From laser cooling to BEC First experiments of superfluid hydrodynamics

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Quantum Fluids course - Complement 1

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Plan

Two possible path towards degeneracy

Condensation criterion: $\rho\lambda^3 \geq 2.612$; $\lambda = \sqrt{\frac{h^2}{2\pi m k_B T}}$

- **High density** : superfluid helium

$$\rho \sim 10^{30}\mathrm{at/m}^3 \qquad T_c = 2\mathrm{K}$$

- **Low temperature** and low density : **ultra-cold gases**

$$\rho \sim 10^{19}\mathrm{at/m}^3 \qquad T_c \sim 1\mu\mathrm{K} \qquad \lambda \sim 10^{-6} m$$

Reach $T = T_c$ and remain a gas !

Polarized H : it seamed a good idea but ...

- Absence of two body bound states
- Bad collisional properties

Extremely diluted gases in a metastable state

Slowing down and cooling atoms

Nobel Pize (1997)

Slowing down : $\langle v \rangle$ **Cooling** : $\langle \Delta v \rangle$

Na@300 K $\langle \Delta v_x \rangle = \langle v_x \rangle = 300$ m/s $\lambda = 5 \times 10^{-11}$m

SLOWING Radiation pressure: **Plane wave** : $\vec{p} = \hbar \vec{k}_L$

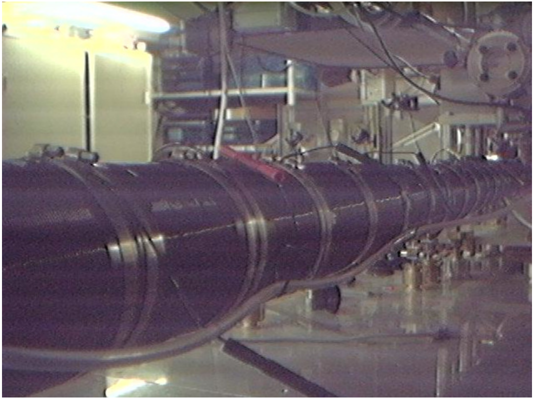
absorption and spontaneous emission Γ

Average force : $\vec{F} \simeq \hbar \vec{k}_L \Gamma$

Deceleration : $\frac{F}{m} \simeq 10^6 \quad \text{m/s}^2 = 10^5 g$

Zeeman slower

Keep the laser on resonance ω_A Zeeman effect in $B(z)$

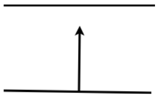


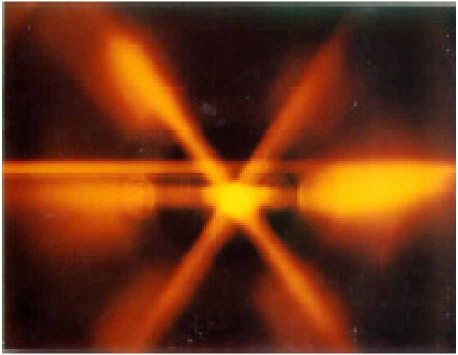
^{4}He* ; ENS

Laser cooling...

- Laser cool and trap the atoms: $\delta_{v=0} = \omega_L - \omega_A < 0$, $F = -\alpha v$

Doppler effect $\rightarrow$ atom preferentially absorbs counterpropagating photons





Optical Molasses: laser beams provide a friction force

Limits of Doppler cooling

detuning: $\delta_{v=0} = \omega_L - \omega_A < 0$; $\qquad \delta_v = \omega_L - \omega_A - \vec{k}_L \cdot \vec{v}$

Excitation rate at rest: $\gamma(\delta) \propto \frac{I_L}{\delta^2 + \Gamma^2/4}$

Average force:

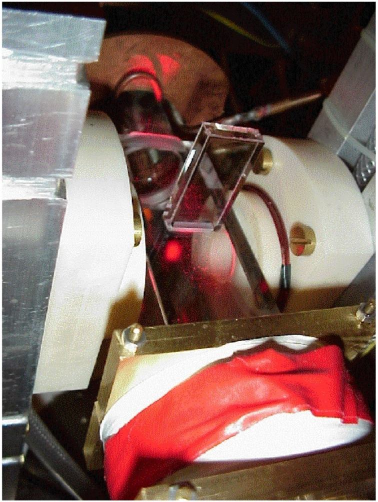
$$F = \hbar k \left[\gamma(\delta - kv) - \gamma(\delta + kv)\right] = -2\hbar k^2 v \gamma'(\delta) = -\alpha v$$

Diffusion of momentum:

$$\Delta p^2 = 2Dt \qquad \mathbf{D} = \hbar^2 k^2 \gamma(\delta)$$

Einstein relation: $\mathbf{k_B T} = \frac{\mathbf{D}}{\alpha} \geq \frac{\hbar\Gamma}{2}$ $\qquad$ **240** μK pour Na.

