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Sustainable Development and Spatial Inhomogeneities

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Abstract Historical data, theory and computer simulations support a connection between growth and economic inequality. Our present world with large regional differences in economic activity is a result of fast economic growth during the last two centuries. Because of limits to growth we might expect a future world to develop differently with far less growth. The question that we here address is: "Would a world with a sustainable economy be less unequal?" We then develop integrated spatial economic models based on *limited* resources consumption and technical knowledge accumulation and study them by the way of computer simulations. When the only coupling between world regions is diffusion we do not observe any spatial unequality. By contrast, highly localized economic activities are maintained by global market mechanisms. Structures sizes are determined by transportation costs. Wide distributions of capital and production are also predicted in this regime.

Keywords Patterns · Environment · Economics · Distributions · Integrated assessment · Energy

1 Introduction

Any geographical map of human economic activities display strong contrasts between world regions. Rich "Industrial regions" concentrate most of wealth, industrial production and human capital, while others are practically depleted of any economic activity. Traditional geography relates these contrasts to physical geography and human history. In view of their ubiquitous character, more recent views developed in Economic Geography for more than 150 years tried interpretations based on self-organisation as discussed in for instance [14]. Mathematical models proposed by economists are General Equilibrium models. By contrast, we view the self organisation and spatial economic patterns as the results of dynamical processes; in that respect, our interpretation goes with those of chemical patterns discussed

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by Prigogine [13] as dissipative structures, Rayleigh-Bénard rolls [5], Turing patterns in Biology [28], patterns in the visual cortex [11] and so on.

We can use historical evidence to support the dynamic hypothesis. As discussed in Bairoch [3] and followers [9], the industrial revolution strongly increased economic disparities. Before the industrial revolution, average wealth of the western world circa 1750 was comparable to the average wealth of Asian countries while this ratio was increased by a ten-fold factor at the end of the XXth century. More details can be found in the first chapter of Economic geography [9].

One of the first attempt to develop the interpretation of economic spatial disparities as the result of dynamical processes was proposed by a set of physicists and geographers in [6, 10, 18, 32, 33]. The used a very simple mathematical model, the so-called AB model, proposed earlier to explain the origin of life in [16, 17, 24, 25]. In the AB model, patterns are the result of the noisy multiplicative growth of the B variable. We will rapidly give the basis and the results of the AB model in the model section.

Our present concern is different: we are interested in the possible existence or survival of economic disparities in a sustainable world with very limited growth. In other words, if growth is absent, are there other positive interaction loops which would support spatial patterns?

We start here from a "renewable resource" perspective. But, rather than trying to describe the dynamics of the transition between a "business as usual" regime to a "sustainable economy" regime as most economists of the global change do [22, 27, 31], we here focus on the future state of the world after the transition to a sustainable economy.

Although there exists a number of models in the economics literature, we take here a physics and engineering point of view which takes into account several technical features which are neglected in most of this literature.

- A standard assumption of economist is the possibility of infinite substitution among production factors such as labour, capital and resources. For instance a firm lacking workers for any reason would be able to reach equivalent production levels by increasing capital investments. This assumption has been strongly criticised by some resource economists [15]. We then propose a production function which does not allow infinite substitution among resource and capital.
- An important aspect of renewable energy resources is that their influx is limited, since they come essentially from solar radiation. Their yield per unit area is further limited by physical constraints [19]. In the model that we propose the source term for energy is finite and kept constant.

Since the issue of interregional equity is central to all negotiations concerning Climate Change, let us stress that our perspective here is not prescriptive, but only descriptive, or rather predictive. We here propose to figure out the consequences of future technical choices in the energy sector on wealth distribution in the World.

The purpose of this communication is to discuss this problem using a simple framework, coupling capital, technology and resource dynamics. Since we want to take into account technological and resource dynamics, each component is simplified to the extreme, without compromising the essentials. We then take a complex system approach: our predictions are restricted to semi-quantitative laws generic to a class of models [1, 29], rather than precise computations valid for one specific model. The advantage of such predictions is their robustness to the specific details of the studied models. The complex system approach is also consistent with the CLAW[1] (a Crude Look At the Whole [12]) perspective on sustainable development advocated by Murray Gell-Mann.

We start by briefly recalling the basics and the results of the AB model which relates growth with spatial inequalities in Sect. 2.1.

The simplest local version of the models are the three differential equation system discussed in Sect. 2.2. The equilibrium solutions of this model correspond to a limited growth. We also discuss in Sect. 2.2 our specific modeling choices with respect more standard approaches in the economics of Global Change.

A more novel aspect of this analysis is the spatial variants presented in Sect. 2.3. We take into account the fact that production is localised in a two dimensional space, as on Earth, and we investigate the spatial repartition of economic activity.

We show, mostly by numerical simulations in Sect. 3 that depending upon the hypotheses on the distribution of resources, passive diffusion or simplified market mechanisms, rather different repartitions can be observed. One of the results is that market mechanisms induce segregation of economic activities.

In Sect. 4 we draw the conclusions in terms of economics and future available technologies.

A short Appendix give the results of more systematic trials in scaling, pattern stability and initial conditions.

2 Models

2.1 The AB Model

The AK model [4] is the most basic model of evolutionary economics, i.e. the part of economics which describes growth and technical change. It is based on a simple differential equation describing the evolution of capital K:

$$\dot{K} = \rho AK - \delta_K K \tag{1}$$

AK is the production function linear in capital and technological coefficient A. A expresses the efficiency of used technology. ρAK represents the fraction ρ of production AK which is re-invested. The second term represents capital depreciation[2] per unit time. The model and its variants have been used extensively in evolutionary economics.

