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# "Spin squeezing for quantum metrology"

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In the two lectures, I will introduce the concept of spin squeezing and its interest for quantum metrology. As a physical example I will consider the one-axis twisting spin squeezing process that occurs in a two-component interacting Bose-Einstein condensate. For this example I will present two-mode analytical calculations and a more advanced analysis of the limitations due to decoherence and finite temperature. Always within the frame of the one-axis twisting Hamiltonian, I will also introduce entangled states beyond spin squeezing, such as Schrödinger cats and their sensitivity to decoherence.

1. Entangled states for quantum metrology
   (a) Spin squeezing
   (b) Frequency measurement by Ramsey sequence
   (c) Frequency measurement using a maximally entangled state
   (d) Phase estimation, Fisher information, Hellinger distance
   (e) Quantum Fisher information

2. Entangled states by interactions in bimodal BEC
   (a) Non-linear Hamiltonian
   (b) Squeezing by non-linear dynamics
   (c) Squeezing limit in presence of decoherence
   (d) Microscopic description: dephasing due to particle losses
   (e) Schrödinger cats by non-linear dynamics
   (f) Fidelity of the cat state generation in the presence of losses
   (g) Fidelity of the cat state generation at finite temperature

### Experimental results

From 2016 Review of Modern Physics, Non-classical states of atomic ensembles: fundamentals and applications in quantum metrology, L. Pezze, A. Smerzi, M. Oberthaler, R. Schmied, P. Treutlein

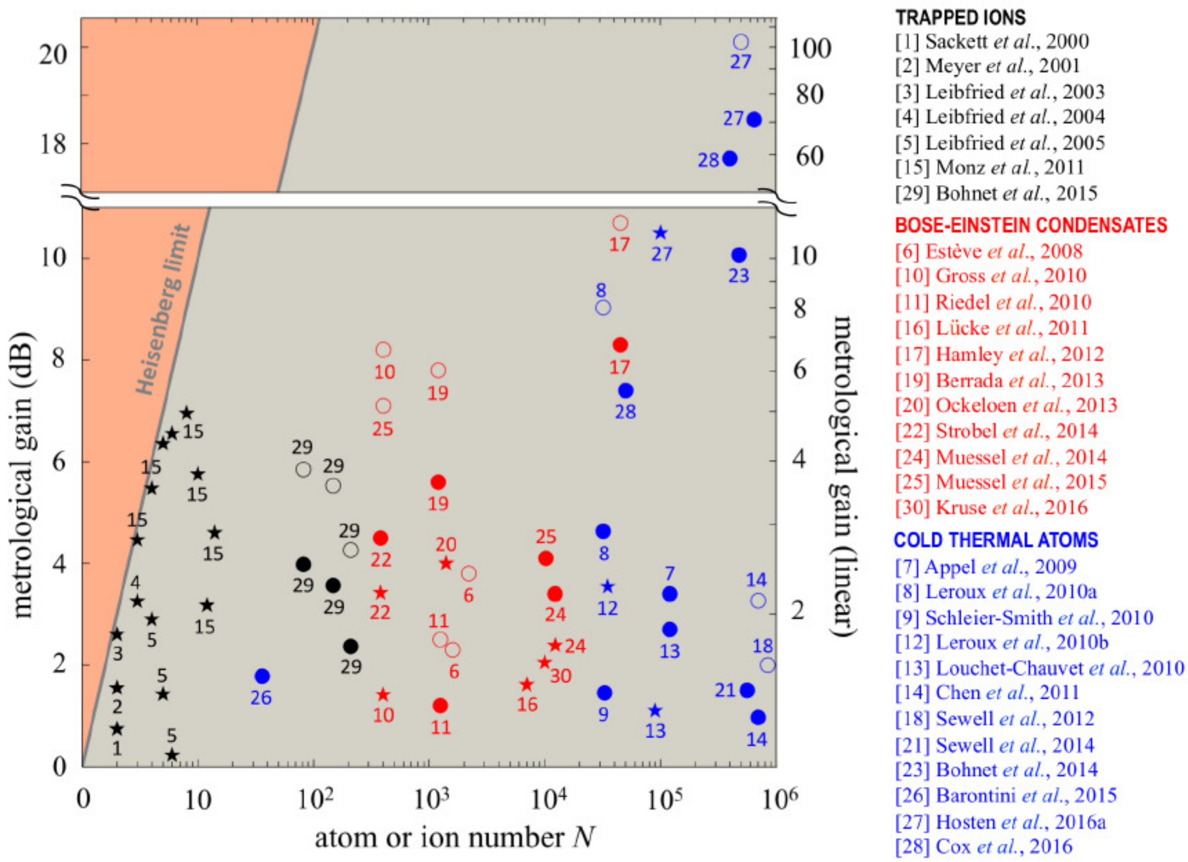


FIG. 2 **Summary of experimental achievements.** Gain over the standard quantum limit $(\Delta\theta_{\mathrm{SQL}})^2 = 1/N$ achieved experimentally with trapped ions (black symbols), Bose-Einstein condensates (red) and cold thermal ensembles (blue). The gain is shown on logarithmic [left, dB, $10\log_{10}(\Delta\theta_{\mathrm{SQL}}/\Delta\theta)^2$] and linear [right] scale. The solid thick line is the Heisenberg limit $(\Delta\theta_{\mathrm{HL}})^2 = 1/N^2$. Stars refer to directly measured phase sensitivity gains, performing a full phase estimation experiment. Circles are expected gains based on a characterization of the quantum state, *e.g.* calculated as $(\Delta\theta)^2 = \xi_{\mathrm{R}}^2/N$, where $\xi_{\mathrm{R}}^2$ is the spin-squeezing parameter, or as $(\Delta\theta)^2 = 1/F_{\mathrm{Q}}$, where $F_{\mathrm{Q}}$ is the quantum Fisher information, see Sec. II. Filled (open) circles indicate results obtained without (with) subtraction of technical and/or imaging noise. Every symbol is accompanied by a number (in chronological order) giving the reference reported in the side table. Here $N$ is the total number of particles or, in presence of fluctuations, the mean total.

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# I Entangled states for quantum metrology

## 1. Spin squeezing

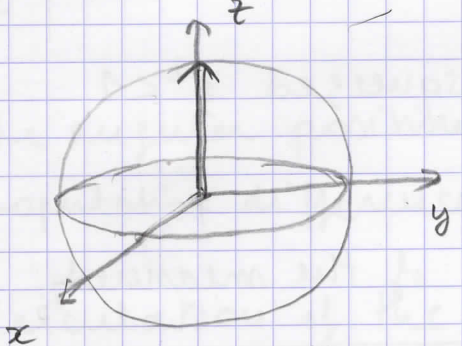
Ensemble of N 2-level atoms $|a\rangle$ $\updownarrow \omega_0$ $|b\rangle$

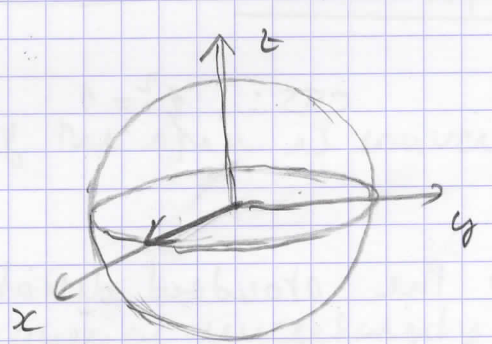
* Collective spin $\frac{N}{2}$ 

$$\hat{S}_x = \frac{1}{2}\sum_{i=1}^{N} |a\rangle\langle b|_i + |b\rangle\langle a|_i$$

$$\hat{S}_y = \frac{1}{2i}\sum_{i=1}^{N} |a\rangle\langle b|_i - |b\rangle\langle a|_i$$

$$[\hat{S}_x, \hat{S}_y] = i\hat{S}_z \quad ; \quad \hat{S}_z = \frac{1}{2}\sum_{i=1}^{N} |a\rangle\langle a|_i - |b\rangle\langle b|_i$$




Bloch sphere

$$|\psi\rangle = |a\rangle^N = \left|\left(\tfrac{N}{2}\right)_z\right\rangle \qquad |\psi\rangle = \left(\frac{|a\rangle + |b\rangle}{\sqrt{2}}\right)^N = \left|\left(\tfrac{N}{2}\right)_x\right\rangle$$

$$\hat{S}_z|\psi\rangle = \frac{N}{2}|\psi\rangle \qquad \text{CSS}$$

$\hat{H} = \hbar\omega_0\hat{S}_z \Rightarrow$ precession of the collective spin around $(Oz)$ at angular frequency $\omega_0$ (atomic clocks)

