depends on polarity



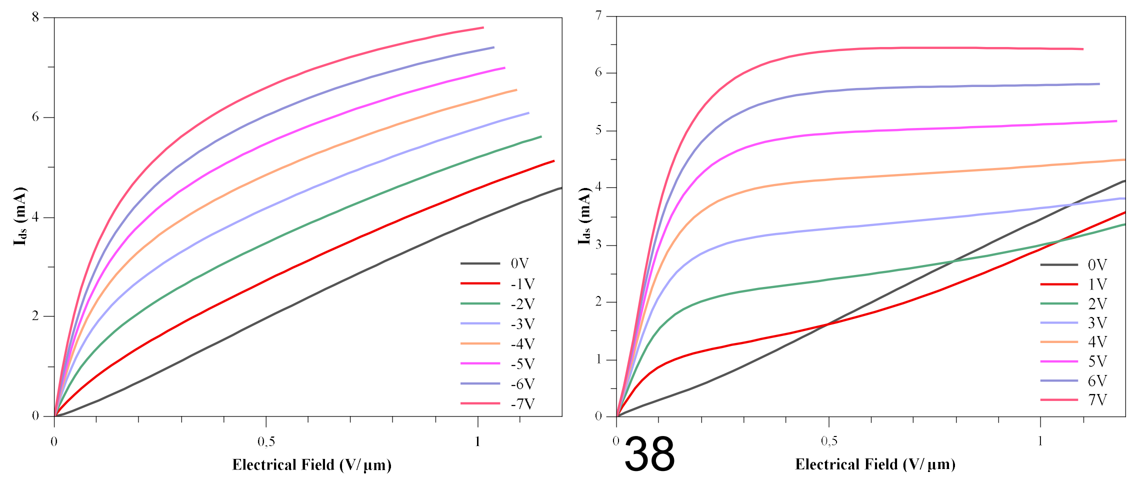
Transconductance



Constant carrier density



Constant gate voltage



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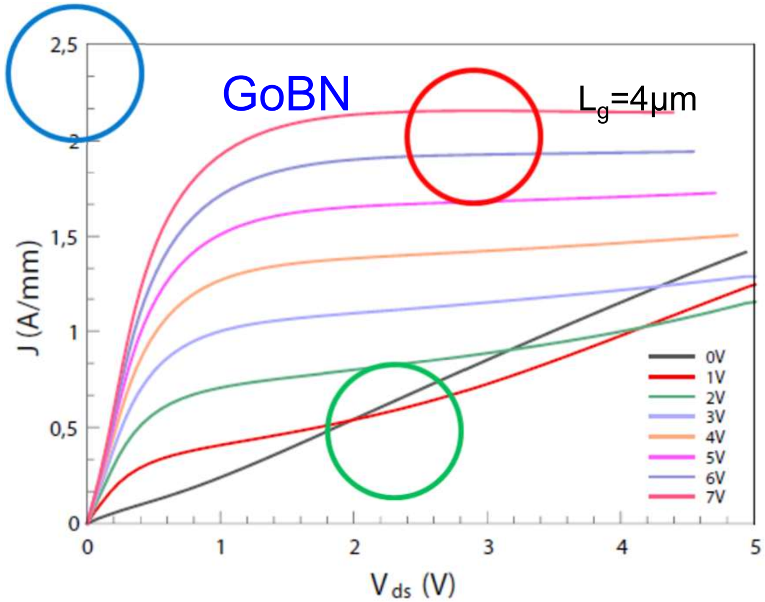
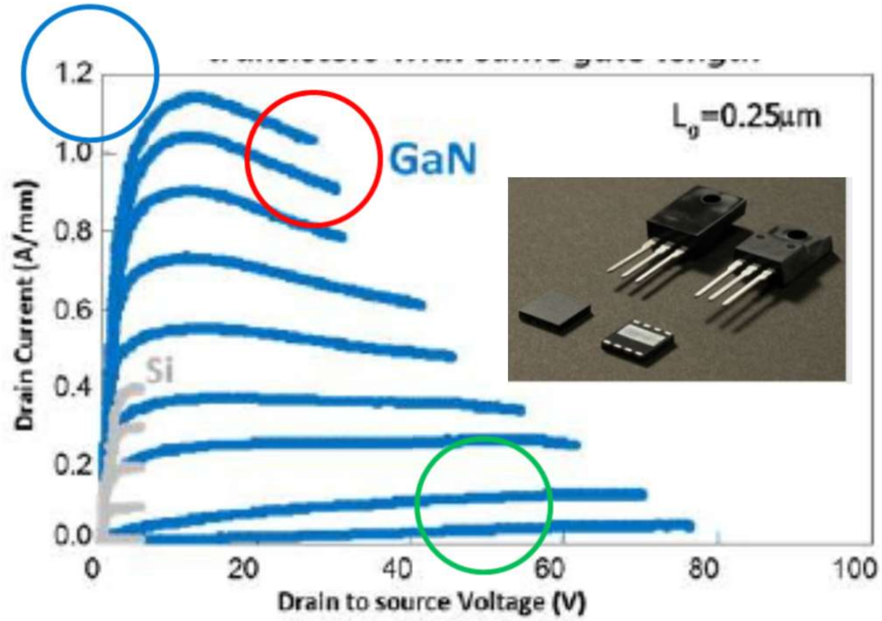
How good is graphene FET ?

GoBN Zener-Klein transistor



Panasonic : X-GaN Power transistor



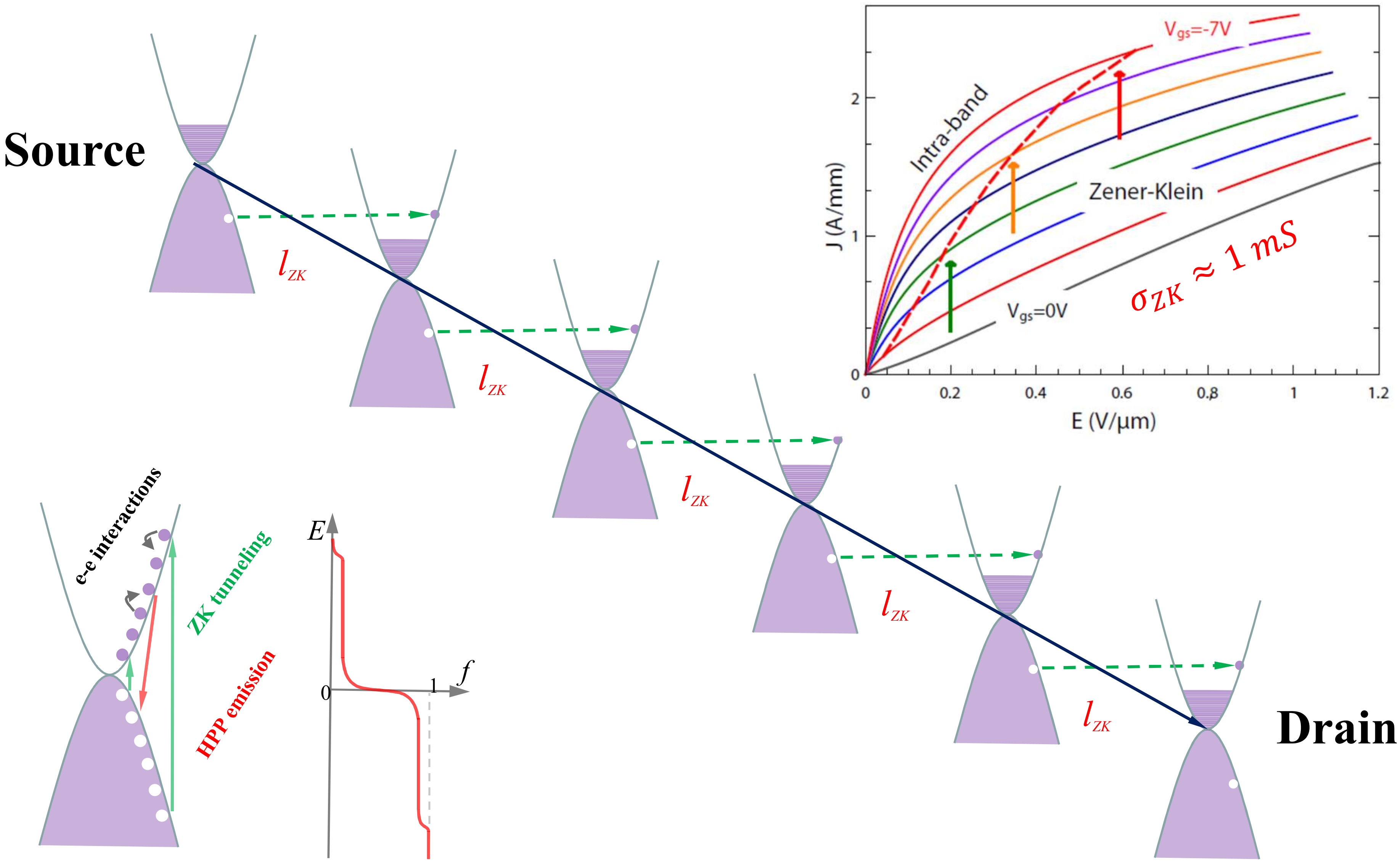
Similar current densities in graphene and GaN

Little thermal degradation of current in graphene

No gap no pinchoff in graphene ⇒ Zener tunneling

H. Xu, Huawei Ltd., private communication

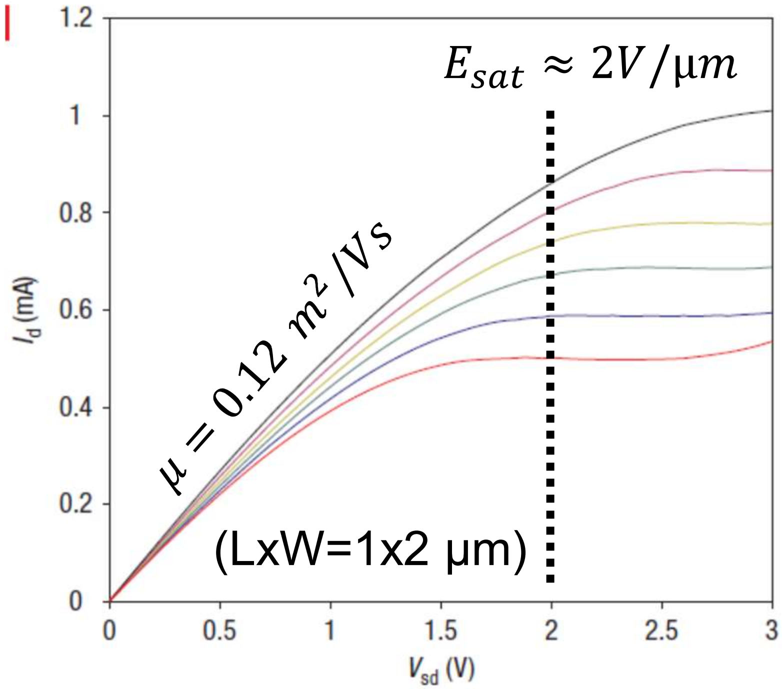
Zener tunneling



Why current saturates ?

Graphene on SiO2

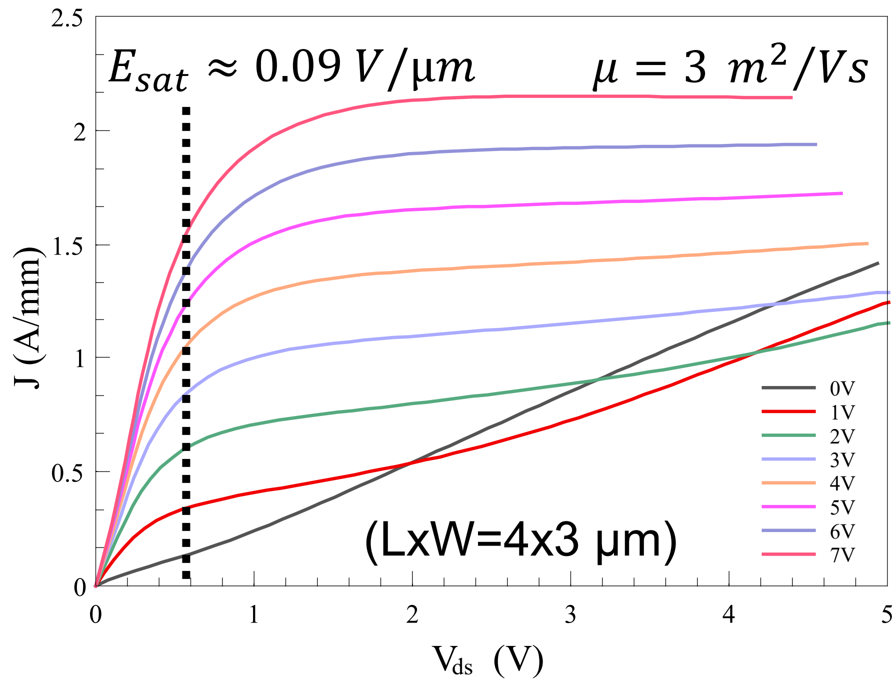
I Meric et al., Nat. Nano (2008)



$\mu E_{sat} \sim 0.25\ 10^6\ m/s$

Graphene on BN

W. Yang et al., Nat. Nano (2018)

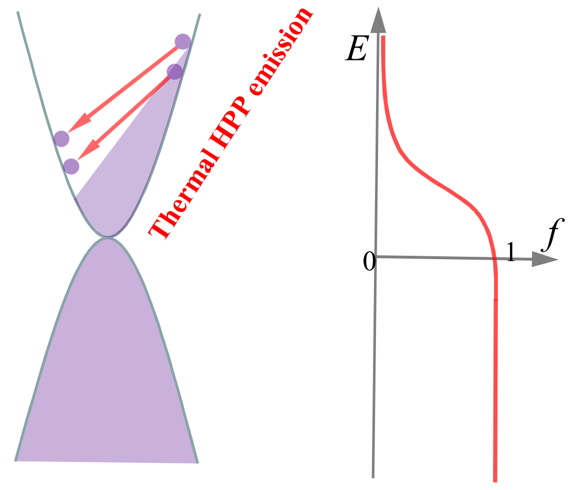


$\mu E_{sat} \sim 0.3\ 10^6\ m/s$

Indication of velocity saturation $v_{sat} = \mu E_{sat} \leq 0.3\ 10^6\ m/s$

Good saturation requires $\mu \geq 10\ 000\ cm^2/Vs$

velocity saturation



Velocity saturation by OP emission :

$$v_{sat} \equiv \frac{J_{sat}}{ne} = \frac{2}{\pi}\frac{\Omega_{OP}}{k_F}$$

$$\frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta = \frac{2}{\pi}$$

$$\hbar\Omega_{OP} \approx 170 - 200\ meV$$

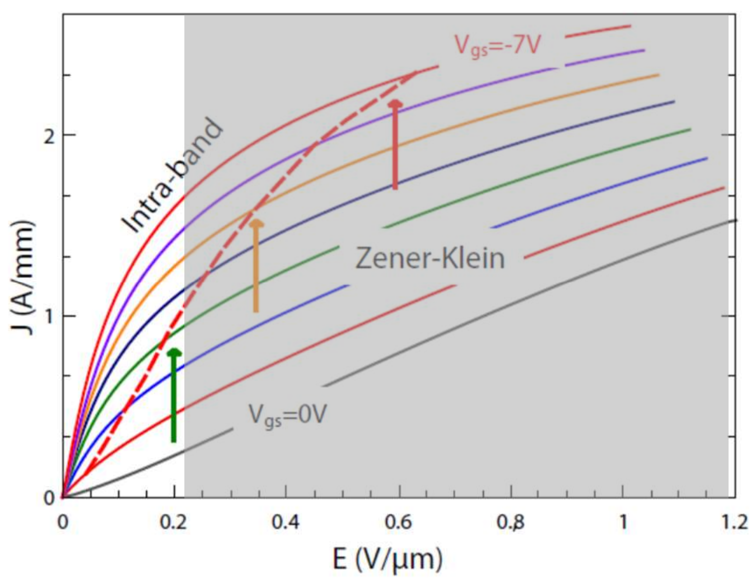
Velocity saturation by substrate phonon emission (remote)

