Distribution of velocities in an avalanche

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For a driven elastic object near depinning, we derive from first principles the distribution of instantaneous velocities in an avalanche. We prove that above the upper critical dimension, $d \geq d_{\rm uc}$, the n-times distribution of the center-of-mass velocity is equivalent to the prediction from the ABBM stochastic equation. Our method allows to compute space and time dependence from an instanton equation. We extend the calculation beyond mean field, to lowest order in $\epsilon = d_{\rm uc} - d$.

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Obtaining a quantitative description of the dynamics during an avalanche is of great importance for systems whose dynamics is governed by jumps, such as magnets, superconductors, earthquakes, the contact line of fluids, or fracture [1–5]. In particular the motion of domain walls (DW) in magnets is important for many applications, such as magnetic recording. It can be measured from the Barkhausen (magnetization) noise [6, 7], which is a complicated time-dependent signal. Its origin is due to an interplay between quenched impurities and the elastic deformation energy which tend to pin the DW, as well as the driving and magnetostatic forces.

A major step forward was accomplished by Alessandro, Beatrice, Bertotti and Montorsi (ABBM) [8] who introduced, on a phenomenological basis, a stochastic equation approximating the DW motion by a single degree of freedom. Although a crude description, this model has been used extensively to compare with experiments on magnets, with success in some "mean-field like" cases, and failure in other [9]. However, no microscopic foundation for the validity of this model exists.

On the other hand, sophisticated field theoretic methods were developed in the last decades to study systems with quenched disorder. In particular, for elastic interfaces, relevant to describe DW motion, functional RG methods (FRG) [1, 10–12] have recently allowed to derive the distribution of quasi-static avalanche sizes [13, 14]. Until now however, no description of the dynamics during an avalanche was available. In fact, since it involves much faster motion than the average driving velocity, it led to difficulties in the early FRG approaches [11].

The aim of this Letter is to show how to compute from first principles the distribution of instantaneous velocities in an avalanche. We study a single elastic interface, of internal dimension d (total space dimension is $D = d+1$) at zero temperature, near the depinning threshold. The method works in an expansion around the upper critical dimension $d_{\rm uc}$, with $d_{\rm uc} = 4$ for standard elasticity, and $d_{\rm uc} = 2$ in presence of long-range elasticity, e.g. arising from dipolar forces. Remarkably, we find that for $d = d_{\rm uc}$ (and above) and in the scaling limit, the n-time probability distribution (with n arbitrary) of the center of mass of the interface is equivalent to that of the ABBM stochastic equation, in terms of renormalized parameters which in some cases can be estimated. The two methods are rather different in spirit, and the identification non-trivial. Our result establishes the universality of the ABBM model for $d \geq d_{\rm uc}$. In addition it allows to resolve the spatial structure, and gives the corrections to ABBM for $d < d_{\rm uc}$.

Here we sketch a very simple derivation, for details and *various subtleties* involved we refer to [15]. Consider the equation of motion, in the comoving frame, for the local velocity of an interface driven at velocity v:

$$(\eta_0 \partial_t - \nabla_x^2)\dot{u}_{xt} = \partial_t F(vt + u_{xt}, x) - m^2 \dot{u}_{xt} \ . \qquad (1)$$

It is obtained by time derivation (noted indifferently $\dot{u}$ or $\partial_t u$) of the standard overdamped equation of motion. Here x is the d-dimensional internal coordinate, $vt + u_{xt}$ the space and time dependent displacement field and η_0 the friction. $F(u,x)$ is the quenched random pinning force from the impurities, with e.g. Gaussian distribution and variance $\overline{F(u,x)F(u',x')} = \delta^d(x-x')\Delta_0(u-u')$. m^2 is the strength of the restoring force $-m^2(u_{xt} - vt)$ (i.e. the mass, or spring constant), which flattens the interface beyond a scale $L_m \sim 1/m$. In the small m, large L_m, limit, studied here, the interface has the roughness exponent ζ of the depinning transition, with $u \sim x^\zeta$ for $x \lesssim L_m$ and $u \sim L_m^\zeta$ for $L > L_m$. For simplicity we chose standard elasticity $\sim \nabla_x^2$, but it can be replaced by an arbitrary elastic kernel as needed in applications [2, 3, 9].

