Adjustment and social choice

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Abstract. We discuss the influence of information contagion on the dynamics of choices in social networks of heterogeneous buyers. Starting from a cellular automata model we show that when buyers and sellers try to adjust bids and asks the tatonement process does not converge to equilibrium at some intermediate market share and that large amplitude fluctuations are actually observed. When the tatonement dynamics is slow with respect to the contagion dynamics, large periodic oscillations reminiscent of business cycles appear.

1 Introduction

Bubbles in financial market are one of the most spectacular stylised fact in contradiction with General Equilibrium Theory. Economists and "econophysicists" have also noticed that the spectral properties of stock, commodities and foreign exchange return series were far for Gaussian noise: return series display scale invariance, a property that physicists and economists relate to as cooperativity (interactions) among the agents. To our knowledge there have been very few explicit models of the phenomenon and our aim in this contribution is to discuss one of the simplest theoretical approach.

Basically our model couples interaction among agents with their tatonement procedure to get "fair" prices. Both processes have attracted the attention of modelers, see e.g. Fölmer (1974), Galam (1982, 1991) and Orlean (1995) for interactions and herding behaviour and Lesourne (1992) for adjustment.

- We suppose that agents are not independent and that their individual choices are influenced by the choice of their neighbours for whatever reason, externalities or information;
- Agents initially have some reservation price that they adjust to match offers.

In a previous attempt to understand these phenomena, we used a percolation model to describe the information contagion dynamics (Solomon etal 2000, Weisbuch and Stauffer 2000, Weisbuch and Solomon 2000, Weisbuch etal 2001). Agents are nodes of a lattice and are susceptible to purchase a

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good when its quality meet their expectations after one of their neighbours purchased. Adjustment of qualities and expectations are based on a standard tatonement process which we describe further in section 2.2. The resulting dynamics were checked by the observation of time series of purchase, patterns of purchase, and Fourier transform of the time series. A distinctive feature of cooperativity was the observed $1/f^2$ spectrum of the purchase time series. Comparable results were obtained by Plouraboué etal (1998) and Steyer and Zimmermann (2000) who reported $1/f$ noise when the slow dynamics is Hebbian learning.

One of the purpose of the present study is to check the genericity of the previously modeled behavior: we only have a very indirect knowledge of the reasons why agents decide to purchase a good or not to purchase it, not to mention the specific algorithm they would use to survey their neighbours before taking a decision. The previously used percolation approach supposes that the purchase by one of their neighbour is sufficient to provide a full knowledge of the quality of the product, while the "counter" (or voting) dynamics presented here implies that agents survey all their neighbours to take some average opinion. There might be specific situations for which one or the other scheme would make more sense, but in general we would like to know how different would the resulting global dynamics be.

The paper first describe the model. We then give simulation results for the two different dynamical regimes. After some study of the influence of parameters we discuss in the conclusion the relevance of the observed dynamics to business cycles and financial markets.

2 The INCA model

2.1 Information contagion

A rather standard model of information contagion is based based on cellular "counters" (also called voters dynamics). Let us consider cellular automata: binary agents occupy sites of a 2-dimensional lattice. The decision rule, buy or not buy, corresponding to state $S_i = 1$ or to state $S_i = -1$,[1] is based on some combination of private information and information coming from neighboring (on the lattice) agents. The private information of agent i is some threshold θ_i, and information incoming from neighbours is simply the sum of their individual states. At each time step one agent randomly selected updates its state by comparing the sum of its neighbours' states to its threshold:

$$S_i(t+1) = 1 \quad \textit{iff} \quad \sum_j S_j(t+1) > \theta_i \qquad (1)$$

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[1] the choice 1 or 0 for buy or not buy, is more standard in economics, but our choice, inspired from physics, respects symmetry and thus makes computations and mathematical expressions simpler.

Otherwise $S_i(t+1) = -1$. Homogeneous counters automata, with identical thresholds, are simple cellular automata which dynamical attractors depend on the amplitude of the threshold as compared to the number of neighbors. Lower threshold (less than -2) give homogeneous attractors with state 1 for all automata, higher thresholds (more than +2) give homogeneous attractors with state -1 for all automata. Intermediate threshold values give coexisting domains of plus and minus ones which size depends upon thresholds and initial conditions (see e.g. Weisbuch 1991 for more details).

