RESERACH SUMMARY 2018

KAY WIESE

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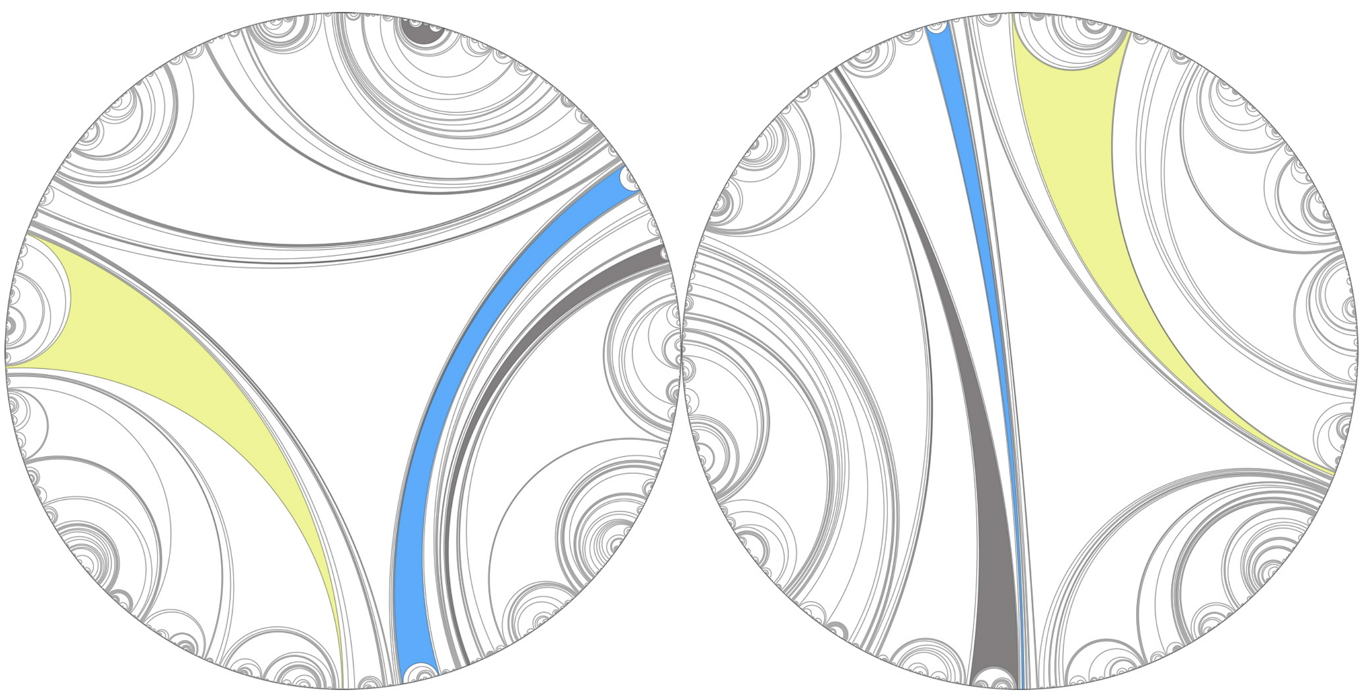
FRANCE



(Graphical representation of RNA folding, presented at SciArt 2011 at KITP, UCSB, Santa Barbara, USA)

Curriculum Vitae

Kay Wiese

Born:

August 7, 1966, son of Prof. Dr. Günther Wiese, professor of law at Universität Mannheim and Elisabeth Wiese, born Momsen.

Positions and long-term mobility:

UNIVERSITÄT HEIDELBERG, September 1987 – January 1993
Degree: "Diplomphysiker"[1], very good, January 1993
Title: 3-loop ζ-function for a supersymmetric non-linear σ-model
Diploma advisor: Prof. Dr. Franz Wegner

SERVICE DE PHYSIQUE THÉORIQUE, C.E. DE SACLAY, France
September 1993 – September 1996
Degree: PhD in physics, très honorable (summa cum laude), September 1996
Title: Statistical mechanics of self-avoiding membranes
PhD advisor: Prof. Dr. François David

UNIVERSITÄT ESSEN, October 1996 – December 2000
Position: Privatdozent[2] (since December 1999)
Degree: Habilitation[3], 1999
Title: Polymerised Membranes
Habilitation advisor: Prof. Dr. H.W. Diehl

ITP, UCSB, SANTA BARBARA, USA, January 2001 – September 2003
Position: Visiting scientist (Heisenberg-grant[4])

LPT, ÉCOLE NORMALE SUPÉRIEURE, PARIS, France, October 2003 – March 2004, and August 2004-December 2004
Position: Invited Researcher (Heisenberg grant).

UNIVERSITÄT KÖLN, Germany, April 2004 – July 2004
Position: Professor (C4), per procurationem of Professor Zirnbauer.

LPT, ÉCOLE NORMALE SUPÉRIEURE, PARIS, France.
Position: Chargé de Recherche de Première Classe (CR1), January 2005 – August 2010.
Position: Directeur de Recherche de Deuxième Classe (DR2), since September 2010.