Near the depinning transition, i.e. at small v, the interface proceeds via avalanches. This is easiest seen in the center-of-mass position $u_t = L^{-d} \int_x u_{xt}$. There is a well-defined quasi-static limit $v = 0^+$ where $u_t = u(w)$, with $w = vt$ the well position. The process $u(w)$ jumps at discrete locations w_i, i.e. $u(w) = L^{-d} \sum_i S_i \theta(w - w_i)$, with S_i the avalanche sizes. Their statistics was predicted via FRG, and checked numerically [13, 14, 17]. There, the bare disorder correlator $\Delta_0(u)$ flows, under coarse graining, to the renormalized one $\Delta(u)$, which, at the depinning transition exhibits a linear cusp $-\Delta'(0^+) > 0$. This cusp is directly related to the moments of the normalized size distribution $P(S)$, via [14]

$$S_m := \frac{\langle S^2 \rangle}{2\langle S \rangle} = \frac{|\Delta'(0^+)|}{m^4} \ . \qquad (2)$$

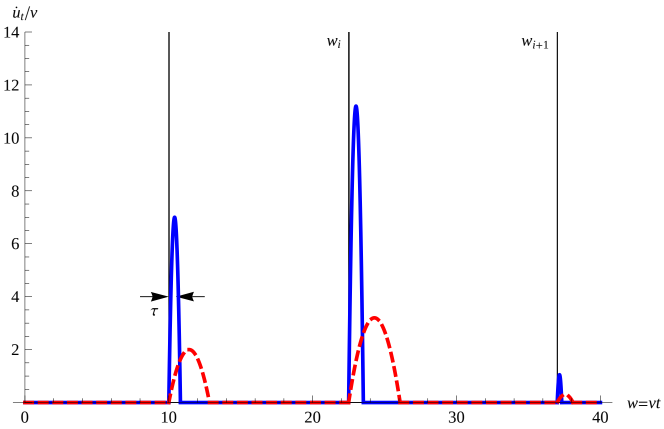

FIG. 1: Schematic plot of the instantaneous velocity (divided by v) as a function of vt for different v. The area under the curve is the avalanche size hence is constant as $v \to 0^+$. The quasi-static avalanche positions w_i are indicated.

$S_m \sim m^{-(d+\zeta)}$ is the large-scale cutoff of $P(S)$. Here we study the dynamics *inside* these avalanches, which occur for small v on a time scale $\tau_m \sim L_m^z \ll \Delta w/v$, where Δw is the typical separation of avalanches in the same space region, and z the dynamical exponent. Hence we are considering small enough v so that avalanches remain well separated, a condition equivalent to $L_m \ll \xi_v$, where ξ_v is the standard critical correlation length [10, 11] near depinning (for $m = 0$). This is illustrated on figure 1.

The information about the dynamics in an avalanche is contained in the n-times cumulants $C_n = \overline{\dot u_{t_1} \dots \dot u_{t_n}}^c$, $n \geq 2$ (with $\overline{\dot u_t} = 0$). In the limit $v \to 0^+$ the product $\dot u_{t_1} \dots \dot u_{t_n}$ vanishes unless all times are inside an avalanche. The probability that exactly one avalanche occurs in a time interval $T < \Delta w/v$ is $\rho_0 v T$, with $\rho_0 = L^d/\langle S \rangle$ the avalanche density per unit w. C_n is thus $O(v)$, rather than $O(v^n)$, the hallmark of a non-smooth motion. In addition, C_n obeys the sum rule $L^{nd} \int_{[-T/2,T/2]^n} \mathrm{d}t_1 \dots \mathrm{d}t_n \overline{\dot u_{t_1} \dots \dot u_{t_n}} = \rho_0 v T \langle S^n \rangle + O(v^2)$. It can be computed perturbatively in the (renormalized) disorder. For $n = 2$ and to lowest order one finds

$$\overline{\dot u_{t_1} \dot u_{t_2}}^c = -L^{-d} \Delta'(0^+) \frac{v}{m^2 \eta} \mathrm{e}^{-\frac{m^2}{\eta}|t_1 - t_2|} \tag{3}$$

where here and below η is the renormalized friction [16]. Integrating over time, one recovers (2).