But we are interested here in inhomogeneous counters with different thresholds: INCA[2] are disordered systems and their dynamical properties reflect their disordered structure: one observes stable dynamical regimes separated by finite width phase transitions for which attraction basins depend up on the particular realisation of disorder. Weisbuch and Boudjema (1999) have shown for instance that even under a favourable average threshold which would predict invasion by +1's in a sea of -1's, the detailed positions of the automata initially at state +1 are important. The phenomenon is well-known in the physics of phase transitions: germs are initially necessary to the growth of a stable phase in conditions of supercooling for instance.

2.2 The adjustment model

Let us now be more specific about the interpretation of the threshold in terms of economics. The threshold which drives the buying behavior of the agent can be seen as a price difference between how much a seller would like to get from a product and how much a buyer is ready to pay for it in the absence of any buyer in the agent neighborhood. A positive threshold would prevent purchase, a negative one would allow it. The presence of other purchasers in the neighborhood would favour purchase. (Of course prices have to be expressed in some units consistent with equation 1. Two units in price difference corresponds to a threshold change of one extra neighbour).

The adjustment process now can be simply stated as:

- When an agent did not purchase upon update ($S_i = -1$), she decreases her threshold with the hope to be able to be in a better condition to purchase next time;
- in the opposite case she increases it.

The algorithm is above described as adjustment of a buyer reservation price, but a symmetrical reasoning for a seller would give the same threshold dynamics.

The adjustment dynamics in the absence of any coupling between agents would be similar to the simple mechanism describe e.g. by Laffond and Lesourne (1992) and would yield a similar clearing of the market as described in Lesourne's book (1992). The difference here is that we are interested in

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[2] INhomogeneous Cellular Automata (Vichniac 1986).

multiple purchases by agents and that we couple adjustment and contagion dynamics.

3   Simulation results

3.1   The slow adjustment regime

Before a full study of parameters and variants let us try to understand the simplest case. An obvious parameter of the model is how fast the threshold is adjusted with respect to the buying propagation dynamics. Let us suppose that at every update, the adjustment is random and uniformly distributed on $[0, q]$. We further refer to q as the adjustment rate. Its magnitude has to be compared with the number of neighbours taken into account in the simulation: we used four. For slow adjustment rate such as $q = 0.1$, we expect the dynamics of adjustment to be slow with respect to the diffusion dynamics. Let us define the relative adjustment rate ρ as the ratio between the average adjustment, here $q/2$, and the difference between thresholds such that isolated individuals among a neighbourhood of either buyers or non-buyers would take an opposite view to their neighbours, here 8. The slow adjustment regime is such that:

$$\rho < \frac{2}{L} \tag{2}$$

where L is the linear dimension of the lattice. The second term is the inverse of the time necessary to propagate a position, buying or not buying, across the net, under the most favourable threshold condition (the term 2 is due to the fact that we use periodic boundary conditions).

The simulation conditions are then:

- A square lattice of dimension L^2 (e.g. 20x20 for figure 1);
- random[3] updating based on the described algorithms;
- The initial configuration of agents is random for the binary state and thresholds are uniformly distributed on [-1,1].

Figure 1 is a time plot of the average state of agents fraction of buyers and average threshold.

The regular oscillations of agents states and average thresholds obtained at large time give some indication of the processes which control the dynamics. Note that both quantities display relaxation rather than sinusoidal oscillations, threshold varying as a triangular wave and purchases more like a square wave which saturates at maximum (all or no agents purchasing). The two quantities have a phase shift of $\pi/2$: extreme variations of the average threshold occur when purchases saturate. These observations plus direct

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[3] at each time step one node randomly chosen is updated according to equation 1 for its state and section 2.2 for its threshold