KITP, SANTA BARBARA, CA, USA, August 2006 – December 2006.
Position: Organizer of the program "Stochastic Geometry and Field Theory: From Growth Phenomena to Disordered Systems".

KITP, SANTA BARBARA, CA, USA, SEPTEMBER 2014 – DECEMBER 2014,
Position: Organizer of the program "Avalanches, Intermittency, and Nonlinear Response in Far-

[1] Diploma in physics; the german "Diplom" is equivalent to a master, and in addition includes a diploma thesis, which takes 15 months.

[2] Comparable to an assistant professor in the US, with the exception that the German system has no tenure track positions.

[3] The German system requires this "second PhD" as a prerequisite to become professor.

[4] Highly prestigious grant, awarded only after habilitation. Funded by DFG, (german science foundation). Covers living costs, travel.

From-Equilibrium Solids"

Prizes and awards:

Outstanding Referee of the Amarican Physical Society, 2017.

Physics prize of the Academy of Sciences in Göttingen, 2000.

Heisenberg-grant[4] from DFG (german national science foundation), 2000.

Vordiplom[5] in physics: very good, Universität Heidelberg, June 1989.

Vordiplom[5] in mathematics: very good, Universität Heidelberg, June 1989.

Abitur[6]: very good (1.0), June 1986.

Member of "Studienstiftung des Deutschen Volkes"[7], 1986-1993.

Member of the German team at the XVII. International Physics Olympics, London, July 1986.

Special prize of the President of the state Baden-Würtemberg, 1986.

2nd prize and the special prize of IBM in the competition "Jugend forscht"[8] for a machine to recognize hand writing, 1988.

Several first and second prizes in "Bundeswettbewerb Mathematik"[9]

Internships (Stagiaires):

Frank Zwielich (1997)
Christian Hagendorf (2006)
Alexander Dobrinevski (2010)
Mathieu Delorme (2012)
Thimotée Thierry (2013)
Zumou Lin (2013)
Federica Gerace (2014)
Duan Zhihao (2014)
Carlo Ferrari (2016)
Assaf Shapiro (2016)
Hadi Haghighi (2017)
Zhaoxuan Zhu (2017)

Diploma-students:

Florian Kühnel, September 2005 – September 2006.

PhD-students (with current occupation):

Henryk Pinnow (1998-2002), now founder and CEO of Likron
Christian Hagendorf, September 2006 – September 2009; codirected with Pierre Le Doussal; permanent research position, Universit catholique de Louvain, Belgium
Alexander Dobrinevski (2010-2013), industry (d-fine)
Mathieu Delorme (2012-2016), professeur agrégé
Thimotée Thierry (since 2013), postdoc in Louvain, Belgium.

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[5] Roughly corresponds to a bachelor.
[6] German high-school finals, after 13 years of school. The maximum score is 1.0.
[7] National German program which supports the best 1% of all students.
[8] "Youth researches": German national competition for high-school students.
[9] German national competition in mathematics for high-school students.

Postdocs (with current occupation):

Andrei Fedorenko, January 2006 – January 2008; now CR1 in CNRS, ENS Lyon
Raoul Santachiara, September 2006 – October 2008; now CR1 in CNRS, LPS Orsay
Aleksandra Petkovic (2010-2012), now Maitre de Confrence, LPT Toulouse
Zoran Ristivojevic (2010-2012), now CR2 in CNRS, LPT Toulouse

Grants/funding

Heisenberg-grant, from DFG[4], 2000.
Marie-Curie incomping fellowship from EU for Andrei Fedorenko, as supervisor, 2005.
ANR-grant for 3 years, together with Pierre Le Doussal, 2005.
ANR-grant for 4 years, together with Alberto Rosso and Pierre Le Doussal, 2009.
Bourse du CNRS for a PhD student (Alexander Dobrinevski), 2010-2013.
ECOS-grant between Bariloche, LPTMS, LPT (Orsay) and LPTENS on *Dynamics of disordered systems in condensed matter*, 2009-2012.
Grant from PSL, project AvaCrack (with E. Bouchaud, P. Le Doussal, L. Ponson), 2013-2016.

Distinguished articles

EPL Best of 2009: *Fluctuation force exerted by a planar self-avoiding polymer* [KW64]
Editor's choice in EPL: *Distribution of velocities in an avalanche* [KW73]
EPL Best of 2012: *Distribution of velocities in an avalanche* [KW73]
Editor's suggestion in Phys. Rev. B: *Equilibrium avalanches in spin glasses* [KW75]
J. Phys. A Highlights of 2017: *Pickands' constant at first order in an expansion around Brownian motion* [KW100]

Organisation of conferences, workshops and seminars

"Séminaire commun de la Fédération LPT-ENS/LPTHE-Jussieu 2004-2013.
"Dynamics and Relaxation in Complex Quantum and Classical Systems and Nanostructures", July 24 – October 6, 2006. Organizers: I. Aleiner, B. Altshuler, V. Falko, A. Ludwig, K. Wiese.
"Stochastic Geometry and Field Theory: From Growth Phenomena to Disordered Systems", Santa Barbara, CA, USA, Aug. 7, 2006 - Dec. 15, 2006. Organizers: I. Gruzberg, P. Le Doussal, A. Ludwig, S. Sheffield, P. Wiegmann, K.J. Wiese.
"Avalanches, Intermittency, and Nonlinear Response in Far-From-Equilibrium Solids KITP, Santa Barbara, CA, USA, Sep. 22, 2014 - Dec. 12, 2014.