To obtain all moments at once, as well as the velocity distribution, we now compute the generating function

$$Z[\lambda] = L^{-d} \partial_v \overline{e^{\int_{xt} \lambda_{xt}(v + \dot u_{xt})}}\Big|_{v=0^+} . \tag{4}$$

The average over disorder (and initial conditions) is obtained from the dynamical action $S = S_0 + S_{\text{dis}}$ of (1):

$$S_0 = \int_{xt} \tilde u_{xt} (\eta \partial_t - \nabla_x^2 + m^2) \dot u_{xt} \tag{5}$$

$$S_{\text{dis}} = -\frac{1}{2} \int_{xtt'} \tilde u_{xt} \tilde u_{xt'} \partial_t \partial_{t'} \Delta(v(t-t') + u_{xt} - u_{xt'}) \tag{6}$$

This yields

$$Z[\lambda] = L^{-d} \partial_v \int \mathcal{D}[\dot u] \mathcal{D}[\tilde u] \, e^{-S + \int_{xt} \lambda_{xt}(v + \dot u_{xt})} \Big|_{v=0^+} \tag{7}$$

with $Z[0] = 0$. We write

$$\begin{aligned} &\partial_t \partial_{t'} \Delta(v(t-t') + u_{xt} - u_{xt'}) \\ &= (v + \dot u_{xt}) \partial_{t'} \Delta'(v(t-t') + u_{xt} - u_{xt'}) \\ &= (v + \dot u_{xt}) \Delta'(0^+) \partial_{t'} \mathrm{sgn}(t - t') + \dots \end{aligned} \tag{8}$$

where we have used that the interface is only moving forward (Middleton theorem [18]). We can thus rewrite the disorder term as $S = S_{\text{dis}}^{\text{tree}} + \dots$, where

$$S_{\text{dis}}^{\text{tree}} = \Delta'(0^+) \int_{xt} \tilde u_{xt} \tilde u_{xt} (v + \dot u_{xt}) \tag{9}$$

is the so-called tree-level or mean-field action [16]. The terms neglected are $O(\Delta''(0^+))$ and higher derivatives, and we have shown that they contribute only to $O(\epsilon)$ to $Z[\lambda]$, hence can be neglected at tree level.

We now study the tree approximation for $Z[\lambda]$, i.e. (7) with S_{dis} replaced by (9). Thus the highly non-linear action (6) has been reduced to a much simpler cubic theory! Even more remarkably, $\dot u_{xt}$ appears only linearly in (9), and viewing $\dot u$ as a response field, the tree level theory is *equivalent to the following non-linear equation*:

$$(\eta \partial_t + \nabla_x^2 - m^2) \tilde u_{xt} - \Delta'(0^+) \tilde u_{xt}^2 + \lambda_{xt} = 0 \tag{10}$$

We denote $\tilde u_{xt}^\lambda$ the solution of this equation for a given source λ_{xt}. Performing the derivative w.r.t v in (7) gives

$$\begin{aligned} Z[\lambda] &= L^{-d} \int_{xt} \lambda_{xt} - \Delta'(0^+) (\tilde u_{xt}^\lambda)^2 \\ &= L^{-d} \int_{xt} (-\eta \partial_t - \nabla_x^2 + m^2) \tilde u_{xt}^\lambda = m^2 L^{-d} \int_{xt} \tilde u_{xt}^\lambda \end{aligned} \tag{11}$$