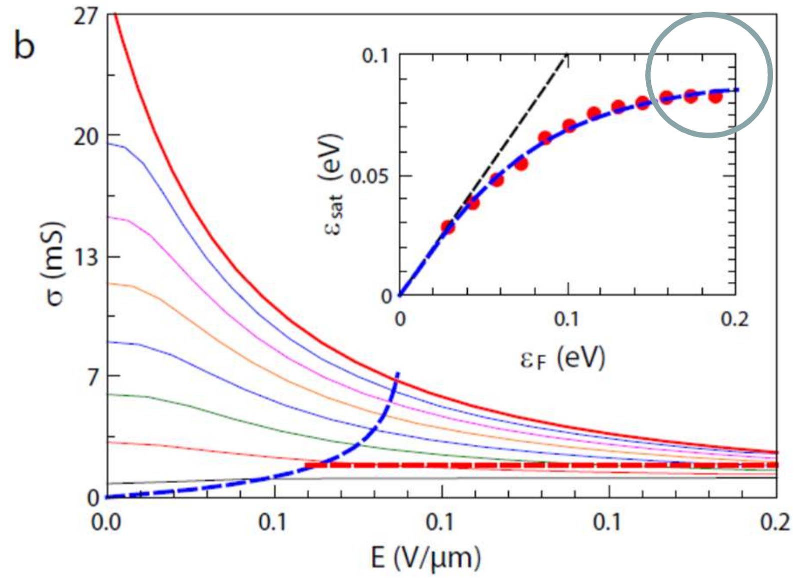
$$v_{sat} \equiv \frac{J_{sat}}{ne} = \frac{2}{\pi}\frac{\Omega_{Substrate}}{k_F} \qquad \leq 0.4 v_F \text{ !!!!}$$

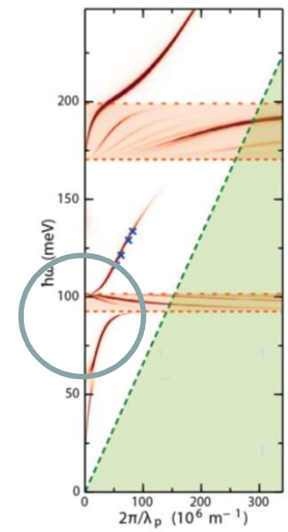
$$\hbar\Omega_{SiO2} \approx 50\ meV, \hbar\Omega_{SiC} \approx \hbar\Omega_{BN} \approx 100\ meV$$

field dependent mobility

100 meV







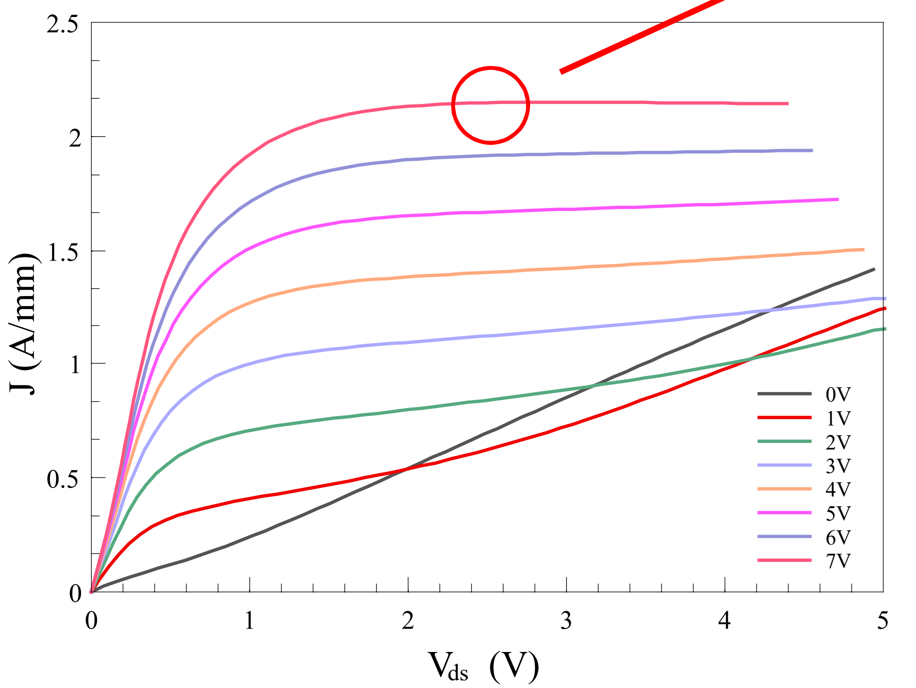
$$J_{sat} = nev_{sat}$$

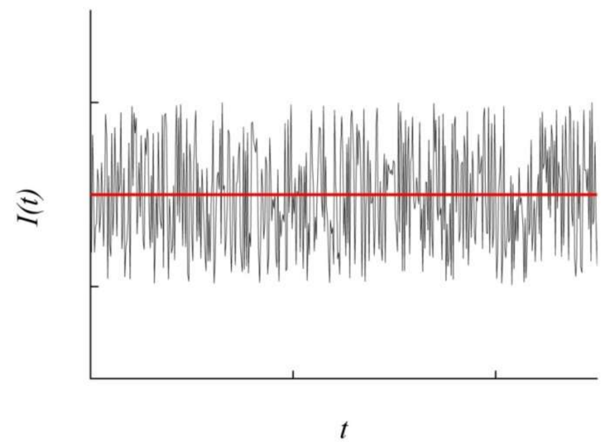
$$\sigma(E) = ne \times \frac{\mu}{\left(1+\frac{E}{E_{sat}}\right)^2}$$

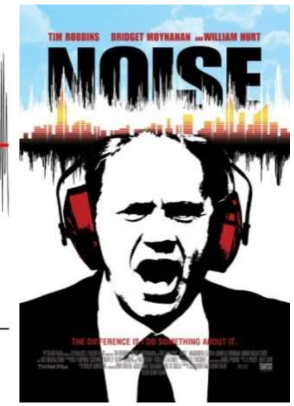
$$\varepsilon_{sat} = \frac{\pi}{2}\hbar k_F v_{sat}$$

W.Yang et al. Nat Nano. (2018)

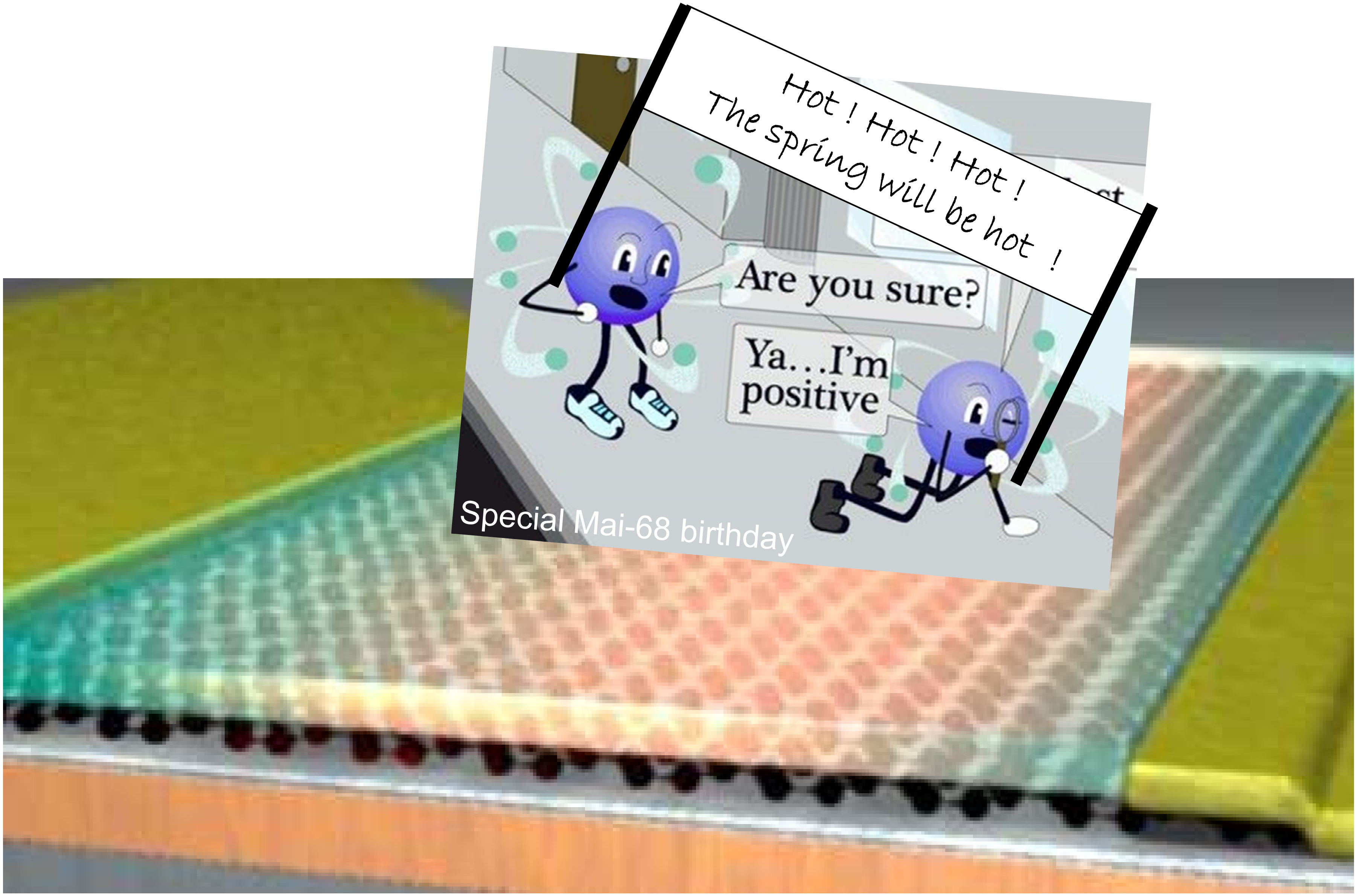
On listening electron noise ......