Work as Reviewer

Scientific Journals: Nature, Nature Physics, Nature Communications, Physical Review Letters, Physical Review E, Physical Review B, European Physical Journal B, Europhysics Letters, Journal of Physics A, Nuclear Physics B, Journal of Statistical Physics, Journal of Statistical Mechanics, Chaos, Solitons & Fractals, Physica A.
Grant Agencies: National Science Foundation (NSF, USA), Deutsche Forschungsgemeinschaft (DFG, Germany), Israel Science Foundation (ISF, Israel), Agence National pour la Recherche (ANR, France), European Research Council (ERC).

Teaching experience

Exercises in Mechanics, summer 1992, Universität Heidelberg

Exercises in Mechanics, winter 1998/1999, Universität Essen

Exercises in Statistical Physics for teachers, summer 1999, Universität Essen

Exercises in Quantum Mechanics, winter 1999/2000, Universität Essen

Lectures on Conformal Field Theory, summer 2000, Universität Essen

Lectures: Principles of non-local field theories and their application to polymerized membranes, Workshop of "Graduiertenkolleg der Universität Heidelberg", September 2000

Lectures on Disordered Systems, winter 2000/2001, Universität Essen

Lectures on "Quantum Phase Transitions", summer 2004, Universität Köln

Lectures on "Selected Topics in Statistical Field Theory", second year of the master program of ENS-ICFP, spring 2018, ENS Paris [Lecture notes]

Outreach, transdisciplinary work

Work at the interface between Art and Science. The exhibition SciArt 2011 was on display for a year at the KITP, Santa Barbara, USA.

Other responsibilities

ACMO, responsable d'hygiène et sécurité du LPTENS, since 2005.

Summary of Past Research

References marked as [KWnn] refer to my list of publications on page 32, references [nnn] refer to the literature, see page 27; note that my list of publications on page 32 includes links to the corresponding pdf files. The project begins on page **??**, a table of contents can be found on page 58. Recent work is explained in more detail. All figures are from my work, if not stated otherwise.

1 Objectives

The aim of my research was and is to develop new analytical methods, either using field theory or other analytical methods, including exact solutions, to obtain a unified and precise description of disordered and complex systems with current applications ranging from condensed matter physics to non-linear and biologically inspired physics.

2 Functional renormalization and applications

2.1 Introduction

The functional renormalization group (FRG) method is an old idea going back to Wilson [1] and Wegner [2]. In 1986 D.S. Fisher realized that this method can fruitfully be applied to a large class of disordered systems [3]. The idea is that it is necessary to follow the full disorder distribution (a function, rather than a single parameter, its strength) under renormalization, i.e. under elimination of the small-scale fluctuations. Typically, the flow equation, say for the force-force correlator $\Delta(u)$, looks like this

$$-\frac{m\partial}{\partial m}\Delta(u) = (\epsilon - 2\zeta)\Delta(u) + \zeta u\Delta'(u) - \frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}u^2}\left[\Delta(u) - \Delta(0)\right]^2 \tag{1}$$ flowDelta

(m is the IR regulator, see below, $\epsilon = 4 - d$ (d=dimension), and ζ is the roughness exponent.) By following the flow for a full function, instead of a single moment (the "disorder strength") the method, encompasses the most severe shortcomings of more traditional methods, as the mean-field treatment invoking replica-symmetry breaking [4], or hierarchical lattices, which do not take into account spatial fluctuations in realistic systems. There are solid reasons to believe (see below) that this method is the currently most promising, both conceptually and for applications. For instance it should ultimately settle such old unsolved problems as: What is the correct description of the ground state and excitations in glasses: Parisi's (multiple pure states) [4], Fisher's (droplets) [5], or (probably) something else, which reduces to the latter two in certain limits.

Working with FRG is difficult for several technical reasons. 1-loop approximations have been developed and used for some time, but recently, in a series of publications [KW20, KW23, KW25-KW27, KW31-KW33, KW35, KW36, KW38, KW40-KW44, KW46-KW48, KW50-KW54, KW57-KW62, KW65-KW67, KW69, KW, KW72-KW74, KW76-KW78, KW81, KW82, KW84, KW85, KW87, KW89, KW91, KW93, KW95, KW98, KW103, KW104,KW105], together with various collaborators, I have laid the foundations for a deeper understanding of the method, and thereby opened the door to further applications. The power of the method is now well established, and we can calculate finer and finer details.

2.2 Basic ideas

The functional renormalization group (FRG) approach to disordered systems studies the flow of the complete disorder correlator (as opposed to its moments) under renormalization, i.e. eliminating the fast degrees of freedom. One observes that after a finite renormalization the disorder correlator becomes non-analytic, and the description in moments breaks down. This non-analyticity is an immediate consequence of the multiple minima of the problem, for an explanation see e.g. section 6 in [KW53], and the upcoming sections 2.3 and 2.5. [KW53] also provides a good introduction and overview. Earlier reviews are [KW26, KW33, KW42].

