Self-organised patterns in production networks

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October 10, 2005

Abstract

Economic firms interact via suppliers/customers interactions. These interactions define a production network. We here study the dynamics of these networks under very simple assumptions and show that they exhibit noticeable patterns of wealth and production. The main dynamical properties of this time/space organisation are studied and shown to be shared by a wide class of models.

keywords: networks, firms, avalanches, production, scale-free distributions, patterns, reaction-diffusion systems.

1 Networks of firms

Economic firms can interact via personnel, knowledge, capital, and market relations. Their obvious connection through production has received far less attention from economists than market interactions, although supply chain problems are at the heart of operational research.

Economic activity can be seen as occurring on an economic network ("the economic web") where firms are represented by vertices and their interactions by edges. In the case of providers/customers interactions these edges are asymmetric.

Taking the view of economics as a generalised supply chain, one is interested in those conditions ensuring a regular flow of goods across the production network, or inversely, to check the frontiers separating a regular flow regime from an avalanche of delivery failures which could damage the economy.

One of the earliest papers on avalanche distribution in economic networks is due to Bak *et al* [1]. It concerns production networks: edges represent suppliers/customers connections among firms engaged in batch production activity. The authors describe the distribution of production avalanches triggered by random independent demand events at the output boundary of the production network.

The recent cascade of bankruptcies which occurred in Eastern Asia in 1997, provoked some research on the influence of the loans network structure on the propagation of "bad debts" and resulting avalanches of bankruptcies ([2],[3]).

These papers ([1],[2],[3]) are not based on any empirical description of the network, but assume a very simple interaction structure: periodic lattice in Bak *et al* paper[1] and star structure in the papers about bankruptcies[2],[3]. They neither take into account price dynamics.

The present paper is along these lines: we start from a very simple lattice structure and we study the consequences of simple local processes of orders/production (with or without failure)/delivery/profit/investment on the global dynamics: evolution of global production and wealth in connection to their distribution and local patterns. In the spirit of complex systems analysis, our aim is not to present specific economic prediction, but primarily to concentrate on the generic properties (dynamical regimes, transitions, scaling laws) common to a large class of models of production networks.

A minimal model of a production network will first be introduced in section 2. Simulation results are presented in section 3. Section 4 is a discussion of the genericity of the observed dynamical behaviour: we summarise the results of several variants of the simplest model.

2 A simple model of a production network

We can schematise the suppliers/customers interactions among firms by a production network, where firms are located at the vertices and directed edges represent the delivery of products from one firm to its customers (see figure 1).

Independent local failures to produce (or to deliver) by a firm might give rise to the propagation of shortage across the production network.

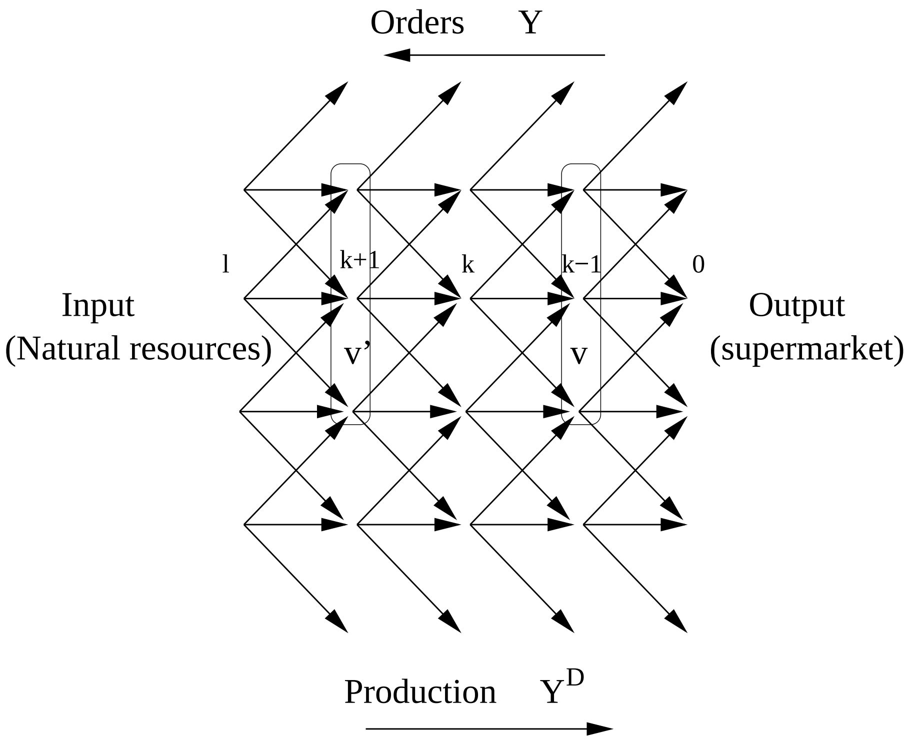

Figure 1: Firms are located at the nodes of the lattice. Delivered production (Y^D) flows from the resource input layer ($k = l$) to the output layer ($k = 0$), orders (Y) flow backward.

Let us start with a simple periodic lattice with three input connections of equal importance and three output per firm. The network is oriented from an input layer (say natural resources) towards an output layer (say the shelves of supermarkets). The longitudinal axis relates to production transfer and the transverse axis can be thought as representing either geographical position or some product space. We here use a one dimensional transverse space to facilitate the representation of the dynamics by two-dimensional patterns, but there is no reason to suppose geographical or product space to be one-dimensional in the real world. We study a 2+1 dimensions space in section 4.

In real economies, the network structure is more heterogenous with firms of unequal importance and connectivity. Furthermore some delivery connections go backwards. Most often these backward connections concern equipment goods; neglecting them as we do here implies considering equipment goods dynamics as much slower than consumption goods dynamics. Anyway, since these backward connections enter positive feedback loops, we have no reason to suppose that they would qualitatively disrupt the dynamics that we further describe.