where we have used equation (10) and, in the last equality, assumed that $\tilde u^\lambda$ vanishes at large t and x. To analyze the result, it is convenient to use dimensionless equations, replacing $x \to x/m$, $L \to L/m$, $t \to \tau_m t$, $v \to v v_m$, $\lambda \to \lambda/S_m$ and $\tilde u_{xt} \to \tilde u_{xt}/m^2 S_m$, where $v_m = S_m m^d/\tau_m$, and $\tau_m = \eta/m^2$. From now on we use these units, and consider the center-of-mass velocity, thus choosing $\lambda_{xt} = \lambda_t$ uniform.

The 1-time probability at time $t = 0$ is given by $\lambda_t = \lambda \delta(t)$ through its Laplace transform

$$\tilde Z(\lambda) = L^{-d} \partial_v \overline{e^{L^d \lambda (v + \dot u)}}\Big|_{v=0^+} . \tag{12}$$

$\dot{u} = \dot{u}_{t=0}$ and the notation $\tilde{Z}$ reminds us that we use dimensionless units. $\tilde{u}_{xt} = \tilde{u}_t$ and we need to solve

$$(\partial_t - 1)\tilde{u}_t + \tilde{u}_t^2 = -\lambda\delta(t) \tag{13}$$

with $\tilde{u}_t \to 0$ at $t = \pm\infty$:

$$\tilde{u}_t = \frac{\lambda}{\lambda + (1-\lambda)e^{-t}}\theta(-t) \tag{14}$$

Inserting into (12) gives

$$\tilde{Z}(\lambda) = \int_t \tilde{u}_t = -\ln(1-\lambda) \ . \tag{15}$$

Calling τ_i the duration of the i-th avalanche out of N, and defining $\langle\tau\rangle := \frac{1}{N}\sum_i \tau_i$ the mean duration, the probability $p_{\rm a}$ that $t = 0$ belongs to an avalanche is $p_{\rm a} = \rho_0 v\langle\tau\rangle$. Hence the total 1-time velocity probability is $P(\dot{u}) = (1-p_{\rm a})\delta(v+\dot{u}) + p_{\rm a}\tilde{P}(\dot{u})$ where $\tilde{P}(\dot{u})$ is the probability given that $t = 0$ belongs to an avalanche. Both $\tilde{P}$ and P are normalized to unity. One notes the two (always) exact relations $\langle\dot{u}\rangle_P = 0$, $p_{\rm a}\langle\dot{u}+v\rangle_{\tilde{P}} = v$. Hence for $v = 0^+$ one has $\rho_0\langle\tau\rangle\langle\dot{u}\rangle_{\tilde{P}} = 1$ and, in dimensionfull units $Z(\lambda) = \frac{1}{m^d v_m}\tilde{Z}(m^d v_m \lambda) = L^{-d}\rho_0\langle\tau\rangle\int \mathrm{d}\dot{u}\,\tilde{P}(\dot{u})(e^{L^d\lambda\dot{u}} - 1)$. We thus obtain, in the slow driving limit, the instantaneous velocity distribution in the range $v_0 \ll \dot{u} \sim \tilde{v}_m$ (v_0 being a small velocity cutoff):

$$\tilde{P}(\dot{u}) = \frac{1}{\rho_0\langle\tau\rangle\tilde{v}_m^2}\,p\Big(\frac{\dot{u}}{\tilde{v}_m}\Big) \ , \qquad p(x) = \frac{1}{x}e^{-x} \ . \tag{16}$$

We defined $\tilde{v}_m = (mL)^{-d}v_m = L^{-d}S_m/\tau_m$. Hence $\langle\dot{u}\rangle_{\tilde{P}} \approx \tilde{v}_m/\ln(\frac{\tilde{v}_m}{v_0})$. Note that (i) $p(x)$ is not a probability, but is normalized by $\int \mathrm{d}x\, x\, p(x) = 1$ (ii) the quantity which is distributed according to $p(x)$ is $x = \tau_m\int_x \dot{u}_{xt}/S_m$, which does not contain the factor L^{-d}.