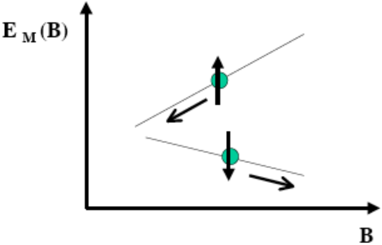
Magneto-optical trap

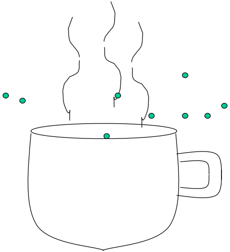


^{6}Li et ^{7}Li ; ENS

Cooling further to reach the degenerate regime

- Laser beams + magnetic field → **Magneto-optical trap** $T \sim 1\text{mK}$ (limit : $T_{rec} = \frac{\hbar^2 k^2}{2M}$)





$\uparrow \vec{B}$

- **Magnetic trap:** In a space-dependent magnetic field $E_m(B) = -\vec{\mu} \cdot \vec{B} = g\mu_B m_j |\vec{B}|$, → Force
  Atoms can be trapped in a minimum of $|\vec{B}|$

- **Evaporative cooling**: Radio frequency $\nu_{rf} = (E_\uparrow - E_\downarrow)(r)/\hbar$ to flip the spin and eject fastest atoms

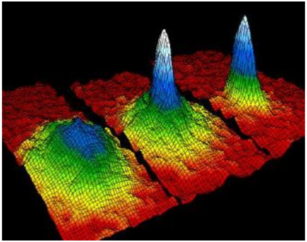
How to “see” a condensate ?

Sizes : ideal gas condensate $\mathbf{a_0}$ and thermal cloud $\mathbf{X}$

$$\mathbf{a_0} = \left(\frac{\hbar}{m\omega}\right)^{1/2} \qquad \mathbf{p_0} = (\hbar m\omega)^{1/2}$$

$$\frac{1}{2} m\omega^2 \langle \mathbf{X}^2 \rangle = \frac{\langle \mathbf{P}^2 \rangle}{2m} = \frac{1}{2} k_B T \qquad \mathbf{X} = \left(\frac{k_B T}{m\omega^2}\right)^{1/2} \qquad \mathbf{P} = (k_B T m)^{1/2}$$

$$\frac{\mathbf{X}}{\mathbf{a_0}} = \frac{\mathbf{P}}{\mathbf{p_0}} = \left(\frac{k_B T}{\hbar\omega}\right)^{1/2} \simeq N^{1/6}$$ à la transition