* Angular uncertainty on the spin position

$$\Delta S_y \, \Delta S_z \geq \frac{1}{2}\left|\langle[\hat{S}_y, \hat{S}_z]\rangle\right| = \frac{1}{2}\left|\langle\hat{S}_x\rangle\right|$$

in $|\psi\rangle = \left|\left(\frac{N}{2}\right)_x\right\rangle$ $\quad \Delta S_z^2 = \Delta S_y^2 = \frac{N}{4} = \frac{\langle S_x\rangle}{2} \Rightarrow$ saturation of inequality

**N.B.** Ind. atoms: $\mathrm{Var}\sum_i s_{zi} = \sum_i \mathrm{Var}\, s_{zi} = N \cdot \frac{1}{4}$

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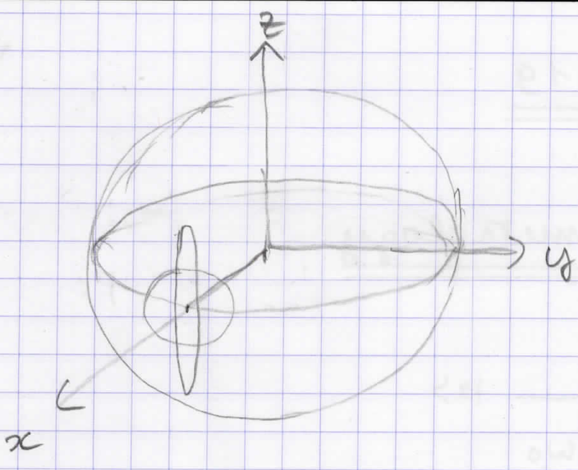



$$\Delta\varphi = \frac{\Delta S_y}{\langle S_x\rangle} = \frac{\sqrt{N}}{2}\frac{2}{N} = \frac{1}{\sqrt{N}}$$

One can reduce $\Delta\varphi$ by having $\Delta S_y^2 < \frac{N}{4}$ for correlated atoms

$$\Delta S_z^2 > \frac{N}{4}$$

$$\langle S_x\rangle \simeq \frac{N}{2}$$

$$\Delta S_y^2 = \underbrace{\sum_i \langle \Delta y_i^2\rangle}_{\frac{N}{4}} + \overbrace{\sum_{i\neq j} \langle \Delta y_i \Delta y_j\rangle}^{<0}$$

"Spin squeezed state"

* Spin squeezing parameter

$$\xi^2 = \frac{N\,\Delta S_\perp^2}{|\langle \vec{S}\rangle|^2}$$

CSS: $\xi^2 = 1$ ; squeezed $\xi^2 < 1$

$\xi$ is related to the standard deviation of the measured frequency in an atomic clock.

Ramsey sequence with precession time T:

$$\Delta\omega_0^{CSS} = \frac{1}{\sqrt{N}\,T} \quad ; \quad \Delta\omega_0^{sq} = \frac{\xi}{\sqrt{N}\,T} \qquad \xi < 1$$

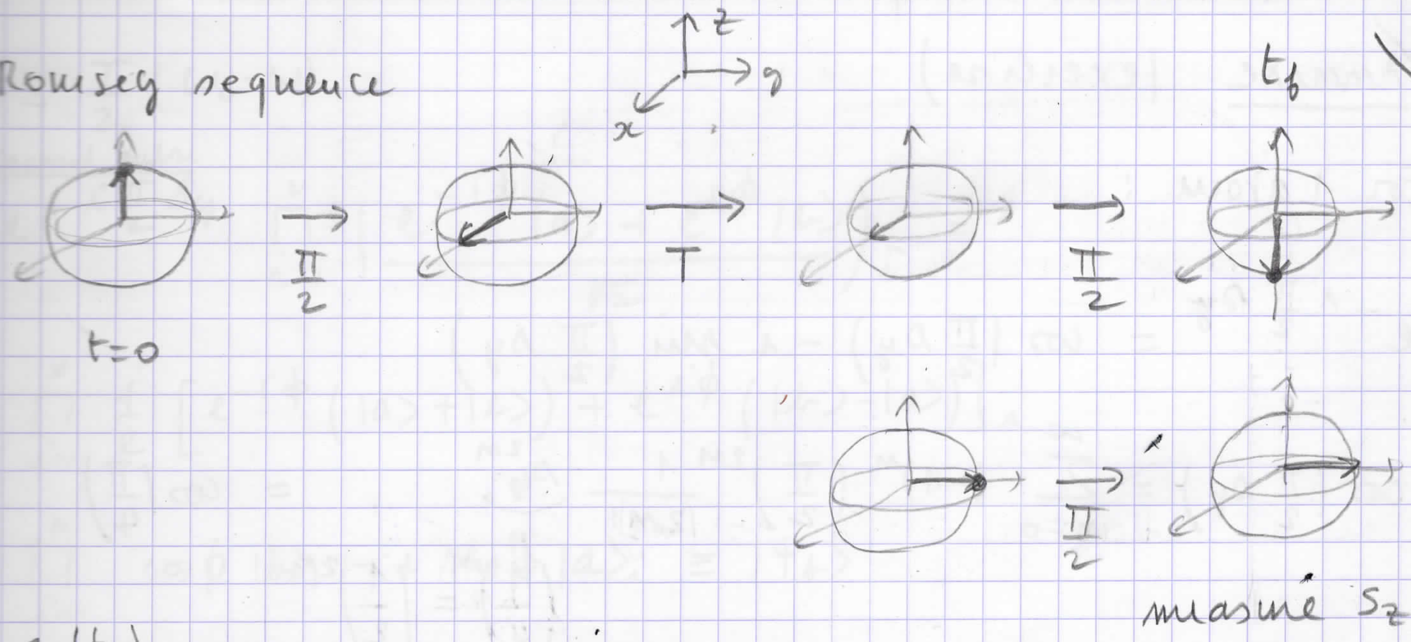
quantum contribution to the "short term stability" in an atomic clock

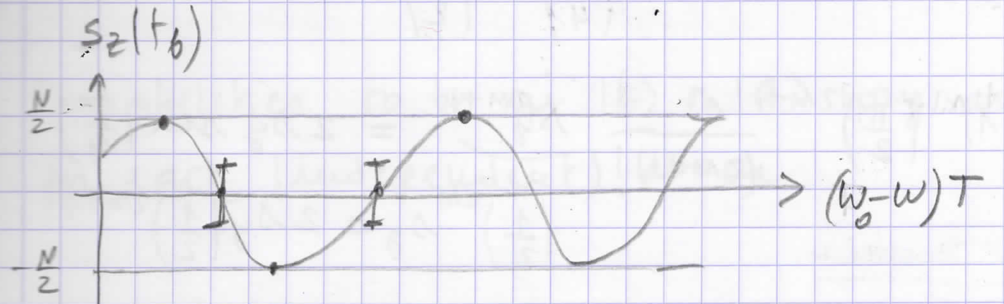
## 2. Frequency measurement by Ramsey sequence

Let us see more in detail how $\omega_0$ is measured in an atomic clock working with independent atoms

[Wineland PRA 50 1050 (1994)]

Ramsey sequence




the angular position of the spin is converted into a population difference

Calculation of the frequency uncertainty

* Initial state

$$|\psi(0)\rangle = \left|\left(\frac{N}{2}\right)_z\right\rangle = \prod_i |a\rangle_i$$

* First pulse

$$e^{-i\frac{\pi}{2}S_y}|\psi(0)\rangle = \prod_i \frac{1}{\sqrt{2}}\left(1-s_+ + s_-\right)_i |a\rangle_i$$

$$= \prod_i \frac{1}{\sqrt{2}}\left(|a\rangle + |b\rangle\right)_i = \left|\left(\frac{N}{2}\right)_x\right\rangle$$

used for one atom :

$$e^{-i\frac{\pi}{2}s_y} = \frac{1}{\sqrt{2}}\left(1-s_+ + s_-\right)$$

proof in the next page

* Free precession

$$e^{-i(\omega_0-\omega)TS_z}\left|\left(\frac{N}{2}\right)_x\right\rangle = \prod_i \frac{1}{\sqrt{2}}\left(e^{-i\phi}|a\rangle + e^{i\phi}|b\rangle\right)_i$$

$$\phi \equiv \frac{T}{2}(\omega_0 - \omega)$$

* Second pulse

$$e^{-i\frac{\pi}{2}S_y} \prod_{i=1}^{N} \left( \frac{e^{-i\phi}|a\rangle + e^{i\phi}|b\rangle}{\sqrt{2}} \right)_i =$$

$$= \prod_{i=1}^{N} \frac{1}{2}\left[ e^{-i\phi}(|a\rangle + |b\rangle) + e^{i\phi}(|b\rangle - |a\rangle) \right]_i$$

$$= \prod_{i=1}^{N} \cos\phi\, |b\rangle_i - i \sin\phi\, |a\rangle_i \equiv |\psi_f\rangle$$