Similarly one obtains the n-time distribution of the center-of-mass velocity solving (13) with $\lambda_t = \sum_{j=1}^n \lambda_j\delta(t-t_j)$, noting $z_{ij} := 1 - \mathrm{e}^{-|t_i - t_j|/\tau_m}$

$$\tilde{Z}_n(\lambda_1,\dots,\lambda_n) = -\ln\Bigg(\sum_{\Lambda\subset\{1,\dots,n\}}\prod_{i\in\Lambda}[-\lambda_i]\prod_{\{i,j\}\subset\Lambda, i<j} z_{ij}\Bigg) \tag{17}$$

For $n = 2$ one finds $\tilde{Z}_2 = -\ln(1-\lambda_1-\lambda_2+\lambda_1\lambda_2 z)$ with $z = 1-\mathrm{e}^{-|t_2-t_1|/\tau_m}$. From this we obtain (i) the probability $q_{12} = v q'_{12}$ that both t_1 and t_2 belong to the same avalanche and the velocity distribution $\tilde{P}$ conditioned to this event:

$$q'_{12}\tilde{P}(\dot{u}_1,\dot{u}_2) = \frac{1}{\tilde{v}_m^3}\,p\Big(\frac{\dot{u}_1}{\tilde{v}_m},\frac{\dot{u}_2}{\tilde{v}_m}\Big) \tag{18}$$

$$p(v_1,v_2) = \frac{e^{-\frac{t}{2}-\frac{v_1+v_2}{1-e^{-t}}}}{(1-e^{-t})\sqrt{v_1 v_2}}\,I_1\Big(\frac{2\,e^{-t/2}\sqrt{v_1 v_2}}{1-e^{-t}}\Big) \tag{19}$$

with $t = |t_2 - t_1|/\tau_m$, $q'_{12}\tilde{v}_m = \ln(1/z)$, and $I_1(x)$ is the Bessel-I function of the first kind. The probability that t_1 but not t_2 belongs to an avalanche is

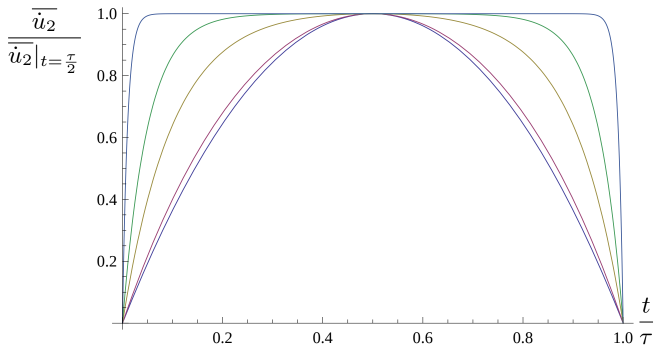

FIG. 2: "Pulse-shape": The normalized velocity at time t in an avalanche of duration τ for $\tau \ll \tau_m$ (lower curve) to $\tau \gg \tau_m$ (upper curve).

$$q'_1\tilde{P}_1(\dot{u}_1) = \frac{1}{\tilde{v}_m^2}\,p\Big(\frac{\dot{u}_1}{\tilde{v}_m}\Big) \ , \qquad p(\dot{u}_1) = \frac{e^{-\dot{u}_1/z}}{\dot{u}_1} \tag{20}$$

with $p'_a = q'_1 + q'_{12}$. Since the probability that there exists an avalanche starting in $[t_1, t_1+dt_1]$ and ending in $[t_2, t_2+dt_2]$ is $-dt_1 dt_2\partial_{t_1}\partial_{t_2}q_{12}$ we obtain the distribution of durations τ as

$$P(\tau) = \frac{1}{\rho_0\tilde{v}_m\tau_m^2}\frac{e^{-\tau/\tau_m}}{(1-e^{-\tau/\tau_m})^2} \ . \tag{21}$$