At each time step two opposite flows get across the lattice: orders are first transmitted upstream from the output layer; production is then transmitted downstream from the input layer to the output layer.

- Orders at the output layer

  In agreement with earlier publications([2],[3]) we suppose that orders are only limited by the production capacity A_{0i} of the firm in position $0, i$, where 0 indicates the output layer, and i the transverse position in the layer.

  $$Y_{0i} = q \cdot A_{0i} \tag{1}$$

  Y_{0i} is the order in production units, and q a technological proportionality coefficient relating the quantity of product Y to the production capacity A, combining the effect of capital and labor. q is further taken equal to 1 without loss of generality.

- Orders

  Firms at each layer k, including the output layer, transfer orders upstream to get products from layer $k+1$ allowing them to produce. These orders are evenly distributed across their 3 suppliers upstream. But any firm can only produce according to its own production capacity A_{ki}. The planned production Y_{ki} is then a minimum between production capacity and orders coming from downstream:

  $$Y_{ki} = min(q \cdot A_{ki}, \sum_{i' \in v_i} \frac{Y_{(k-1)i'}}{3}) \tag{2}$$

  v stands for the supplied neighborhood, here supposed to be the three firms served by firm k, i (see figure 1).

  We suppose that resources at the input layer are always in excess and here too, production is limited only by orders and production capacity.

- Production downstream

Starting from the input layer, each firm then starts producing according to inputs and to its production capacity; but production itself is random, depending upon alea. We suppose that at each time step some catastrophic event might occur with constant probability $\mathcal{P}$ and completely destroy production. Failures result in canceling production at the firm where they occur, but also reduce production downstream, since firms downstream have to reduce their own production by lack of input. These failures to produce are uncorrelated in time and location on the grid. Delivered production Y_{ki}^d by firm k, i given by:

$$Y_{ki}^d = (\sum_{i' \in v_i'} Y_{(k+1)i'}^d \cdot \frac{Y_{ki}}{\sum_{i'' \in v_{i'}} Y_{ki''}}) \cdot \epsilon(t) \quad (3)$$

It depends upon:

- production delivered upstream from its delivering neighborhood v_i';
- whenever any of the firms $i' \in v_i'$ at level $k+1$ is not able to deliver according to the order it received, it delivers downstream at level k to its delivery neighbourhood $v_{i'}$ in proportion of the initial orders it received, which corresponds to the fraction term;
- $\epsilon(t)$ is a random term equals to 0 with probability $\mathcal{P}$ and 1 with probability $1 - \mathcal{P}$.

The propagation of production deficit due to local independent catastrophic event is the collective phenomenon we are interested in.

- Profits and production capacity increase

  Production delivery results into payments without failure. For each firm, profits are the difference between the valued quantity of delivered products and production costs, minus capital decay. Profits Π_{ki} are then written:

$$\Pi_{ki} = p \cdot Y_{ki}^d - c \cdot Y_{ki}^d - \lambda A_{ki} \quad (4)$$

  where p is the unit sale price, c is the unit cost of production, and λ is the capital decay constant due to interest rates and material degradation. We suppose that all profits are re-invested into production. Production capacities of all firms are thus updated according to:

$$A_{ki}(t+1) = A_{ki}(t) + \Pi_{ki}(t) \quad (5)$$

- Bankruptcy and re-birth.

  We suppose that firms which capital becomes negative go into bankruptcy. Their production capacity goes to zero and they neither produce nor deliver. In fact we even destroy firms which capital is under a minimum fraction of the average firm (typically 1/50). A re-birth process occurs for the corresponding vertex after a latency period: re-birth is due to the creation of new firms which use the business opportunity to produce for the downstream neighborhood of the previously bankrupted firm. New firms are created at a unique capital, a small fraction of the average firm capital (typically 1/25). (Adjusting these capital values relative to the average firm capital $< A >$ is a standard hypothesis in many economic growth models: one supposes that in evolving economies such processes depend upon the actual state of the economy and not upon fixed and predefined values).

The dynamical system that we defined here belongs to a large class of non linear systems called reaction-diffusion systems (see e.g. [8]) from chemical physics. The reaction part here is the autocatalytic loop of production and capital growth coupled with capital decay and death processes. The diffusion part is the diffusion of orders and production across the lattice. We can a priori expect a dynamical behaviour with spatio-temporal patterns, well characterised dynamical regimes separated in the parameter space by transitions or crossovers, and scale free distributions since the dynamics is essentially multiplicative and noisy. These expectations guided our choices of quantities to monitor during simulations.

3 Simulation results

3.1 Methods and parameter choice

Unless otherwise stated, the following results were obtained for a production network with 1200 nodes and ten layers between the input and the output.

Initial wealth is uniformly and randomly distributed among firms:

$$A_{ki} \in [1.0, 1.1] \tag{6}$$

One time step corresponds to the double sweep of orders and production across the network, plus updating capital according to profits.

The simulations were run for typically 5000 time steps.

The figures further displayed correspond to:

- a capital threshold for bankruptcy of $< A > /50$;
- an initial capital level of new firms of $< A > /25$;

Production costs c were 0.8 and capital decay rate $\lambda = 0.2$. In the absence of failures, stability of the economy would be ensured by sales prices $p = 1.0$. In fact, only the relative difference between these parameters influences economic growth (or decay).

Most simulations were monitored online: we directly observed the evolution of the local patterns of wealth and production which our choice of a lattice topology made possible. Most of our understanding comes from these direct observations. But we can only display global dynamics or static patterns in this manuscript.