The AB model uses is a discretised version of Eq. (1) where variable name K is changed for B. Variables A and B have integer values and the dynamics involves transition probabilities in the population of A and B species occupying lattice cells. It has been originally proposed by [16, 17, 24, 25] to discuss the origin of life in terms of auto-catalysis: the chemical reactions involving the replication of DNA for instance use DNA template as an input and their output is an increase in DNA content. In the AB model, species B is multiplied by the reaction and A the second chemical species plays the role of a catalyst. The spatial version of the model shows that self-replication can be locally maintained with B growing

[1] According to Gell-Mann, the use of very simple assessment models, the Crude Look At the Whole, is well adapted to policy makers who don't care about details.

[2] When firms invest in production rather than in buying stock, part of the capital *decreases* in time because for instance of machines maintenance.

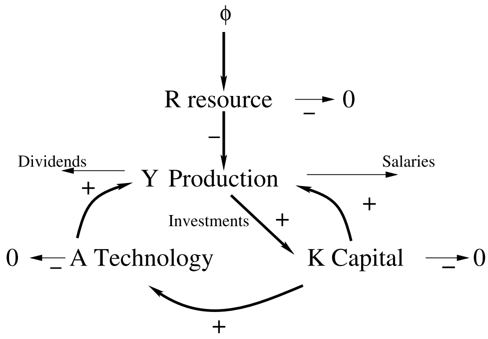

Fig. 1 Scheme of interactions. Each *signed arrow* corresponds to a term in the system (2)–(5). Natural Resources are supplied at a constant rate ϕ. A, K and R (or C a part of R), contribute to production Y. Production is re-invested partly in Capital, K, but consumes resources R. A fraction of capital K is invested in R&D to increase A. A, K and R also decay linearly, as represented by the *arrows* towards 0

exponentially, even when average A concentration would not be sufficient to sustain growth in a homogeneous vessel.

The application of the AB model to other fields of research, ecology and economics is discussed in [6, 10, 18, 32, 33]. The "chemistry of economic reactions" in geographical space is the creation of wealth: capital K is multiplied similarly to B thanks to technology A. Not surprisingly equivalent results were obtained: concentration of economic activity in favoured regions, those where skilled workers or cultural factors are present. The empirical analysis is based on the transition in Eastern Europe in the nineties after the fall of the Soviet Union and of the Popular Democracies. Poland for instance, displays geographical patterns of economic activities, with very active production spots in contrast to economically depleted regions.

In conclusion the simplest *unlimited growth* model, based on multiplicative noisy processes, yields large distribution of wealth and production and contrasted spatial patterns of economic activity.

2.2 The Homogeneous Model of a Resource Limited Economy

Let us start with a local description of economic processes as if all quantities were available everywhere on earth without transportation cost. We then use a simple ordinary differential equation model which variables are energy resources R, capital K and technical knowledge A. The economic sector is reduced to a single product and the model can be considered as an extension of the class of AK models used in the description of technical progress (Evolutionary economics). These 3 variables are coupled by processes such as production, which uses resource, technical progress improving A thanks to capital investments etc. as described in Fig. 1 and Eqs. (2)–(5).

We now use three-variable differential equations which do not take into account possible spatial inhomogeneities. The results of this simple model will be used as a reference to interpret the results of the spatial models.

Natural Resources R are supplied at a constant rate ϕ. Part of the resource, C for consumption, is used for production. Since Sustainable energies are difficult to store, we introduce a fast decay term $\delta_R R$ (think of electricity which is the intermediate form of the energy resource). In our model, natural resources are a limiting factor, a hypothesis that applies well to the energy sector and to traditional industries.

Production depends upon three factors, technology A, capital K and used resource C. Production capacity includes a variable technical coefficient A; its variations represent technical progress, driven by capital investment. Production also depends upon the availability of resources. Several authors have stressed the importance of physical resources in production basing their analysis on empirical evidence gathered during the 20th century such as [15]. We suppose that a constant fraction of profits is re-invested in capital. Profits are the difference between production in monetary units and costs.

The dynamics of the technical coefficient is first driven by investment of capital in research and development which increases the technical coefficient in proportion to the available capital. We also take into account the limits of technology. For instance, the best solar technology cannot yield more kilowatts than what is received from the sun. (The idea of an upper limit to A is not common in Evolutionary Economics [21], and contradicts in some sense our present experience of technical innovation in Information and Communication Technologies, the media industries, financial services etc. But unlimited growth of technology coefficients is not the case for extraction and energy industries [19], which concern us in this paper.) The role of the K_1 term in the denominator of Eq. (2) is to generates the upper bound for the technical coefficient. The proportional decay term corresponds to the loss of technicity (e.g. because of retirement of skilled workers) in the absence of maintenance (by e.g. training).

Figure 1 shows the 'chemistry' of interactions among the three factors: A Technology, K Capital and R Resource. Even without mathematics, such an interaction scheme interpreted in terms of system dynamics predicts the possibility of multi-stationarity (several attractors) because of the two positive feedback loops between capital and production on the one hand, and technology, production and capital on the other hand.[3]

A general formulation gives the following ODE system:

$$\dot{A} = \mu \frac{K}{K + K_1} - \delta_A A \tag{2}$$

$$\dot{K} = \rho\big(Y(A, K, C) - p.C - \delta_K K\big) \tag{3}$$

$$\dot{R} = \phi - C - \delta_R R \tag{4}$$

$$C = C(A, K, R) \tag{5}$$

where Y is production, depending upon A, K and C the level of resource used for production. δ_A, δ_K, δ_R are linear decay terms, p is the unit price of the resource, and ϕ the constant resource influx. ρ is the fraction of re-invested profits and μ the initial growth rate of the technical coefficient.