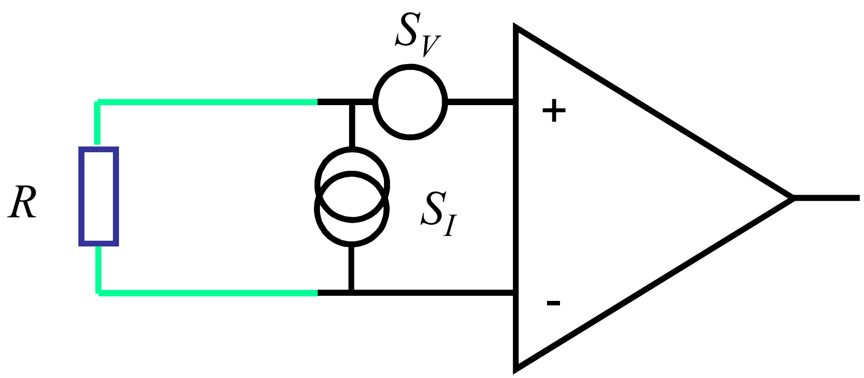


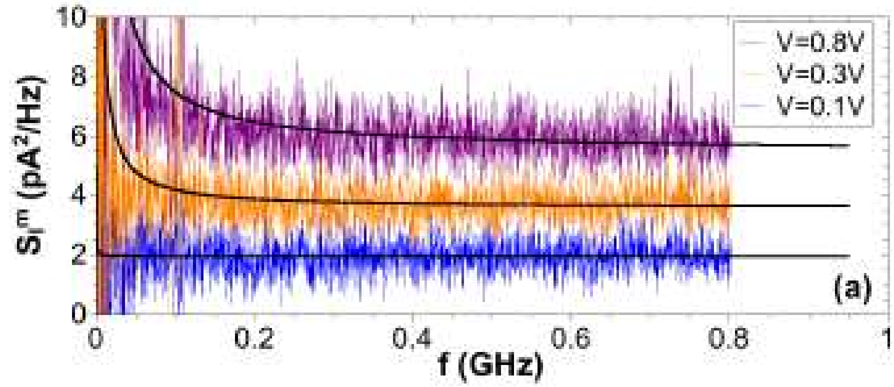
...... you eventually learn something



Electrical noise

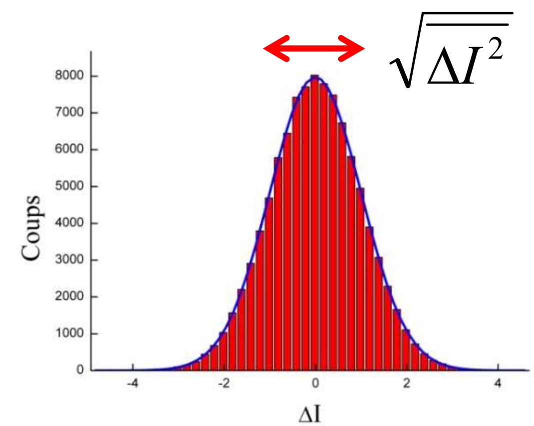
Noise of an amplifier





$$S_V^{in} = S_V + R^2 S_I$$

Statistical distribution

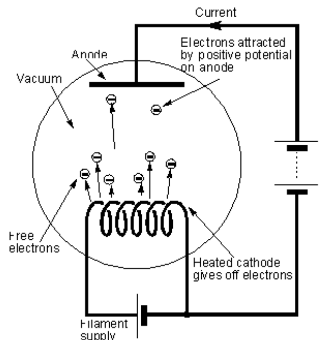


Fluctuations : $\Delta I(t) = I(t) - \overline{I(t)}$

Noise spectrum : $\overline{\Delta I^2(t)} = \int S_I(\nu)\, \Delta\nu$

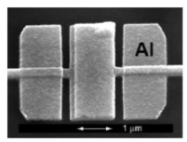
Shot-noise

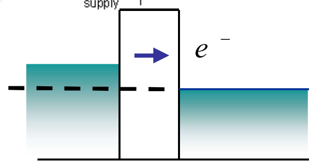
Vacuum tube



or

Tunnel junction

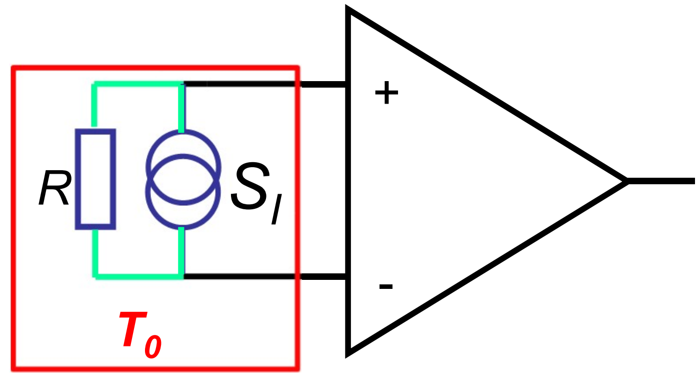




W. Schottky

$$S_I = 2e\bar{I}$$

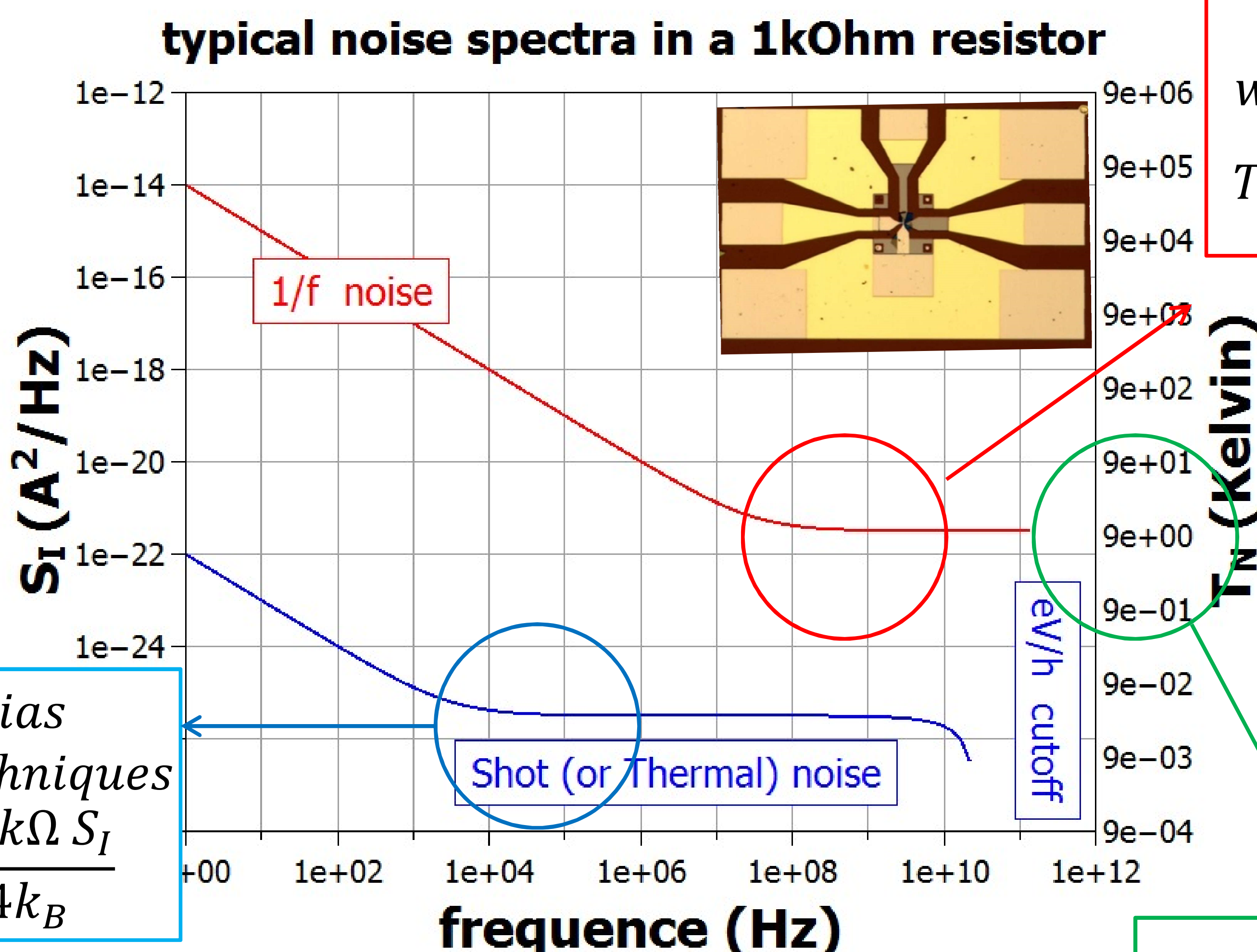
Thermal noise



J.B. Johnson

$$S_I = 4\,k_B T_0 / R$$

Measuring noise



High bias wave guide

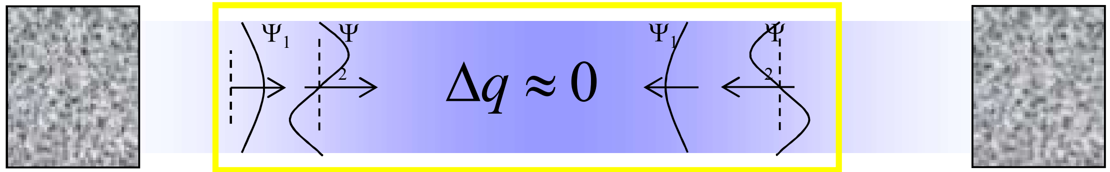
$$T_N \equiv \frac{50\Omega\, S_I}{4k_B}$$

Low bias lockin techniques

$$T_N = \frac{10k\Omega\, S_I}{4k_B}$$

I need cryogenic LNAs !

The Fano factor of quantum conductors



Conductance is transmission

Noisy scattering

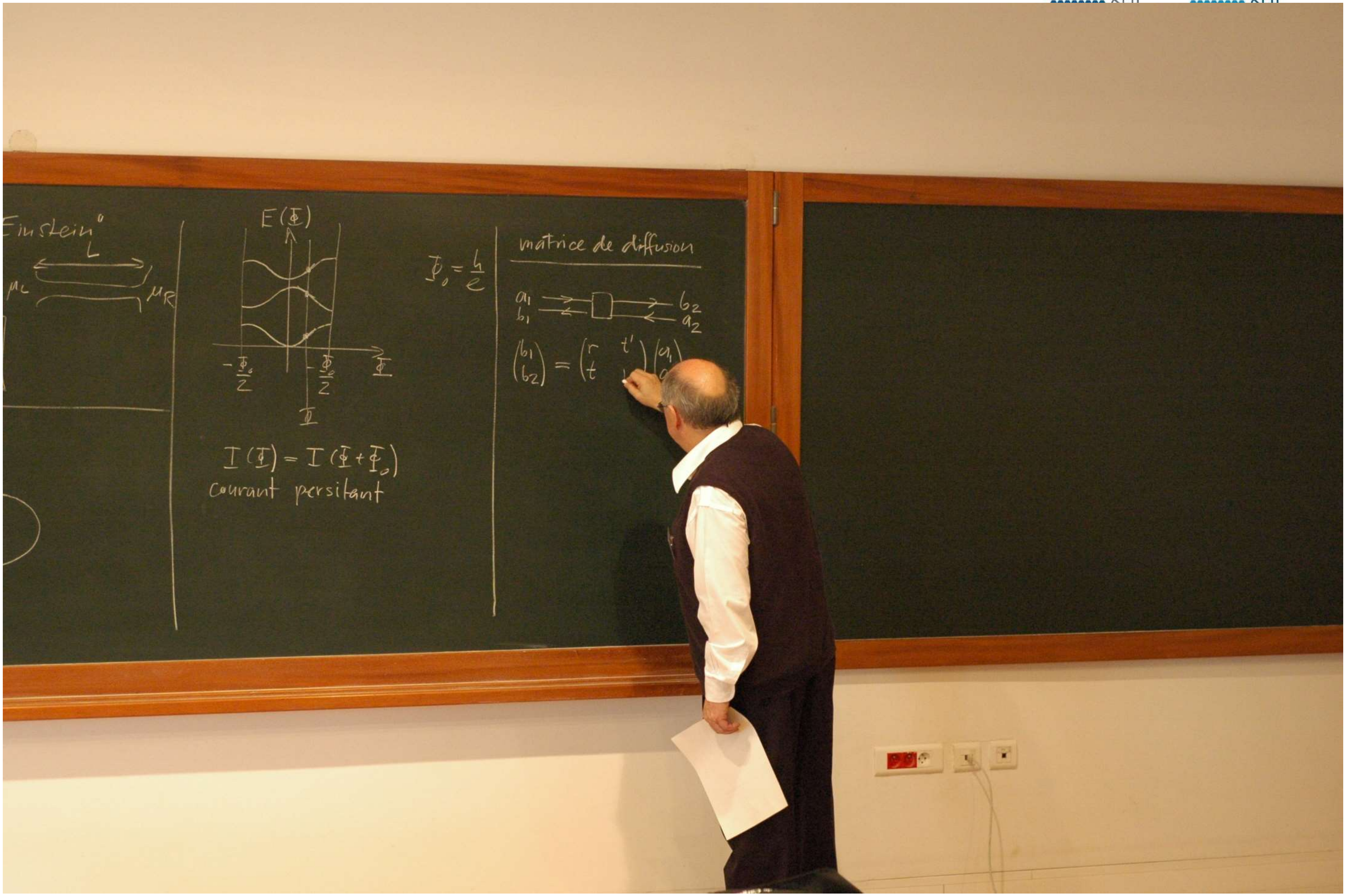
$$G = 4\frac{e^2}{h}\sum_{1}^{N} T_n$$

$$S_I = 2eI\,\frac{\sum T_n\,(1 - T_n)}{\sum T_n} = 2eI \times \text{"}Fano\text{"}$$

R. Landauer and M . Büttiker

Fano factor F<1 in mesoland with $\mathrm{F} = \frac{1}{3}$ for a diffusive metal

Markus was in Cargèse in 2008

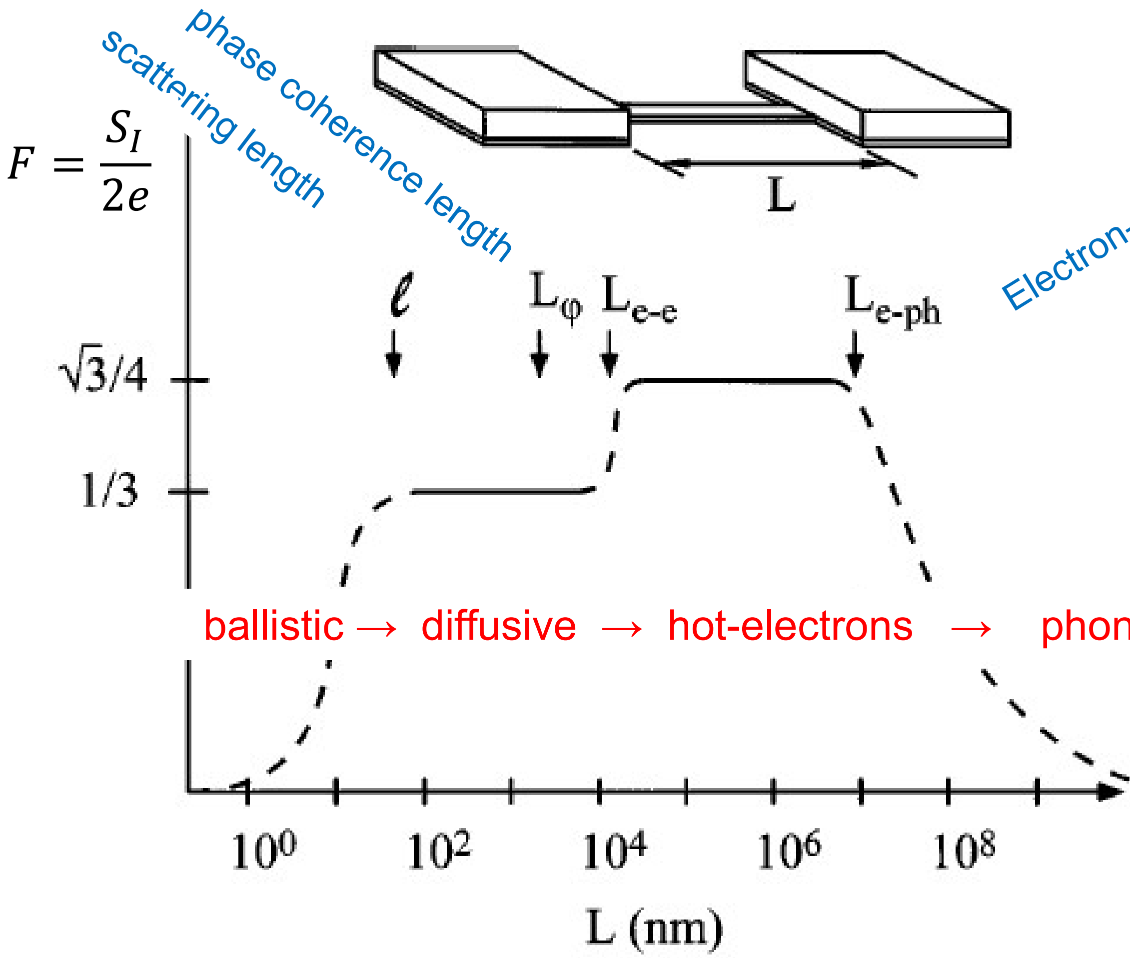


Gilles was there ……. and learned form Markus

Increase the sample Length, Temperature or Bias

$$L_{e-e} \propto \frac{1}{(k_B T)^2}, \frac{1}{(eV_{ds})^2}$$

$$F = \frac{S_I}{2e}$$



ballistic → diffusive → hot-electrons → phonons → macroscopic world

A.H. Steinbach et al., PRL1996 : Observation of Hot-Electron Shot Noise in a Metallic Resistor

Thermal vs charge transport

Electron density

$$n = \int_0^\infty N(\varepsilon) f(\varepsilon) d\varepsilon$$

electronic energy

$$u = \int_0^\infty N(\varepsilon) f(\varepsilon) \varepsilon d\varepsilon = \mu_F + \frac{\pi^2}{6} k_B^2 T^2 N(\varepsilon_F)$$

Entropy / heat capacity :

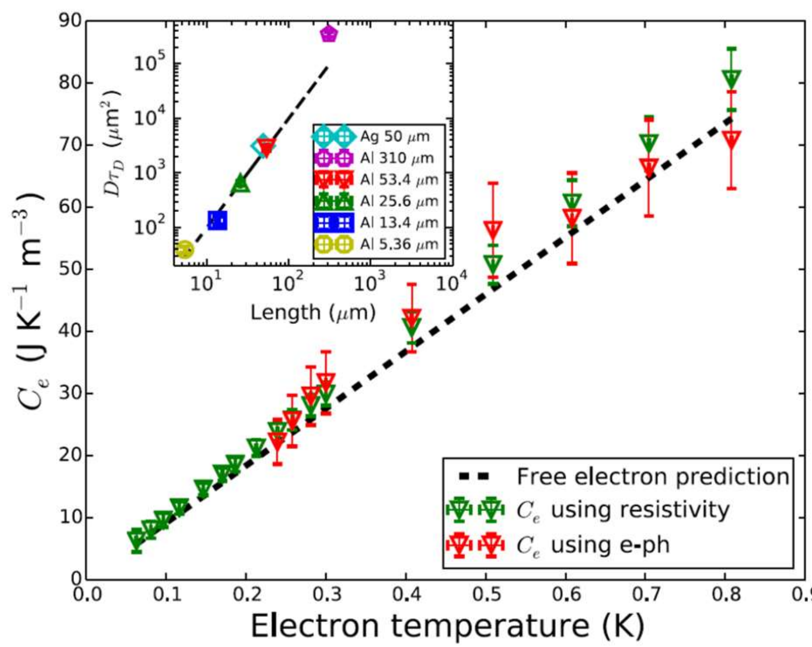
$$C_e = L_o T C_Q(\varepsilon_F) \quad \left(L_o = \frac{\pi^2 k_B^2}{3e^2} = 24{,}4\ nW\Omega K^{-2} \right)$$

Einstein/Onsager relation :

$$J_Q \equiv -\kappa \nabla T = -L_o T\ \sigma\ \nabla T = -L_o T \big(C_Q D \big)\ \nabla T$$