3.2 Economic performance: the breakeven transition

The capital dynamics being essentially exponential, the parameter space is divided in two regions, where economic growth or collapse are observed. These regions are separated by a breakeven manifold. The performance of the economic system can then be tested by checking which prices correspond to breakeven. Drawing the breakeven manifolds in the parameter space allows to compare the influence of failure probability, sale price versus production cost, network depth, time lag between bankruptcy and rebirth and so on.

Figure 2 display the breakeven manifold in the failure probability $\mathcal{P}$ and sale price p plane for different values of the network depth. The growth regime is observed in the low $\mathcal{P}$ and high p region, the collapse regime in the high $\mathcal{P}$ and low p region.

At low failure probability, the breakeven prices follow a linear relation:

$$p = c + \lambda + \frac{l}{2} \cdot \mathcal{P} \tag{7}$$

where l is the total number of layers. (The $\frac{l}{2}$ comes from the fact that the integrated damage due to an isolated failure is proportional to the average number of downstream layers). At higher values of $\mathcal{P}$, interactions among firms failures are important, hence the non linear increase of compensating prices.

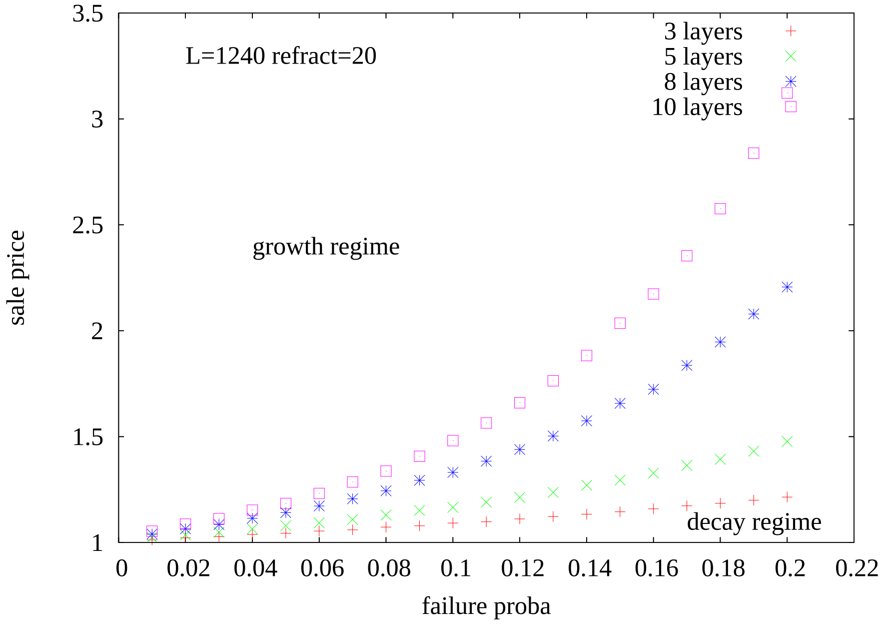

Figure 2: Regime diagram in the sale price versus probability of failure plane. The time lag between bankruptcy and re-birth is 20. Two regions of growth and economical collapse at large times are separated by lines which position are fixed by simulation parameters. The red '+' line was obtained for a 3 layers net, the green 'x' line for a 5 layers net, the blue '*' line for an 8 layers net, and pink square line for a 10 layers net.

Breakeven manifolds are a simple test of the economic performances of the network: when performances are poor, the compensating sales price has to be larger. We checked for instance that increasing the bankruptcy threshold and new firms initial capital increase global economic performance. On the other hand, increasing the time lag between bankruptcy and the apparition of new firms increase breakeven sale prices in the non-linear region.

Most further results, dynamical and statistical, are based on runs close to the breakeven price in order to avoid systematic drifts and recalibrations.

3.3 Time evolution

The simplest way to monitor the evolution of the system is to display the time variations of some of its global performance. Figure 3 displays the time variations of total wealth A, total delivered production Y^d, and the fraction of active firms for a 1200x10 lattice, with a probability of failure of 0.05 and a sale price of 1.185. Time lag between bankruptcy and and new firm creation is either 1 (for the left diagram) or 5 (for the right digram).

The features that we here report are generic to most simulation at breakeven prices. During the initial steps of the simulation, here say 1000, the wealth distribution widens due to the influences of failures. Bankruptcies do not occur as observed by checking the number of active firms, until the lowest wealth values reach the bankruptcy threshold. All quantities have smooth variations. Later, for $t > 1000$ one observes large production and wealth fluctuations characteristic of critical systems.

For larger time lag (5) between bankruptcy and firm re-birth, bankruptcies can cascade across the lattice and propagate in both network directions as checked by the decrease in the number of active firms on the right diagram of figure 3 and the patterns of figure 5..

A surprising feature is that avalanches of bankruptcies are not correlated with production level. Even when only one tenth of the firms are active, the total production is still high. In fact, most of the total production is dominated by large firms, and avalanches which concern mostly small firms are of little consequence for the global economy.