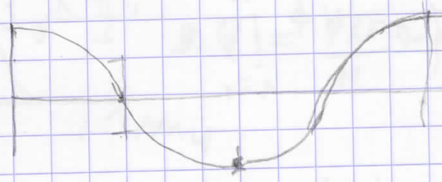
Probabilities to be in $|b\rangle$ or $|a\rangle$ vary as $\cos^2\phi$ and $\sin^2\phi$ for each (independent) atom

* population measurement

$$\langle \psi_f | S_z | \psi_f \rangle = \sum_{i=1}^{N} \frac{1}{2} \{ (\cos\phi \langle b| + i \sin\phi \langle a|)_i \cdot$$
$$\cdot (-\cos\phi |b\rangle - i \sin\phi |a\rangle)$$

$$= \sum_{i=1}^{N} \frac{1}{2}(-1)(\cos^2\phi - \sin^2\phi) = -\frac{N}{2}\cos(2\phi)$$

$$\boxed{\langle S_z \rangle (T) = -\frac{N}{2}\cos(\omega_0 - \omega)T}$$



$$\Delta S_z^2 = \sum_i \langle s_{zi}^2 \rangle - \langle s_{zi} \rangle^2 = \frac{N}{4} - \frac{N}{4}\cos^2(\omega_0 - \omega)T$$

$$\boxed{\Delta S_z = \sqrt{\frac{N}{4}}\, |\sin(\omega_0 - \omega)T|}$$

$$\Delta \omega = \frac{\Delta S_z}{\left| \frac{\partial \langle S_z \rangle}{\partial \omega_0} \right|} = \left| \frac{\sqrt{\frac{N}{4}} \sin(\omega_0 - \omega)}{\frac{N}{2} T \sin(\omega_0 - \omega)} \right| = \frac{1}{\sqrt{N}\,T}.$$

Usually exps work on the point of steepest slope.

With a spin squeezed state $\Delta\omega = \frac{\xi}{\sqrt{NT}}$

## 3. Frequency measurement using a maximally entangled state

$$|\psi_M\rangle = \frac{|(\frac{N}{2})_z\rangle + |(-\frac{N}{2})_z\rangle}{\sqrt{2}} = \frac{|N,0\rangle + |0,N\rangle}{\sqrt{2}}$$

We cannot use this state in a standard Ramsey interferometer, another measurement will be used.

$$|\psi(a)\rangle = e^{i\frac{\pi}{2}J_y}|\psi_M\rangle \xrightarrow{\frac{\pi}{2}} |\psi_M\rangle$$

* free precession $\phi \equiv \frac{T}{2}(\omega_0-\omega)$

$$e^{-i(\omega_0-\omega)TS_z}|\psi_M\rangle = \frac{1}{\sqrt{2}}\left\{e^{-iN\phi}|(\tfrac{N}{2})_z\rangle + e^{iN\phi}|(-\tfrac{N}{2})_z\rangle\right\}$$

* second $\frac{\pi}{2}$ pulse

$$|\psi_b\rangle = \frac{1}{\sqrt{2}}\left\{e^{-iN\phi}\prod_{i=1}^{N} e^{-i\frac{\pi}{2}S_{yi}}|a\rangle_i + e^{iN\phi}\prod_{i=1}^{N} e^{-i\frac{\pi}{2}S_{yi}}|b\rangle_i\right\}$$

$$= \frac{1}{\sqrt{2}}\left\{e^{-iN\phi}\prod_i\left(\frac{|a\rangle+|b\rangle}{\sqrt{2}}\right)_i + e^{iN\phi}\prod_i\left(\frac{|b\rangle-|a\rangle}{\sqrt{2}}\right)_i\right\}$$

* parity measurement $\tilde{O} = \prod_{i=1}^{N} \overbrace{2S_{zi}}^{\sigma_{zi}}$

$m_a + m_b = N$

$m_a - m_b = 2m$

$$\tilde{O}|m_z\rangle = 1^{m_a}(-1)^{m_b} = (-1)^{\frac{N}{2}-m_z}$$

$$\tilde{O}\prod_i\left(\frac{|b\rangle-|a\rangle}{\sqrt{2}}\right)_i = \prod_i(-1)\left(\frac{|b\rangle+|a\rangle}{\sqrt{2}}\right)_i = (-1)^N\prod_i\left(\frac{|b\rangle+|a\rangle}{\sqrt{2}}\right)_i$$

$$\langle \tilde{O} \rangle_b = \frac{1}{2}\{e^{2iN\phi} + e^{-2iN\phi}\}(-1)^N = (-1)^N \cos(N(\omega_0-\omega)T)$$ 6

$$\boxed{\langle \tilde{O} \rangle_b = (-1)^N \cos(N(\omega_0-\omega)T)}$$

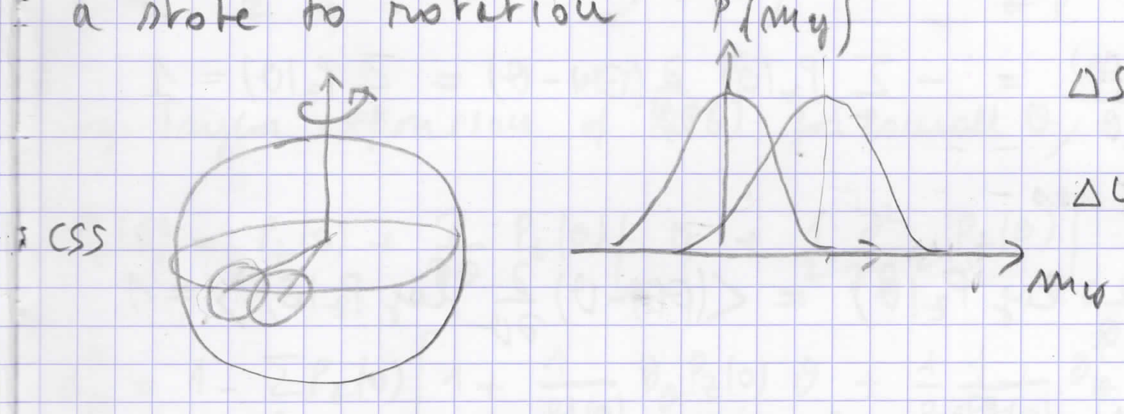
N.B. $\tilde{O}^2 = \mathbb{1} \Rightarrow \Delta\tilde{O}^2 = 1 - \cos^2(N(\omega_0-\omega)T)$

$$\boxed{\Delta\tilde{O} = |\sin(N(\omega_0-\omega)T)|}$$

$$\Delta\omega = \frac{\Delta\tilde{O}}{\left|\frac{\partial\langle\tilde{O}\rangle}{\partial\omega}\right|} = \left|\frac{\sin(N(\omega_0-\omega)T)}{NT\sin(N(\omega_0-\omega)T)}\right| = \frac{1}{NT}.$$

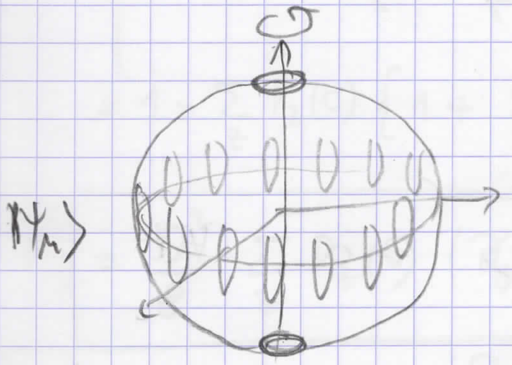
we find, with respect to the CSS, an enhancement factor N in front of the frequency.