We choose a production function:

$$Y = \frac{AKC}{\alpha K + C} \tag{6}$$

which results in saturation of production in the presence of either an excess of resource or an excess of capital. We suppose that a machine is able to process a given quantity of energy for production; excess of resource is useless, but the machine remains idle when resource is insufficient. We thus abandon the hypothesis of perfect substitution among production

[3] The scheme also allows to predict the possible emergence of "dissipative structures" in space [13]: a constant influx of energy is dissipated after going through intermediate stages including positive loops involved in the production process.

factors commonly used by most economists. The chosen expression for Y still maintains constant return to scale.[4]

The amount of resource to be used in production $C(A, K, R)$ is computed from profit Π maximisation:

$$\Pi = Y(A, C, R) - p.C \tag{7}$$

where p is the price of the resource. With $Y = \frac{AKC}{\alpha K + C}$ optimal resource consumption is:

$$C_{opt} = K\left(\sqrt{\frac{\alpha A}{p}} - \alpha\right) \tag{8}$$

As a matter of fact, the level of available resources might be less that optimal; in that case, since natural resource level R cannot become negative, the actual consumption level is:

$$C = \min(C_{opt}, R) \tag{9}$$

2.2.1 Dimensional Analysis and Parameter Reduction

R and C are energies and ϕ is an energy flux, i.e. a power.

A has the dimension of a unitary cost in the expression of production. It correspond to the value, in monetary units, of the good produced with one unit of energy.

Since the system has 3 variables plus time, we can use parameter changes to obtain a simpler mathematical system. We take $\alpha = 1$ in the equations: capital is expressed in the unit corresponding to the processing of one energy unit per unit time. Further simplification are possible, for instance taking $\rho = 1$ by changing the time unit by a factor ρ. We will not do so in the present text to keep more explicit our choices of decay factors. But we used these simplifications in order check the parameter space.

The present ODE formalism is a simpler description than the optimisation approach commonly used in the economics of Global Change. Our basic assumption is that constant fractions of profits, themselves supposed proportional to production, are distributed in productive capital (increasing K) and in consumption (salaries and dividends).

Let us summarise at this stage the main differences in assumptions between "Climate Change" economics [22, 27, 31] and our approach:

– Models of climate change are models of the transition towards a greener economy under some environmental constraint, for instance a 2-degree temperature increase. They are optimal control models: they investigate economic trajectories optimising a constant discount rate utility function. But there is little agreement on which discount rate to choose or even whether a finite discount rate is a reasonable assumption. By contrast, we are here interested in a stabilised situation, after the transition(s). We then do not pretend to give any results pertinent to the transition itself. In the period of time we are considering, our simple assumption of a constant fraction of re-invested profits rather than a constant discount rate of consumption used in "Climate Change" economics corresponds to an equivalent degree of simplification.

[4] This expression from economics simply means that production scales linearly with the productions factors K and C: a firm with twice as much capital using twice as much resource has a twice as much output.

– Constant return to scale and perfect substitution among production factors hypotheses make Cobb-Douglas production functions [8] a preferred choice among economists. On the other hand, economists of renewable resources (see e.g. [7]) use production functions bilinear in resource and effort (effort includes both Capital and Labour). In our view, the first choice applies when the resource is abundant and when the limitation is on the side of capital, but the second choice is more appropriate when the resources are rare. Specifically, in this paper, all plots are drawn using a production function defined by Eq. (6); i.e. based on constant return to scale but abandoning the idea of perfect substitution, thus reflecting the limitation imposed by the scarcity of the resource on production.
– As in Evolutionary Economics [21], we take into account technological progress. But in the case of energy production, there are physical limits which we here introduce. These limits might come from thermodynamics, from laws of conservation of energy, or simply from the surface occupied by power stations: a thorough discussion is given in [19].

2.2.2 Algebraic Solution of the Homogeneous Model

With a choice of $\frac{AKR}{K+R}$ as the production function Y the differential system 2–5 has several stationary points. In the region of large capital, resource consumption C is limited by the availability of the resource. We can then replace C by R and the simplified equations for the stationary point are written:

$$R_0 = \frac{\phi}{1+\delta_R} \tag{10}$$

$$\frac{A_0 x}{1+x} = \frac{p}{x} + \delta_K x \tag{11}$$

$$\mu \frac{K_0}{K_0+K_1} = \delta_A A_0 \tag{12}$$

where $x = K_0/R_0$. The second degree equation relating $x = K_0/R_0$ and A_0:

$$\delta_K x^2 + (p + \delta_K - A_0)x + p = 0 \tag{13}$$

has two roots x_1 and x_2 which expression is simplified when[5] $(A_0 - (p+\delta_K))^2 \gg 4p\delta_K$

$$x_1 = \frac{A_0}{\delta_K} \tag{14}$$

$$x_2 = \frac{p}{A_0} \tag{15}$$

Only the first root corresponds to a stable positive fixed point and we finally obtain:

$$K_0 = \frac{\phi\mu}{(1+\delta_R)\delta_K\delta_A} - K_1 \tag{16}$$

$$A_0 = \frac{\mu}{\delta_A}\left(1 - K_1 \frac{(1+\delta_R)\delta_K}{\phi}\right) \tag{17}$$

[5] Since production is a concave function with respect to C, a positive profit is only made when production increases faster than cost for low values of C. $p < A$ is then a necessary condition for a positive profit. The expressions obtained under the assumption $A_0 \gg p$ remain valid for a large price domain as checked by numerical simulations.

Surprisingly resource cost p appears only as a first order correction to K_0:

$$K_0 = \frac{\phi\mu}{(1+\delta_R)\delta_K\delta_A} - K_1 - \frac{\phi p}{(1+\delta_R)\delta_K} \tag{18}$$

although it controls how fast capital grows.

K_0 is in first approximation the ratio of the product of source terms over the product of decay terms.