Time lag and network depth separate two distinct dynamical regimes: one with no avalanches at small time lag and network depth; nearly

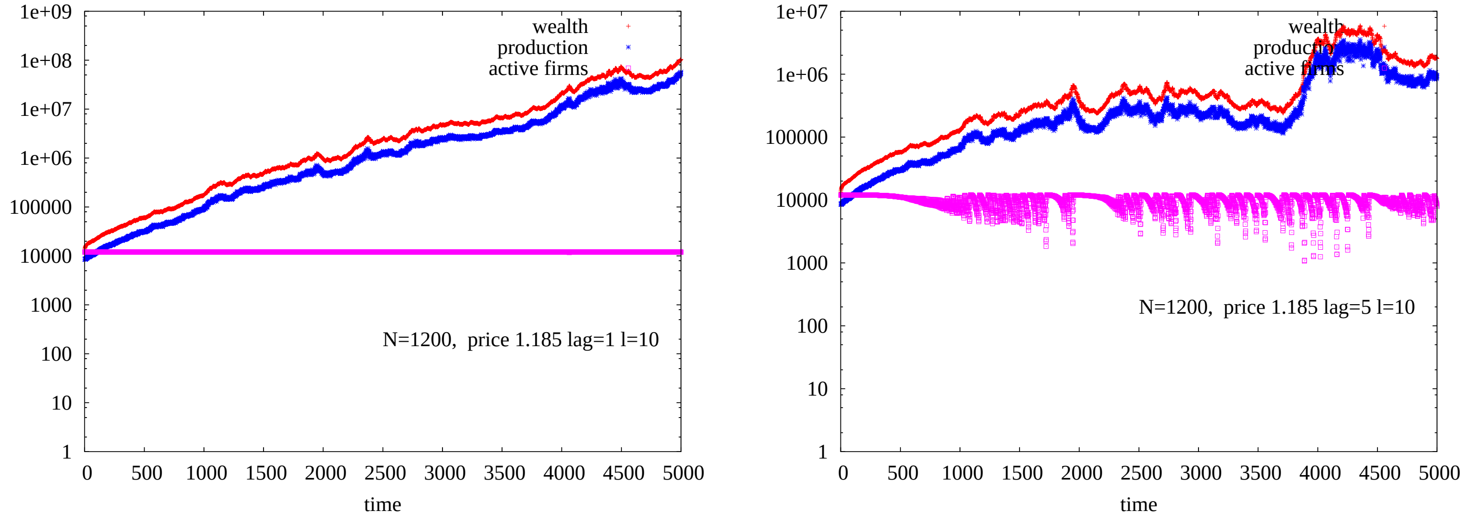

Figure 3: Time evolution of wealth (red '+'), production (blue '*'), and active firms (magenta empty squares). The network has 10 layers, 1200 firms per layer, $\mathcal{P} = 0.05$ (the failure probability). The left diagram corresponds to a small time lag (1) between bankruptcy and firm re-birth, right diagram corresponds to a larger time lag (5). Vertical scale is logarithmic.

all firms are active (with non-zero production capacity). By contrast in the avalanche regime the number of active firms can be strongly reduced (by 90 perc. in the worst cases). The transition is rather abrupt: the time averaged fraction of bankrupted firms changes from zero in the absence of avalanches to a typical magnitude of 10 perc. in the avalanche regime. The avalanche regime appears for time lags equal or larger than 2 for 6 layer nets, 3 for 4 and 5 layer nets and 4 for 3 layer nets.

A similar avalanche dynamics is reported in Battiston etal[4] who study a related model where bad debts induce higher credit and production costs which might result into avalanches of bankruptcies.

3.4 Wealth and production patterns

Like most reaction-diffusion systems, the dynamics is not uniform in space and display patterns. The wealth and production patterns displayed after 5000 time steps on figure 4 and 5 were obtained for $\mathcal{P} = 0.05$. They reflect wide distributions and spatial organisation. In these diagrams, production flows upward. The upper diagram displays wealth A and the lower one production Y_d. The intermediate bar is the colour scale, black=0, violet is the maximum wealth or production. (We in fact displayed square roots of A and Y_d in order to increase the visual dynamics of the displays; otherwise large regions of the patterns would have been red because of the scale free distributions of A and Y_d, see further).

The important result is that although random production fluctuations are uncorrelated, the inherent multiplicative (or autocatalytic) process of production + re-investment coupled with local diffusion results in a strong metastable local organisation: the dynamics cluster rich and productive firms in "active regions" separated by "poor regions" (in red or black).

Only larger time lags allow bankruptcies avalanches represented on figure 5 by black regions in the production and wealth diagrams.

These patterns are evolving in time, but are metastable on a long time scale, say of the order of several 100 time steps as seen on the succession of production patterns at different steps of the simulation as one can observe on figure 6.

The relative importance of active (and richer) regions can be checked by a Zipf plot[5]. We first isolate active regions by "clipping" the downstream (along k axis) integrated wealth at a level of one thou-

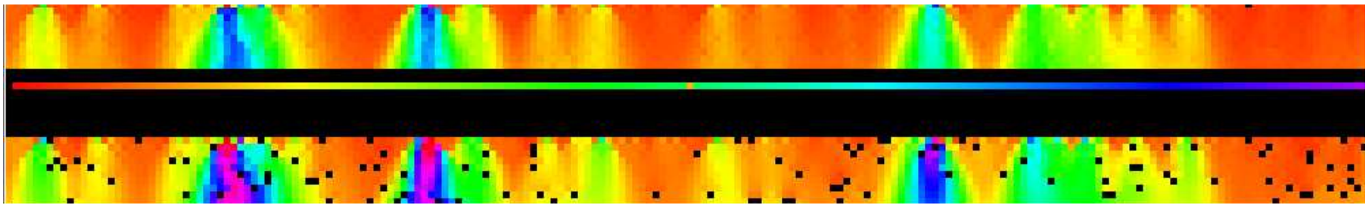

Figure 4: Patterns of wealth(upper pattern) and production (lower pattern) after 5000 iterations steps with the parameter set-up of figure 3 (left) (time lag =1), for a 200x10 lattice. For both patterns the output layer is the last one above. The intermediate line is the colour code, with minimal amplitude at the extreme left. We observe alternation of highly productive regions (in pink, blue and green colour), with less active regions (in red). Local production failures represented by black dots are randomly distributed across the production pattern.