I have used this method to study a number of problems (Collaboration with Pierre Le Doussal). The first are the equilibrium properties of an elastic manifold in disorder, for which I have been able to obtain results at 2-loop order [KW20, KW35, KW43], in quite good agreement with numerical simulations. This had been a challenge for some time, since beyond leading order derivatives of the disorder-correlator at the non-analyticity appear, which led many researchers to question the method. I have been able to show that a consistent theory can be constructed beyond leading order, with quantitative predictions for experiments, as e.g. the roughness exponent [KW20, KW35] or even the complete distribution of the interface width [KW31, KW32] or threshold forces [KW48]. Those have now been verified in a number of numerical simulations[6, 7, 8, 9] and [KW31] and even in an experiment [10]. I have recently been able to obtain the FRG flow-equations for the equilibrium statics at 3-loop order [KW103,105].

Considering dynamics, a critical force is necessary before the elastic manifold starts to move. At this threshold, a new universality class emerges, which is different from the static one discussed above [KW20, KW25]. It had been stated by Narayan and Fisher [11] that the roughness of the interface is exactly $\zeta = (4-d)/3$ ($d =$ dimension of the manifold), to all orders in perturbation theory. While this result is correct for the equilibrium properties of Random Field disorder, numerical evidence raised doubts about its validity for the depinning transition. Even more, their theory is unable to distinguish between statics and driven dynamics, and thus account for the non-reversibility of the latter. Together with Pierre Le Doussal and Pascal Chauve, I have been able to show that there are indeed corrections at 2-loop order, which correct the roughness exponent upwards, and render the disorder correlator non-potential, as expected on physical grounds [KW20, KW25]. Our numerical predictions were later verified by Rosso and Krauth [6], and by Rosso, Hartmann, and Krauth [8].

Universal quantities are not restricted to critical exponents, but include more complex observables as the distribution of the interface width [KW31, KW32]. Equivalently intriguing is the case of depinning in an anisotropic medium [KW27], which bares some relation to directed percolation.

In [KW41] I have discussed the use of the supersymmetry method for elastic manifolds. One explicitly sees that when the cusp appears at the Larkin length, then supersymmetry is broken. Interestingly, when the Gaussian variational method with replica-symmetry breaking can be used — this is the case for number

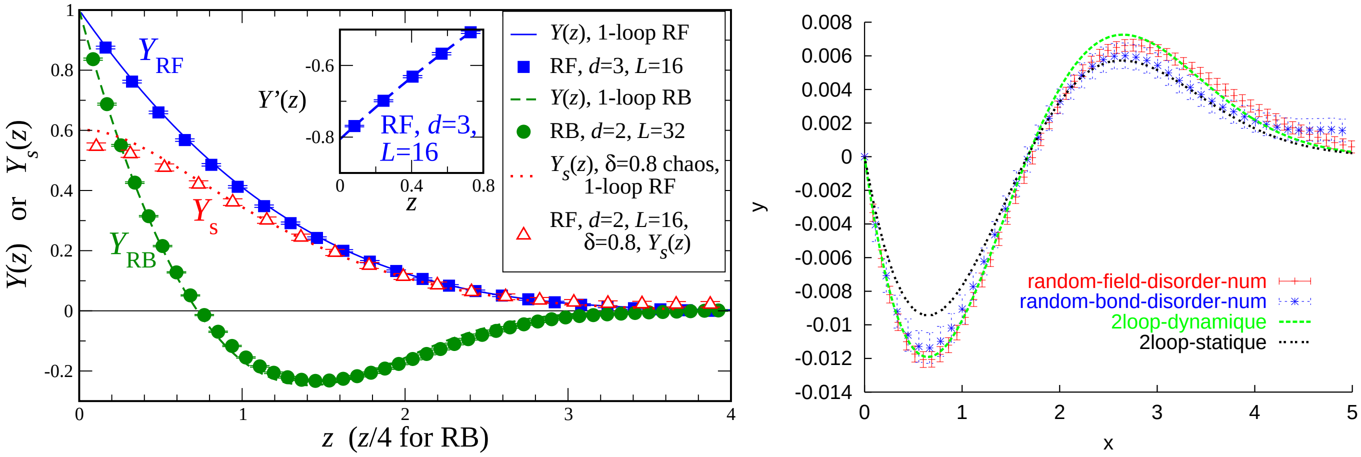


Figure 1: Left: rescaled disorder correlator $Y = \Delta/\Delta(0)$ of the statics, from numerics in [KW46], for random bond (RB) and random field (RF) disorder. Solid lines are one loop predictions (inset: the cusp). Right: $Y - Y_{1\text{loop}}$ for the depinning, from [KW52]. Scale is much enhanced and one loop prediction is substracted to compare with two loop (shown). Difference with statics is resolved.

of components $N \to \infty$ which makes the Gaussian variational method exact — then cusp, supersymmetry breaking and replica-symmetry breaking all appear at the same scale [KW41, KW57].

