Social Percolators and Self Organized Criticality

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Abstract. We discuss the influence of information contagion on the dynamics of choices in social networks of heterogeneous buyers. In the case of non-adaptive agents, the dynamics results in either the contagion process being stuck and very few agents actually buying (flops) or in a 'hit' where most agents a priori interested in getting the product actually buy it. We also show that when buyers and sellers try to adjust bids and asks the tatonement process does not converge to equilibrium at some intermediate market share and that large amplitude swings are actually observed across the percolation threshold.

1 Introduction

A number of recent studies (see e.g. Albert and Barabasi 2002) concern the structure of social networks although the subject itself has more than half century a history. Computers and computer networks have made data gathering and processing easier, hence the availability of more comprehensive data and new models of network structures. The study of what dynamics occur on a network and of its functional role, in term of a social institution the network can provide has not proceeded as fast, which is not uncommon in science: characterising dynamics is often more difficult than taking statistics. In the case of social networks, the problem is made harder because of the difficulty of simply modeling the local interactions occurring along the edges connecting agents.

A very simple process of data and information exchange among economic or political agents in a social context, is the idea that connected agents exchange pieces of information. Agents never possess a complete information about the world and can only take decision based upon incomplete information (and non-perfect information processing abilities). But they try to gather complementary pieces of information from their "neighbours" (agents to which they are connected).

One of the most studied dynamics on network is then the dynamics of epidemics, the "pathogen" being either a true pathogen as in epidemiology, or a rumor, an opinion, some piece of information etc.

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Economists base the theory of social choice upon the idea that economic decisions are taken under the influence of other agents. They use the simplest mode of interaction among agents imitation. A number of economists characterized the dynamic outcome of such processes for identical agents: under a wide range of hypothesis, one observes homogeneity of agent choices. These dynamics are often described as herding behavior and empirical phenomena such as bubbles in financial markets are interpreted along these lines.

When one tries to take into account the fact that economic agents are not identical, the situation gets more intricate and uniformity of choices is not the standard macro-behaviour as we can observe in real life. Since macroscopic behaviour of disordered systems can often display phase transitions, as it is the case for percolation, it is tempting to suggest that some surprising "stylised facts" observed in socio-economic systems could be explained by the same features. The purpose of this contribution is to shed some light on empirically observed phenomena with the help of two concepts proposed in natural sciences, percolation (Broadbent and Hammersley 1957, Stauffer and Aharony 1994) and self organized criticality (Bak and Tang 1989).

A stylised fact observed in a number of economic and social phenomena could be denoted "hits or flops". In situation such as:

- markets such as toys and gadgets (Farrell 1998);
- the movie industries;
- the adoption of technological changes, or political and economical measures,
- the political arena where voters choose parties and political options.

one observes a sharp contrast between between hits which market share is a large fraction of the market and flops with a nearly negligible success. One possible interpretation is the occurrence of percolation across a social network which we discuss in this contribution.

The next section concerns passive actors with fixed preferences for buyers (or adopters or voters) facing passive sellers (or decision makers or parties) with fixed offers: we then consider a "one shot" possible purchase or adoption situation.

But in the case of a series of purchases on different occasions, one would expect changes in the hits and flops situation in the presence of adaptive sellers which would want to avoid flops (!) and would be happier if they could obtain sales hits at a lower production cost. Buyers could also adjust their preference according to whether they were able to get what they wanted in the past. One of the question we raise in the later sections is whether a "tatonement" processes would restore equilibrium and balance between supply and demand. In fact, we will show by computer simulation that the resulting dynamics is characteristic of self organized criticality.