More generally, we can ask what is the sensitivity of a state to rotation




$\Delta S_y = \frac{\sqrt{N}}{2}$

$\Delta\varphi = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$




The state moves at a much thinner scale $\Delta\varphi \sim \frac{1}{N}$

(and clearly the variances and mean of $\vec{S}$ are not enough to characterize it)

## 4. Phase estimation, Fisher information, Hellinger distance

* Cramer Rao bound

$\rho \to \rho_\theta = e^{-i\hat{A}\theta}\rho\, e^{i\hat{A}\theta} \Rightarrow$ measure an observable $\hat{Z}$

Let $P_z|\theta) \equiv P(z|\theta)$ the conditional distribution of the random variable $z$ for a value of $\theta$

And let $\Theta(z)$ be an UNBIASED ESTIMATOR of $\theta$ :

$$\boxed{\langle\Theta\rangle \equiv \sum_z \Theta(z)\, P_z|\theta) = \theta}$$

$$\begin{cases} \langle\Theta\rangle = \theta \\ \sum_z P_z|\theta) = 1 \end{cases} \Rightarrow \begin{cases} \frac{\partial\langle\Theta\rangle}{\partial\theta} = 1 \\ \frac{\partial}{\partial\theta}\sum_z P_z|\theta) = 0 \end{cases}$$

$$\Rightarrow 0 = \frac{\partial}{\partial\theta}\left[\sum_z \Theta(z)P_z|\theta) - \theta\sum_z P_z|\theta)\right] = \frac{\partial}{\partial\theta}\left[\sum_z (\Theta(z)-\theta)\, P_z|\theta)\right]$$

$$\Rightarrow \sum_z (\Theta(z)-\theta)\frac{\partial P_z(\theta)}{\partial\theta} = -\sum_z P_z|\theta)\frac{\partial}{\partial\theta}(\Theta(z)-\theta) = \sum_z P_z|\theta) = 1$$

we can rewrite it as

$$\sum_z (\Theta(z)-\theta)\, P_z|\theta)\frac{\partial}{\partial\theta}\log P_z|\theta) = \langle(\Theta(z)-\theta)\frac{\partial}{\partial\theta}\log P_z|\theta)\rangle = 1$$

Using the Cauchy-Schwarz inequality

$$\langle f g\rangle^2 \leq \langle f^2\rangle\langle g^2\rangle$$

$$\boxed{\begin{array}{l} \langle(f+tg)^2\rangle = \langle f^2\rangle + t^2\langle g^2\rangle + 2t\langle fg\rangle \geq 0 \quad \forall t \\ \Rightarrow \Delta \leq 0 \qquad \cancel{4}\langle fg\rangle^2 - \cancel{4}\langle f^2\rangle\langle g^2\rangle \leq 0 \end{array}}$$

$$\langle(\Theta(z)-\theta)^2\rangle\langle\left(\frac{\partial}{\partial\theta}\log P_z|\theta)\right)^2\rangle \geq 1$$

$$\boxed{\Delta\Theta^2 \geq \frac{1}{F}} \qquad \boxed{F = \sum_z P_z(\theta)\left(\frac{\partial}{\partial\theta}\log P_z(\theta)\right)^2}$$

variance of the estimator            Fisher information

CRAMER-RAO BOUND

Indeed F gives information about how fast $P_z(\theta)$ changes with $\theta$. This is clearly pointed out by $d_H^2$

* Hellinger distance $d_H$

$d_H$ is also accessible in expts [Strobel et al SCIENCE (2014)] [Chalopin et al NAT COMM 4955 (2018)] vrais spin $\frac{N}{2}=8$ Dy sensitivity to magnetic field enhanced by a factor 14 (16 is the Heisenberg limit)

For the two prob. distrib: $p_0 = P_z(0)$ and $p_\theta = P_z(\theta)$

$$d_H^2 = \frac{1}{2}\sum_z\left(\sqrt{P_z(0)} - \sqrt{P_z(\theta)}\right)^2 = 1 - \sum_z\sqrt{P_z(0)\,P_z(\theta)}$$

by Taylor expansion of $P_z(\theta)$ for small $\theta$,

$$P_z(\theta) = P_z(0) + \frac{\partial}{\partial\theta}P_z(\theta)\Big|_0\,\theta + \frac{1}{2}\frac{\partial^2}{\partial\theta^2}P_z(\theta)\Big|_0\,\theta^2 + \dots$$

$$d_H^2 = 1 - \sum_z P_z(0)\left[1 + \frac{1}{P_z(0)}\partial_\theta P_z(0)\,\theta + \frac{1}{2}\frac{1}{P_z(0)}\partial_\theta^2 P_z(0)\,\theta^2 + o(\theta^2)\right]^{1/2}$$

$$= 1 - \sum_z P_z(0)\left\{1 + \frac{\theta}{2}\frac{1}{P_z(0)}\partial_\theta P_z(0) + \frac{\theta^2}{4}\frac{1}{P_z(0)}\partial_\theta^2 P_z(0) - \frac{\theta^2}{8}\left(\frac{1}{P_z(0)}\partial_\theta P_z(0)\right)^2 + o(\theta^2)\right\}$$

$$d_H^2 = \frac{\theta^2}{8}\sum_z P_z(0)\left(\partial_\theta \log P_z(0)\right)^2 + o(\theta^2)$$

$$\boxed{d_H^2 = \frac{F}{8}\,\theta^2 + o(\theta^2)}$$

N.B. $\sum_z P_z(0) = 1 \Rightarrow \partial_\theta \sum_z P_z(\theta) = 0$

$\partial_\theta^2 \sum_z P_z(\theta) = 0$

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## 5. Quantum Fisher information

An upper bound of F is obtained by maximizing F over all possible measurements

$$F \leq \max_{meas} F \equiv F_Q$$

$F_Q$ depends only on $\hat{\rho}_\theta \Rightarrow F[\hat{\rho}_\theta]$

$$\boxed{\Delta\Theta^2 \geq \frac{1}{F_Q[\hat{\rho}_\theta]}}$$ QUANTUM CRAMER RAO

in the eigenbasis of $\hat{\rho}_\theta = \sum_k q_k |k\rangle\langle k|$

$$\boxed{F_Q[\hat{\rho}] = \sum_{\substack{k,k' \\ q_k+q_{k'}>0}} \frac{2}{q_k+q_{k'}} \left|\langle k'|\partial_\theta \hat{\rho}_\theta|k\rangle\right|^2}$$

For us $\partial_\theta \hat{\rho}_\theta = e^{-i\hat{A}\theta}\left[-i\hat{A}\hat{\rho} + \hat{\rho}\, i\hat{A}\right] e^{i\hat{A}\theta}$

$$\boxed{\partial_\theta \hat{\rho}_\theta = i(\hat{\rho}_\theta \hat{A} - \hat{A}\hat{\rho}_\theta)}$$

$$F = \sum_{k,k'} \frac{2}{q_k+q_{k'}} \langle k|\partial_\theta \hat{\rho}_\theta|k'\rangle\langle k'|\partial_\theta \hat{\rho}_\theta|k\rangle$$

$$= \sum_{k,k'} \frac{2}{q_k+q_{k'}} \left[i(q_k - q_{k'})\langle k|\hat{A}|k'\rangle\right]\left[i(q_{k'}-q_k)\langle k'|\hat{A}|k\rangle\right]$$

$$= \sum_{\substack{k,k' \\ k\neq k'}} \frac{2(q_k-q_{k'})^2}{q_k+q_{k'}} \langle k|\hat{A}|k'\rangle\langle k'|\hat{A}|k\rangle$$

For a pure state $q_{k_0} = 1$

$q_{k\neq k_0} = 0$

$$F = \sum_{k' \neq k_0} 2 \langle \psi_0 | \hat{A} | k' \rangle \langle k' | \hat{A} | \psi_0 \rangle +$$

$$+ \sum_{k \neq k_0} 2 \langle k | \hat{A} | \psi_0 \rangle \langle \psi_0 | \hat{A} | k \rangle$$

$$F = 4 \sum_{k} \left[ \langle \psi_0 | \hat{A} | k \rangle \langle k | \hat{A} | \psi_0 \rangle - \langle \hat{A} \rangle^2 \right] = 4 \, \Delta A^2 .$$

pure state: $\boxed{F_Q = 4 \Delta A^2}$

N.B. $\mathrm{tr}\left[ B(\hat{A}) e^{-i \hat{A} \theta} \hat{\rho} e^{i \hat{A} \theta} \right] = \mathrm{tr}\left[ B(\hat{A}) \hat{\rho} \right] \Rightarrow F_Q$ ind. of $\theta$.