2.3 Measuring the cusp

Central to the functional RG approach is the disorder correlation function $R(u)$, which defines the correlations of the disorder potential $V(x, u)$, and which may be taken to be smooth. Functional RG states that after some finite renormalization, at the Larkin length, the effective $R(u)$ acquires a cusp in its second derivative, and that this cusp is central to overcoming the seemingly exact predictions of dimensional reduction [12, 13, 14]. However it was unclear how $R(u)$ could be measured. Only recently, a precise definition as an *observable* has been given [15]. Since this method will prove central for the following, let us briefly explain it.

The energy of a configuration $u(x)$ is given by

$$\mathcal{H}^w[u] = \int_x \frac{1}{2}\Big[\nabla u(x)\Big]^2 + \frac{m^2}{2}\Big[u(x) - w\Big]^2 + V\Big(x, u(x)\Big) \,. \tag{2}$$

The first term is the elastic energy, the last one the disorder. In order to render the problem well-defined, one needs a confining potential $m^2[u(x) - w]^2/2$, where w can be chosen freely. Thermal fluctuations play no role at large distances, so one needs to find the configuration $u_w(x)$ which minimizes $\mathcal{H}^w[u]$. Its center of mass $u_w := \frac{1}{L^d}\int_x u_w(x)$ explicitly depends on the chosen w. Quite remarkably, u_w can easily be measured in an experiment *and* encodes much of the physics of the model, as we demonstrate now:

As mentioned above, the central object of functional RG is the renormalized disorder correlator. Noting $\Delta_0(u)$ the bare *disorder-force correlator*, defined by $\Delta_0(u - u')\delta(x - x') := \overline{V'(x, u)V'(x', u')}$, the renormalized one $\Delta(u)$ can be obtained via functional RG as the solution of a differential flow-equation for $\Delta(u)$, starting with initial condition $\Delta_0(u)$; see (1) for this equation at 1-loop order. Quite remarkably, the renormalized disorder-force correlator satisfies the following *exact* relation:

$$\Delta(w - w') = \frac{L^d}{m^4}\,\overline{(u_w - w)(u_{w'} - w')} \,. \tag{3}$$

This relation holds both for the statics, and at depinning (one has to use the connected expectations there). It allows for very precise measurements.

With A. Middleton we have applied powerful algorithms to find exact ground states for interfaces (i.e. $N = 1$) [KW46] and measured from them the FRG function $\Delta(u)$. The result is presented on figure 1 (left). The agreement with the 1-loop prediction is spectacular, and can further be improved, with high numerical statistics, using the analytic 2-loop result. The difference between statics and depinning is resolved and compares well with the prediction. The hallmark of the FRG, the linear cusp, is observed and confirmed.

We proceeded similarly for the driven case and measured numerically, with A. Rosso [KW52], the FRG function at the depinning fixed point. The deviations from the 1-loop result for $\Delta(w)$, calculated from the field theory, and the 2-loop correction are shown in figure 1.

These high-precision tests of the FRG give confidence that this unconventional field theory correctly describes glass phases of disordered elastic systems. This is further confirmed in toy models for a particle dragged through disorder of various correlations [KW51, KW62].

In the next section, we will discuss an experimental test.

2.4 Contact-line depinning: First experimental confirmation of the cusp, and remaining challenges

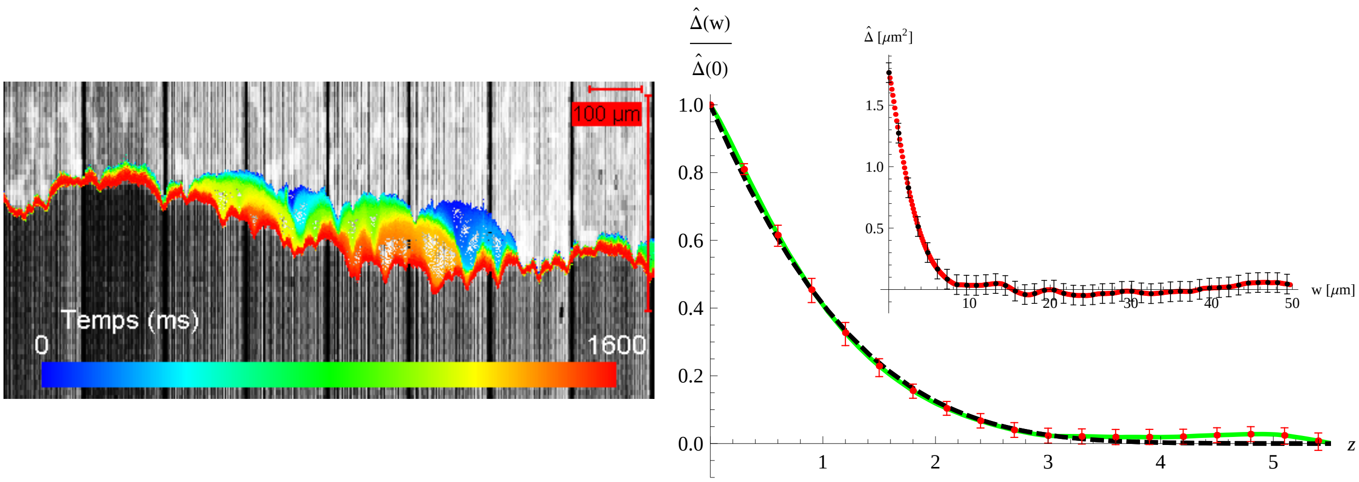
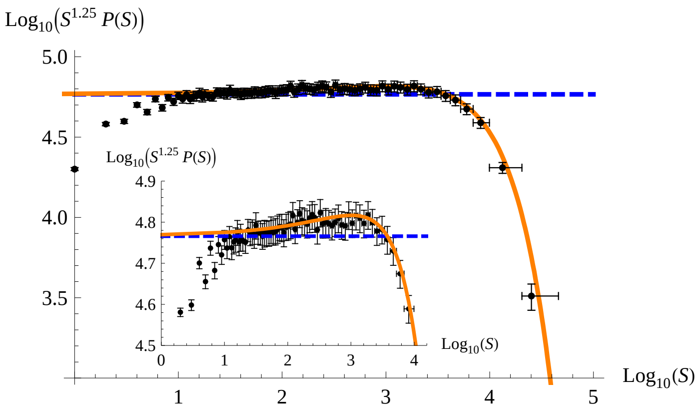