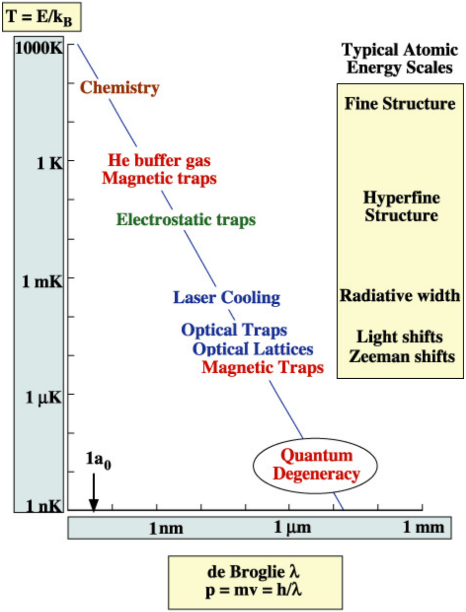


Atomic BEC 1995-2007

1995 First observation of BEC in atomic gases in JILA and MIT ...

Nobel Prize 2001: E. Cornell, C. Wieman, W. Ketterle

Energy scales



Typical numbers for a BEC

Size: $\Delta x = 100\,\mu$m

Number of atoms: $N = 10^6$

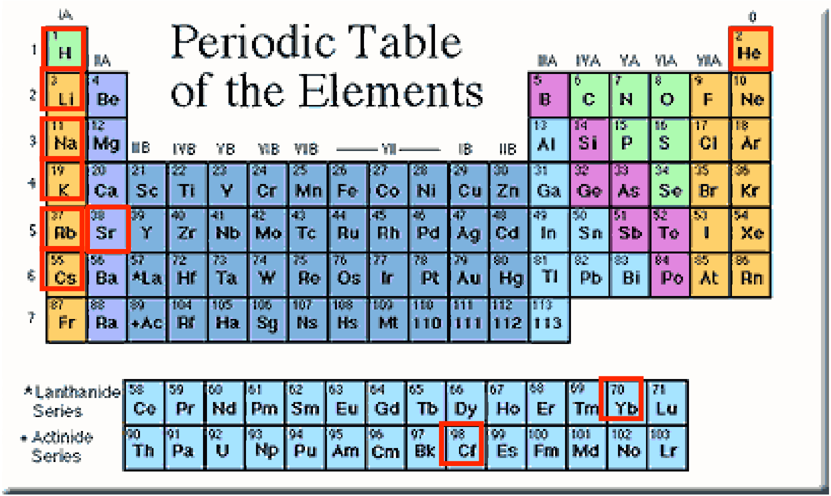
Temperature $T = 100$ nK
$\left(\lambda_{th} = \sqrt{\frac{2\pi\hbar^2}{mk_BT}}\right)$

Density $\rho = 10^{19}$ at/m^3
$(\rho\lambda_{th}^3 > 1)$

Lifetime $\tau = 100$ s

Figure for Na atoms from Burnett et al., Nature (2002)

Elements already condensed

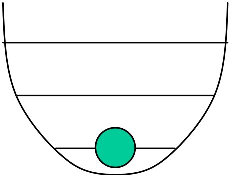


Bose versus Fermi statistics

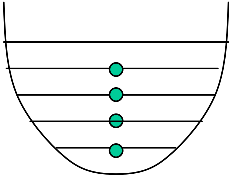
$$\rho\lambda_{th}^3 \sim 1 \quad ; \quad \lambda_{th} = \sqrt{\frac{2\pi\hbar^2}{mk_B T}}$$

In an harmonic potential: $U(\vec{r}) = \frac{1}{2}m\omega^2 r^2$

Bose



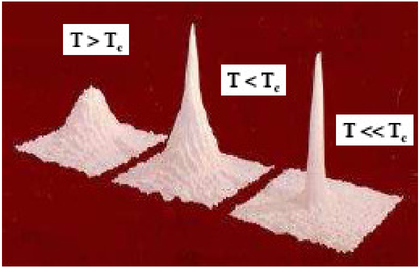
Fermi

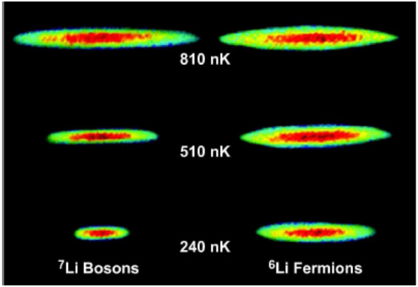


$$N = \left(\frac{k_B T_c}{\hbar\omega}\right)^{1/3} 1.202$$

$$E_F = (6N)^{1/3}\hbar\omega$$

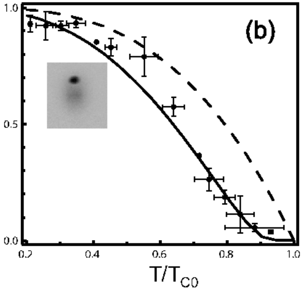
BEC and degenerate Fermi gas in an harmonic trap





- **Non condensed cloud**: $\frac{1}{2}m\omega^2X^2 = \frac{1}{2m}P^2 = \frac{1}{2}k_bT$
- **BEC** Narrow peek both in real space and in momentum space
- **Fermi gas**: Pauli exclusion principle $\rightarrow$ Fermi pressure

condensed fraction



Jila (1996) and
Institut d'Optique (2004)

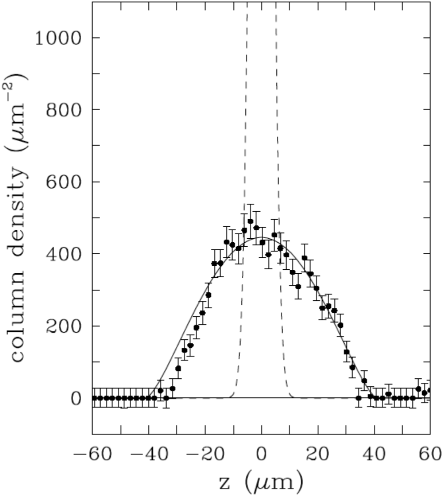
For an ideal gas

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_C}\right)^3 \qquad k_B T_C = \hbar\bar{\omega}\left(\frac{N}{1.202}\right)^{1/3}$$

Effect of interactions (mean field in the trap)

$$\frac{\delta T_C}{T_C} \simeq \mathbf{C}\, a\,; \qquad \mathbf{C} < 0$$

Thomas-Fermi profile



Interactions affect strongly the condensate shape

Simplest description of a BEC with interactions

- **N particles in one mode**

  All the atoms share the same one-body wavefunction $\phi(x,t)$.