Particle diffusion $\quad \frac{\partial n}{\partial t} + D\ \Delta n \neq 0$

Heat diffusion $\quad \frac{\partial T}{\partial t} + D\ \Delta T \neq 0$



Heat and charge obey same diffusion equation

E. Pinsolle B. Reulet, PRL2016: Direct Measurement of the Electron Energy Relaxation Dynamics in Metallic Wires

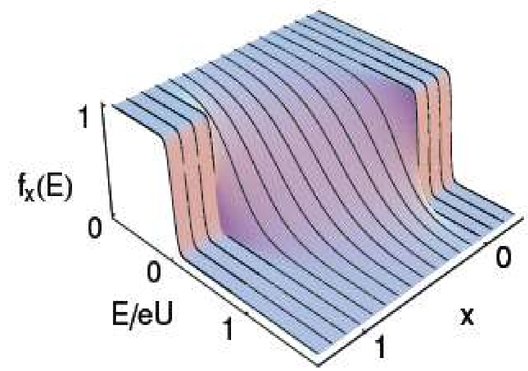
Bertrand in Cargèse (2008)

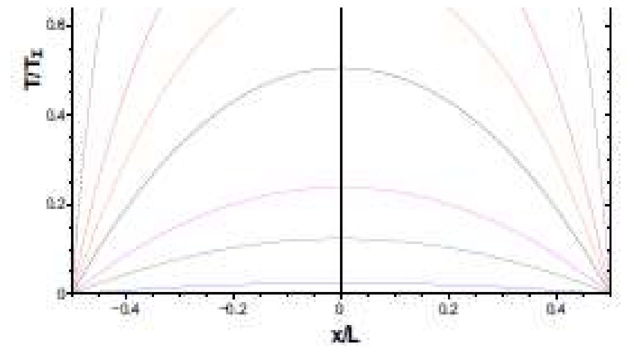
Self heating and thermal shot noise

Moderate bias : self-heated electrons are cooled by heat conduction to the leads

Diffusive regime : potential drop is linear (Omhs law) $\varphi(x) = \frac{V_{ds}}{2} - (2x/L - 1)V_{ds}$

$$f(\varepsilon, x) = \left[1 + \exp\left(\frac{\varepsilon - e\varphi(x)}{k_B T_e(x)}\right)\right]^{-1}$$





Stationnary conditions : $\frac{L_o}{2} \frac{\partial^2 {T_e}^2(x)}{\partial x^2} = -E^2$ $\quad T_e(x) = \frac{V_{ds}}{2\sqrt{L_o}}\sqrt{1 - 4x^2/L^2}$

Average temperature

$$k_B \langle T_e(x) \rangle = eV_{ds} \frac{\sqrt{3}}{8}$$

Thermal shot noise

$$S_I = 4Gk_B\langle T_e(x)\rangle = 4GeV_{ds}\frac{\sqrt{3}}{8} = 2eI_{ds} \times \frac{\sqrt{3}}{4}$$

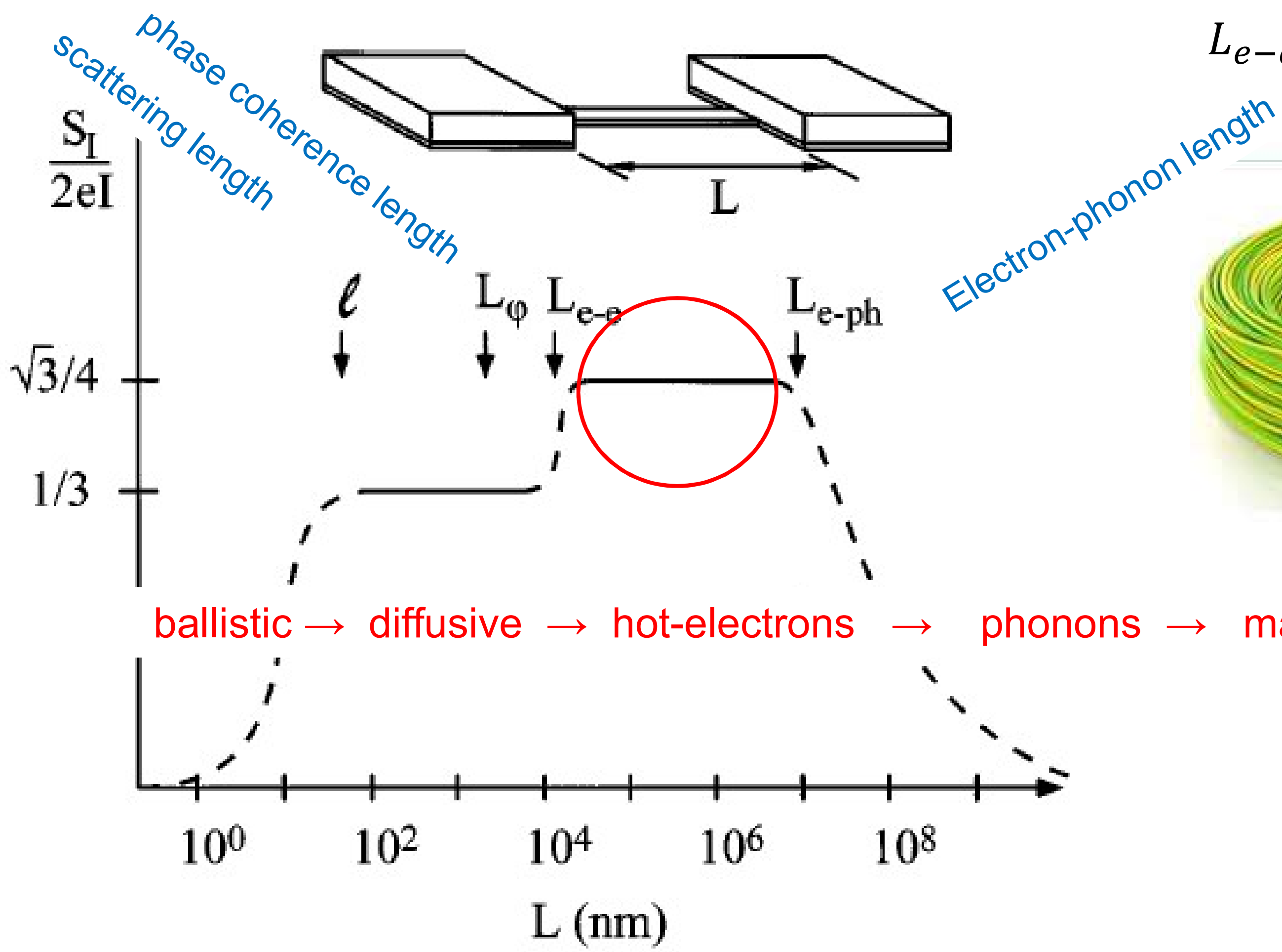
Hot electron Fano factor : $F = \frac{\sqrt{3}}{4}$

Wiedemann Franz cooling power : $P_{WF} = \frac{64}{3\sigma} \times k_B^2\left({T_e}^2 - {T_{bath}}^2\right)$

Conductors : from quantum to macro

… just increase the sample length L

$$L_{e-e} \propto \frac{1}{(k_B T)^2}, \frac{1}{(eV_{ds})^2}$$

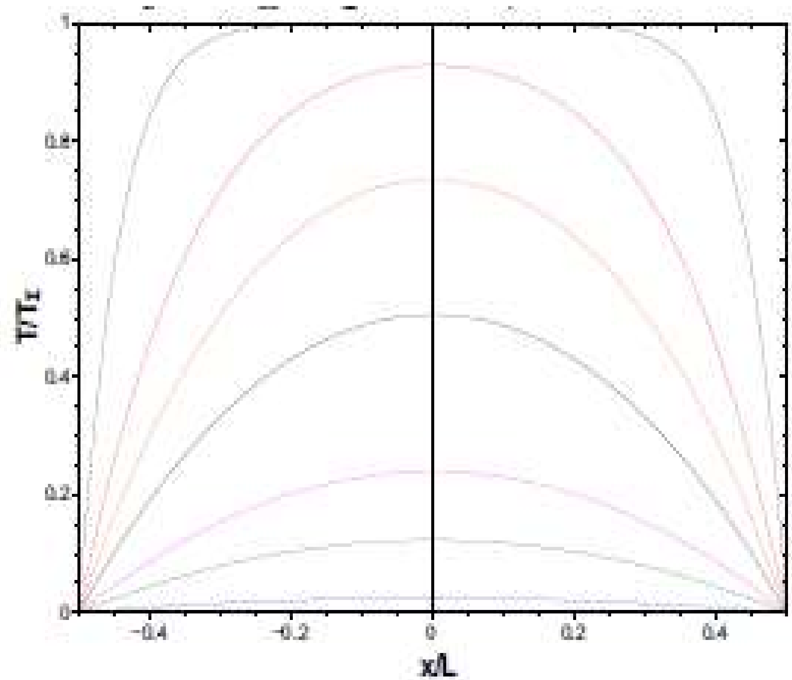


A.H. Steinbach et al., PRL1996 : Observation of Hot-Electron Shot Noise in a Metallic Resistor

Phonon cooling

$$\frac{\sigma L_o}{2}\frac{\partial^2 {T_e}^2(x)}{\partial x^2} = -\sigma E^2 + P_{ph} = -\sigma E^2 + \Sigma\left({T_e}^4 - {T_{ph}}^4\right)$$

Temperature maximum : $T_\Sigma^2 = V_{ds}/\sqrt{LW\Sigma R}$

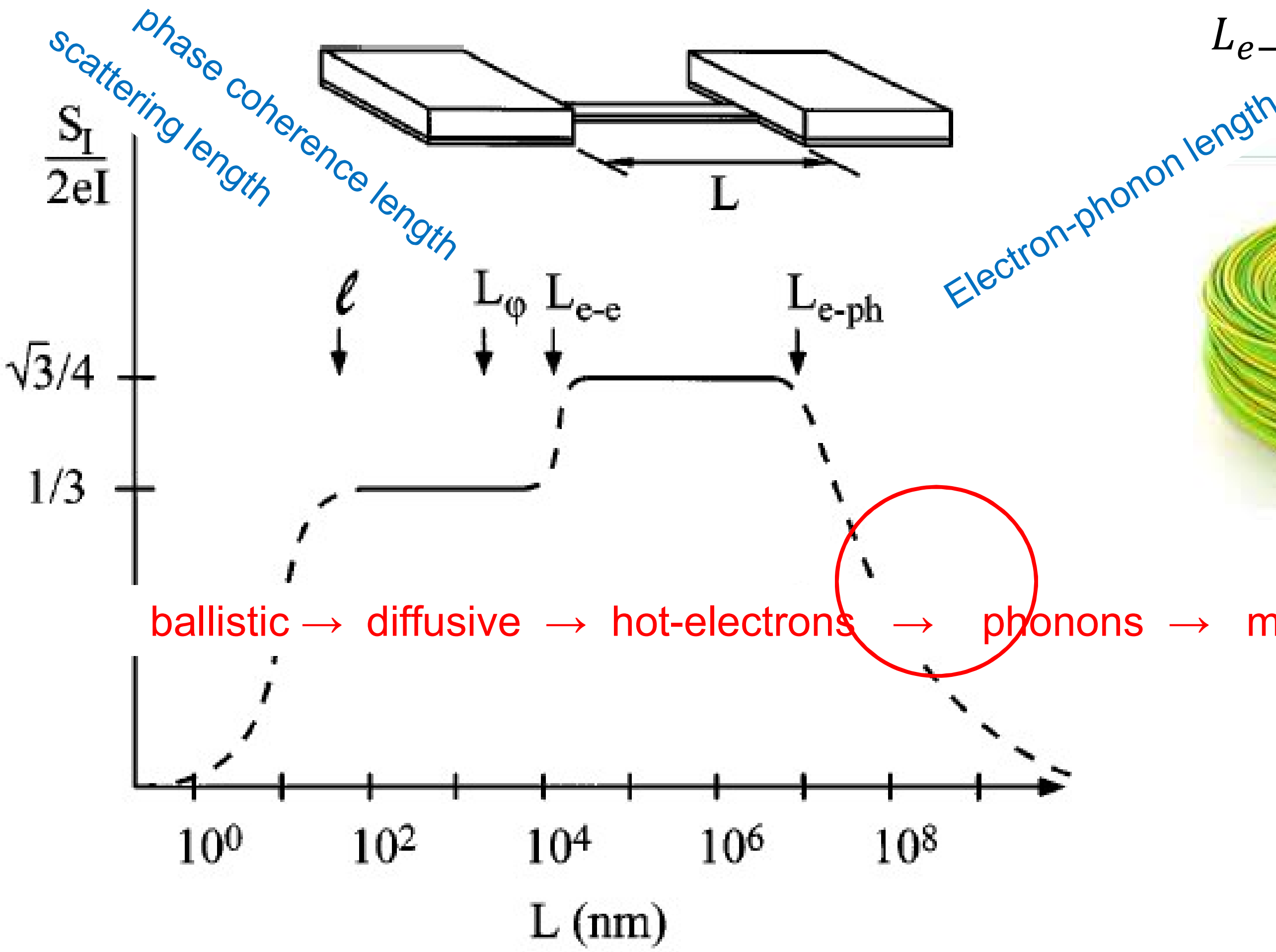


Low-T phonon Fano factor $F \propto 1/\sqrt{V_{ds}}$

Analytical solution in A. Betz thesis 2013, appendix D : https://tel.archives-ouvertes.fr/tel-00784346/

… just increase the sample length L

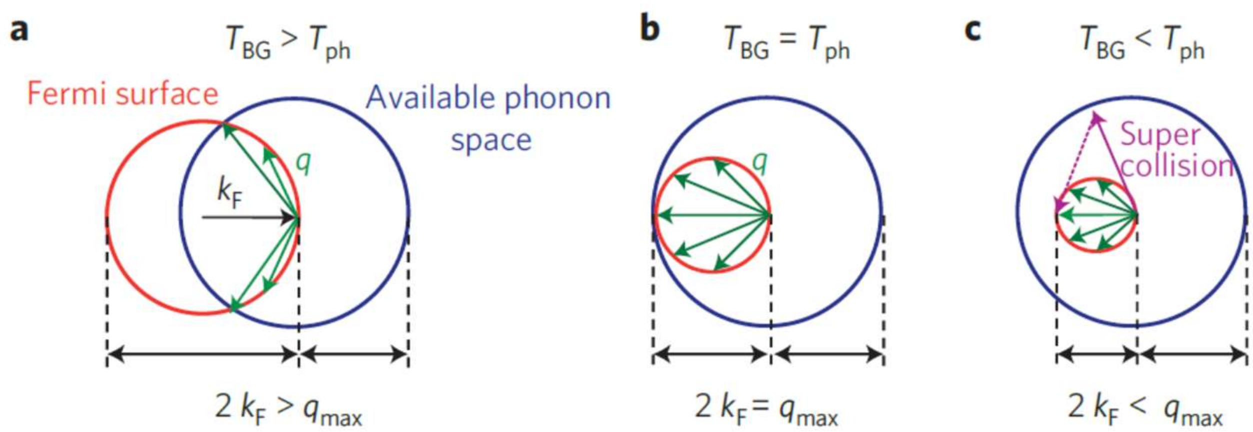
$$L_{e-e} \propto \frac{1}{(k_B T)^2}, \frac{1}{(eV_{ds})^2}$$



A.H. Steinbach et al., PRL1996 : Observation of Hot-Electron Shot Noise in a Metallic Resistor

phonon relaxation in graphene (theory)