2 Social percolation

2.1 Simple models

Although the present discussion could apply to a number of equivalent situations, let use the case of a market with buyers and sellers to be more specific. We first start from the idea that agents lack full information about a product before buying it. But agents are not isolated. They are members of social networks: any of them is related to a number of other agents, called in our framework neighbours. An agent can get lacking information about a product from those of her neighbours who bought the product since they were able to use it. A simple hypothesis is that information is reducible to a scalar quantity that we call the quality q of the product, transferable from any agent who purchased to all her neighbours. On receiving the information, an agent can then take a decision about buying herself. In fact all agents might not want to buy a product of quality q: information is only one necessary condition to take a decision; if it happens that the information from their neighbours is "the quality of the product is less than what you demand", the agents take a negative decision. A simple way to implement such a decision process is to suppose that agents i each have individual preferences p_i. Heterogeneous agents of course have different preferences. The two conditions for purchase can be written as:

An agent will buy a product (or adopt a new technology, or vote for a party) when:

- she obtains information about the quality q from one of her neighbours who made a positive choice;
- if $q > p_i$.

As presently posed the problem becomes equivalent to what physicists call site percolation (Stauffer and Aharony 1994). Let us consider a network, lattice or with random connections, with a random uniform distribution of p_i's. If one starts from initial conditions with a large majority of undecided agents (by lack of information) and a small fraction of "early birds" who have already taken a positive decision, contagion proceeds across the net from newly decided agents to their neighbours. The early birds could be individuals who got the product for free or who happened to get direct relevant information in the case of markets, promoters of a new technology in case of the adoption of new technologies or party members in politics. Waves of purchase (or adoption) propagate across the network, new purchasers carrying the information at each time step to their neighbours. We can predict a priori that none of the agents with preferences larger than the quality of the product will participate to the above described contagion process across the social network; they can then be removed as totally inactive. In fact for random distribution of p_i's the outcome of the process is:

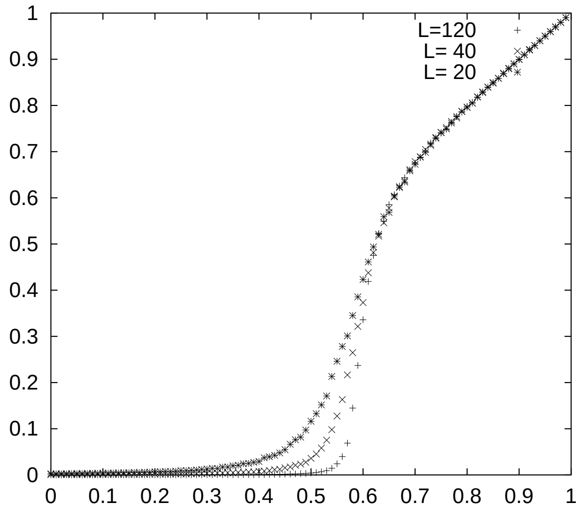

Fig. 1. Fraction of purchasers as a function of the percentage of agents with preferences smaller than the quality of the product for square lattices of linear dimension 20, 40 and 120.

- surprisingly predictable: the fraction of actual buyers (or adopters) only depends upon the fraction f of agents with
$$p_i < q$$
and not upon the initial distribution of early birds[1];
- surprisingly non-linear: the fraction of actual buyers is nearly a step function of fraction f with a threshold p_c.

Figure 1 summarizes simulations on square lattices of linear size 20, 40 and 120. f is varied from 0 to 1 by steps of 0.01. Each point correspond to an average over 1000 random samplings. Periodic boundary conditions were used. The initial condition is one early bird in the center of the lattice. The steepness of the percolation transition depends upon L: the slope would be vertical for infinite L.

The comparison of f to the percolation threshold p_c on the social network allows to predict the success or failure of the product with a very high probability:

[1] Still prediction is difficult in the transition region, with outcome fluctuating largely for different realizations of the sampling of preferences. Furthermore what we mean by independence of initial distribution, refer to genericity: the property is true with a probability converging to one for neworks of infinite size and for random samplings.