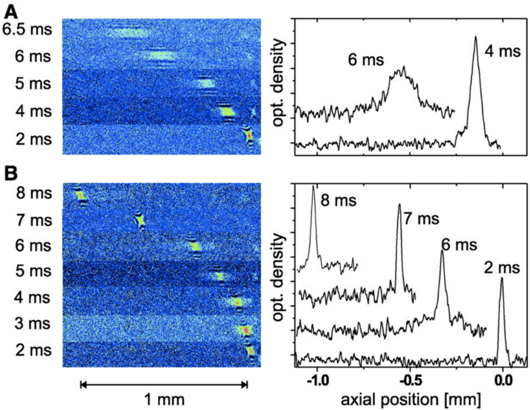
$$i\hbar\partial_t\phi = \left[-\frac{\hbar^2\Delta}{2m} + U(x) + gN|\phi(\vec{r},t)|^2\right]\phi$$

  Time-dependent Gross-Pitaevskii equation

- **Interactions**
  s-wave collisions between ultra-cold atoms modeled by a contact potential $g = \frac{4\pi\hbar^2}{m}a$, a=s-wave scattering length

Matter waves Solitons

Non-linear Shrödinger equation with $a < 0$ (attractive interactions) in a 1D geometry has solitonic solutions.



(ENS) A magnetic field is used to change the sign of a before releasing the condensate in a 1D waveguide + expelling potential.

Sound Waves in a uniform BEC

Initially $\vec{v} = 0$ and $\rho = \rho_0$. Sudden switch-on of a perturbing potential δU

Linearized equations: $\rho = \rho_0 + \delta\rho, \quad \vec{v} = \delta\vec{v}$

$$\frac{\partial \vec{v}}{\partial t} = -\frac{\vec{\mathrm{grad}}}{m}[\delta U + g\delta\rho] \quad ; \quad \frac{\partial \delta\rho}{\partial t} + \mathrm{div}[\rho_0 \vec{v}] = 0$$

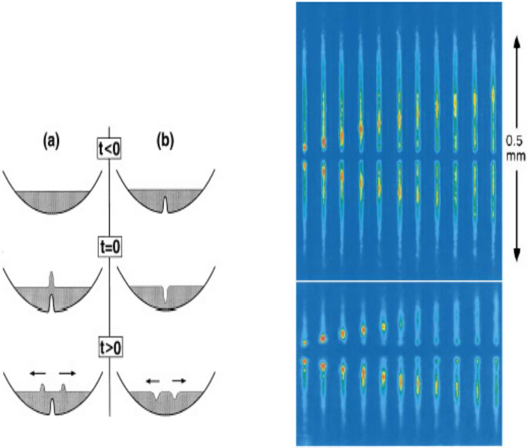
$$\boldsymbol{\delta U = 0} \quad \rightarrow \quad \frac{\partial^2 \rho}{\partial t^2} - \mathrm{div}[\frac{\rho_0 g}{m}\vec{\mathrm{grad}}\delta\rho] = 0$$

$$\rho_0 = \mathrm{const} \quad \rightarrow \quad \frac{\partial^2 \rho}{\partial t^2} - \frac{\boldsymbol{\rho_0 g}}{\mathbf{m}}\Delta\delta\rho = 0$$

$$\Delta\delta\rho - \frac{1}{\mathbf{c^2}}\frac{\partial^2 \rho}{\partial t^2} = 0$$

Speed of sound: $c = \sqrt{\frac{g\rho_0}{m}} = \sqrt{\frac{\mu}{m}}$

Sound waves in the BEC



between images $\delta t = 1.3$ ms. $c \sim 2\text{cm/s}$

What happens when we set a BEC into rotation ?