$T_{BG} = 2s/v_F \; T_F$ $\qquad$ $T_{BG}(n_{12}) \approx 54\ K$



AC phonon resistivity (*Hwang-DasSarma, PRB 2008*)

A $\rho_{ph}(T_{ph} \ll \theta_{BG}) = \frac{12\zeta(4)D^2}{\rho_m e^2 \hbar^4 s^5 v_F^2 k_F{}^3} \times k_B^4 T_{ph}{}^4$

B $\rho_{ph}(T_{ph} \gg \theta_{BG}) = \frac{\pi D^2}{4\rho_m e^2 \hbar s^2 v_F{}^2} \times k_B T_{ph}$

AC phonon cooling power (*Viljas-Heikkila, PRB 2010*)

C $P_{ph}(T_{ph} \ll \theta_{BG}) = \frac{\pi^2 D^2 |\varepsilon_F|}{15\rho_m \hbar^5 s^3 v_F^3} \times k_B^4 (T_e{}^4 - T_{ph}{}^4)$

D $P_{ph}(T_{ph} \gg \theta_{BG}) = \frac{D^2 k_F{}^4}{2\pi \rho_m \hbar^5 v_F^6} \times k_B (T_e - T_{ph})$

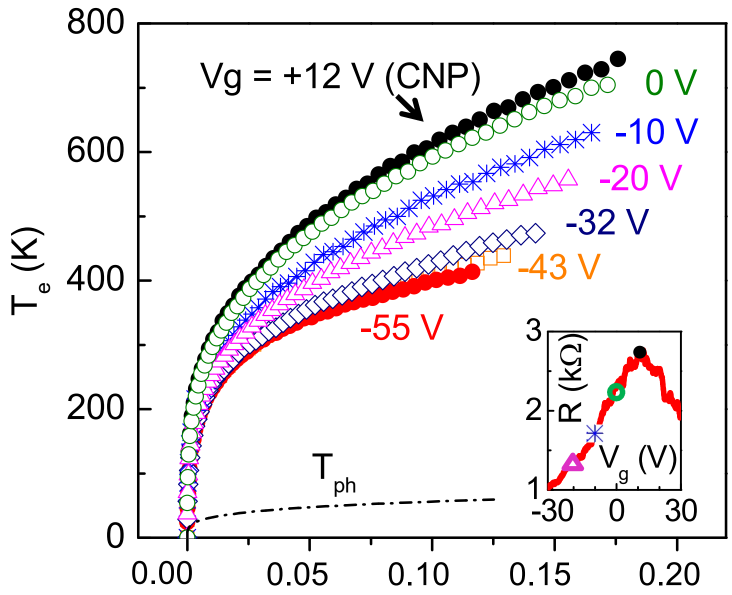
Supercollision cooling (*Song-Levitov, PRL (2013) :*

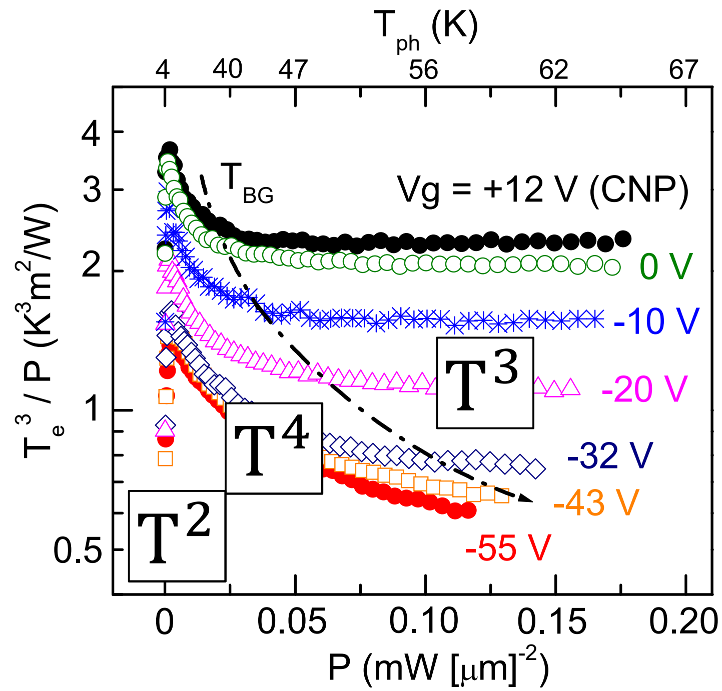
E $P_{SC} = \frac{1}{k_F l_e} \times \frac{9{,}62\, D^2 \varepsilon_F{}^2}{4\pi^2 \rho_m \hbar^5 s^2 v_F^4} \times k_B^3 (T_e{}^3 - T_{ph}{}^3)$

Wiedemann Franz cooling :

$P_{WF} = \frac{64}{3\sigma} \times k_B^2 (T_e{}^2 - T_{bath}{}^2)$

phonon cooling : experiment





Wiedemann Franz cooling ($P \propto T^2$) :

Betz et al., PRL2012, K.C. Fong et al. PRX2012, Crossno et al., Science 2016, etc…..

AC phonon cooling ($P \propto T^4$) ~~($P \propto T$)~~

Betz et al., PRL2012, K.C. Fong et al. PRX2012, McKitterick et al., PRB2016

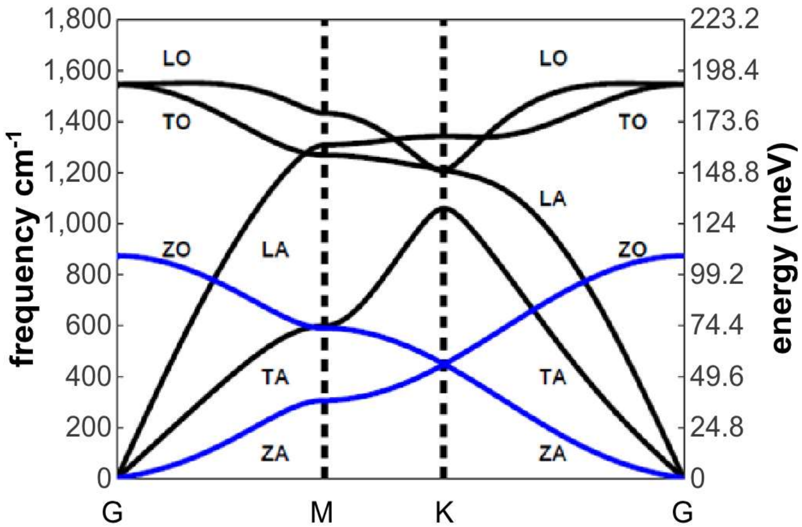
Supercollision cooling ($P \propto T^3$) :

Betz et al., Nat. Phys. 2013; Graham et al., Nat. Phys. 2013; Laitinen et al., NanoLett. 2014, ……..

Summary of electron cooling pathways

WF	Low-T AC	Super-collisions	OP	Hyperbolic cooling	QHE
$P \propto T^2$	$\propto T^3$	$\propto T^4$			
$\leq 10^8\ W/m^2$	$10^8\ W/m^2$	$3\ 10^8\ W/m^2$			

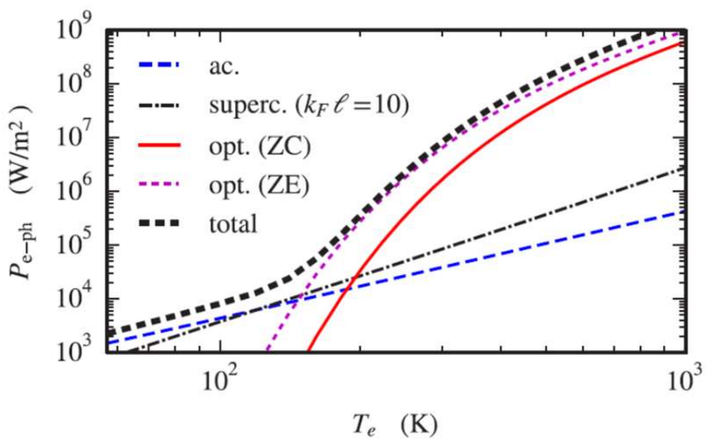
Optical phonon cooling (suspended graphene)

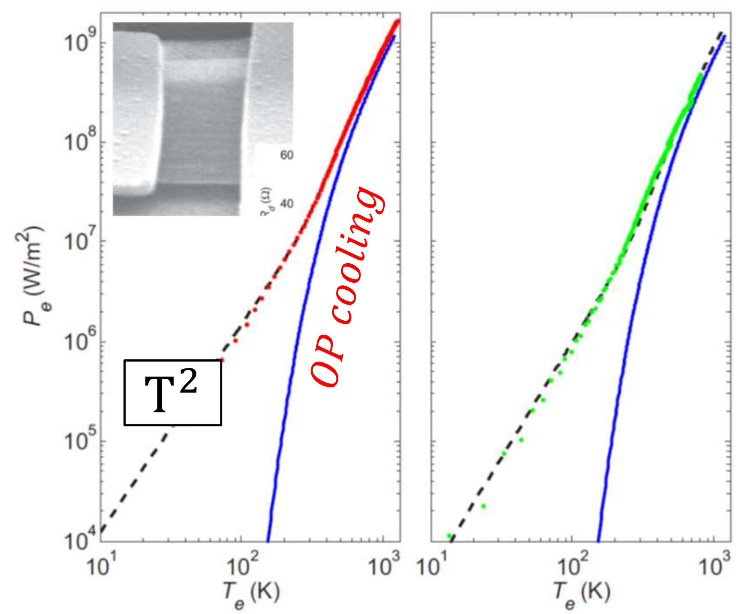


OP phonon : activated and strong (flat band)

$P_{hBN} \sim 1\ 10^{9}\ \ W/m^{2}$

$$P_{OP} = \frac{18\hbar^2\Omega_0^2\gamma'^2_0\hbar\gamma_1}{\pi\rho_m\hbar^4 v_F^4} \times [n_e(\Omega_{OP}) - n_{OP}(\Omega_{OP})]$$

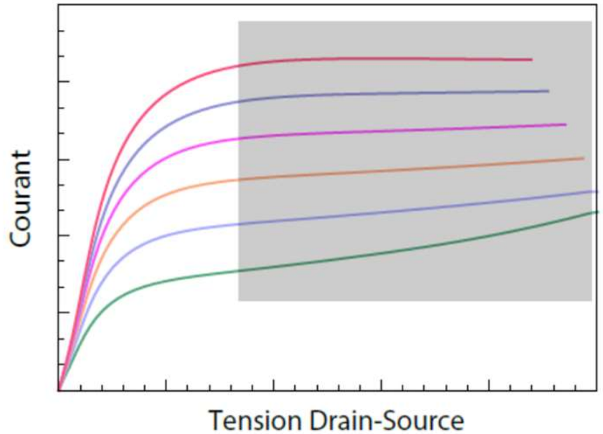


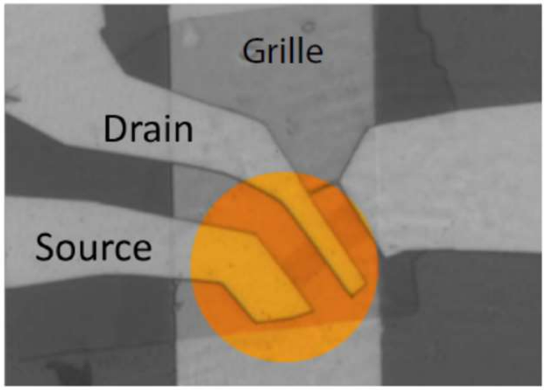


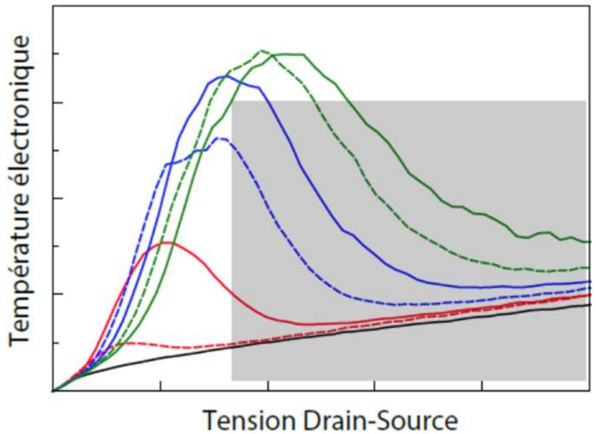
Laitinen et al., PRB-R (2015): Coupling between electrons and optical phonons in suspended bilayer graphene

Super efficient BN substrate phonon cooling

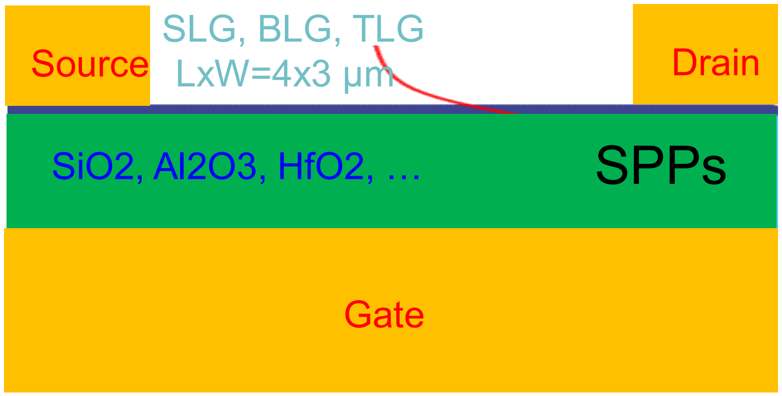
New and efficient cooling in hBN supported graphene in the current saturation regime

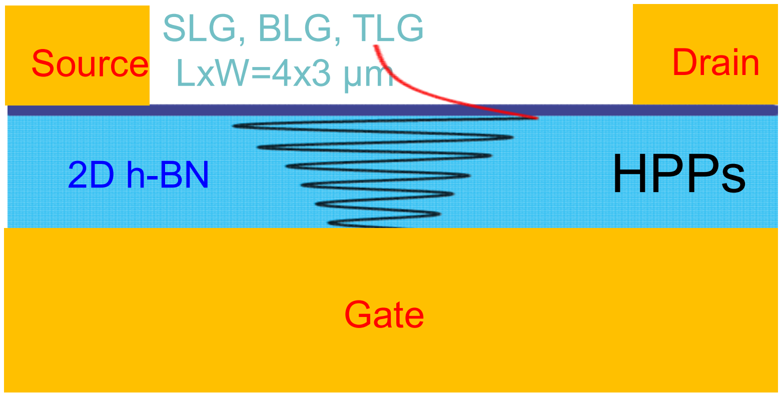






Surface Phonon polariton (SPPs) Hyperbolic Phonon polaritons (HPPs)