- When $f > p_c$ most sites with preferences smaller than q are invaded, which results in a large success for the seller (or the promoters of the new technology, or the political party presenting the platform). The fraction of actual buyers is nearly equal to f in this regime as seen in figure 1.
- By contrast when $f < p_c$, contagion soon stops and the product is a commercial failure with very small market share.

Let us recall some well known figures (Stauffer and Aharony 1994) in the case of uniform random distributions of p_i's: for a square lattice with four neighbours per site $p_c = 0.593$, with eight neighbours $p_c = 0.407$, and for a random network with large connectivity k, $p_c = 1/(k-1)$ (the connectivity of a network is the number of neighbours per site). The above contrasted dynamic behaviours resulting from the necessity of contagion for adoption of a new product are very different from the prediction of "perfectly rational economics" which would yield a market share exactly equal to f.

This contrasted behaviour between the two dynamical regimes is a generic property independent of the details of the network connectivity. Furthermore it is not restricted to the hypothesis of unique criterion for agent's decision. In management science, a standard approach to decision making is multi-criteria analysis. Several criteria can influence agent decisions: they might take into account not only immediate or delayed profits but also time spent, risks, or even non-economic considerations such as moral, aesthetic or social values. The qualitative behavior that we above described, hits or flops, is also observed for multi-criteria analysis whatever procedure is used to process the criteria, as long as we suppose that all the information necessary for decision is transmitted at the same time by the neighborhood.

3 Adjustment meta-dynamics

Among the situations that we try to model some are occurring rarely such as major technological changes, while some others are recurrent as in the movie industry: people often visit movie theaters (or restaurants) in their home town, and producers are producing new movies for the market at a frequency high enough for both parties to adjust supply and demand according to their previous experience. In fact, one should distinguish among two regimes according to whether the percolation process is faster or slower than the tatonnement process.

3.1 Slow adjustment

In the case of the movie industry, a director is producing movies at a slow rate with respect to the time it takes for the movies to eventually percolate across the social network of movie-goers. Readjustment is slower than percolation. The phenomenon can be formalised (Salomon etal 2000) as:

- after opportunities during which they went to the movie, the agents will be more demanding and typically increase their expectations (here the preferences p_i); on the opposite, those who did not go, lower their preferences.
- after hits (resp. flops) the movie producers will decrease (resp. increase) the quality q of the produced movie(s), in their effort to remain above the threshold while minimizing expenses.

We have made computer simulations based on a fast contagion process leading eventually to percolation, embedded in a slower adjustment process as described above: we iterated a series of steps each one composed of a contagion process which was left evolving until percolation or its absence was checked, and of the resulting adjustment process. The early birds were agents with $p_i < q$ aligned along one side of the square lattice. Percolation (resp. its absence) was considered to be achieved when one site of the purchasing cluster reached the other side (resp. when the contagion process stopped earlier). All details concerning the algorithms that were used, including the Leath algorithm, are described in Solomon et al (2000).

After a transient adjustment period, we observed an alternation of hits and flops. Such a dynamics is often described as self organised criticality: the adjustment process brings and maintains the system parameters p_i's and q in the neighborhood of the percolation threshold.

The simulations were done on square lattices. We restricted ourselves to the simplest dynamics: the quality of the movie increased by δq if no cluster spanned from top to bottom, while it decreased by δq otherwise. The viewer's preference p_i, initially distributed randomly between 0 and 1, changed by $\pm\delta p$ depending on whether i went to the movie or not.

The results can be summarized as :

- Adaptive movie quality: for fixed agent preferences, $\delta p = 0$, and for adaptive quality $\delta q > 0$, we observed that the quality q moves to the usual percolation threshold; in this limit, the dynamics of our system is reminiscent of self-organization mechanisms arising in thermal critical phenomena [11], where a suitable feedback mechanism may push the temperature towards the critical temperature.
- Adaptive customer preferences: $\delta p > 0$ but fixed quality $\delta q = 0$, the p_i distribution drifts towards a single peak centered on the fixed q value, taken equal to 0.5 (no percolation) or 0.593 (some percolating clusters).
- When both δp and δq are positive, p_i and q drift towards $p_c = 0.593$, even if the initial q was 0.5. Our dynamic percolator thus drifts towards the standard percolation threshold of 0.593, exhibiting self-organized criticality (for longer times the p_i and q may drift away together).