With our choice of decay terms, $\delta_R \gg \delta_K \gg \delta_A$, numerical integrations show that equilibrium of the resource is first achieved, in times of the order of $1/(1+\delta_R)$ while the system (after times of the order of $1/\delta_K$) evolves in the (A, K) space along the slow manifold of equation:

$$K = A\frac{C_0}{\delta_K} \tag{19}$$

2.2.3 Adding Noise to the Model

All economic systems are subject to fluctuations in production, deliveries and market prices. We checked by simulations that the results are little changed by introducing a noise level of 10 % on three parameters μ, ρ and price p. At each integration step, the parameter values are random numbers drawn from a normal distribution with the mean and standard deviation given as arguments.

Facing a fluctuating signal such as profit, from which an agent has to take a decision such as investment, a good averaging technique to minimise the consequences of noise is to take a moving average. Capital dynamics equation (3) shows precisely that Capital is the moving average of Profit with a decay factor of δ_K, which is relevant in such a case. This is the reason why we use Capital K as the signal for agents decisions to invest in R&D so as to increase A as we already did in the equation describing the dynamics of A in Eq. (2).

2.3 The Spatial Models

Both variants are coupled map lattices. They use the same local dynamics as Eqs. (2)–(6) with fluctuating μ, ρ and price p parameters. Only the nature of coupling neighbouring cells differs.

2.3.1 The Spatial Model with Diffusion Coupling

Let us see what happens when spatial dynamics are introduced in the model. For a physicist, the most direct and natural way to couple time and space dynamics is via diffusion terms. We will here first check this approach, although its economic interpretation is not as straightforward as in physical sciences:

- Diffusion of technology A is a standard assumption in the sociology of innovation [23].
- The outsourcing of production inputs by firms to external providers in their neighbourhood is one of the basis of the diffusion term for K. The decrease of moral hazard is one more reason to invest in one's neighbourhood. Transportation costs might also play a role etc.
- Diffusion of the resource R is a simple representation of local market mechanisms among neighbouring regions which would tend to equilibrate local disparities in R.

The corresponding dynamics are a reaction-diffusion system and we use coupled map lattices for numerical simulations on a square lattice with cyclic boundary conditions.

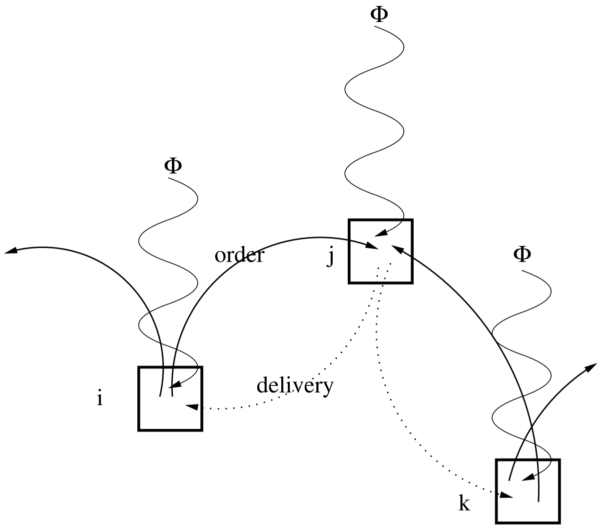

Fig. 2 The market process: Cell i sends orders to e.g. cell j. Upon receiving orders cell j delivers according to orders and availability of resource. All cells receive constant resource flux Φ from the sun

2.3.2 The Spatial Model with Trade Dynamics

From an economic perspective, trading among possibly distant providers and energy users is a more sensible mechanism than passive diffusion. Simulations of trade in a variety of situations such as stock exchange, Forex, perishable goods, etc. has been an active field of research in the recent years. Some economists, empiricists as well as theoretists, often noticed that the trading rules, the institutions, have a very important role in shaping the market structure, as opposed to the vision of fluid exchanges guided by the Adam Smith Invisible Hand. This is especially true in markets of perishable goods [30], and electricity, the secondary form of renewable energy, is certainly a perishable good. One could imagine to incorporate in our model models of markets based on Agents methods, but present attempts with these methods yield impressive simulation times and do not even guarantee convergence for any reasonable number of agents [20].

We rather take the view of Bounded Rationality advocated by e.g. Herbert Simon [26]: because of limits in available information and cognitive processing, human agents base their decisions on standard routines which have proven their efficiency according to collective experience. Although slightly more intricate to model than the full rationality and optimisation used by standard economists, bounded rationality can be expressed in simple routines to be incorporated in mathematical model which we here demonstrate.

We then use the following simplified description of a market (see Fig. 2). Cells are both providers of resources and users of resources for production. The trade algorithm is the following:

– The user in cell i "computes" its total consumption C_i based on profit optimisation: Profit Π_i is given by:

$$\Pi_i = \frac{A_i K_i C_i}{K_i + C_i} - p.C_i \tag{20}$$

Optimal profit is obtained when:

$$C_i = K_i\left(\sqrt{\frac{A_i}{p}} - 1\right) \tag{21}$$

– The user i fractionates[6] her order to different neighbouring providers j, including cell i, according to their attractivity function $f(K_j, d_{ij})$. The fraction f of orders to cell j depends upon its economic power represented by K_j and upon its distance d_{ij}. f is normalised such that:

$$\sum_j f(K_j, d_{ij}) = 1. \tag{22}$$

Orders C_{ij} from user i are then distributed to providers j according to:

$$C_{ij} = f(K_j, d_{ij}).C_i \tag{23}$$

The total amount of distributed orders is C_i.

– Providers j deliver in proportion to their actual resource level R_j and the total received orders; when they don't have enough resource to deliver all orders, they deliver:

$$C_{ji} = R_j \frac{orders_{ij}}{\sum_j orders_{ij}} \tag{24}$$

Otherwise they deliver the full orders.