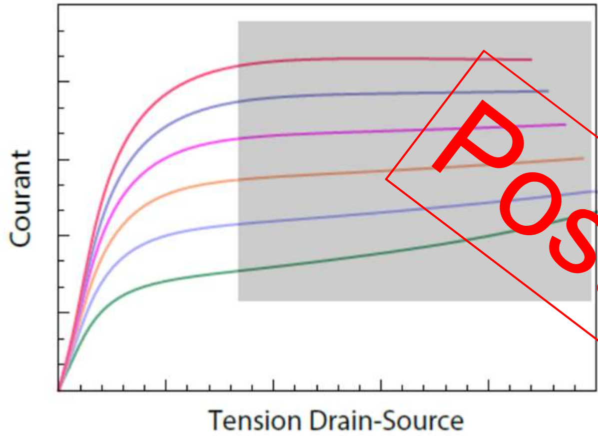


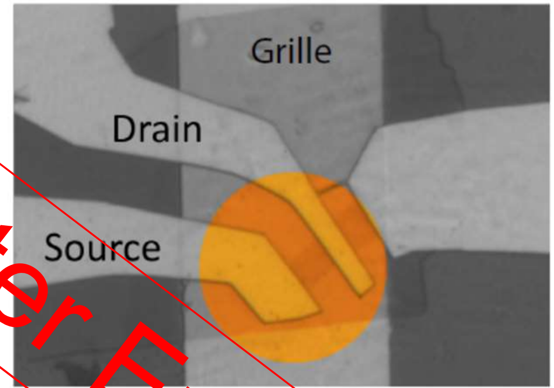
$P_{hBN} \sim 3\ 10^9 \quad W/m^2$

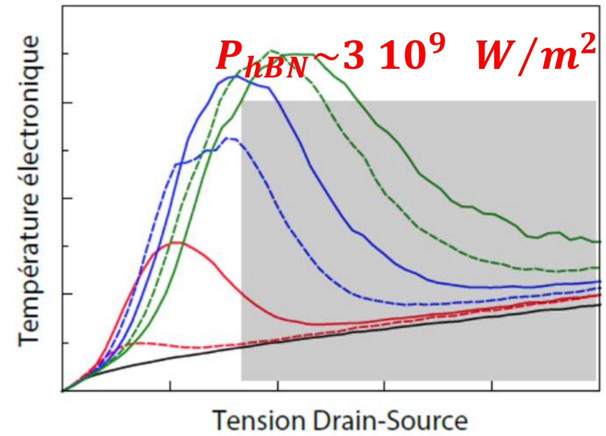
W. Yang, Nature Nanotech. 13, 47 (2018)
K.J. Tielrooij et al., Nature Nanotech. 13, 41 (2018)
A. Principi et al., Phys. Rev. Lett. 118, 126804 (2017)

Substrate phonon scattering

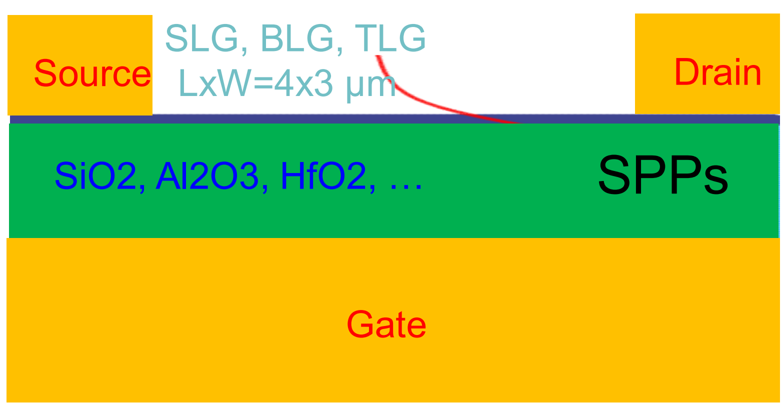
New and efficient cooling in hBN supported graphene in the current saturation regime



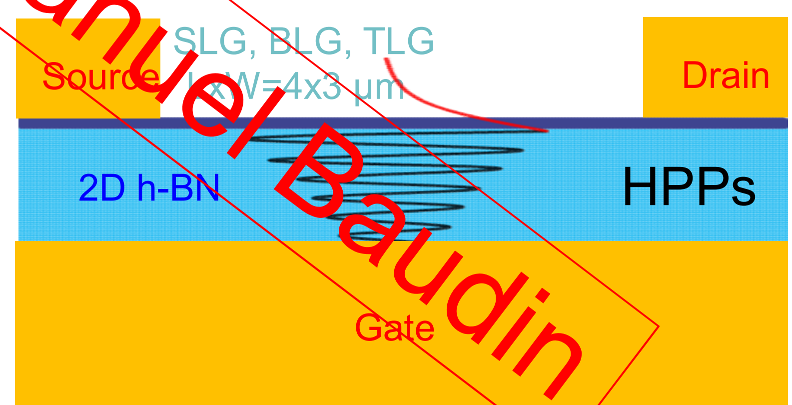




Surface Phonon polariton (SPPs)

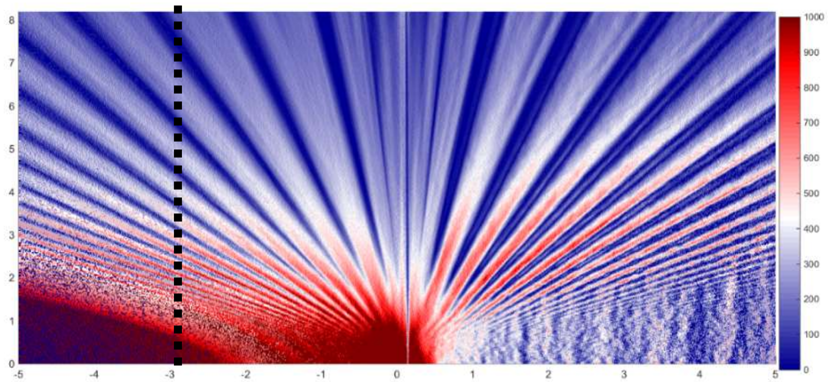


Hyperbolic Phonon polaritons (HPPs)



W. Yang, Nature Nanotech. 13, 47 (2018)
K.J. Tielrooij et al., Nature Nanotech. 13, 41 (2018)
A. Principi et al., Phys. Rev. Lett. 118, 126804 (2017)

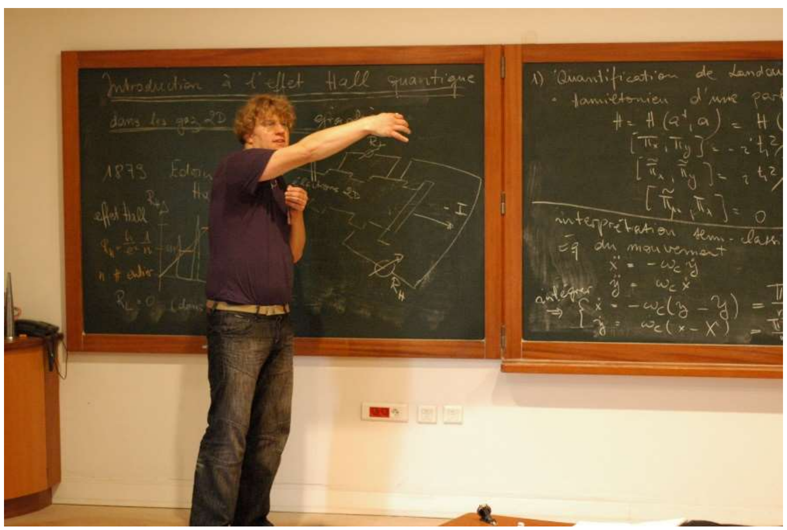
Suppression of phonon scattering (QHE)



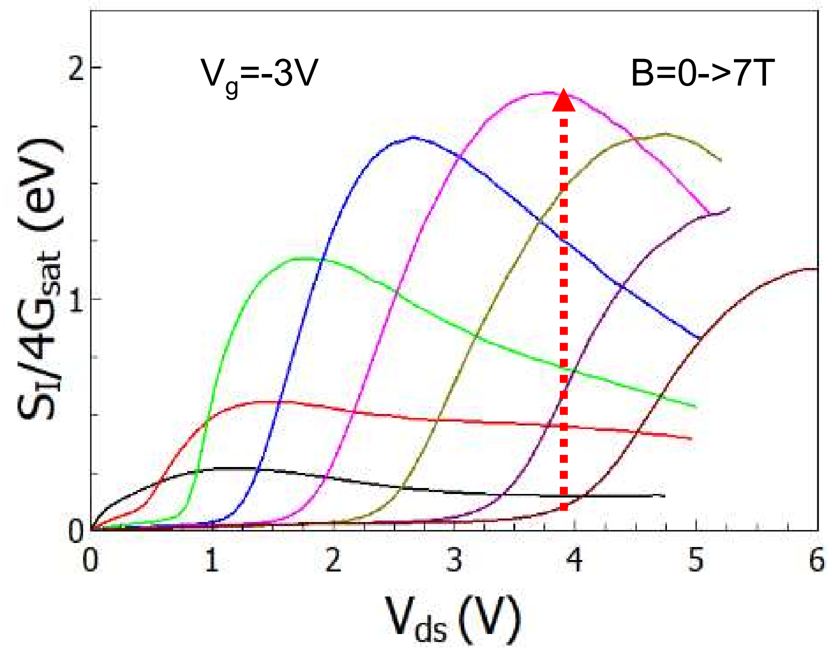
V_g=-3V

Discreteness of Landau Levels turns phonon scattering to resonnant : $q_{phonon} = 2k_F$

$P_{ph}(QHE) \to 0$ and $k_B T_e \sim eV_{ds}/2$



Mark Goerbig in Cargèse (2008)



W. Yang et al, Drift induced collective breakdown of quantum Hall effect in graphene, in preparation (2018)

Make our planet green again

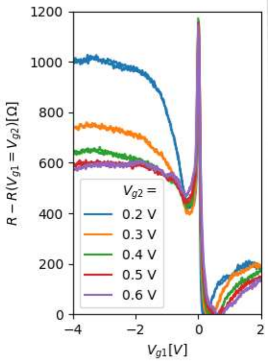
WF	Low-T AC	Super-collisions	OP	Hyperbolic cooling	QHE
$P \propto T^2$	$\propto T^3$	$\propto T^4$	$\propto exp[T/\Omega_{OP}]$	population inversion	forbidden
$\leq 10^8\ W/m^2$	$10^8\ W/m^2$	$3\ 10^8\ W/m^2$	$10^9\ W/m^2$	$3\ 10^9\ W/m^2$	0

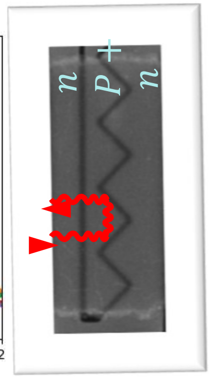
Outline

I. Low-field : from DC to high frequency
- Field-effect, density of states, conductivity,
- Scattering, mean free-path and mobility
- Quantum capacitance and kinetic inductance

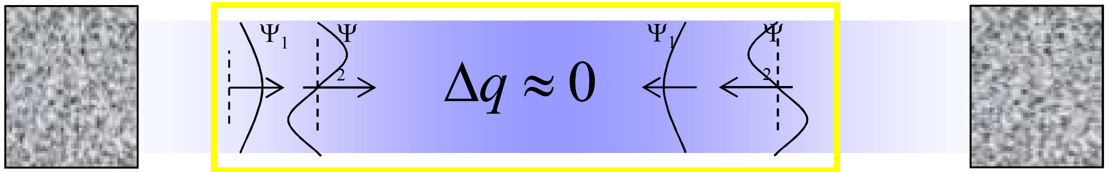
II. High-Field : transport
- Current saturation by optical phonon scattering
- Hot electrons effects and phonon relaxation

III. Ballistic's
- Landauer conductance and shot noise
- Klein tunneling across p-n junctions
- Dirac Fermion optics devices





Dirac Fermion waves



Fermi momentum : $k_F = \sqrt{\pi n}$ $\lambda_F = 10 - 100\ nm$

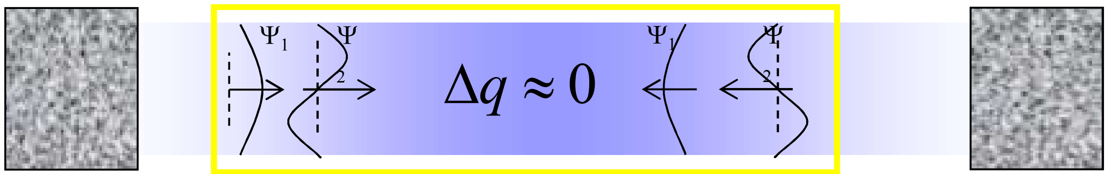
Number of modes: $W = M\, \lambda_F / 2$ $\mathrm{M}(n_{12})/W = 56\ \mu m^{-1}$

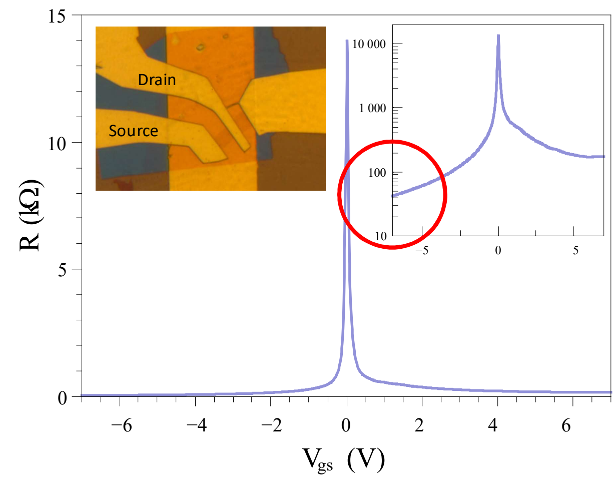
Conductance is transmission : $G = M \times R_{Landauer} = \frac{4e^2}{h} \times \frac{k_F W}{\pi}$

2-Terminal Landauer contact resistance : $\mathrm{R}(n_{12})\mathrm{W} = 114\ \Omega$

Phonon limited ballistic length : $l_b \leq l_{mfp}(T) = \frac{\pi\hbar\sigma_{ph}(T)}{k_F}$ $l_{ph}(n_{12}) \approx 0.7 \times \frac{300K}{T}\ \mu m$

Playing with Dirac Fermion waves





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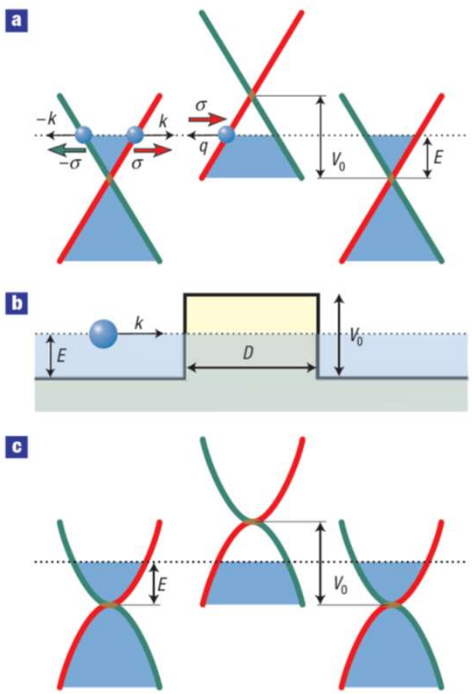
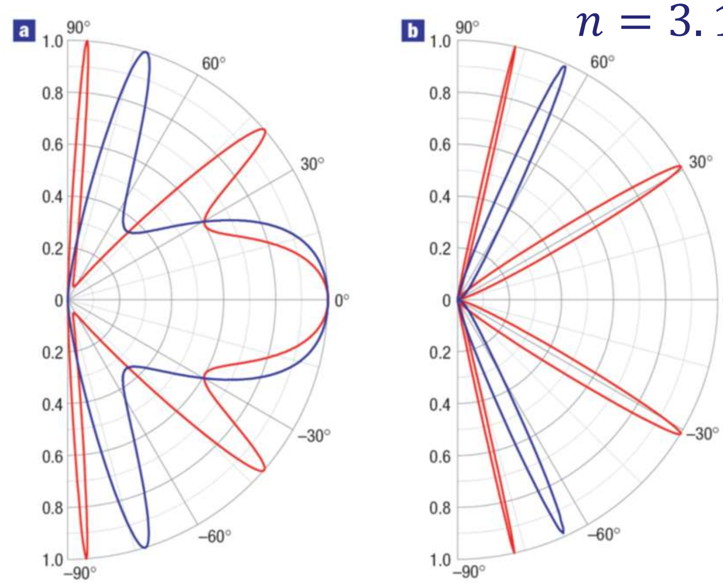
Klein tunneling in p-n-p transistor theory