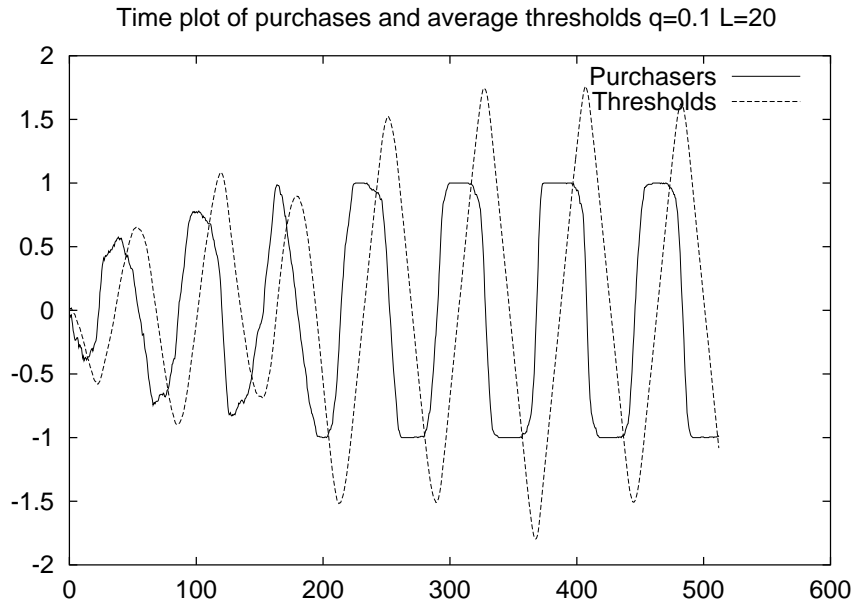Time plot of purchases and average thresholds q=0.1 L=20

Fig. 1. Time evolution of the average state of agents and average threshold, in the slow adjustment regime. (average state=1, everyone buys, average state=-1, no-one buys.). Unit time correspond to updating each site once on average).

online observation of the lattice dynamics for purchase and threshold can be easily understood.

Once the lattice is in a saturated condition, say everyone buying, an isolated agent who would choose not to buy needs a threshold much higher than if she were surrounded by non-buyers. The system has to "wait" until thresholds which were low during the rise of the purchasing behaviour rise again to allow the apparition of isolated non-buyers. Hence the straight part of the average threshold evolution corresponding to its slow and regular increase. But as soon as isolated non-buyers are present, their neighbours need a lower threshold to switch to no-purchase; a wave of no-purchase propagates accross the lattice. Hence the fast switch observed on the purchase time evolution: online observation display the growth of non-purchaser germs surrounded by disappearing domains of purchasers. One single sweep from purchasing to non-purchasing is the equivalent of a phase transition for which germs are needed (first order phase transitions). The phenomenon is symmetrical for purchase and no purchase, hence the observed oscillations.

3.2   The fast adjustment regime

With large networks and fast adjustment rates, the global synchrony between all agents on the lattice is destroyed. Agent states (purchase/no-purchase) and thresholds display small homogeneous domains on the lattice. Because of the randomness of the updating process, some agents easily reach "eccentric" (opposed to their neighbours) positions and many domains out of phase start growing in different places[4]. This behaviour is observed with online simulations and displayed on figure 2 at time 100 000.

The change of dynamical regime with adjustment rates q is very smooth and correspond to what physicists call a crossover rather than a phase transition.
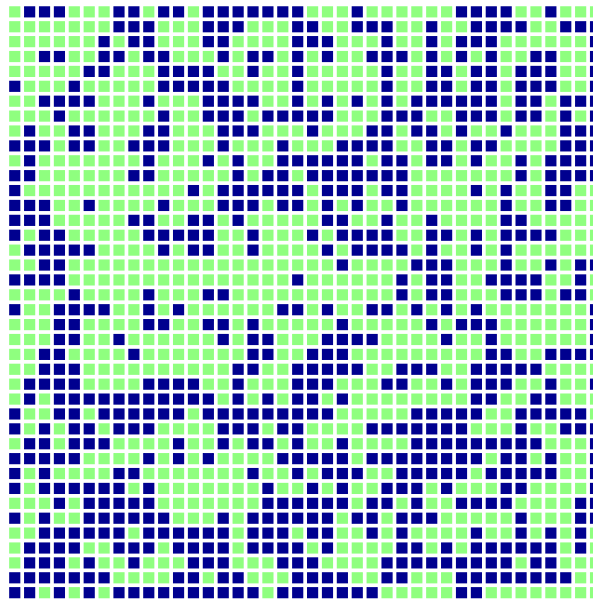

Fig. 2. Pattern of behaviour at time 100 000 in the fast adjustment regime. Adjustment rate q is 0.7. Grey squares correspond to buyers, black squares to non-buyers. (In the slow adjustment regime domains would be much larger. Sometimes, consensus accross the lattice is achieved).

Because of domains asynchrony, oscillation are relatively smaller and less regular in amplitude in the fast adjustment regime than in the slow adjustment regime as observed in figure 3. Saturation of the (no)purchaser fraction is never reached.