Several attractivity function f can be imagined. We chose *a priori* linear dependence in K_j[7] and exponential decay in d_{ij}, on the basis of an optimisation between the extra cost of non-local purchase and the gain in information thanks to exploration of several providers as in [30]. Taking price as linearly increasing over distance, the combined profit $\hat{\Pi}$ is written:

$$\hat{\Pi} = \sum_j \frac{A_i K_i C_{ij}}{K_i + \sum_j} C_{ij} - (p_0 + t_c d_{ij}).C_{ij} - \frac{1}{\beta}\sum_j C_{ij}\log(C_{ij}) \tag{25}$$

where $\frac{1}{\beta}$ is the value of information,[8] t_c is transportation cost and the last term is the entropy of the distribution of orders, equivalent to Shanon information. Deriving with respect to C_{ij} with $\sum_j C_{ij}$ held constant one obtains the expression of C_{ij}, from which deduce f:

$$f(d_{ij}) = \frac{\exp(-\beta t_c d_{ij})}{\sum_j \exp(-\beta t_c d_{ij})} \tag{26}$$

[6] In fluctuating economic conditions, a customer never knows precisely whether her order to a provider will be delivered. She then distributes orders to different providers to minimise risks. This is often described by economists as the exploration/exploitation compromise: exploit available knowledge about best providers but keep on exploring other possibilities.

[7] Actual profit could be a rational choice for attractivity. We already said that in noisy environments, actors rather guide their choice on a moving average of profit upon characteristic time $\frac{1}{\delta_K}$, which is precisely K according to Eq. (3).

[8] Noisy environments induce losses when resources are not delivered. Checking more expensive non-local providers for supply is an insurance against such losses; β is thus related to the cost of these failures of delivery.

Table 1 Table of parameters used in the reported results. μ and $K1$ are resp. the transfer rate of capital to the technology coefficient and its limiting factor, ρ is the fraction of re-invested profit and ϕ is the influx of resource per cell. A uniformly distributed noise level of 0.1 is applied to variables μ, ρ and p_0, the base price of the resource. δ are decay rates. Orders to neighbours are decreased by a factor f per unit distance. The time unit is one year

Parameters	Values
μ	0.3
$K1$	100.
ρ	0.2
ϕ	10
p_0	10
δ_A	0.03
δ_K	0.15
δ_R	0.3
f	0.5
noise	0.1

f thus depends upon distance, but is independent of K_j. The chosen attractivity function is often called a logit function [2] by economists which justify this choice by a 'Shaking Hand' hypothesis: economic actors are supposedly driven away from optimality because of stochastic processes.

The above algorithm corresponds to some procedural rationality balancing the exploration/exploitation compromise. It is not optimal: more efficient algorithms could be imagined such that agents would learn how much providers can deliver and choose accordingly. The above described procedure is simple and adequate to the sophistication of the present model. And furthermore, procedures improved by learning would re-enforce the concentration phenomena that we report in the results section.

Let us note that the procedures that we have chosen for trade can be interpreted as a kind of "active transport" where capital attracts resource to be used to increase even more capital. The trade and capital production processes thus allow to re-establish a positive reaction loop.

3 Simulation Results

All reported simulations use the same local dynamics as Eqs. (2)–(6) with fluctuating μ, ρ and price p parameters.

Most spatial simulations reported here were run on a 50×50 square lattice with Von Neumann neighbourhood (4 neighbours, N, S, E, W) and periodic boundary conditions (cells at the upper (resp. right) edge of the lattice are connected to cells at the lower (resp. left) edge). They usually last until time $= 1000$, with a time step of 0.001. Smaller time steps yields equivalent results, but larger time steps favour spurious checkerboard structures well-known in cellular automata with parallel iteration. Longer simulations on larger lattices gave equivalent results described in the Appendix. Initial conditions were random values of A, K and R distributed according to a Poisson distribution with average values computed from Eqs. (10), (16) and (17). The role of initial conditions is discussed in the Appendix. We run a number of simulations with different values of the parameters, and variants of the production functions, but the reported figures were made with the set of parameters figuring in Table 1 and a production function given by Eq. (6) ($Y = \frac{AKR}{K+R}$).

3.1 Local Diffusion of Resource R

When all variables A, K and R dynamics are driven by passive diffusion dynamics, their distribution is narrow peaked and no spatial structure is observed. The interpretation of this

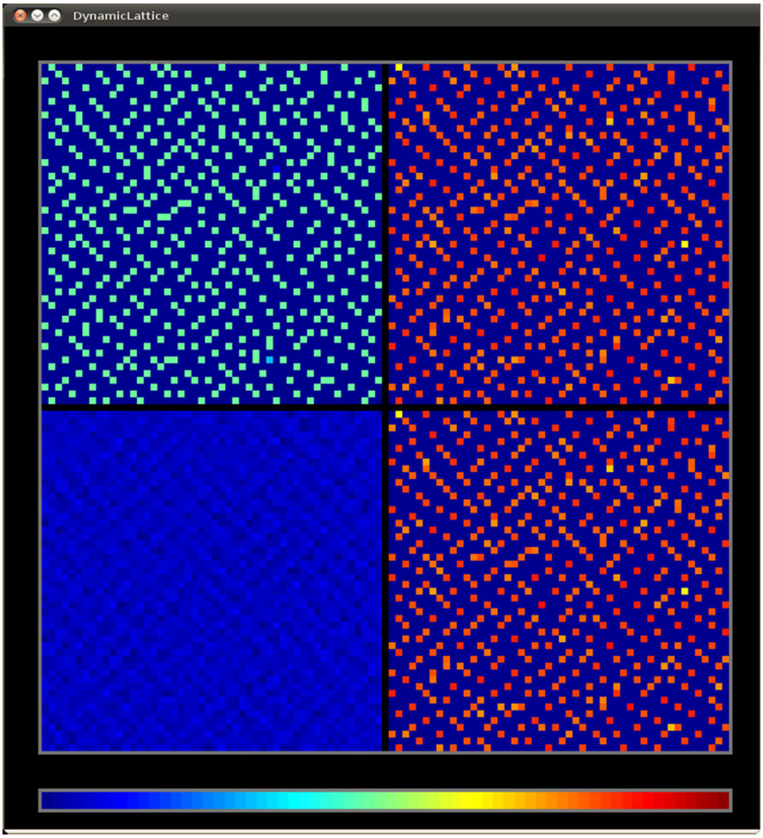

Fig. 3 Spatial patterns at time 1000 of Technical knowledge A, Capital K, resource R and Production displayed clockwise starting from the upper left square. The colour scale is logarithmic: from 0 (*dark blue*) to 100,000 (*dark red*). The same *lighter cells* on the A, Capital K and Production patterns are active spots while the *dark blue regions* are depleted of any economic activity (Color figure online)