SLG / BLG



Klein tunneling   anti-Klein tunneling



$n = 3.10^{12}$

$n = 1.10^{12}$

$p = 0.5\ 10^{12}$

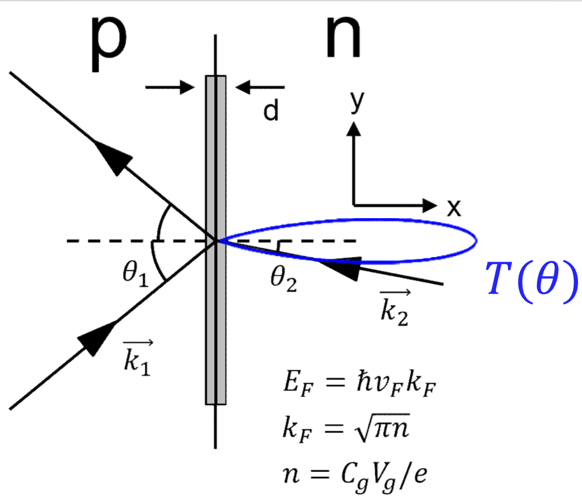
Transmission amplitude pattern is ruled by pseudo-spin conservation

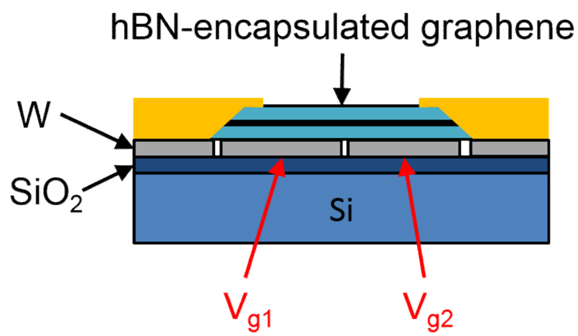
Secondary lobes are artefacts of sharp junction limit (not seen in experiment)

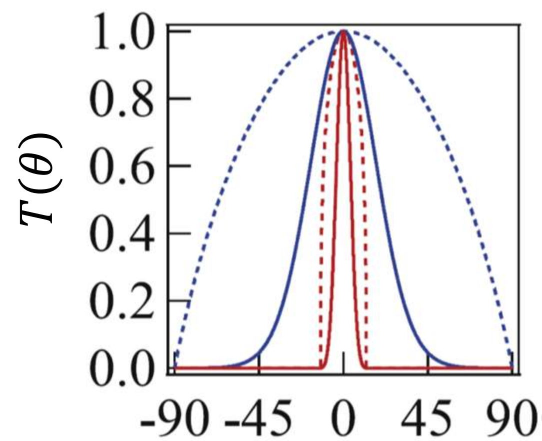
Transport averages over incidence angle according to $\langle T(\theta)\cos\theta\rangle$

Katsnelson-Novoselov-Geim, Nat. Phys. (2006) Chiral tunneling and the Klein paradox in graphene

Dirac Fermion optics





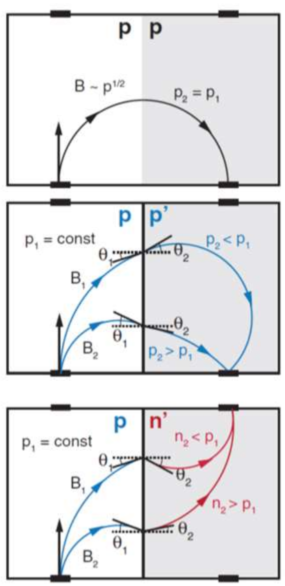


	Photon optics (3D)	Dirac fermion optics (2D)
Medium	transparent	ballistic
Phase velocity	$3 \cdot 10^8\ ms^{-1}$	$10^6\ ms^{-1}$
Snell-Descartes	$n_1 sin\theta_1 = n_2 sin\theta_2$	$E_{F1} sin\theta_1 = E_{F2} sin\theta_2$
Critical angle	$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$	$\theta_c = \arcsin\left(\frac{E_{F2}}{E_{F1}}\right)$
Fresnel relation	$R_s = \left\lvert\frac{n_1 cos\theta_i - n_2 cos\theta_t}{n_1 cos\theta_i + n_2 cos\theta_t}\right\rvert^2$	$T(\theta) = e^{-\pi\frac{2d}{\lvert k_1 - k_2\rvert}k_2^2 \sin^2\theta}$

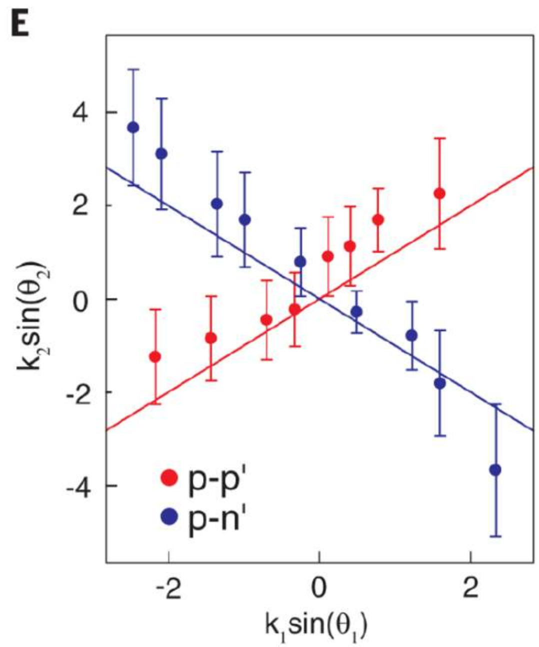
Smooth junction model ($k_F d \geq \pi$): Cheianov et al., PRB ***74*** *(2006) 041403(R)*
abrupt junction model: J. Cayssol et al., PRB ***79*** *(2009) 075428*

Dirac Fermion refraction at a p-n junction

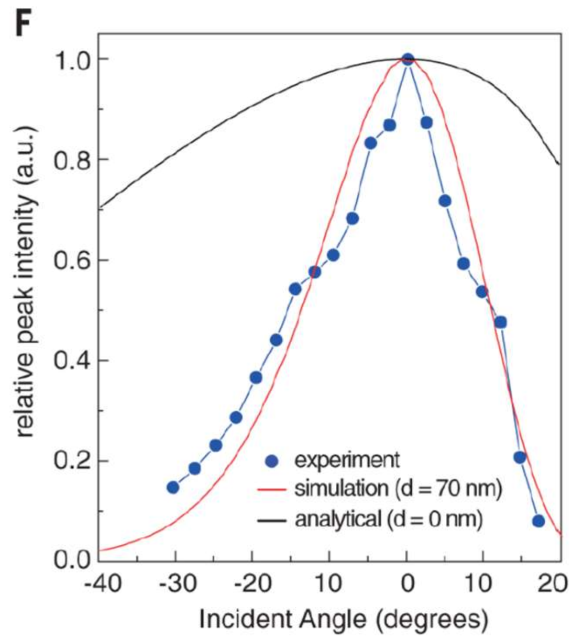
Magnetic focussing



Snell-Descartes



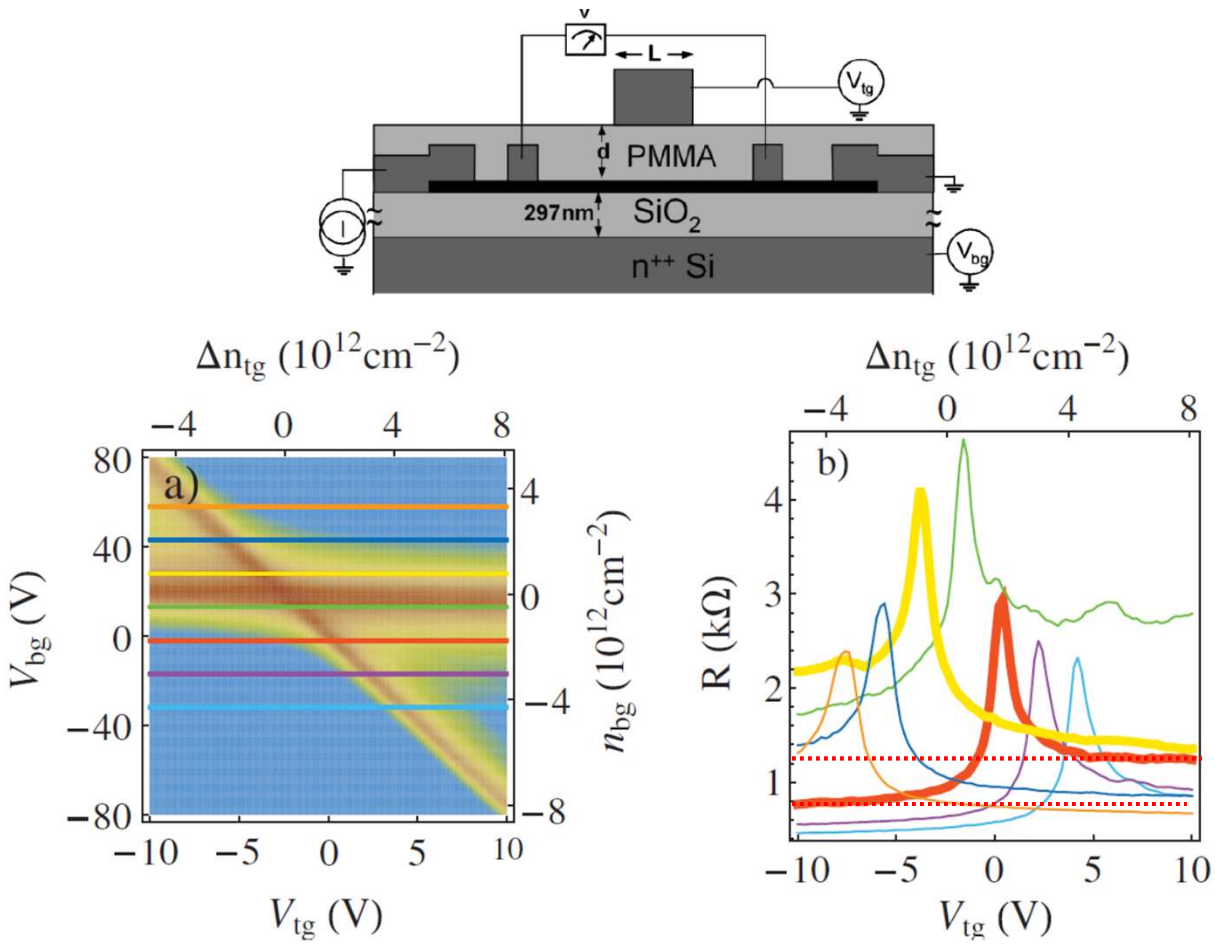
Fresnel



S. Chen et al., Science 353 (2016) Electron optics with p-n junctions in ballistic graphene
G.H. Lee et al., Nat. Phys. (2015) Observation of negative refraction of Dirac fermions in graphene

Klein tunneling in p-n-p barrier

A potential barrier (p-n-p transistor) has a finite resistance (Klein paradox)

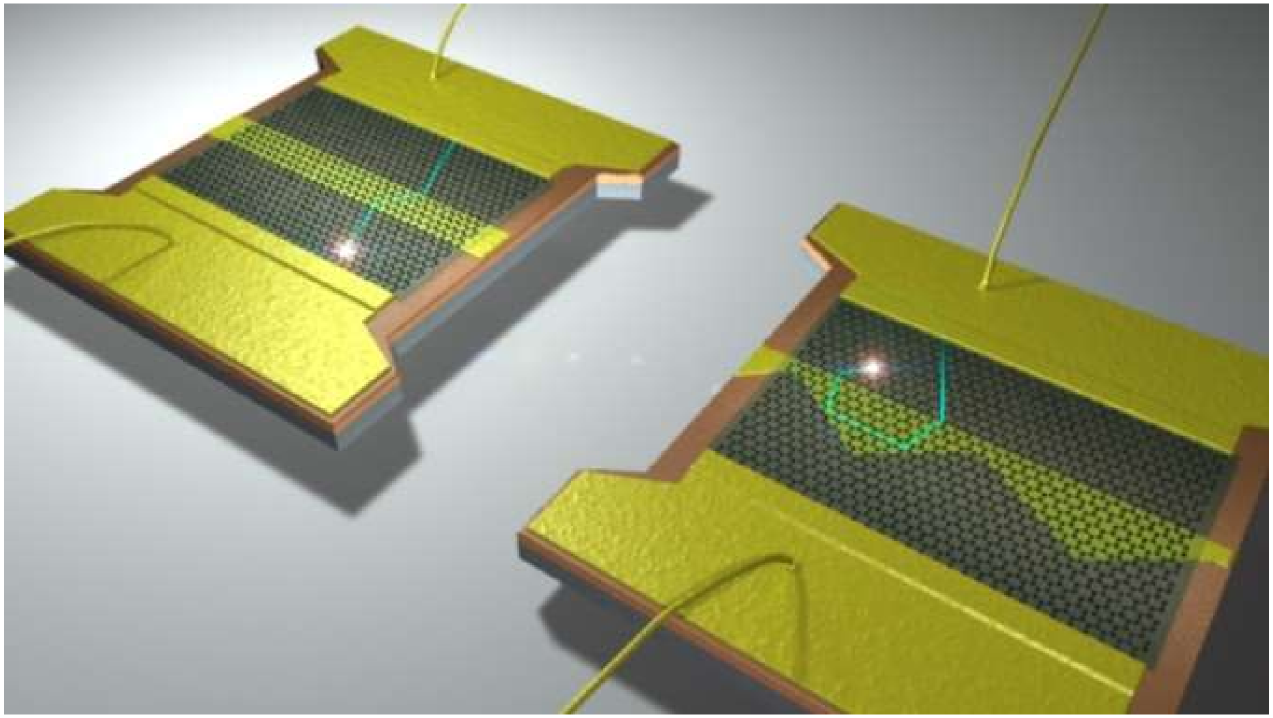


$R_{pnp} \approx 500\Omega$

N. Stander, et al., PRL 2009 : Evidence for Klein Tunneling in Graphene p-n Junctions

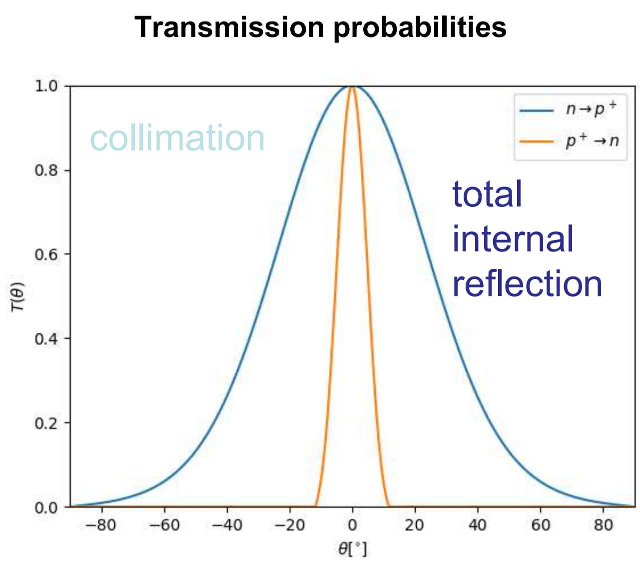
Is it possible to stop the race of Dirac Fermions ?