Figure 2: Left: a contact line in a wetting experiment at LPSENS (courtesy E. Rolley). Avalanches are shown in colors; they are reproducible. Right, inset: The disorder correlator $\Delta(w)$ for H_2/Cs, with error bars estimated from the experiment. Main plot: The rescaled disorder correlator $\Delta(w)/\Delta(0)$ (green/solid) with error bars (red). The dashed line is the 1-loop result; from [KW67].

Interestingly, the setting with a confining parabolic potential, which allows to test the FRG in numerical simulations, is realized in many experiments, e.g. magnetic domain walls in a magnetic field with a gradient, or submitted to dipolar demagnetizing fields, or fracture with imposed external deformations.

Together with my experimental colleagues from LPS-ENS, E. Rolley and Sebastien Moulinet, I have studied contact line depinning [KW67]. In this system, capillarity, i.e. gravity and surface tension, act as a quadratic well pulling the elastic contact line of the fluid. We considered three experimental systems: Water climbing on a glass plate as the level of the fluid is raised in the reservoir, where controlled disorder can be built in by lithography. This experiment is hampered by apparent long-range correlations of the disorder, induced by the glass plate. A better system is isobutanol on a silicon wafer with silanized patches, where the patch positions are chosen randomly in the computer, and then transferred via lithographic techniques. The advantage of this setup is that it is easily implemented. However the ratio of capillary length to disorder length, which determines how far the FRG flow has advanced towards the fixed point, is rather small. The best experimental system therefore turned out to be liquid hydrogen on a rough Cesium substrate.



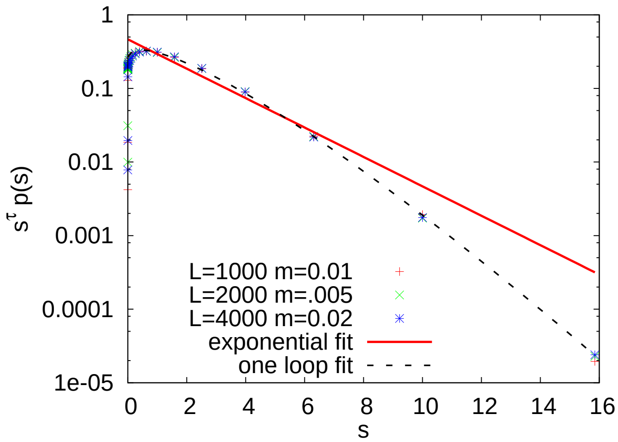


Figure 3: Left: Size distribution $P(S)$ of static avalanches in $d = 2$ showing the scale-free regime $S^{-\tau}$ with $\tau = 1.25$ from numerics, compared to the FRG prediction [KW60]. Inset: vertical blow-up of main plot. Right: Comparison of theory and simulations for depinning in $d = 1$. The solid red curve is a scale-free comparison to $P(s) \sim s^{-\tau} e^{-s}$, with $\tau = 1.08$, whereas the dashed line is a scale-free comparison to the 1-loop result with the same τ. One sees that even the shoulder (left plot) and the predicted tail exponent $\delta \approx 7/6$ in $d = 1$ (right figure) agrees with the simulations [KW66].

Avalanche motion is observed, see left of Fig. 2. On the right of the same figure is presented the first ever experimental measurement [KW67] of $\Delta(u)$. Both the raw data (inset) as the rescaled scale-free function (main plot) are shown. The data (solid green line with error bars) and the 1-loop result (dashed line) are in quite good agreement. (2-loop corrections are smaller than the error bars, thus can not be resolved.)