absence of this spatial homogeneity is simply that the two positive loops of Fig. 1 do not give rise to instabilities because the technical efficiency is near saturation and any further increase of capital is limited by the availability of the resource.

3.2 Non-local Trade of Resource R

Trade dynamics described in Sect. 2.3.2 is specified by a trade neighbourhood. We have chosen a maximum 'Manhattan' distance for trade of 5 cells, corresponding to a diamond shape neighbourhood of 61 cells. In fact the important parameter is the ratio of the transportation cost over information cost which defines an inverse characteristic length $t_c.\beta$ according to Eq. (26). It characterises the exponential decrease of orders with unit distance by a constant factor $f = \exp -(t_c.\beta)$. Figures 3, 4, 5 and 6 were obtained with $f = 0.5$.

Figure 3 displays the spatial patterns of the technical coefficient, the capital, the resource and the production. They are organised in very active spots (around 500) surrounded by less productive zones. Patterns obtained at time 1000 are disordered; patterns taken after a longer integration time ($t = 10{,}000$) display more regularity (Fig. 7).

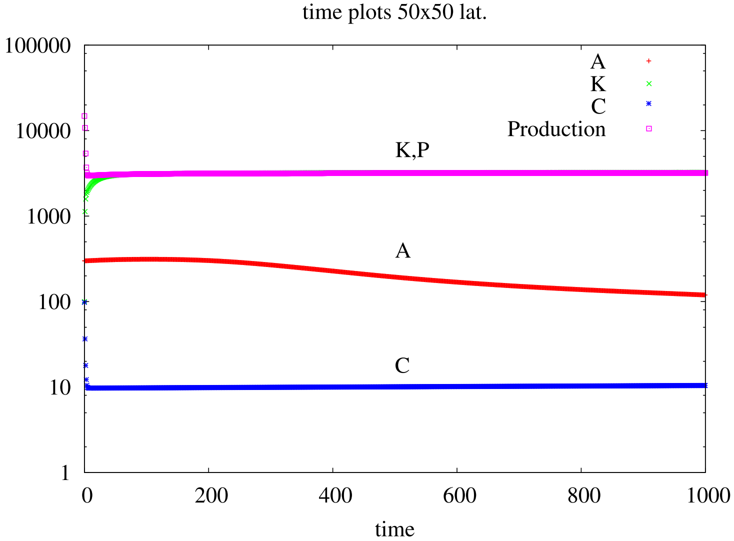

Fig. 4 Time variation of average Technical knowledge A, resource consumption C, production P and capital K

The time variations of the average quantities displayed in Fig. 4 is not very different from those observed in the homogeneous system and in the system with only passive diffusion (not presented here). The only noticeable difference is the decrease of the average technical coefficient A since technicity is not maintained in the absence of production in depleted regions. We checked that the asymptotic average values of capital, resource and production are quite robust with respect to noise or to the spatial decay factor f: this is because the total production and capital correspond to the exploitation of all available resource flux by those active firms which have reached the limit technological coefficient $\frac{\mu}{\delta_A}$.

The amplitude of spatial differences appears clearly when histograms are drawn (Fig. 5). The log-log plots[9] indicate scale-free distributions on several order of magnitude. These wide distributions of capital and production reflects their multiplicative (or auto-catalytic) dynamics.

Cumulative distribution function (Fig. 6) display more clearly the bimodal character of the distribution of Production. A large fraction of the sites have nearly zero production. This fraction increases with time from $2/5$ at time $t = 400$ to $4/5$ at time $t = 8000$; it seems to saturate around $4/5$. The parameters have been chosen such that the time unit is one year. The distribution of active sites production extends over one order of magnitude. As observed in Fig. 6, production difference slowly increases with time. The rich sites saturates early in wealth and production, and due to the slower dynamics of the technical coefficient A, more and more depleted sites are slowly driven to collapse. The collapse of A in the depleted regions is the main factor responsible for the metastability of the patterns.

Increasing noise levels by a factor 2 does not really influence the results.

On the other hand changing market spatial decay constant f from 1 to 0.25 changes the relative number of active spots as observed in Fig. 7 where f varies from 1 to 0.25.

[9] Since some variables also take 0 values they are translated by 0.1 to figure on the log-log plot.