Figure 5: Patterns of wealth(upper pattern) and production (lower pattern) after 5000 iterations steps with the parameter set-up of figure 3 (right) (time lag is 5). The same alternation of active and less active regions is observed, but with a larger time lag (5), we also get large zones of bankrupted firms in black.

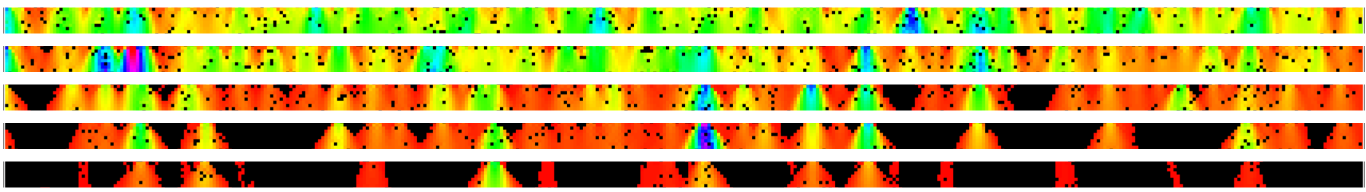

Figure 6: Successive patterns of wealth after 250, 750, 1250, 1750 and 2250 time steps with the parameter set-up of figure 3 (right, time lag = 5) for a 1200x10 lattice.

sandth of the total production. We then transversally (along i axis) integrate the wealth of active regions and order these regional wealths to get the Zipf plots (fig.7).

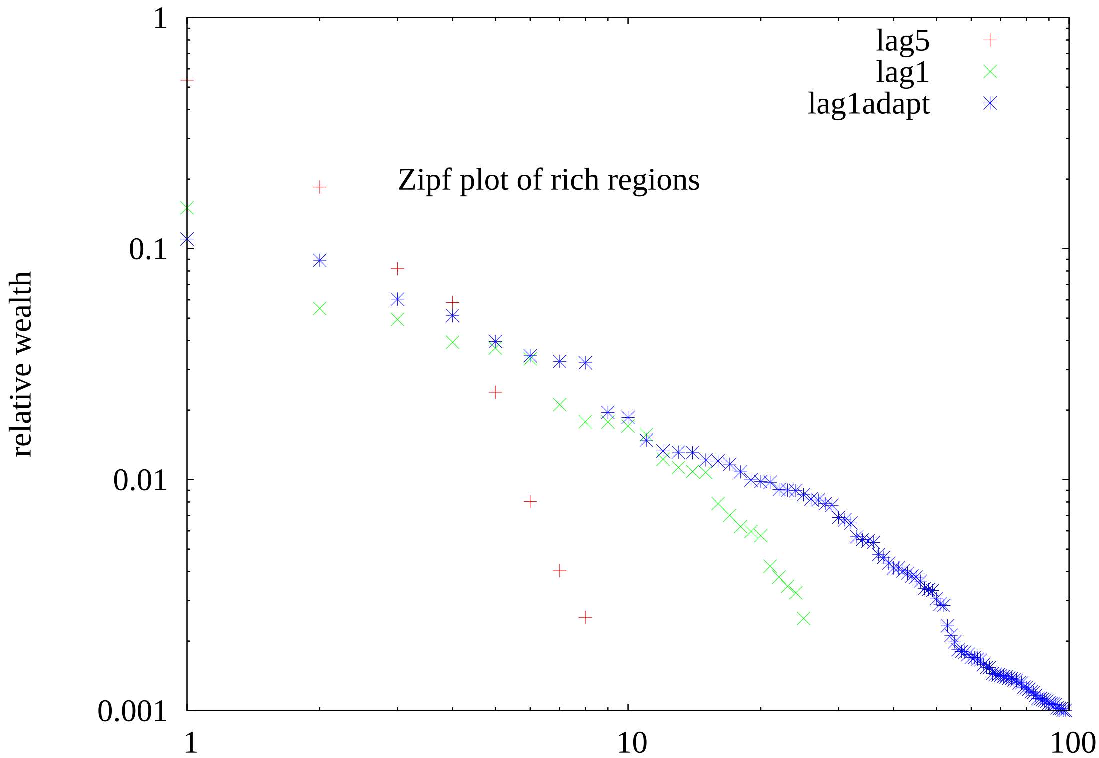

Figure 7: Zipf plot of wealth of the most active regions for the standard and adaptive firms models (cf. section 4). The vertical axis display the production relative to the total production and the horizontal axis the rank of the firm. The red '+' correspond to the standard model with time lag = 5, green 'x' to time lag = 1, and blue '*' to the adaptive firms model with time lag = 1.

All 3 Zipf plots display some resemblance with standard Zipf[5] plots of individual wealth, firm size and city size. For the model discussed here, the size decrease following approximately a power law. The apparent exponent is one when the time lag is 1. It is much higher when the time lag is 5.

Zipf plots of output ($k = 0$) active regions (not shown here) display the same characteristics.

When the time lag is 5, the most productive region accounts for more than 50 perc. of total production. The figure is 18 perc. for the second peak. The distribution typically is "winner takes all". The equivalent figures when the time lag is 1 are 10 and 8.5 perc..

In conclusion, the patterns clearly display some intermediate scale organisation in active and less active zones: strongly correlated active regions are responsible for most part of the production. The relative importance of these regions obeys a Zipf distribution.

3.5 Wealth and production histograms

The literature on multiplicative random dynamics [6, 7, 8] and the direct observation of wealth and production patterns would lead us to predict a scale free distribution of wealth and production, i.e. with no characteristic scale apparent from the distribution, as opposed for instance to Gaussian distributions.

The cumulative distribution functions (cdf) of firms wealth observed on figure 8 are indeed wide range and do not display any characteristic scale: the data on wealth and production (not shown) were taken for the same conditions as the previous figures at the end of the simulation, i.e. after 5000 time steps. The medium range of the cdf when time lag is 1 (figure 8a) extends on one and a half decade with an apparent slope of 1 ± 0.05 in log-log scale.