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[4] By contrast, in the slow adjustment regime, many time steps are needed to sweep from the lower to the upper threshold, and the standard deviation of the threshold is reduced by the summing process, thus allowing synchrony of agents behaviour.
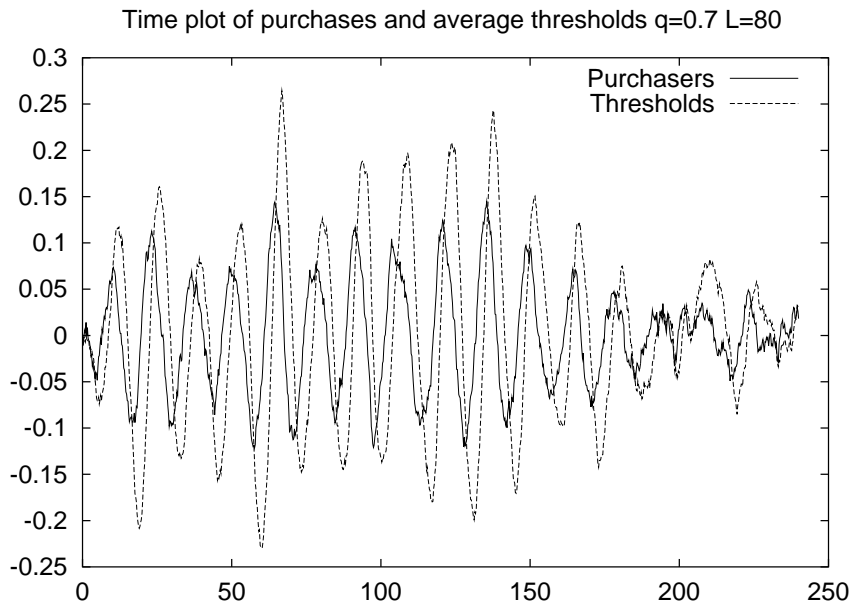
Time plot of purchases and average thresholds q=0.7 L=80

Fig. 3. Time evolution of the average state of agents and average threshold in the fast adjustment regime. Oscillation are smaller and less regular in amplitude than in the slow adjustment regime.

Another way to monitor inhomogeneity is to check the spatial autocorrelation function of states.

$$C(d) = \frac{\sum_i S(i)S(i+d) - (\sum_i S(i))^2}{L^2} \tag{3}$$

where i is the position of lattice sites and d the translation distance. L^2 is the number of agents.

Figure 4 displays the autocorrelation function for different lattice sizes in the fast adjustment regime when the adjustemnt rate $q = 0.7$. Its fast decay, over some 4 lattice sites, tells us that the average linear dimension of purchasing or no-purchasing domains is of order 4 to be compared with the patterns observed on figure 2. A striking result is that although lattice linear sizes change by a factor 8, the autocorrelation function are very similar, implying that the average size of domains is independent of lattice size in the fast adjustment regime. In other word, figure 2 is statistically representative of any part of a larger lattice.

Scaling

The system has only two parameters, L the lattice dimension and the average adjustment rate ρ. We would like to know how the characteristic variable of the dynamics, frequency and amplitude of the oscillations, and