Let us consider the frequency measurements of § 2 and 3

$$\hat{H} = \hbar(\omega_0 - \omega) \hat{S}_z \qquad e^{-\frac{i}{\hbar} \hat{H} T} = e^{-i \theta \hat{A}} \quad \text{with} \begin{cases} \hat{A} = \hat{S}_z \\ \theta = (\omega_0 - \omega) T \end{cases}$$

* in the state $|(\frac{N}{2})_x\rangle$

$$F_Q = 4 \Delta S_z^2 = 4 \frac{N}{4} = N \Rightarrow \Delta\theta \geq \frac{1}{\sqrt{N}} \; ; \; \Delta\omega \geq \frac{1}{\sqrt{N} T}$$

meaning that the considered measurement saturates the QCR bound for the given state.

* in the state $|\psi_M\rangle$

$$F_Q = 4 \Delta S_z^2 = 4 \frac{N^2}{4} = N^2 \, ,$$

Indeed $\langle S_z \rangle = 0$ and

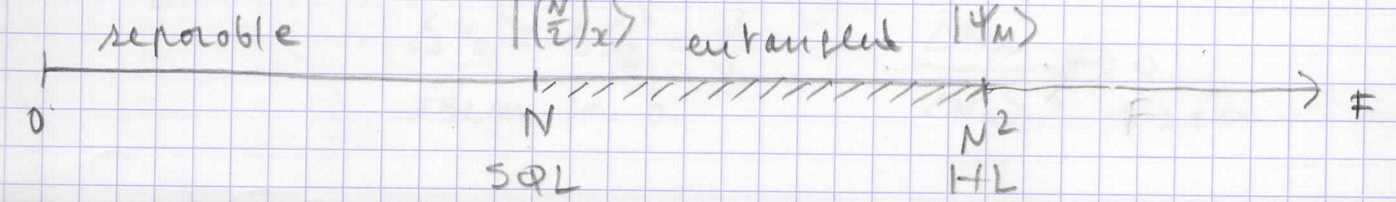
$$\frac{1}{2}\left(\langle \tfrac{N}{2}| + \langle -\tfrac{N}{2}|\right) \hat{S}_z^2 \left( |\tfrac{N}{2}\rangle + |-\tfrac{N}{2}\rangle \right) = \frac{1}{2}\left( \frac{N^2}{4} + \frac{N^2}{4} \right) = \frac{N^2}{4} .$$

$$\Delta\theta \geq \frac{1}{N} \; ; \; \Delta\omega \geq \frac{1}{NT}$$

here also the considered measurement saturates QCR

* One can relate $F_Q$ to the "entanglement" in the system

For a system of $N$ particles and $\hat{\rho}_\theta = e^{-i\theta \hat{S}_\alpha} \hat{\rho} \, e^{i \theta \hat{S}_\alpha}$ ; $\alpha = x, y, z$



# II Entangled states by interactions in bimodal BEC

## 0. Non linear hamiltonian

In order to obtain highly entangled states with a relatively large $N$, BEC are good candidates.

BEC in 2 modes $a, b$ (typically, 2 internal states and same spatial mode) $T=0$.

$N$ indistinguishable bosons in 2 modes: basis $\{|N_a, \hat{N_b}\rangle\}$ $N_b = N - N_a$

$[a, a^+] = [b, b^+] = 1$

$\hat{S}_x = \frac{1}{2}(a^+b + b^+a)$ ; $\hat{S}_y = \frac{1}{2i}(a^+b - b^+a)$ ; $\hat{S}_z = \frac{1}{2}(a^+a - b^+b)$

"phase state" $|\phi\rangle \equiv \frac{1}{\sqrt{N!}}\left(\frac{e^{i\frac{\phi}{2}}a^+ + e^{-i\frac{\phi}{2}}b^+}{\sqrt{2}}\right)^N |0\rangle$

$|(\frac{N}{2})_x\rangle \equiv |\phi = 0\rangle$ ; $\phi$ = relative phase $\in [-\pi, \pi]$

states on the equator of the Bloch sphere with $|\langle\vec{S}\rangle| = \frac{N}{2}$

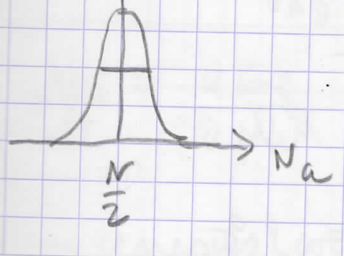
N.B

$\{|N_a, N-N_a\rangle\}$ is the basis for the fully symmetrized subspace of N-spin $d = 2^N$ ; $d_S = N+1$

In the angular momentum language is $\{|J = \frac{N}{2}, M_J\rangle\}$

We will show that, in presence of interactions, the factorized state $|\phi\rangle \to$ entangled state.

* Particle number distribution in $|\phi = 0\rangle$



$\frac{\Delta N_a}{\langle N_a\rangle} = \frac{\sqrt{N}/2}{N/2} = \frac{1}{\sqrt{N}}$ indeed $\boxed{\begin{array}{l} N_a = \frac{N}{2} + S_z \\ N_b = \frac{N}{2} - S_z \end{array}}$

$\Delta N_a^2 = \Delta S_z^2 = \frac{N}{4}$ idem for $b$.

$\frac{\Delta N_a}{\langle N_a\rangle} \xrightarrow[N \to +\infty]{} 0$

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## * Expansion of the energy

$$E(N_a, N_b) = \bar{E} + \frac{\partial E}{\partial N_a}\left(N_a - \frac{N}{2}\right) + \frac{\partial E}{\partial N_b}\left(N_b - \frac{N}{2}\right) +$$

$$+ \frac{1}{2}\left\{ \frac{\partial^2 E}{\partial N_a^2}\left(N_a - \frac{N}{2}\right)^2 + \frac{\partial^2 E}{\partial N_b}\left(N_b - \frac{N}{2}\right)^2 + 2\frac{\partial^2 E}{\partial N_a \partial N_b}\left(N_a - \frac{N}{2}\right) \cdot \left(N_b - \frac{N}{2}\right)\right\} + \dots$$

$$E(N_a, N_b) = \bar{E} + (\mu_a - \mu_b)\hat{S}_z + \frac{1}{2}\left\{\partial_{N_a}\mu_a + \partial_{N_b}\mu_b - \partial_{N_a}\mu_b - \partial_{N_b}\mu_a\right\} S_z^2$$

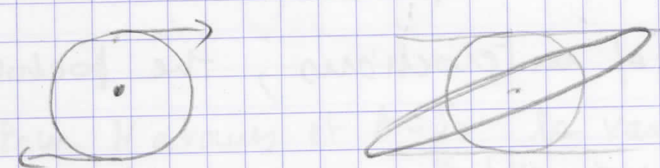
$$\hat{H} = (\mu_a - \mu_b)\hat{S}_z + \hbar\chi \hat{S}_z^2 \qquad \chi = \frac{1}{2\hbar}\left(\partial_{N_a}\mu_a + \partial_{N_b}\mu_b - 2\partial_{N_a}\mu_b\right)$$

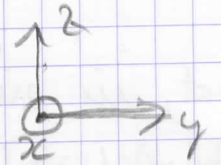
in the symmetric case

$$\boxed{\hat{H}_{NL} = \hbar\chi \hat{S}_z^2} \qquad \underline{\text{N.B.}} \quad [\hat{H}, \hat{N}_a] = [\hat{H}, \hat{N}_b] = 0$$

$\hat{H}_{NL}$ creates squeezing and also maximally entangled states starting from $|\phi\rangle$:

## 1. Squeezing by NL dynamics






$$H_{NL} = \hbar\chi \underbrace{\hat{S}_z}_{\omega_z} \hat{S}_z$$

homogeneous BEC, symmetric spatially sep. modes $\begin{cases} g_{aa} = g_{bb} = g \\ g_{ab} = 0 \end{cases}$

$$\mu_a = \frac{g N_a}{V} \qquad \mu_b = \frac{g N_b}{V} \qquad \Rightarrow \chi = \frac{g}{\hbar V} \xrightarrow[TL]{} 0$$

$$N\chi = \frac{\rho g}{\hbar}$$

finite at the T.L.