This observed dependence of the wealth cdf, log normal at lower A values followed by power law at intermediate A values, is partly consistent with expressions derived for pdf in the literature[6, 7, 8] on coupled differential equations with multiplicative noise.

At higher wealth, the straight line giggles and drops much faster: this is because of the underlying region structure. The last 80 perc. of the wealth is concentrated in two rich regions and its distribution is dominated by local diffusion phenomena in these regions.

The departure form the standard smooth distribution is even more noticeable when avalanches are present. The large wealth shoulder is bigger (95 perc. of production) and the first point at zero wealth stands well above the rest of the distribution: it corresponds to those 50 perc. of the firms which are momentarily bankrupted. The fraction of bankrupted firms fluctuates in time and so does the slope of the linear segment (both fluctuations are correlated since the slope of the linear segment depends upon the number of firms in the distribution).

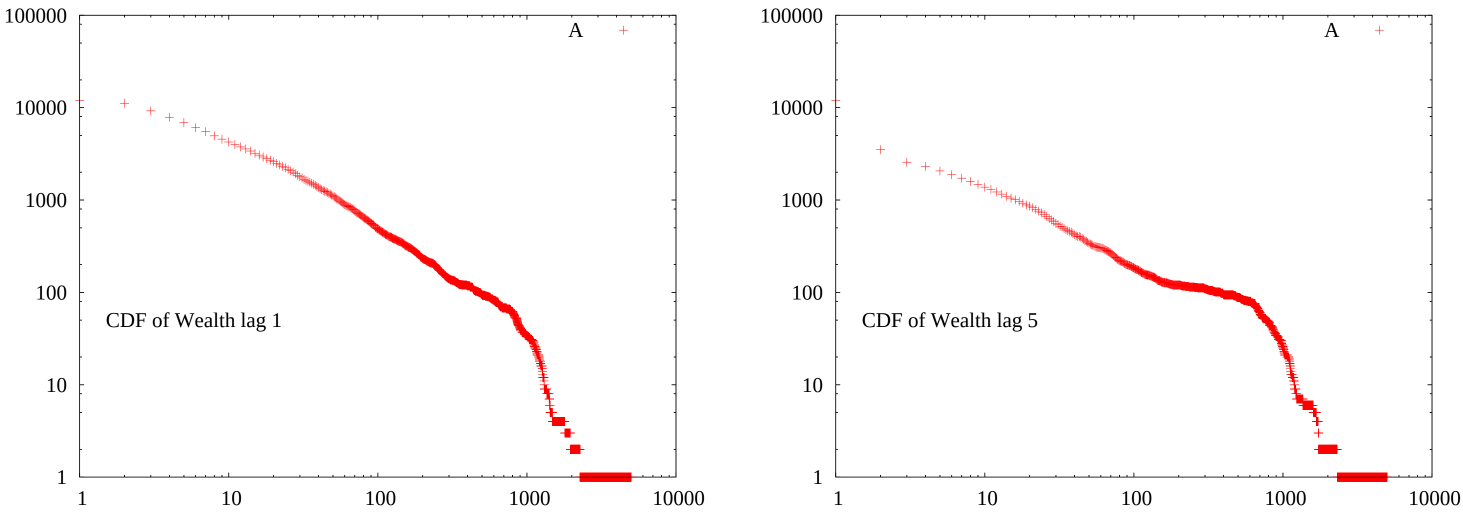

Figure 8: Cumulative distribution of wealth (red '+') after 5000 iteration steps. Parameter choices are the same as the previous figures.

In conclusion, the observed statistics largely reflect the underlying region structure: at intermediate levels of wealth, the different wealth peaks overlap (in wealth, not in space!) and we observe a smooth cdf. At the large wealth extreme the fine structure of peaks is revealed.

4 Model extensions and conclusions

The simple model of production networks that we proposed presents some remarkable properties:

- Scale free distributions of wealth and production.
- Large spatial distribution of wealth and production.
- A few active regions are responsible for most production.
- Avalanches of bankruptcies occur for larger values of the time lag between bankruptcy and firm re-birth. But even when most firms are bankrupted, the global economy is little perturbed.

Are these properties generic to a large class of models?

We checked four variants of the original model, starting with more realistic production costs:

- Influence of capital inertia: production costs don't instantly readjust to orders; capital and labour have some inertia which we modeled by writing that productions costs are a maximum function of actual costs and costs at the previous period.
- Influence of the cost of credit: production failures increase credit rates and production costs.

The simulations confirm the genericity of the semi-quantitative properties (patterns and cdf). Only breakeven prices are increased.

The third variant is a model with "adaptive firms". The lattice connection structure supposes a passive reactive behaviour of firms. But if a firm is consistently delivering less than the orders it receives, its customers should order less from it and look for alternative suppliers. Such adaptive behaviour leading to an evolutive connection structure would be more realistic. We checked an adaptive version of the model by writing that orders of firm i are proportional to the production capacity A of the upstream firms connected to firm i. Simulations gave qualitative results similar to those obtained with fixed order ratios.