For small durations $\tau \ll \tau_m$, $P(\tau) \approx \frac{1}{\rho_0\tilde{v}_m\tau^2}$, cut off at $\tau \approx \tau_0$. This gives $\langle\tau\rangle = \frac{1}{\rho_0\tilde{v}_m}\ln(\frac{\tau_m}{\tau_0})$ in good agreement with the above, using $\ln(\frac{\tau_m}{\tau_0}) \approx \ln(\frac{\tilde{v}_m}{v_0})$. Note that $q'_{12}\tilde{P}(0^+,0^+)$ is proportional to the probability that an avalanche starts at t_1 and ends at t_2.

The "shape" of an avalanche with duration τ can then be extracted from the probabilities at 3 times $(t_1,t_2,t_3) = (0,t,\tau)$ setting $\dot{u}_1 = \dot{u}_3 = 0^+$. From the generating function (17) for 3 times, the probability distribution for the intermediate-time velocity is $P(\dot{u}_2) = b^2\dot{u}_2 e^{-\dot{u}_2 b}$, with $\tilde{v}_m b := \frac{1}{z_{12}} + \frac{1}{z_{23}} - 1$ resulting in the average "shape"

$$\overline{\dot{u}_2} = \frac{2}{b} = \tilde{v}_m\frac{4\sinh\big(\frac{t}{2\tau_m}\big)\sinh\big(\frac{\tau}{2\tau_m}\big[1-\frac{t}{\tau}\big]\big)}{\sinh\big(\frac{\tau}{2\tau_m}\big)} \ . \tag{22}$$

This interpolates from a parabola for small $\tau \ll \tau_m$ to a flat shape for the longest avalanches (see Fig 2.). This result holds for an interface at or above its upper critical dimension, which previously was used [7] on the basis of the ABBM model.

We now clarify the relation to the phenomenological ABBM theory [8]. The latter models the interface as a single point driven in a long-range correlated random-force landscape, $F(u)$, with *Brownian* statistics. It amounts to suppressing the space dependence in (1), hence corresponds in our general model to the special case $d = 0$ and $\Delta_0(0) - \Delta_0(u) = \sigma|u|$. The instantaneous velocity $\mathsf{v} = \dot{u}_t + v$ satisfies the stochastic equation

$\eta d\mathsf{v} = m^2(v - \mathsf{v})\mathrm{d}t + \mathrm{d}F$ where $\overline{\mathrm{d}F^2} = 2\sigma \mathsf{v}\mathrm{d}t$, with associated Fokker-Planck equation

$$\eta \partial_t Q = \partial_{\mathsf{v}} \left[\frac{\sigma}{\eta} \partial_{\mathsf{v}} (\mathsf{v} Q) + m^2 (\mathsf{v} - v) Q \right] \qquad (23)$$

for the velocity probability $Q \equiv Q(\mathsf{v}, t|\mathsf{v}_1, 0)$. For $v > 0$ it evolves to the stationary distribution $Q_0(\mathsf{v}) = v_m^{-v/v_m} \mathsf{v}^{v/v_m - 1} e^{-\mathsf{v}/v_m} / \Gamma(v/v_m)$ with $v_m = S_m/\tau_m$ and here $S_m = \sigma/m^4$ and $\tau_m = \eta/m^2$. For $v = 0^+$ one recovers (16), up to a normalization which entails a small-scale cutoff. Similarly for $v = 0^+$ one finds the propagator $Q(\mathsf{v}, t|\mathsf{v}_1, 0) = v_m^{-1} \tilde{Q}(\frac{\mathsf{v}}{v_m}, \frac{t}{\tau_m} | \frac{\mathsf{v}_1}{v_m}, 0)$ with

$$\tilde{Q}(v_2, t|v_1, 0) = v_1 e^{v_1} \left[p(v_1, v_2) + \frac{1}{v_1} e^{-\frac{v_1}{1 - e^{-t}}} \delta(v_2) \right], \quad (24)$$