We thus observe large-amplitude variations of the fraction of movie goers which is reminiscent of the fat tails observed in the distribution of return in financial markets. The above analysis is consistent with the interpretation of fat tails as due to strong cooperative effects, (here the constraints imposed by information contagion on the buy/sell decisions of the agents), in the neighbourhood of a transition[2]. Variants of this model are discussed in Ahmed and Abdusalam (2000), Goldenberg etal (2000), Huang (2000), Das Gupta (2000) and Weisbuch and Stauffer (2000).

3.2 Fast adjustment

In the present case, we suppose that the time necessary for the information to propagate across the social net is larger than the time between successive purchases by the same buyer. Although purchases are repetitive, we further suppose that after purchasing a good, a buyer is not active during some kind of a refractory period m in analogy with the phenomena observed in nerve tissues or chemical reactions. The rationale for this hypothesis in economics is the existence of some consumption time for a commodity or some decay time for an investment between successive purchases. The regime of fast adjustment is observed when the refractory period is smaller than the time it takes for the purchase front to sweep across the lattice. In the iterated percolation model three conditions are then necessary for purchase by an agent:

- The agent has not taken any decision (to purchase or not) for a period at least equal to the refractory period m;
- he gets some private information from one of his neighbours;
- the quality q of the product is higher than his preference/ expectations p_i.

We further add preferences[3] adjustment dynamics to the above decision process. We suppose that agents which purchased before the refractory period increase their preference coefficient for the next period by choosing p_i through a random sampling between p_i and 1. Alternatively, we suppose that agents who did not purchase before the refractory period decrease their preference coefficient for the next period by choosing p_i through a random sampling between 0 and q. Those agents which did not receive the information because they might have been screened by refusers don't readjust their preference.

In the case of repetitive purchases, waiting until one of your neighbours purchases still makes sense for the interpretation of contagion based on positive externalities. For the interpretation based on information propagation, waiting for private information from a neighbour after the first purchase might seem unnecessary, since the agent should already know about q. The idea here is that q might change through some adjustment process of the

[2] We do not imply here that the General Equilibrium theory is wrong, but simply that some of its assumptions about convexity don't apply to situations where the decision process gives such large variations as observed around the percolation transition

[3] quality adjustment discussed later does not change the results

producer, hence the necessity for a prospective buyer to wait for up-to-date private information before any decision. Since only the sign of the quantity $q - p_i$ matters for the decision process, readjusting randomly p_i should be equivalent to readjusting randomly both q and p_i. We ran most simulations with fixed q and tested the hypothesis on equivalence by a few runs with adaptive q.

Simulations set-up The model defined above was run on square lattices with four neighbours and periodic boundary conditions. We ran many numerical simulations (see Weisbuch and Solomon 2000). We used the Leath algorithm to follow the time evolution of purchases. In the description of the results, one time step corresponds to the propagation of information and of the purchasing process from all purchasers at the current time step to all the immediate neighbours (at the next time step). The following quantities were monitored:

- Time evolution of purchases at each time step (figure 2).
- Patterns of purchase in the network at given time steps (figure 3).
- Average fraction of actual purchases at any given time step. (These average fractions were taken after an initial growth process of 100 time steps).
- The purchase process eventually comes to an end if no purchase is made during a time step. We have measured the average final time whenever the purchase process actually stopped, and the frequency of such occurrences. Averages were most often made over 5 000 runs. (The averages for the fractions of purchase and of agents willing to purchase are only taken for the set of non-stopping runs).
- For the set of non-stopping runs we averaged the power spectrum of purchases (figure 4).