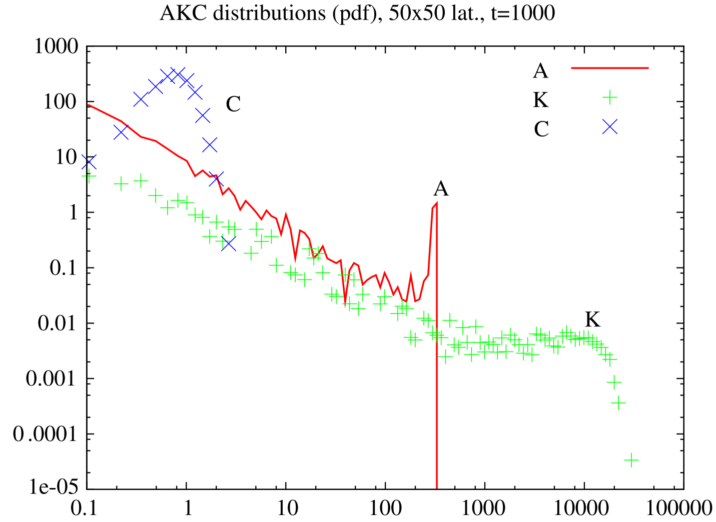

Fig. 5 Partial distributions functions of technical coefficient A, resource consumption C, and capital K, in log-log coordinates. Production distribution is not displayed for clarity but it is quite similar to capital distribution

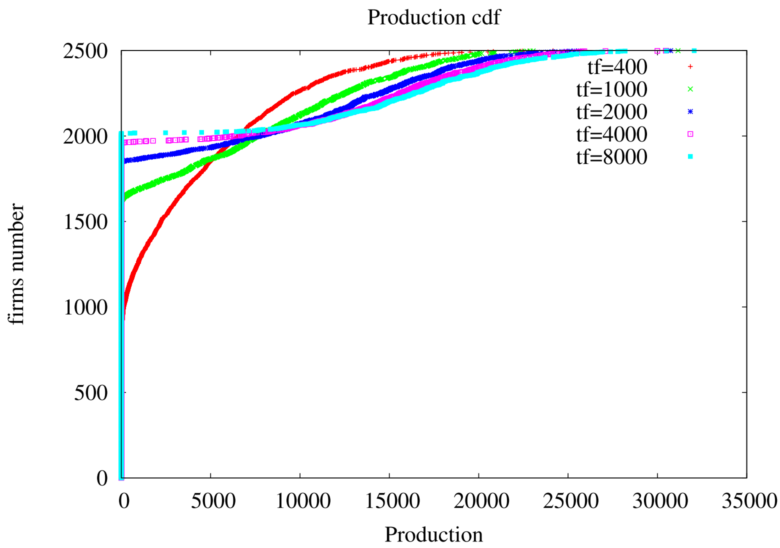

Fig. 6 Evolution of the cumulative distribution function of Production at large times

Since f is a coupling constant directly correlated with the interaction length $1/\beta t_c$ by expression (26) we might expect that a further decrease in f would bring the system in the homogeneous regime. This is indeed the case when $f = 0.1$ or less; the transition occurs around $f = 0.2$ for which mixed patterns are observed.

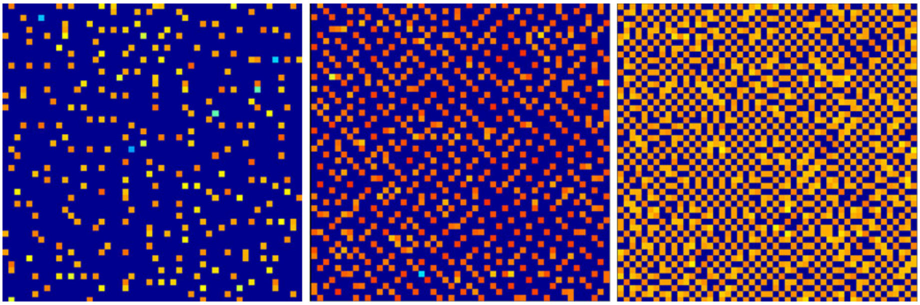

Fig. 7 Production patterns at large integration times, 10,000, for different market spatial decay constants 1.0, 0.5 and 0.25 from left to right

4 Conclusions

Even such a rudimentary model allows semi-quantitative conclusions. Let us remind our hypotheses:

1. Limitation on resource influx, technological coefficients and substitution of production factors.
2. Market mechanisms in favour of the concentration of production to richest producers.

The first hypothesis limits the instabilities introduced by the positive loops production-profit-capital investment-production … and production-profit-R&D investment-production … The second hypothesis on the fact that capital attracts resources is sufficient to re-establish one active positive loop. This loop results in spatial inequalities which are further re-inforced by the depletion of technology in the already capital depleted regions. Our guess is that if we would include other similar positive loops regarding the attraction of capital and skilled workers by rich regions we would re-inforce this mechanism.

The elementary processes behind the pattern structure thus differs from Turing instability where the positive loop is in the chemical reaction part; in our case, reactions saturate and the instability comes from the market counter-diffusive part.

We tried to use a more or less realistic set of parameters. Most of them only change the magnitude of the averaged variables A, K, R and C, or how fast the systems converges. The crucial parameter is the size of the market in the vicinity of a customer/provider. This size depends upon one reduced parameter, the ratio of information price to transportation cost. It appears as the exponent of the function describing how fast orders decay with distance. Fast (resp. slow) decay with distance results in a large (resp. small) number of production centers. Resource is concentrated in production centers while no-producing regions are also depleted in capital, technicity and available resource for their few machines. If coupling is too weak, $f \leq 0.2$, spatial structures disappear and homogeneity is again observed.

Capital and production distributions are bi-modal, with a very large peak at zero capital and production. The active centers are distributed according to a nearly uniform distribution with a sharp cut-off; their distribution extends typically over one order of magnitude.

Let us now try to translate our results into the real world and its future.

We insisted on physical constraints. Let us notice that our results also depends on what economists call institutions: for instance how is the market organised. We based our analysis on a stylised representation of the rules used by agents in a market of perishable goods. Such rules are fundamental in creating (or not) positive loops in the attribution of resources. They are other institutional rules practised in our world and even more can be imagined. For instance, auctions, export taxes, credit facilities, planning by a central institution as it was the case in Soviet Union etc.