and $p(v_1, v_2)$ given in Eq. (19). $\tilde{Q}(v_2, t|v_1, 0)$ is solution of (23) with $Q(\mathsf{v}_2, 0^+|\mathsf{v}_1, 0) = \delta(\mathsf{v}_2 - \mathsf{v}_1)$. The piece $\sim \delta(v_2)$ corresponds to avalanches which have already terminated at time t, and is necessary for Q to conserve probability. The *joint distribution* $\tilde{Q}(v_2, t|v_1, 0) \frac{1}{v_1} e^{-v_1}$ reproduces the 1-time and 2-times probabilities given in Eqs. (18) and (20), up to a global normalization. More generally, since $\mathsf{v(t)}$ is a Markov-process, the n-time velocity probability obtained from (10) is $q'_{1p} \tilde{P}(\dot{u}_1, \ldots, \dot{u}_n) = \frac{1}{\dot{u}_1} e^{-\dot{u}_1} \prod_{j=1}^{n-1} Q(\dot{u}_{j+1} t_{j+1} | \dot{u}_j t_j)$.

Several remarks are in order. The first one is specific to the ABBM model: Since it is the zero-dimensional limit of (1), the dynamical-action method can be applied. Hence we just found that for the ABBM model at $v = 0^+$ *the tree approximation is exact*. In the field theory it means that the effective action Γ equals the bare action S, and there are no loop corrections. Hence $\Delta'(u) = \Delta'_0(u) = -\sigma\, \mathrm{sgn}(u)$ is an exact FRG fixed point (with $\zeta = 4 - d$) as noted in [14]. Crucial for this remarkable property is that the force landscape is a Brownian, and even in $d = 0$, this is not valid for any other, e.g. shorter ranged, force landscape. In that sense, the model proposed by ABBM [8], although unnatural from a microscopic point of view, appears extremely judicious.

Second, since a realistic interface in a short-ranged random force is described for $d \geq d_{\mathrm{uc}}$ by the tree approximation, we proved that the temporal correlations of its center-of-mass velocity for $v \to 0$ are given by the ABBM model. Only two parameters enter, η and S_m, which in $d = 4$ acquire a logarithmic dependence on m [14].

Third, it is not expected that $S = \Gamma$ extends to finite driving velocity $v > 0$; hence whether the phenomenology of the ABBM model with an avalanche exponent τ dependent on v has anything to do with realistic interface motion remains an open question.

Fourth, the present theory allows to go *beyond* the ABBM model in several ways: In $d \geq 4$, the non-linear equation (10) allows to study the full time- and space-dependence of velocity correlations, as was done for the statics in [14]. Second, including loop-corrections allows to compute corrections in a systematic expansion in $d = 4 - \epsilon$ [15]. The main result for the 1-time velocity distribution for $v_0 \ll v \ll \tilde{v}_m$ is to first order in ϵ

$$P(v) \sim 1/v^a \ , \qquad a = 1 - \epsilon(1 - \zeta_1)/3 + O(\epsilon^2) \quad (25)$$

i.e. $a = 1 - \frac{2}{9}\epsilon$ for a non periodic interface, and $a = 1 - \frac{\epsilon}{3}$ for a charge density wave (CDW). The large-cutoff scale is given by $\tilde{v}_m$ with $\eta_m \sim m^{2-z}$, $z = 2 - \frac{2}{9}\epsilon$ for non-periodic disorder and $z = 2 - \frac{\epsilon}{3}$ for CDW [10–12].

To conclude, we introduced a general method to compute both spatial and temporal velocity correlations in an avalanche. Its tree-approximation is exact at and above the upper critical dimension $d \geq d_{\mathrm{uc}}$. There the center-of-mass motion is equivalent to the phenomenological ABBM model. This establishes the range of validity of the latter. For $d < d_{\mathrm{uc}}$ corrections are calculated in a controled expansion in $\epsilon = d_{\mathrm{uc}} - d$.

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