Annexe

weakly interacting 2 component Bose gas

$$\hat{H} = \int d^3r \left\{ \sum_{\varepsilon=a,b} \left[ \hat{\Psi}_\varepsilon^\dagger h_\varepsilon \hat{\Psi}_\varepsilon + \frac{1}{2} g_{\varepsilon\varepsilon} \hat{\Psi}_\varepsilon^\dagger \hat{\Psi}_\varepsilon^\dagger \hat{\Psi}_\varepsilon \hat{\Psi}_\varepsilon \right] + \right.$$
$$\left. + g_{ab} \hat{\Psi}_b^\dagger \hat{\Psi}_a^\dagger \hat{\Psi}_a \hat{\Psi}_b \right\}$$

$$[\hat{\Psi}_\varepsilon(\bar{r}), \hat{\Psi}_\varepsilon^\dagger(\bar{r}')] = \delta(\bar{r}-\bar{r}')$$

$$g_{\varepsilon\varepsilon'} = \frac{4\pi\hbar^2}{m} a_{\varepsilon\varepsilon'}.$$ $a_{\varepsilon\varepsilon'}$ s-wave scattering lenght for one atom in $\varepsilon$ - one in $\varepsilon'$.

collisional interactions between cold atoms that occur only in s-wave are modeled by a zero-range potential.

$\hat{\Psi}_a(\bar{r}) = \phi_a(\bar{r})\, \hat{a}_\phi$ ; $\hat{\Psi}_b(\bar{r}) = \phi_b(\bar{r})\, \hat{b}_\phi$ pure condensates

$$H_0 = \sum_\varepsilon N_\varepsilon \left[ \int \phi_\varepsilon^* h_\varepsilon \phi_\varepsilon + \frac{g_{\varepsilon\varepsilon}}{2} (N_\varepsilon - 1) \int |\phi_\varepsilon|^4 \right] + g_{ab} N_a N_b \int |\phi_a|^2 |\phi_b|^2$$

Minimisation of the Energy functional $E[\phi_\varepsilon, N_a, N_b]$ with $(N_\varepsilon - 1) \to N_\varepsilon$ gives the GPEs

$$\left[ h_\varepsilon + g_{\varepsilon\varepsilon} N_\varepsilon |\phi_\varepsilon|^2 + g_{\varepsilon\varepsilon'} N_{\varepsilon'} |\phi_{\varepsilon'}|^2 \right] \phi_\varepsilon = \mu_\varepsilon \phi_\varepsilon$$

with:

$$\boxed{\mu_\varepsilon = \int \phi_\varepsilon^* h_\varepsilon \phi_\varepsilon + g_{\varepsilon\varepsilon} N_\varepsilon \int |\phi_\varepsilon|^4 + g_{\varepsilon\varepsilon'} N_{\varepsilon'} \int |\phi_{\varepsilon'}|^2 |\phi_\varepsilon|^2}$$

$$\mu_a = \frac{dE}{\partial N_a} \quad \text{as} \quad \frac{\delta E}{\delta \phi_a} = \frac{\delta E}{\delta \phi_b} = 0$$

homogeneous case: $\mu_\varepsilon = g_{\varepsilon\varepsilon} \frac{N_\varepsilon}{V} + g_{\varepsilon\varepsilon'} \frac{N_{\varepsilon'}}{V}$

$\phi_\varepsilon = \frac{1}{\sqrt{V}}$

13/ $\hat{H} = \hbar\chi\,\hat{S}_z^2$

$$i\hbar\,\dot{\hat{S}}_y = [\hat{S}_y, \hbar\chi\hat{S}_z^2] = i\hbar\chi\,(\hat{S}_z\hat{S}_x + \hat{S}_x\hat{S}_z) \simeq i\hbar\chi N\hat{S}_z$$

where, close to the fully polarized state $\hat{S}_x \simeq \langle\hat{S}_x(0)\rangle \equiv \frac{N}{2}$

$$\hat{S}_y(t) = \underbrace{\hat{S}_y(0)}_{\sim\frac{\sqrt{N}}{2}} + N\chi t\,\underbrace{\hat{S}_z(0)}_{\sim\frac{\sqrt{N}}{2}}$$

* $\hat{S}_y$ becomes "a copy" of $\hat{S}_z$ when $N\chi t \gg 1 \Leftrightarrow \chi t \gg \frac{1}{N}$

* The rel. phase is blurred when $\frac{\Delta S_y}{\langle S_x\rangle} \simeq 1$

$$\sqrt{N}\chi t \simeq 1 \Leftrightarrow \chi t \simeq \frac{1}{\sqrt{N}}$$

$\Rightarrow$ The squeezing time is such that $\boxed{\frac{1}{N} \ll \chi t_{sq} \ll \frac{1}{\sqrt{N}}}$

In fact one can analytically solve the model to find
[Kitagawa Ueda PRA 47 5138 (1993) ; Sinatra et al. Front. Phys 7 86 (2012)]

$$\chi t_{best} \overset{N\gg1}{=} 3^{1/6}\frac{1}{N^{2/3}} \quad ; \quad \xi^2(t_{best}) \overset{N\gg1}{=} \frac{3^{2/3}}{2}\frac{1}{N^{2/3}}$$

$\Updownarrow$

$$\frac{\rho g}{\hbar}t_{best} \simeq 3^{1/6}N^{1/3} \quad \text{(diverges in the TL)}$$

$\xi^2 \to 0$ in the T.L. but this conclusion changes in presence of decoherence.

## 2. Squeezing limit in presence of decoherence

$$H = \hbar\chi\,(S_z^2 + D S_z) \qquad \text{dephasing model}$$

$$\langle D\rangle = 0 \quad ; \quad \frac{\langle D^2\rangle}{N} \xrightarrow[N\to\infty]{} \varepsilon_{noise}$$

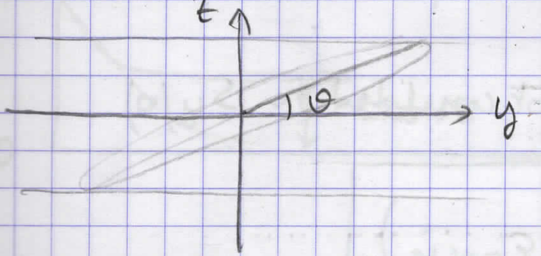
the random variable D, constant in time and varying from shot to shot represents a dephasing environnement due to $\begin{cases} \to T \neq 0 \\ \to \text{particle losses} \end{cases}$

$$\cancel{i\hbar}\, \dot{\hat S}_y = \cancel{i\hbar}\, \chi \left( \hat S_z \hat S_x + \hat S_x \hat S_z + D \hat S_x \right)$$

$$S_y(t) = S_y(0) + \frac{N\chi}{2}\left(2 S_z(0) + D\right) t$$

$$\boxed{S_y(t) = S_y(0) + S_y^{\text{lead}}\, t} \qquad \boxed{S_y^{\text{lead}} = \frac{\rho g}{2\hbar}\left(2 S_z(0) + D\right)}$$

Let us calculate the squeezing parameter




$$S_{\vec n} = \vec n \cdot \vec S \quad ; \quad \vec e_n = \cos\theta\, \vec e_y + \sin\theta\, \vec e_z$$

$$\hat S_{\vec n} = \cos\theta\, \hat S_y + \sin\theta\, \hat S_z$$

Smallest variance $\Delta S^2_{\perp min}$ in the z-y plane $\langle S_z \rangle = \langle S_y \rangle = 0$

$$\Delta S_{\vec n}^2 = \cos^2\theta \langle S_y^2 \rangle + \sin^2\theta \langle S_z^2 \rangle + \cos\theta \sin\theta \langle \{S_y, S_z\} \rangle$$

we use: $\sin 2\theta = 2\cos\theta \sin\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$1 = \cos^2\theta + \sin^2\theta$

$$\Delta S_{\vec n}^2 = \frac{1}{2}\left[ (\cos 2\theta + 1) \langle S_y^2 \rangle + (1 - \cos 2\theta) \langle S_z^2 \rangle + \sin 2\theta \langle \{S_y, S_z\} \rangle \right]$$

$$= \frac{1}{2}\left[ \langle S_y^2 \rangle + \langle S_z^2 \rangle + \cos 2\theta \left( \langle S_y^2 \rangle - \langle S_z^2 \rangle \right) + \sin 2\theta \langle \{S_y, S_z\} \rangle \right]$$

$$0 = \frac{\partial \Delta S_{\vec n}^2}{\partial \theta} \Rightarrow -AS + BC = 0 \quad ; \quad A, B > 0$$

$$S = \frac{B}{A} C \quad ; \quad C^2 + S^2 = 1 \quad ; \quad C^2 \left(1 + \frac{B^2}{A^2}\right) = 1$$

$$C = \pm \frac{A}{\sqrt{A^2 + B^2}} \quad ; \quad S = \pm \frac{B}{\sqrt{A^2 + B^2}}$$