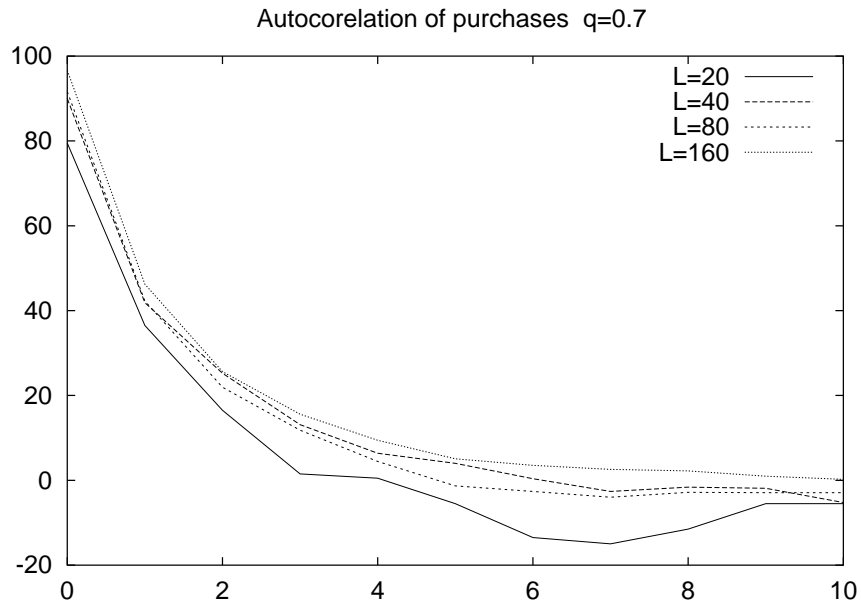Autocorrelation of purchases q=0.7

Fig. 4. Spatial autocorrelation function of the average fraction of buyers for lattice sizes varying from 20x20 to 160x160. Abscissa is distance in units of lattice spacing. Note that the correlation length, given by the distance at which the autocorrelation is zero is nearly invariant with lattice size.

their space Dependance through the autocorrelation function, vary with L and ρ.

Frequencies are surprisingly stable over time and from sample to sample as opposed to magnitudes. A direct measure on time plots of oscillations shows that periods T vary as:

$$T \simeq \frac{10L^2}{q} \qquad (4)$$

This result has a very simple interpretation. The factor L^2 is simply the number of agents. The period is close to the number of agents multiplied by a time which scales with average time it takes for threshold to switch between extremal values of -4 and 4. The threshold dynamics is the rate limiting step of the overall dynamics.

As seen on figure 3, amplitudes display a lot of variations. A simple way to average them on time is to measure power, namely the time averaged squared amplitudes. Even with time averaging over some 800 periods, power values had to be further averaged over several runs (9 in our measurements) to further reduce noise. A first result is that for larger values ρ values, $\rho > 5$, average power scales as $L^2 = N$ the number of agents. If agents behaviour

were oscillating in phase, we would expect power to scale in N^2. The scaling in N implies that N/s patches of constant size s oscillate independently giving:

$$P \sim \frac{N}{s} P_s \sim Ns \sim \frac{N}{q^2} \tag{5}$$

where P_s is the power of one patch, proportional to s^2. This interpretation is consistent with our interpretation of autocorrelation measurements and the observation of small domains.

Figure 5 display the rescaled inverse power (i.e. $\frac{P}{N}$) as a function of q the maximum adjustment rate ($q = 8\rho$) for N varying from 400 to 6400. The collapse of the three curves above $q = 0.6$ is good, the quadratic scaling in q is approximate.
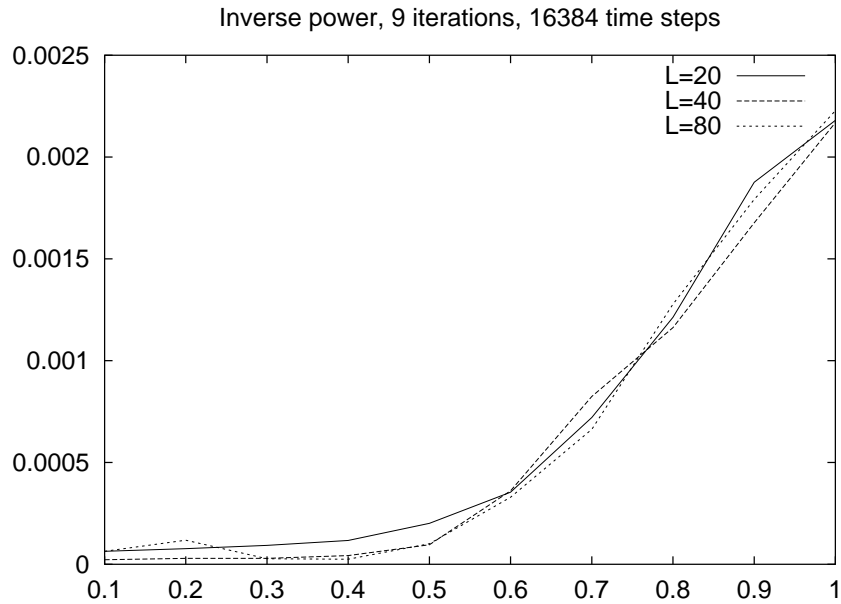
Inverse power, 9 iterations, 16384 time steps

Fig. 5. Rescaled inverse power in the fast adjustment regime, for several network sizes, $L = 20, 40, 80$ as a function of the adjustment rate q. When $q > 0.5$ one observes a good collapse of simulation data for the rescaling in N and a quadratic variation in q.