Understanding this experiment has been a longstanding challenge, notably because the measured roughness exponent $\zeta \approx 0.5$, is larger than both the two-loop FRG and the numerics [7]. Hence, with E. Raphael and R. Golestanian, we have included the next-order correction to the Joanny-de-Gennes theory and found that indeed it can, if it is sufficiently large, yield a larger roughness; although we have not been able to find, via extended FRG, a stable fixed point [KW40]. Understanding better the non-linearities in this problem may thus be crucial, and in collaboration with Costas Bachas we have developed a method, based on conformal transformations, to obtain these corrections in a systematic way [KW47]. This model is also of interest to string theory. Surprisingly, a thorough review of the literature reveals that the elastic energy to quadratic order in deformations appears to have been treated only neglecting gravity [16], or in presence of gravity only in the case where the equilibrium configuration of the interface is flat and horizontal, as for a vertical wall and a contact angle θ of $90°$ [17, 18]. My work [KW69] closes this gap.

In the same work [KW69] is observed a *strong* dependence of the avalanche-size distribution (see next section) on the precise form of the elastic energy. In view of these observations it would be important to reanalyze all existing experiments. Are they really in the scaling regime? To reinforce this question, let us mention that a smaller roughness of about 0.4, consistent with the simplest theory, has indeed been observed in two experiments on fracture which should be in the same universality class [19, 20].

2.5 Distributions of avalanches

The cuspy nature of $\Delta(w)$ at $w = 0$, visible in analytical calculations and in experiments, see figure 2, is a direct consequence of the existence of avalanches. Indeed, since fluctuations of the interface away from the parabola position are bounded, and since there is no renormalization of the elastic part of the energyg, their center of mass satisfies the *exact* result [KW60]

$$S_m := \frac{\langle S^2 \rangle}{2\,\langle S \rangle} = \frac{|\Delta'(0^+)|}{m^4} \ . \tag{4}$$

It had been a challenge for many years to extract the avalanche-size distribution from the field theory, and thus to go beyond toy-models of a single degree of freedom, or ABBM [21, 22]: a particle in a force-field which itself is a random walk. We obtained analytically the distribution of avalanche sizes and its cumulants within an $\epsilon = 4 - d$ expansion from a tree and 1-loop resummation, using FRG techniques [KW65]. These results have now been tested both for the statics [KW60] and the driven dynamics [KW66]. Due to the efficiency of the algorithm in the latter case, even the compacted exponential tail with $\delta \approx \frac{7}{6}$ in $d = 1$ agrees with the numerical data, see figure 3. In order to compare to experiments [KW67], it was important to have a precise formula for the interface energy in presence of gravity. To my astonishment, this had only be done for the case of a flat equilibrium profile, i.e. when the fluid does not climb up on the wall. The general case is much more complicated, but amenable to an analytical solution [KW69].

2.6 Further developments for statistics of avalanches for pinned elastic objects

Extending the results mentioned in the previous section, I have obtained several further results for the statistics of avalanches:

The first realization was that at the upper critical dimension, avalanches behave as in the *Brownian force model* (BFM), introduced in [KW76]: In this model, each degree of freedom (each point on the elastic manifold) sees an independent random-force field, which itself is a random walk. It is the generalization of the the so-called Alessandro-Beatrice-Bertotti-Montorsi (ABBM) model [21, 22] a model of a single degree of freedom, to an elastic manifold. I have shown that the center of mass of the elastic manifold has the same statistics as in the ABBM model. It also generalizes classical results on the Burgers equation by the mathematicians Carraro and Duchon [23, 24, 25] to extended objects.

Second, one can generalize the elastic energy in equation (2) to an N-component field $\vec{u}(w)$ [KW74]. I have calculated the avalanche-size distribution for this case at the tree level. The result becomes exact for an isotropic random-force field with correlations $[\vec{F}(w_1) - \vec{F}(w_2)]^2 = |w_1 - w_2|$, for which one obtains the full avalanche-size distribution (e.g. in the transverse direction) [KW74].

2.7 Dynamcis of avalanches for pinned elastic objects, and avalanche shape

The other topic in which a break-through has been achieved is the dynamics of an avalanche. Previously, this was only possible for a toy-model, i.e. a particle in a force field which itself is a random walk (ABBM model [21, 22]). We can now address a plethora of observables, as e.g. the distribution of durations in an avalanche given its size, or the velocity after a kick [KW77]. Below the upper critical dimension, calculations become much more involved [KW73, KW82] (the long version has 61 pages in PRE).

In recent years, the question of the temporal velocity average *shape* of an avalanche has gained importance, and was studied both in experiments and in simulations, see e.g. [26]. While this function is well-known for the ABBM model, a simple parabola (for small mass), no analytical results were available beyond mean-field [27]. Its temporal symmetry is actually quite surprising: From numerical simulations for, say, a discrete spin model, we know that an avalanche starts with a single spin flip, which then triggers more and more adjacent spins, until a whole region is active. At a later stage these *active* regions may split and become disjoint. Still, for BFM type disorder, the shape is *time-reversal symmetric*. It should though be mentioned that local observables are not time-inversion symmetric [KW82]. It was a major achievement of my PhD student Alexander Dobrinevski to tackle this question analytically [KW87]. Together, we calculated the $\mathcal{O}(\varepsilon)$, i.e. 1-loop, correction to the *avalanche shape* at fixed duration T, both for SR and LR