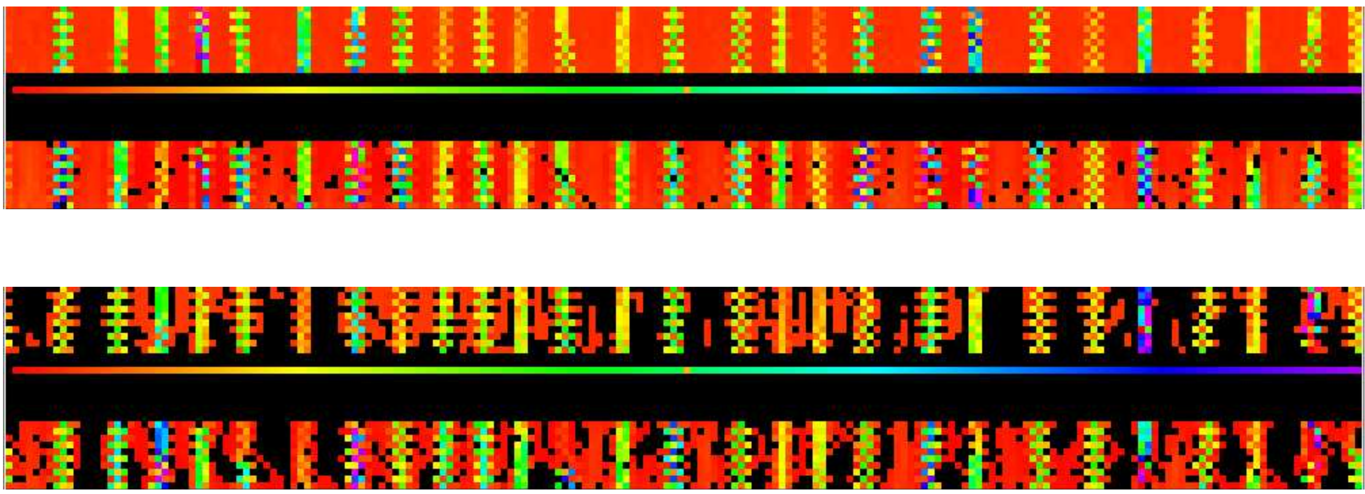

Figure 9: Wealth and production patterns for a network of "adaptive" firms. The conventions and parameters are the same as for figures 3, 4 and 5, for a 200x10 lattice. Time lag is 1, the two upper patterns correspond to $t = 1500$, the lower ones were taken when $t = 1998$.

We observe that adaptation strongly re-enforce the local structure of the economy. The general picture is the same scale free distribution of production and wealth with metastable patterns. Adaptivity of firms gives a more efficient economy with lower breakeven prices. Due to the strong local character of the economy:

- Avalanches of production are observed (see figure 9), even when time lag is short (time lag of 1).
- The spatial periodicity of the active zones is increased (see figure 9 with larger density of smaller active zones). But again the activity distribution among zones is like "winner takes all" (see the Zipf plot of figure 7).

A fourth variant which we tested is increasing dimensionality from 1+1 dimension to 2+1 d lattices (one production axis and two transverse axes). We know from physical systems that one dimensional systems have peculiarities with respect to higher dimensions: to say shortly, they can remain disordered on a much wider range of parameters. We then checked the dynamics of 2+1 dimension production networks: each firm is connected with 9 inputs from firms upstream + 9 output downstream located on 2d lattices. Simulations where run on a 50x50x5 network, at the breakeven price, with a time lag of 1 period. Most features, such as time evolution, probability distribution function of wealth and production, and patterns (see figure 10) are similar to 1+1 d results.

Simulations display patterns with spikes of wealthier and more active region, which is what we expected from empirical observation in e.g. geographical economics (rich and active regions) or vegetation patterns (shrubs in semi-arid regions[9]); but from a general reaction-diffusion perspective, patterns such as spiral waves or stripes (zebra structures) could have been predicted as well. In fact, local stable spikes are computed in analogous systems with short range excitation and long range inhibition[10]: in our case the equivalent of local excitation is the lateral diffusion of orders and the equivalent of long term inhibition is the bankruptcy process which suppresses firms according to a global criterion.

2+1 d systems seem less sensitive to failures: the breakeven prices were lower (1.06 and 1.075 for resp. adaptive and non adaptive firms) reflecting the fact that higher dimensions offer more alternative pathways to production following local failures. Nevertheless avalanches can still be observed.

In conclusion, this admittedly very simplified model - lattice connection structure, Mickey Mouse economics with no trading prices -