Figure 6 displays the Fourier power spectrum of the time series of agent states when $q = 1$. The large peak around abscissa 30 corresponds to a frequency of 10 iterations per agent. At larger frequencies, the long tail corresponds to a $1/f^2$ noise. Small scale correlations in agents behaviour due to local imitation processes are responsible for this long tail. For lower values of

the maximum adjustment rate q, the importance of the peak with respect to the $1/f^2$ noise is increased.
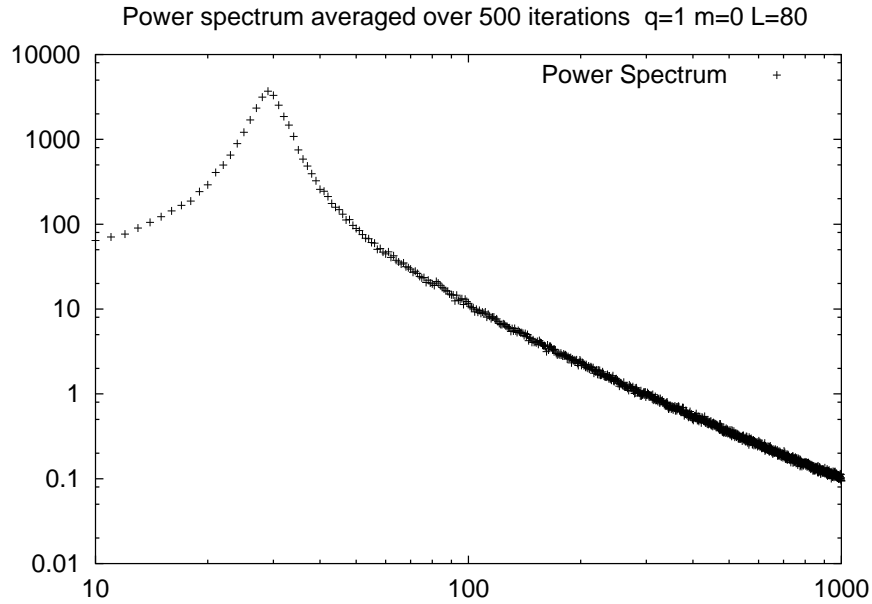

Fig. 6. Power spectrum in the fast adjustment regime, for a large network ($L = 80$) and fast adjustment ($q = 1$) . The frequency scale correspond to 320 updating per agent on average for one frequency unit

4   Conclusions

The obtained results were based on very simple assumptions on the economic network structure and on the imitation and adjustment process. But these results, especially the $1/f^2$ noise, should not depend upon the details of these assumptions. Let us give some directions about the generality of our hypotheses.

- We based the "voting" process on information processing, but this process can be also be accounted for on the basis of "positive externalities". Agents can experience increase in the utility of equipments when their neighbours also own such equipments.
- Who are the agents? The discussion implicitly assumes for simplicity reasons that agents are individuals, but the same reasoning could apply to firms taking decisions on purchasing goods or equipment or even making

strategic decisions. In this respect the size of the network (number of firms) would be much smaller which could move the dynamics towards the slow adjustment regime.

- The network topology A lattice is an extremely regular network which allows nice pattern observation, but which cannot be considered as a good model of a socio-economic network. In fact a lattice shares with real networks the property of having many short loops, which is not the case of random nets. Anyway the imitation model can be extended to structures with inhomogeneous local connectivity, small worlds or scale free networks, by rewriting equation 1 using fraction of sites with positive or negative state rather than direct summation.
- We discussed random updating of agent states, but one can also introduce other conditions, such as propagation of a purchase wave as in the Weisbuch and Solomon (2000), Weisbuch etal (2001) percolation model for which $1/f^2$ noise was also observed.

Let us now come to the observations.

- The $1/f^2$ noise was expected: such fat tails have been consistently reported in empirical data from financial markets. The commonly admitted reason for the fat tails are interactions among agents, although specific models have seldom been proposed.
- The periodic oscillations were unexpected, although their origin becomes pretty evident after observation. The most interesting interpretation in real life are business cycles. In this framework the agents are firms and the network is the "economy": the set of production, trade and services which form the economic network. We here have a possible microscopic theory of business cycles which does not suppose any external trigger such as innovation cycles often suggested by macro-economists. We probably have to take into account some specific features of economic networks such as the anisotropic character of connections (producers/users interactions are different from competition interactions) to get more precise predictions but some results such as the increase of the amplitude of activity variation with coupling are already within the framework of the present model.

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