The simulations were made on square lattices of size L equal to 40, 60, 80 and 160. We used periodic boundary conditions. The refractory period m was varied from 4 to L. For m larger than L the purchase process stops after one sweep of the network by the purchase front (see further) by lack of potential purchasers. Preferences p_i were initially randomly distributed (uniform distribution). One single run normally goes on for 4096 time steps: this is more than necessary to take fast Fourier transform since coherence is lost after roughly $2L$ time steps.

Typical results for a representative set of parameters Let us analyse a first experimental condition with $L = 40, m = 10$, runtime of 4096 steps and averages taken on 500 runs. The figure 2 displays a typical time plot of sales. One can notice large fluctuations with short (around 25 steps) time scale correlations, very different from white noise.

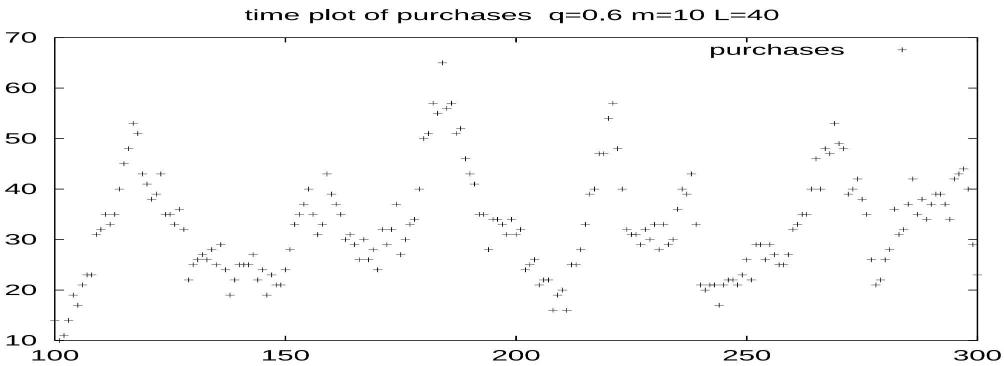

Fig. 2. Time plot of purchases after 100 time steps for a 40×40 square lattice, with a refractory period of 10 and quality 0.6 slightly above the percolation threshold.

A typical pattern displaying the states of the agents is shown in figure 3. White cells correspond to "awake" agents that are ready to make a choice when triggered by eventual active neighbours. Their previous choices occurred more than m time steps ago. Other cells made their choice more recently: black cells correspond to agents who purchased and grey cells to agents who refused to purchase. On average (500 runs and 4096 time steps) 2 percent of the agents (O(1/L)) purchase at any time step. They are located on the black positions at the edges of the dark clusters, forming a disconnected purchasing front.

On-line monitoring of the patterns dynamics shows the displacement of the purchasing fronts across the lattice. The typical time scale, around 25, observed in figure 2 for the low frequency of the oscillation is similar to the time it takes to a front to move across the lattice and it scales with L.

We observed an average fraction of 0.593 potential purchasers, very close to the percolation threshold, which explains why quite often (19 percent of observed runs) the runs stop at an average time of 358, i.e. after a few sweeps across the lattice.

Finally, figure 4 displays the power spectrum of purchases averaged on those 81 percent of the 5000 runs which carried across 4096 time steps without stop. We observe a power law with a -2 exponent between periods 4 and 40. The flat spectrum at larger periods corresponds to the loss of coherence for times larger than the sweep time of fronts across the lattice.

Influence of simulation parameters and model variants Systematic tests of simulation parameters and model variants were made.