Let us now come back to the physics and economy of renewable energy sources:

- Renewable energy sources goes with higher transportation costs than the fossile resources we presently use: wind mills and photo-voltaic cells generate electricity which transportation is more costly than oil, gas or uranium, because of losses and difficulties in storage. Intermittency results in higher values of β, which combined with transportation costs reduce interaction ranges.
- If nuclear energy were made available from fast breeders, we would again be in a low transportation cost situation with little limit on the resource and we might expect a stronger concentration of economic activities. Other technologies such as high T_c superconductor technology might also favour low transportation costs.

In conclusion, the use of wind mills and photo-voltaic cells would results in closer production zones than presently. Uranium and thorium with fast breeders although not renewable, would maintain a larger dispersion of big production zones, for the time period during which they would be available-say of the order of thousands years, which makes some time to refine models and/or evolve new institutions.

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Appendix

We here report the results of numerical tests to check the meta-stability of the observed patterns, possible scaling effects and the role of initial configurations.

5.1 Scaling

The system does no display noticeable side effects. We tested the number of active cells with $A > 100$ for varying lattice sizes after 10000 iteration steps. The fraction of active sites was respectively 19.8 % for a 30×30 cell lattice, 19.1 % for 50×50 cells and 19.6 % for 100×100 cells. We found that the observed difference in percentage to be not significative.

5.2 Patterns Stability

Because the system is noisy we used the Pearson correlation coefficient to measure the correlation between a pattern of capital K taken at time 5000 and patterns observed at further times until $t = 10{,}000$. We use a 50×50 network. The Pearson coefficient is measured by:

$$r(\tau) = \frac{\sum_i (x_i(t) - \overline{x(t)})(x_i(t+\tau) - \overline{x(t+\tau)})}{\sqrt{\sum_i (x_i(t) - \overline{x(t)})^2}\sqrt{\sum_i (x_i(t+\tau) - \overline{x(t+\tau)})^2}} \tag{27}$$

The Pearson coefficient decreases from 1 to 0.99 in 5000 time steps which demonstrates a long term stability after quasi equilibrium has been reached. We also checked the stability of the correlations between one cell and its neighbours at varying distances over 5000 time steps and similarly checked stability (Fig. 8).

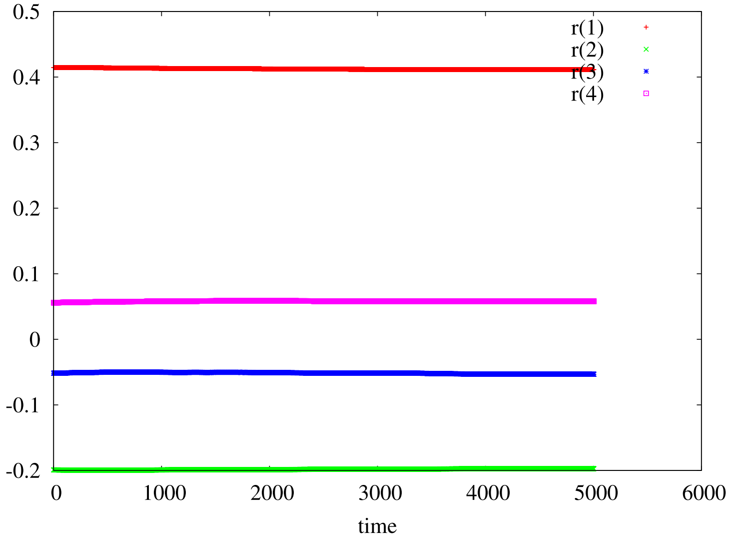

Fig. 8 Time evolution of capital spatial correlations at varying distances $r(1)$, $r(2)$, $r(3)$ and $r(4)$

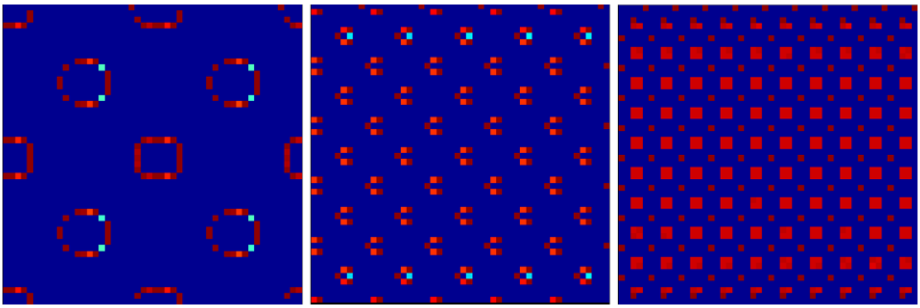

Fig. 9 Production patterns at large integration times, 1000, for different periodic initial conditions

5.3 Influence of Initial Conditions

A choice of initial conditions is arbitrary and we reported in Sect. 3 results obtained from random initial conditions without any spatial correlation. On the other hand, since the present state of the world is already structured in industrial regions, the possible importance of initial structures is worthwhile to study. We then ran simulations with initial sinusoidal patterns of A and K such that:

$$A(0) = 1 + 50\big(\cos(kx)\cos(ky)\big) \tag{28}$$

$$K(0) = 1 + 250\big(\cos(kx)\cos(ky)\big) \tag{29}$$

with several wave vectors $k = 2n\pi/L$, $n = 2, 5, 10$ and lattice size $L = 50$ (Fig. 9). Resource initial distribution were random.

The fraction of active sites is respectively 6.16, 11.18 and 19.32 % for $n = 2, 5, 10$. Not surprisingly in view of the previously observed metastability, initial conditions do play a role in the final aspect of the patterns. The meso-scale initial periodicity is maintained while the micro structure of the localised peaks of activity reflects the character of the dynamics, independently from the initial conditions.

One might be tempted to investigate further the transition from present socio-economic patterns to future conditions with different energy sources. Let us remind that we are not able to describe the dynamics of the slow technological change that would drive the transition. The above set of remarks does not allow precise predictions: what we can still conclude is that some memory of the present large scale spatial structures could be maintained.

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