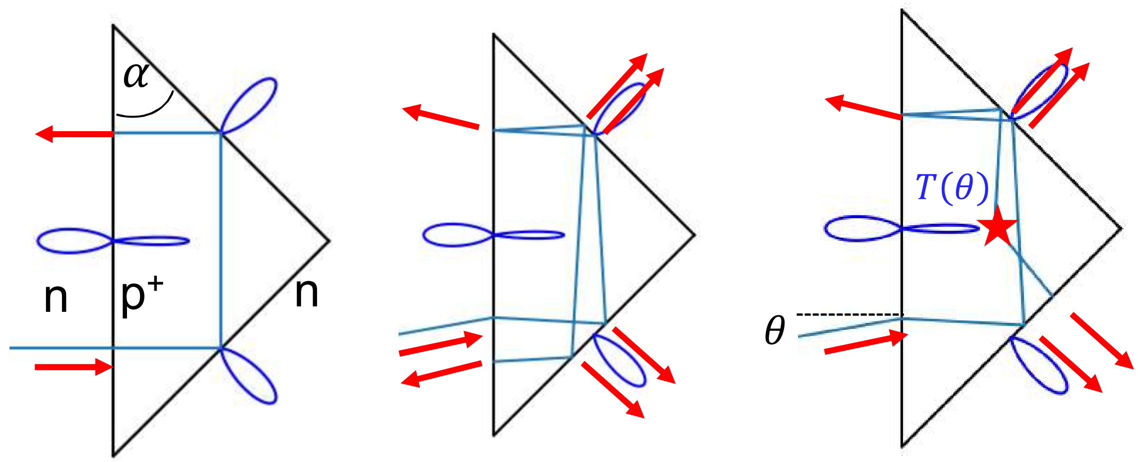
…… Yes, using a Dirac Fermion reflector



Q. Wilmart et al., 2D Materials (2014) Klein tunneling transistor in ballistic graphene..

...... Yes, using a Dirac Fermion reflector

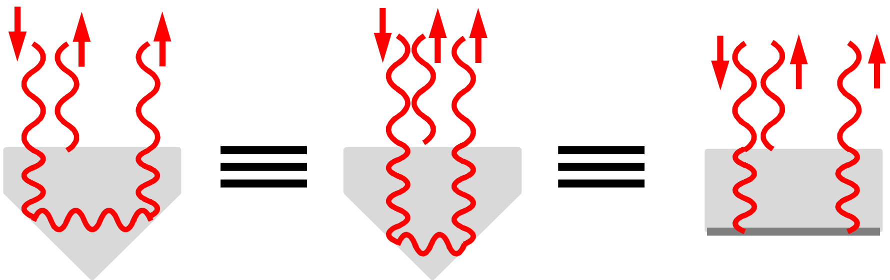


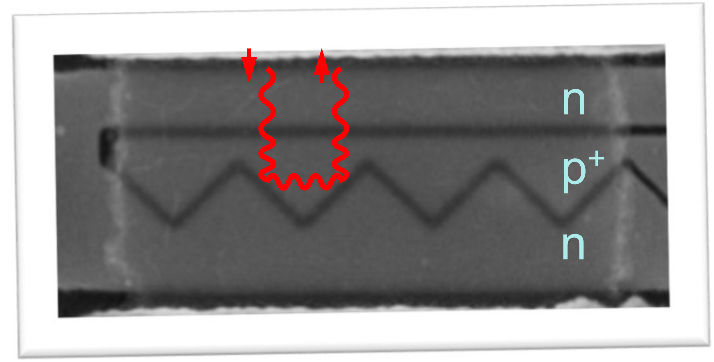


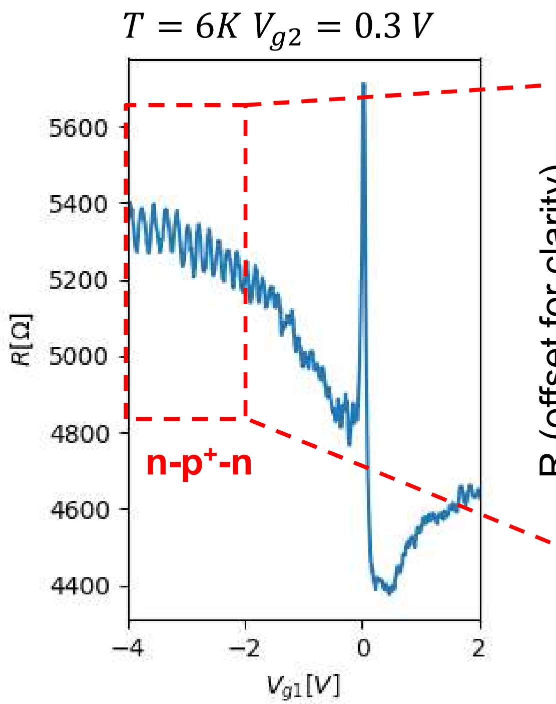
Bicycle photon reflector

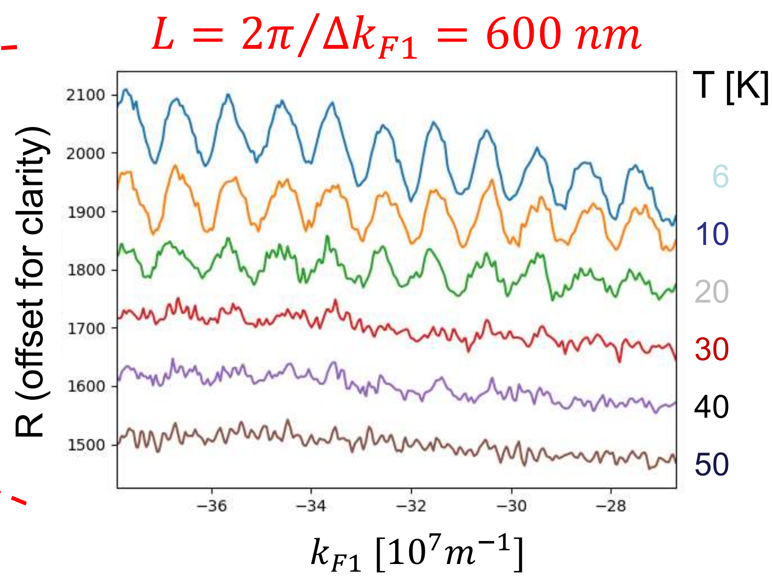
Q. Wilmart et al., 2D Materials (2014) Klein tunneling transistor in ballistic graphene.
S. Morikawa et al., Semicond. Sci. Technol. (2017) DF reflector by graphene sawtooth-shaped n-p-n junctions.

The Dirac Fermion reflector: coherent regime





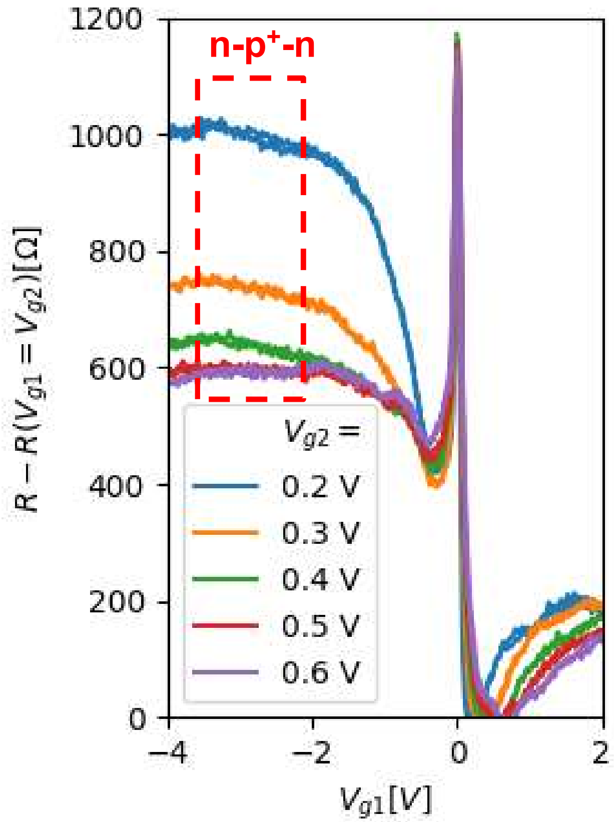




$$T_{smear} = \frac{h v_F}{2 k_B L} = 40\ K$$

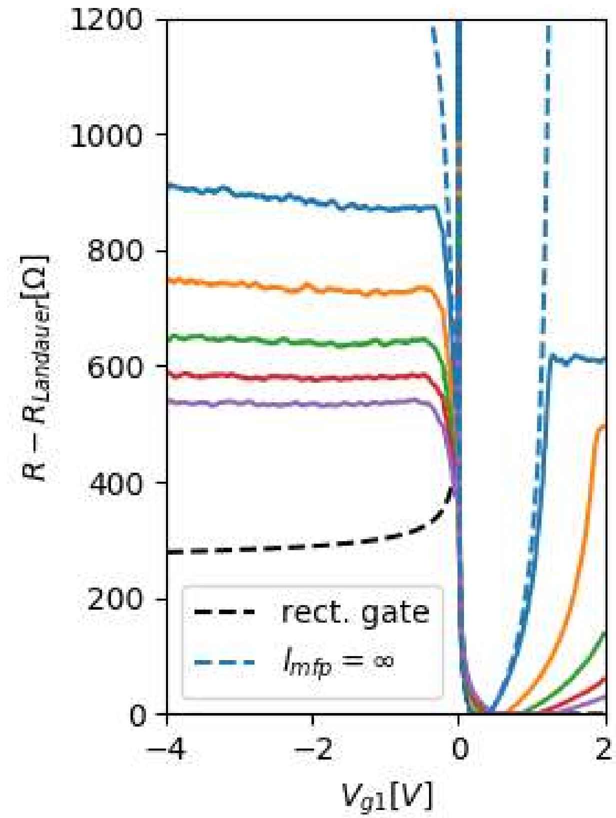
H. Graef, D. Mele et al., in preparation (2018)

DC experiment at 60K



Scattering simulation

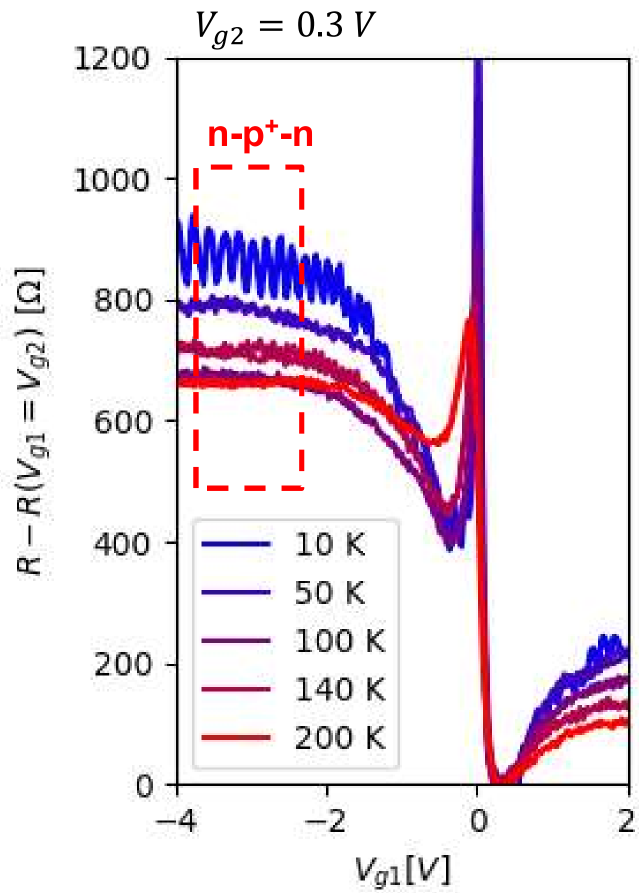
$l_{mfp} = 2.4\ \mu m$

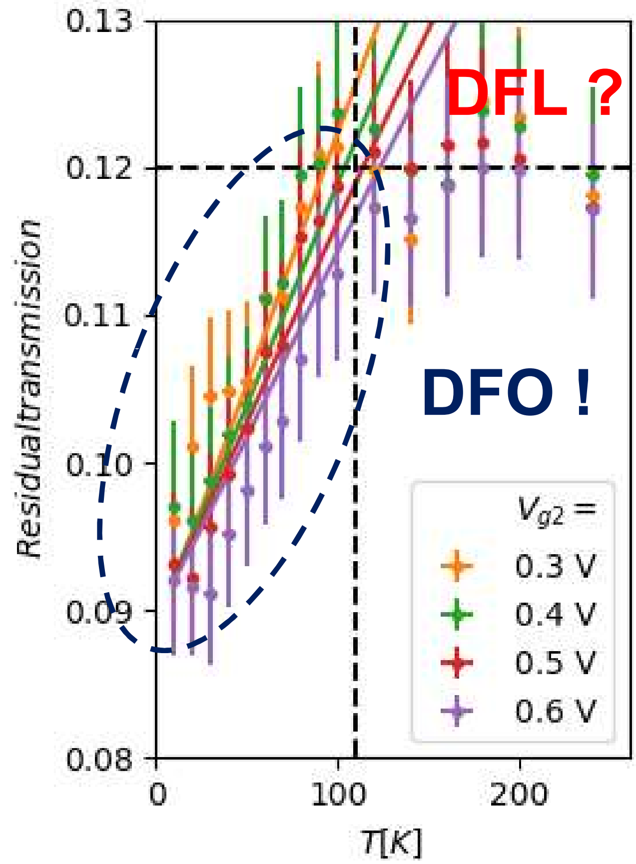


Significant reflection effect (resistance plateau) obeying DF optical index dependence

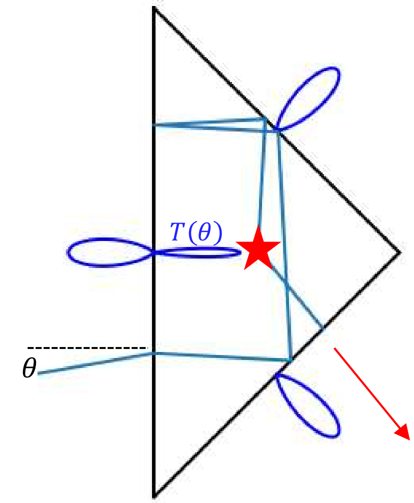
H. Graef, D. Mele et al., in preparation (2018)

Effect of AC phonon scattering





$$\frac{1}{R} = \frac{4e^2}{h}\frac{k_{F2}W}{\pi} < T(\theta) >_\theta$$



$$T_{DFR} = \int_{-\pi/2}^{+\pi/2} T(\theta)\big(1 - T(\theta)\big)^{l_{phonon}/L} \cos(\theta) d\theta \qquad T(\theta) = e^{-\pi\frac{2d}{|k_1 - k_2|}k_2^2 \sin^2\theta}$$

Dirac Fermion Reflectors work up to 100K !
New physics above 100 K ? Viscous Dirac Liquid ?

Take home messages

- Graphene as a 2D plateform
- Graphene/BN : the paradim of a Dirac fluid
  - Minute coupling to the lattice
  - Dirac Fermion gas at low temperature
  - Viscous liquid above 100K
- Prominent, and tunable, hot electron effects
  - Possibility of radiative relaxation in Zener regime
- Applications: electronic and optoelectronic devices
- Good compliance to exotic electronic orders
  - Spin transport
  - Superconductivity