For a rigid body:

$$\vec{v} = \vec{\Omega} \times \vec{r} \qquad \vec{\mathrm{curl}}\, \vec{v} = 2\vec{\Omega} \qquad \oint \vec{v} \cdot \vec{dr} = \int_S (\vec{\mathrm{curl}} \vec{v}) \cdot \vec{n} = 2\,\Omega\, S$$

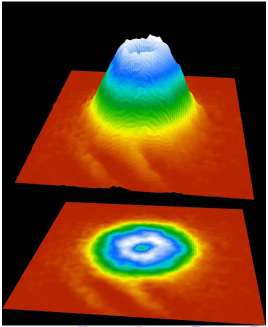
For a condensate:

$$\psi(\vec{r}) = \sqrt{\rho(\vec{r})} e^{i\phi(\vec{r})} \qquad \vec{v} = \frac{\hbar}{m} \vec{\mathrm{grad}} \phi \qquad \vec{\mathrm{curl}}\, \vec{v} = 0 \quad \text{(if } \phi \text{ is regular)}$$

When set into rotation the superfluid creates Vortices !

Phase singularities

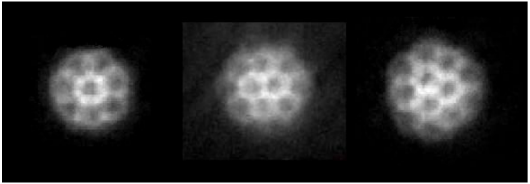
$$\oint \vec{v} \cdot \vec{dr} = \frac{\hbar}{m} \oint \vec{\mathrm{grad}} \phi = \frac{\hbar}{m} 2\pi s = s \frac{h}{m} \qquad s \in \mathbb{Z}$$



Density of vortices $\frac{N_v}{S} = 2\Omega \frac{m}{h}$.

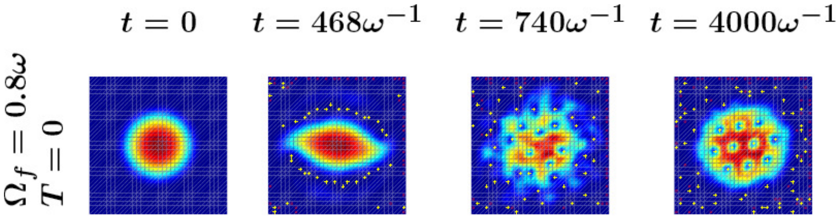
Vortex lattices in the ENS experiment

- **Experiment**: Instability $\Omega > 0.75\omega$, crystallisation of a lattice



Madison et al. PRL (2000)

- **Classical field Simulation** $i\hbar\frac{\partial\psi}{\partial t} = (-\frac{\hbar^2\Delta}{2m} + V(\vec{r}) + g|\psi|^2)\psi$



$\rightarrow$ **Non linear multi mode physics**

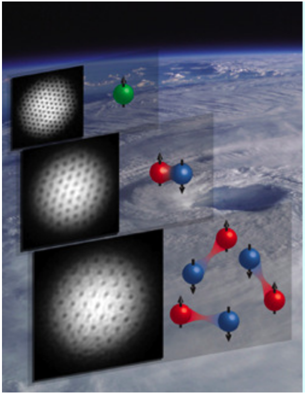
Lobo et al. PRL (2004)

Tune the atomic interactions

From **ideal gas** to a **strongly interacting** system using an external magnetic field (Feshbach resonance)

Superfluidity in the BEC-BCS crossover

Fermi gas in two spin states with attractive interactions



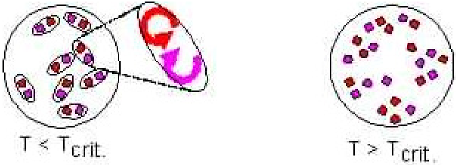
- Direct evidence of superfluidiy across the BEC-BCS regimes
- Regular lattice ↔ constant vorticity: $h/2m$ per vortex

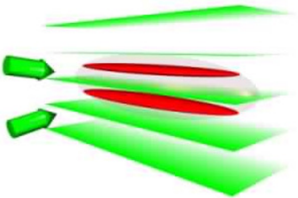
Zwierlein et al. Nature (2005)
(MIT W. Ketterle)

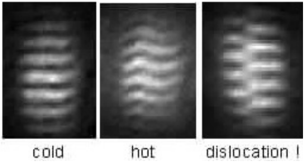
Shape the trapping potential

It is possible to **confine the movement** of the atoms in **1D** or **2D**: change the spatial dimension

Proliferation of vortices associated to the BKT transition in 2D







ENS Hadzibabic et al. Nature (2006)