The quality parameter q

When q is less than the percolation threshold, purchases stop very early since there is no percolation across the lattice. When q is higher than the

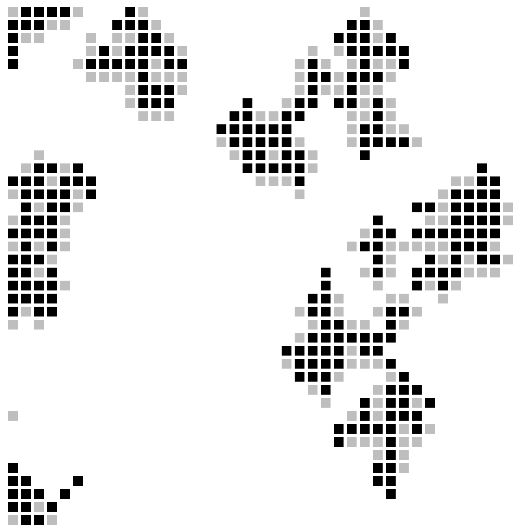

Fig. 3. Pattern of purchases on a 40 × 40 square lattice. White cells correspond to "awake" agents that are ready to make a choice when triggered by their active neighbours. Black cells correspond to agents during their refractory period who purchased and grey cells to agents who refused to purchase.

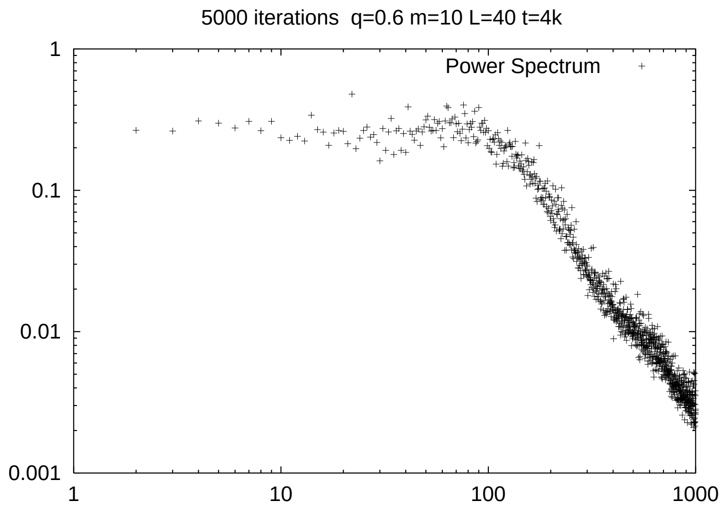

Fig. 4. Log-log plot of the power spectrum of purchases computed on a time span of 4096 time steps and averaged over 3300 runs. 1000 on the x axis correspond to 4 periods, 100 to 40 periods. The high frequency (above 100) spectrum obeys a power law with a -2 exponent.

percolation threshold, the dynamics soon re-shuffle the distribution of individual preferences p_i such that the fraction of agents with preferences below q gets closer to the percolation threshold.

The refractory period m

At low values of m, the sweeping process of the purchase front across the lattice is less dominant and a lot of re-purchasing can occur locally. Purchase rate is doubled at $m = 4$ with respect to $m = 10$ and nearly all runs persist for long time, except 9 percent which stop very early after 7 time steps on average. The power spectrum does not display a well characterized power law.

Variants of the initial model

As mentioned in the introduction, we also tested the equivalence between the model with fixed q and a model with adaptive q to test the equivalence of dynamic properties of both model. We supposed that after each time step, a producer changes the quality of the proposed product by a "tatonnement" process: when a sale is made (resp. refused), she decreases q by δq (resp. increases) through equation:

$$q(t+1) = q(t) \pm \delta q. \tag{1}$$

We tried $\delta q = 0.0001$ and $\delta q = 0.00001$ corresponding to adjustments of roughly 0.1 and 0.01 per lattice sweep for q. The obtained Fourier spectra display the same scaling behaviour as for fixed q.

In the small world (Watts and Strogatz 1998) variant a small fraction (typically a few per thousand) of the connections were randomly assigned across the whole lattice without taking into account any notion of neighbourhood. The results were qualitatively analogous to those obtained when all connections are periodic.