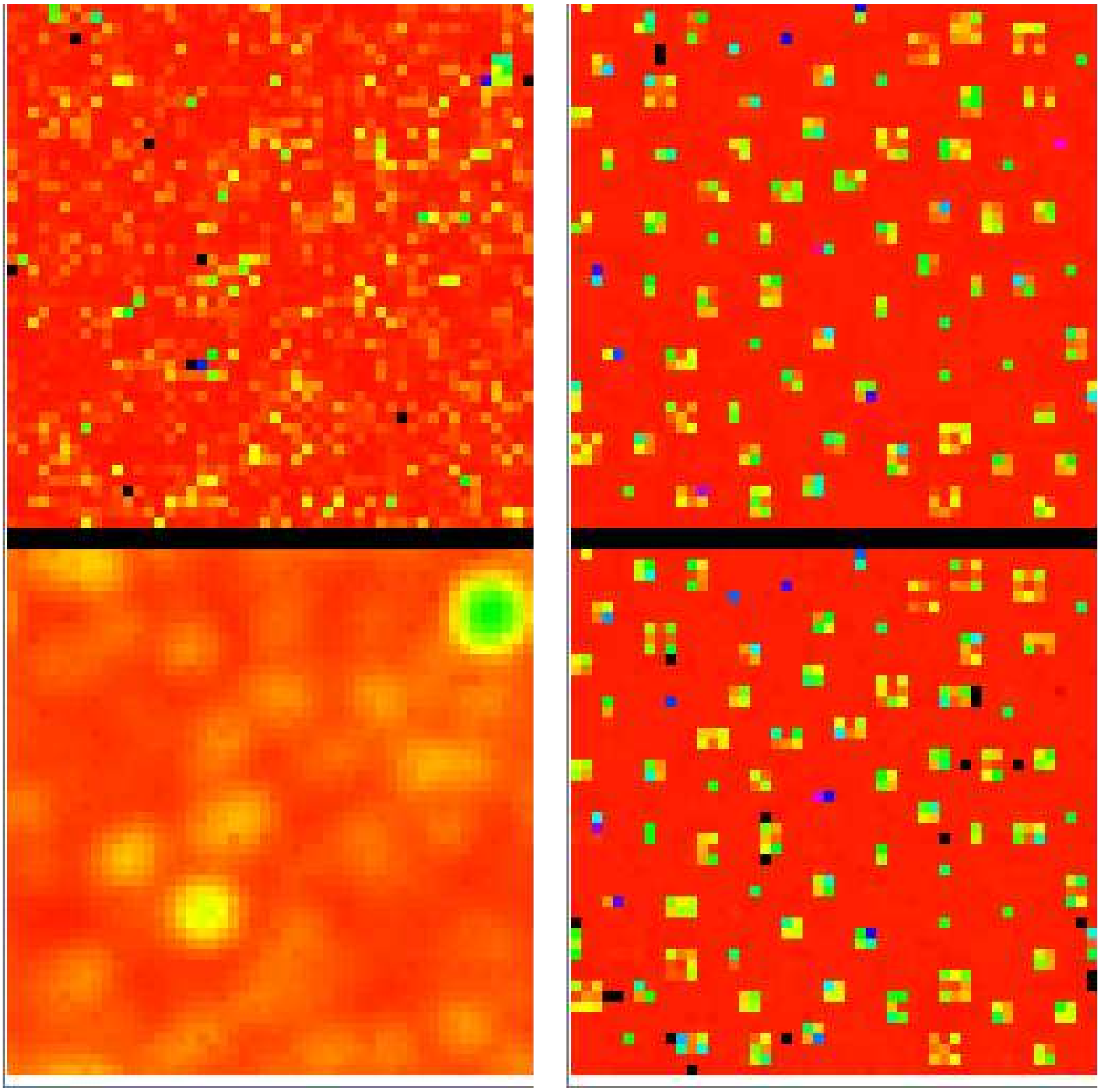

Figure 10: Patterns of wealth at the output (upper patterns) and input (lower patterns) layers of 2+1 d production networks. The left panel displays a network of non-adaptive firms and the right panel adaptive firms. Parameters are the same except for the price (resp. 1.06 and 1.075 for resp. adaptive and non adaptive firms). The same difference between adaptive (fig. 9) and non-adaptive firms (fig 4 and 5) patterns can be observe in 2+1 d as in 1+1 d networks, such as the higher spatial frequencies of peaks, and the cylindrical rather than volcano vertical shapes for adaptive firms.

give a new and direct explanation for the strong localisation of economic activities. The standard economic literature, as presented for instance in the book by Fujita and Thisse[11], attributes localisation to exogenous heterogeneities at market equilibrium, while the class of models presented in this paper attributes it to random fluctuations amplified by growth without any a priori inhomogeneity.

Acknowledgments: We thank Stefano Battiston, Bernard Derrida and Sorin Solomon for illuminating discussions and the participants to CHIEF Ancona Thematic Institute, especially Mauro Gallegati. The Laboratoire de Physique Statistique is associated with CNRS (URA 1306), ENS and Paris 6 and 7 Universities. CHIEF was supported by EXYSTENCE network of excellence, EC grant FET IST-2001-32802. This research was also supported by COSIN FET IST-2001-33555, E2C2 NEST 012975 and CO3 NEST 012410 EC grants.

References

[1] Bak P., Chen K., Scheinkman J. Woodford M.: Aggregate Fluctuations from Independent Sectoral Shocks: Self-Organized Criticality in a Model of Production and Inventory Dynamics. Ricerche Economiche, 1993;47:3-30.

[2] Stiglitz J., Greenwald B.: Towards a New Paradigm in Monetary Economics, (Raffaele Mattioli Lectures). Cambridge University Press, Cambridge UK, 2003.

[3] Delli Gatti D., Di Guilmi C., Gaffeo E., Giulioni G., Gallegati M., Palestrini A.: A new approach to business fluctuations: heterogeneus interacting agents, scaling laws and financial fragility. Journal of Economic Behavior & Organization, 2005; 56: 489-512.

[4] Battiston S., Delli Gatti D., Gallegati M., Greenwald B., Stiglitz J.E.,(2005) Credit Chains and Bankruptcies Avalanches in Supply Networks, submitted.

[5] Zipf, G.K.: Human behavior and the principle of least effort. Addison-Wesley, Cambridge MA, 1949.

Mandelbrot B.: Adaptation d'un message sur la ligne de transmission. I Quanta d'information. Comptes rendus (Paris): 1951; 232:1638-1740.

[6] Kesten, H. Random difference equations and renewal theory for products of random matrices. Acta Math.,1973; 131: 207-248.

[7] Bouchaud JP., Mezard M.: Wealth condensation in a simple model of economy, Arxiv preprint cond-mat/0002374, 2000 - arxiv.org

[8] Shnerb N. M., Louzoun Y., Bettelheim E., Solomon S.: The importance of being discrete - life always wins on the surface. 1999, http://xxx.lanl.gov/pdf/adap-org/9912005

[9] Shnerb N.M., Sarah P., Lavee H., Solomon S.: Reactive glass and vegetation patterns, Phys. Rev. Lett. 2003; 90:038101.

[10] Ermentrout B., Neural networks as spatio-temporal pattern-forming systems, Reports on Progress in Physics, 1998; 61:353-430.

[11] Fujita M., Thisse J.F.: Economics of Agglomeration. Cambridge University Press, Cambridge UK, 2002.