15/

$$\Delta S_{\perp,min}^2 = \frac{1}{2}\left[\langle S_y^2\rangle + \langle S_z^2\rangle - \sqrt{\left(\langle S_y^2\rangle - \langle S_z^2\rangle\right)^2 + \langle\{S_y, S_z\}\rangle^2}\right]$$

Let $A = \langle S_y^2\rangle - \langle S_z^2\rangle$ ; $B = \langle\{S_y, S_z\}\rangle$

$$\Delta S_{\perp,min}^2 = \frac{1}{2}\left[A + \frac{N}{2} - \sqrt{A^2 + B^2}\right]$$

$$\xi^2 = \frac{N\,\Delta S_{\perp,min}^2}{\left(\frac{N}{2}\right)^2} = \frac{4}{N}\Delta S_{\perp,min}^2 = 1 + 2\left(\frac{A}{N} - \sqrt{\left(\frac{A}{N}\right)^2 + \left(\frac{B}{N}\right)^2}\right)$$

$$\frac{A}{N} = \frac{1}{N}\left[\cancel{\frac{N}{4}} + \langle(S_y^{lead})^2\rangle t^2 - \cancel{\frac{N}{4}}\right] = \frac{\langle(S_y^{lead})^2\rangle}{N} t^2$$

N.B. $S_y^{lead}$ is const. of motion and not correlated to $S_y(0)$

$$\boxed{\frac{A}{N} = \alpha t^2 \quad \text{with} \quad \alpha = \left(\frac{\rho g}{2k}\right)^2(1 + \varepsilon_{noise})}$$

$$\frac{B}{N} = \frac{1}{N}\langle S_y S_z(0) + S_z(0) S_y\rangle = \frac{1}{N}\langle S_y^{lead} S_z(0) + S_z(0) S_y^{lead}\rangle t$$

$$= \frac{1}{\cancel{N}}\left(\frac{\rho g}{2k}\right)\cancel{N} t$$

N.B. $\langle S_y(0) S_z(0)\rangle = 0$

$$\boxed{\frac{B}{N} = \beta t \quad \text{with} \quad \beta = \left(\frac{\rho g}{2k}\right)}$$

$$\xi^2 = 1 + 2\left(\alpha t^2 - \sqrt{\alpha^2 t^4 + \beta^2 t^2}\right)$$

$$= 1 + 2\left[\alpha t^2 - \alpha t^2\left(1 + \left(\frac{\beta}{\alpha}\right)^2\frac{1}{t^2}\right)^{1/2}\right]$$

$$= 1 + 2\left[\cancel{\alpha t^2} - \alpha t^2\left(\cancel{1} + \frac{1}{2}\left(\frac{\beta}{\alpha}\right)^2\frac{1}{t^2} + o\left(\frac{1}{t^2}\right)\right)\right]$$

$$\boxed{\xi^2 = 1 - \frac{\beta^2}{\alpha} + o(1)}$$

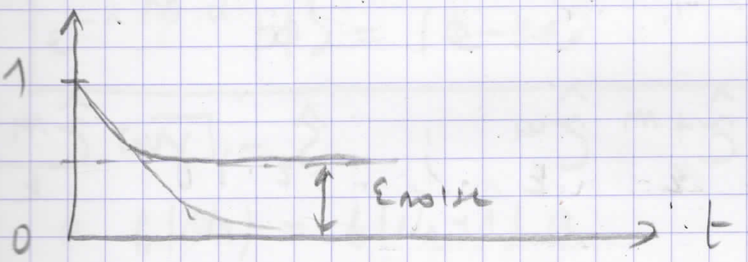
$\chi t \gg \frac{1}{N}$

$\updownarrow$

limit taken for : $\left(\frac{\beta}{\alpha}\right)\frac{1}{t} \ll 1 \iff \frac{\alpha t}{\beta} = \frac{\rho g}{2\hbar} t \gg 1$

$\xi^2 \to$ constant in the TL

$$\xi^2 \xrightarrow[t\to+\infty]{} 1 - \frac{\cancel{\left(\frac{\rho g}{2\hbar}\right)^2}}{\cancel{\left(\frac{\rho g}{2\hbar}\right)^2}(1+\varepsilon_{noise})} = \varepsilon_{noise}$$




$\varepsilon_{noise} \iff$ lost fraction at $t_{sq}$

$\varepsilon_{noise} \iff \simeq \frac{\langle N_{nc}\rangle}{N}$ for $T > \rho g$

## 3. Microscopic description: dephasing due to particle losses

Consider two spatially separated BEC $|\psi(0)\rangle = |\phi=0\rangle = |(\frac{N}{2})_x\rangle$

$\chi = \frac{\partial_{N_a}\mu_a}{\hbar}$ (symmetric)

1-2-3-body losses : open system $\Rightarrow$ ME Lindblad form :

$$\frac{d\hat{\rho}}{dt} = \frac{1}{i\hbar}[\hat{H},\hat{\rho}] + \sum_{m=1}^{3}\sum_{\varepsilon=a,b}\gamma^{(m)}\left[\hat{c}_\varepsilon^m \rho\, \hat{c}_\varepsilon^{+m} - \frac{1}{2}\{\hat{c}_\varepsilon^{+m}\hat{c}_a^m, \rho\}\right]$$

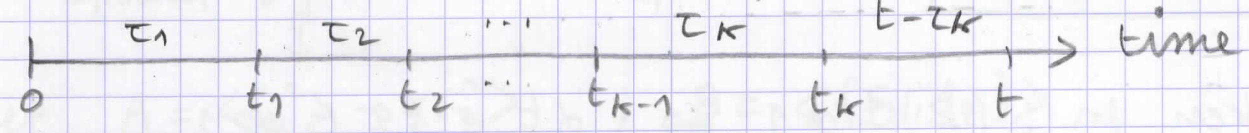
Interaction description

idem for $\hat{a}^+, \hat{b}, \hat{b}^+$

$$\tilde{\rho} = e^{\frac{i}{\hbar}\hat{H}t}\rho\, e^{-\frac{i}{\hbar}\hat{H}t} \quad ; \quad \hat{a} = e^{\frac{i}{\hbar}\hat{H}t}\hat{a}\, e^{-\frac{i}{\hbar}\hat{H}t}$$

$$\frac{d\tilde{\rho}}{dt} - \cancel{\frac{1}{i\hbar}[\tilde{\rho},\tilde{H}]} = \cancel{\frac{1}{i\hbar}[\tilde{H},\tilde{\rho}]} + \sum_m\sum_\varepsilon \gamma^{(m)}\left[\tilde{c}_\varepsilon^m \tilde{\rho}\, \tilde{c}_\varepsilon^{+m} - \frac{1}{2}\{\tilde{c}_\varepsilon^{+m}\tilde{c}_\varepsilon^m, \tilde{\rho}\}\right]$$

17/ Monte Carlo wave functions approach:




- deterministic evolution of a pure state during $\tau_K$

$$|\psi(t_K^-)\rangle = e^{-\frac{i}{\hbar} H_{eff}\, \tau_K} |\psi(t_{K-1})\rangle$$

- quantum jump at $t_K$.

$$|\psi(t_K)\rangle = \hat{S}_\varepsilon(t_K)\, |\psi(t_K^-)\rangle$$

$$\hat{H}_{eff} = -\frac{i\hbar}{2} \sum_{m=1}^{3} \sum_{\varepsilon=a,b} \gamma^{(m)}\, \hat{\tilde{c}}_\varepsilon^{+m}\, \hat{\tilde{c}}_\varepsilon^{m} \quad ; \quad \hat{S}_\varepsilon = \sqrt{\gamma^{(m)}}\, \hat{\tilde{c}}_\varepsilon^{m}$$

$$\langle \hat{O} \rangle = \sum_K \int_{0<t_1<t_2\cdots<t_K<t} dt_1\, dt_2 \cdots dt_K \sum_{\{\varepsilon_j\}} \langle\psi(t)|\, \hat{O}\, |\psi(t)\rangle$$

Sum over all trajectories of the non normalized state vector
[Yun Li et al. PRL 100 210401 (2008)]