We also tried to change the readjustment process of those agents which refused to purchase. Rather than randomly redistribute them in the segment $[0, q]$, we redistributed them in the segment $[0, p_i]$. The same dynamics were observed.

Finally another change was a slower adjustment of preferences: purchasers preferences being redistributed in $[p_i, p_i + \alpha(1 - p_i)]$ and non purchasers' preferences in $[p_i, (1 - \alpha)p_i]$ with $0 < \alpha < 1$. Once more we did not observe big changes.

4 Conclusions

In conclusion, we have seen that information contagion processes do not average but rather amplify at the "macro" level the consequences of the heterogeneity of individual agents.

In the absence of any readjustment dynamics of prices and preferences, contrasted behavior of either hits or flops are observed on either side of the percolation threshold. The imitation or polling processes, such as Polya urns, often described in the economics literature, also result in contrasted behavior depending upon initial conditions. On the opposite, for the contagion dynamics described here, no sensitivity to initial conditions is observed except in the neighborhood of the threshold.

Rather than driving the economic systems towards equilibrium with small eventual fluctuations, readjustment dynamics brings it close to the percolation transition resulting in large non-Gaussian fluctuations. In the case of slow readjustment, the system alternates from a flop regime with no contagion to the hit regime where the purchasing cluster percolate across the network.

Let us stress that although most simulations reported here were done on square lattices, we have no reason to doubt from the validity of the qualitative results for all sort of networks such as random nets or small world. The percolation transition is a generic property observable on any kind of networks.

One might also ask what are the assumptions that are critical for the observed dynamics. Randomness of the initial distribution of preferences is important. We also suppose that early birds are a minimal fraction of all possible buyers, and that their distribution is random, with no correlation with the distribution of preferences.

The percolation model is not the only instance of disorder in dynamical systems. A more general approach is that of INCA (Inhomogeneous Cellular Automata, Weisbuch 1990). Rather than a contact process such as one buying neighbour being a condition for purchase, in voters (or polling) models, an agents polls his neighbours and after comparing the number of buyers to a threshold, decide to purchase when this number is larger than the threshold. Inhomogeneity is introduced by having specific thresholds for each agents. Neural nets and zero temperature spin glasses are variants of this approach. The percolation contact process would correspond to a threshold for adoption being either one adopting neighbour or a threshold higher than the number of neighbors.

The INCA model gives rise to crossover transitions between growth and no growth of a buying process according to the parameters characterising the distribution of thresholds (Weisbuch and Boudjema 1999). We might then expect that the adjustment mechanism that we used for the percolation model would also display non-gaussian noise. We performed preliminary simulations based on the following adjustment process:

- increase the threshold for purchase after purchase;
- decrease the threshold for purchase after refusal.

Non gaussian fluctuations of the fraction of purchasing agents are indeed observed, but no clear scaling law appear.

Finally, along the same ideas, an alternative mechanism was proposed by Plouraboué etal (1998) and Steyer and Zimmermann (2000) to explain long term correlations is also based on information diffusion on a social network but implies a Hebbian learning mechanism: connections among neighbours are of varying intensity, and are reinforced when agents take similar decisions. The decision process of individual agents is based on polling their neighbours rather than simply being triggered by any active neighbour. Steyer and Zimmermann report in the presence of Hebbian learning an even more correlated behaviour with a $1/f$ power spectrum instead of our $1/f^2$ result.

Our conjecture based on the several sets of simulations and on reasonable arguments is that long term correlations and non-exponential distribution of fluctuation sizes should be expected whenever decision processes are based on information contagion and seller/customer adaptation. Lux and Ausloos (2001) express the same idea by referring more generally to interactions among economic agents as the generating mechanism. Reciprocally, when long term correlations are observed, an information contagion process is a possible explanation for the effect.

These observations about dynamics can be used not only to describe empirical phenomena, but also to build "selling" strategies for sellers or policy implementers.

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