↳ analytic results

Losses randomly kick the relative phase

$$\tilde{c}_a(t)|\phi\rangle_N = e^{iS_z^2 \chi t}\, a\, e^{-iS_z^2\chi t}|\phi\rangle_N \quad ; \quad m = N_a - N_b$$

$$= e^{i\frac{\chi t}{4}m^2}\, a\, e^{-i\frac{\chi t}{4}m^2}|\phi\rangle_N \quad ; \quad a\, f(m) = f(m+1)\, a$$

$$= e^{-i\frac{\chi t}{4}[\cancel{m^2}+1+2m-\cancel{m^2}]} \sqrt{\frac{N}{2}}\, e^{i\frac{\phi}{2}}|\phi\rangle_{N-1} \quad ; \quad a|\phi\rangle_N = \sqrt{\frac{N}{2}}\, e^{i\frac{\phi}{2}}a$$

$$= \sqrt{\frac{N}{2}}\, e^{-i\frac{\chi t}{4}}\, e^{i\phi}\, |\phi-\chi t\rangle_{N-1} \quad ; \quad e^{-i\alpha m}|\phi\rangle_N = |\phi-2\alpha\rangle_N$$

Squeezing: $\chi t \ll 1$ ; CAT: $\chi t \simeq \pi$ !! $\left(\chi t_{cat} = \frac{\pi}{2}\right)$

## 4. Schrödinger cats by NL dynamics

$$H = \hbar \chi S_z^2 \qquad \boxed{t_{ch} = \frac{\pi}{2\chi}} \Rightarrow e^{-\frac{i}{\hbar}\hbar\chi S_z^2 t} = e^{-i\frac{\pi}{2}S_z^2}$$

$$e^{-i\frac{\pi}{2}\left(N_a - \frac{N}{2}\right)^2} = e^{-i\frac{\pi}{2}\left(N_a^2 + \frac{N^2}{4} - N_a N\right)} = e^{-i\frac{\pi}{2}N\left(\frac{N}{4} - N_a\right)} e^{-i\frac{\pi}{2}N_a^2}$$

* First factor: $e^{+i\frac{\pi}{2}N\left(N_a - \frac{N}{2} + \frac{N}{4}\right)} = e^{i\frac{\pi}{2}\frac{N^2}{4}} e^{i\frac{\pi}{2}NS_z}$

* Second factor, we use the identity for $m \in \mathbb{Z}$

$$e^{-i\frac{\pi}{2}m^2} = \frac{1}{\sqrt{2}}\left[e^{-i\frac{\pi}{4}} + e^{i\pi\left(m + \frac{1}{4}\right)}\right]$$

$$e^{-i\frac{\pi}{2}N_a^2} = e^{-i\frac{\pi}{4}}\frac{1}{\sqrt{2}}\left[1 + i\, e^{i\pi\left(S_z + \frac{N}{2}\right)}\right]$$

$$\Rightarrow e^{-i\frac{\pi}{2}\hat{S}_z^2} = e^{i\frac{\pi}{2}\frac{N^2}{4}} e^{i\frac{\pi}{2}N\hat{S}_z} e^{-i\frac{\pi}{4}}\frac{1}{\sqrt{2}}\left[\mathbb{1} + i\, e^{i\frac{\pi}{2}N} e^{i\pi\hat{S}_z}\right]$$

$N = 0[4]$ $\quad N = 4q$, $N^2 = 16q^2$, $2S_z = 2N_a - N$ even, $S_z$ integer

$$\boxed{e^{-\frac{i}{\hbar}H_{NL}t_{ch}}|0\rangle_N = e^{-i\frac{\pi}{4}}\frac{1}{\sqrt{2}}\left[|0\rangle_N + i\,|\pi\rangle_N\right] \quad ; \quad (N = 0[4])}$$

the loss of a single particle kills the superposition!

$$\hat{a}|\psi_{cat}\rangle \propto \left[|0\rangle_{N-1} + i\,e^{i\frac{\pi}{2}}|\pi\rangle_{N-1}\right] = \left[|0\rangle_{N-1} - |\pi\rangle_{N-1}\right]$$

$$\hat{b}|\psi_{cat}\rangle \propto \left[|0\rangle_{N-1} + i\,e^{-i\frac{\pi}{2}}|\pi\rangle_{N-1}\right] = \left[|0\rangle_{N-1} + |\pi\rangle_{N-1}\right]$$

## 5. Fidelity of the cat state generation in presence of losses

$$\mathcal{F} = \mathrm{Tr}\left[\hat{\rho}(t_{ch})\,|\psi_{cat}\rangle\langle\psi_{cat}|\right]$$

As the average of any observable, $\mathcal{F}$ is obtained by averaging the stochastic (non normalized) pure state expectation $\langle \psi(t) | \tilde{O} | \psi(t) \rangle$ over all possible stochastic realisations.

→ The only trajectory contributing to $\mathcal{F}$ is the one with no losses!

$$|\psi_0(t_{ch})\rangle = e^{-\frac{i}{\hbar} H_{eff} t_{ch}} |\phi=0\rangle_N$$

1BL $\quad \hat{H}_{eff} = -\frac{i\hbar}{2}\gamma \hat{N} + \hat{H}_{NL}$ (Schrödinger picture)

$$|\psi_0(t_{ch})\rangle = e^{-\frac{1}{2}\gamma \hat{N} t_{ch}} \, e^{-\frac{i}{\hbar}\hat{H}_{NL} t_{ch}} |\phi=0\rangle_N \quad ; \quad [\hat{N}, \hat{H}_{NL}] = 0$$

$$= e^{-\frac{1}{2}\gamma N t_{ch}} |\psi_{cat}\rangle$$

$$\mathcal{F} = \mathrm{Tr}\left[\hat{\rho}_0(t_{ch}) |\psi_{cat}\rangle\langle\psi_{cat}|\right] = e^{-\gamma N t_{ch}}$$

The fidelity decays $N$ times faster than the particle loss rate!

## 6. Fidelity of the cat state generation at finite temperature

Quasi particles in the gas constitute a dephasing environment for the condensate ⇒ we want NO QP!

1. $\boxed{g=0}$ $\boxed{T \ll T_c}$ ideal gas BEC in internal state a
2. $\frac{\pi}{2}$ - pulse : ideal gas phase state $|\phi=0\rangle_N$
3. adiabatically raise of $g$ ($g_{aa} = g_{bb}$ ; $g_{ab} = 0$)
4. evolve to the cat state

20/ (5) adiabatically decrease g.
$\Rightarrow$ ideal gas in $|\Psi_{cot}\rangle$

If $|\Psi(0^-)\rangle = |\Psi_G\rangle$ is prepared at $T=0$ $\Rightarrow U(t_{cot})|\Psi_G\rangle = |\Psi_{cot}\rangle$

If $|\Psi(0^-)\rangle \perp |\Psi_G\rangle$ (1 atom out of the BEC) $\Rightarrow U(t_{cot})|\Psi(0^-)\rangle \perp |\Psi_{cot}\rangle$

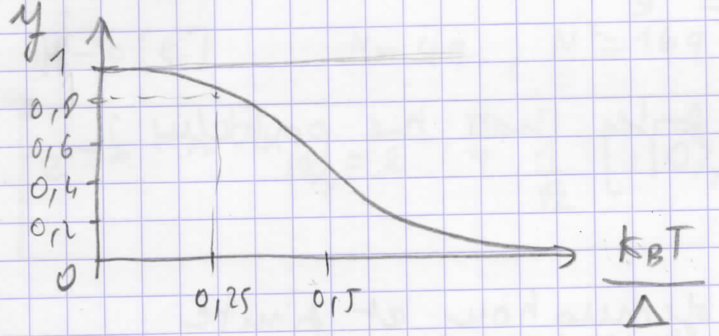
(unitary evolution preserves orthogonality)

So if the system is prepared at very low but finite T:

$$\mathcal{F}(t_{cot}) = \langle \Psi_G | \hat{\rho}_T | \Psi_G \rangle = \prod_{\vec{k} \neq \vec{0}} \mathcal{P}_{\vec{k}}^{th}(n_k = 0)$$

$\mathcal{P}_k^{th}(n_k = n) = \frac{1}{Z} e^{-\beta E_k n}$ thermal probability of having n quasiparticles in mode k

$$Z = \sum_{n=0}^{\infty} \left(e^{-\beta E_k}\right)^n = \frac{1}{1 - e^{-\beta E_k}} \quad ; \quad \left(\bar{n} = \frac{1}{e^{\beta E_k} - 1}\right)$$




$$\Delta = \frac{\hbar^2}{2m}\left(\frac{2\pi}{L}\right)^2$$

$L = 2\,\mu m$ $\quad \frac{\Delta}{4} = 14\,nK$

$^{87}Rb$

[ Pawlowski et al. PRA 95 (2017) ].

Articles

- Frontiers (2012)
- Nascimbene (2018)
- Treutlein (2010)