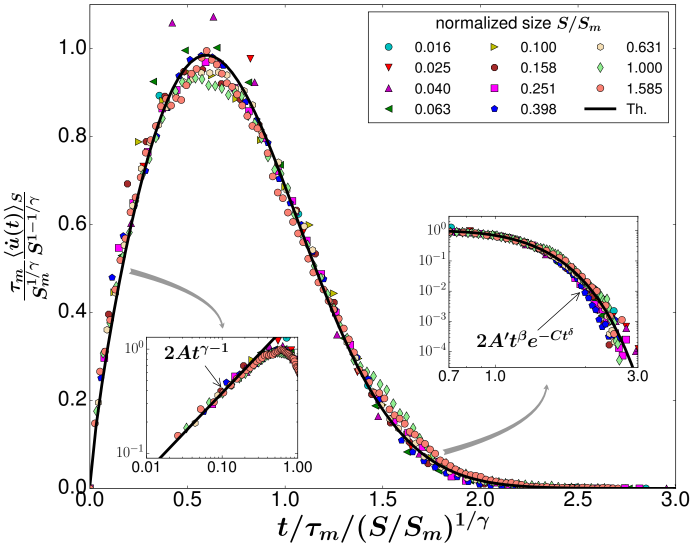


Figure 4: Scaling collapse of the average shape at fixed avalanche size $\langle \dot{\mathrm{u}}(t)\rangle_S$, according to our theoretical results [KW87] plotted on Fig. 5, in the FeSiB thin film. The continuous line is the prediction for the universal SR scaling function, in the limit of short avalanches. The insets show comparisons of the tails of the data with the predicted asymptotic behaviors. For details see [KW94].

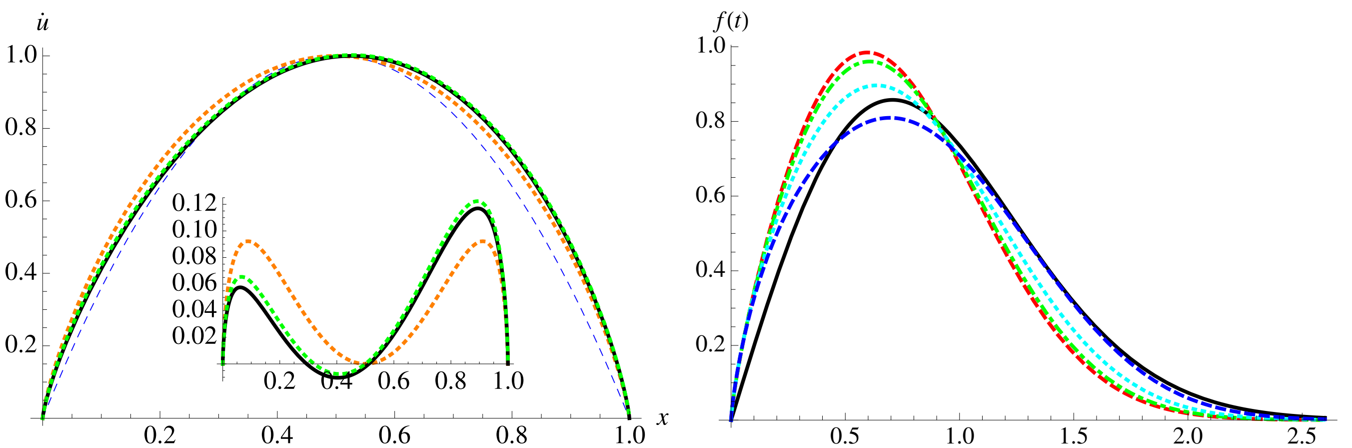


Figure 5: Left: (Universal) normalized shape of an avalanche (of short duration $T \ll \tau_m$), for an interface ($d = 2$) with SR elasticity. Plotted is the total velocity $\dot{u}(t)$ at time $t = xT$, normalized to unit maximum (black thick solid line), compared to: (i) the MF shape $4x(1-x)$ (blue, dashed, thin line); (ii) a symmetric scaling-ansatz $\dot{u} \sim [Tx(1-x)]^{1-\frac{\epsilon}{9}}$ (orange, dot-dashed, thick); (iii) the approximation $\langle \dot{u}(t = xT)\rangle_T \simeq [Tx(1-x)]^{\gamma-1}\exp(\mathcal{A}[\frac{1}{2}-x])$ (green dots), close to the exact result. Inset: *ibid.* with the MF shape subtracted. Right: The shape at fixed size. Mean field (black solid line). The remaining curves are for $d = 2$: small $S/S_m = 0^+$ limit (red dashed) and $S/S_m = 1$, 10, 30 (green dot-dashed, cyan dotted, and blue dashed).

elasticity. The exact expression,[KW87] is asymmetric. The asymmetry $\mathcal{A} \approx -0.336(1-d/d_c)$ is negative for d close to d_c, skewing the avalanche towards its end, as observed in numerical simulations in $d = 2$ and 3 [L. Larson, private communication], see figure 5 (left). We finally introduced and calculated the shape *at fixed avalanche size*, which was for the first time measured experimentally in [KW94], a beautiful verification of the theory.

We have also studied the motion of an elastic object driven in a disordered environment in presence of both dissipation *and inertia* [KW78]. Inertia renders the system history dependent due to oscillations and

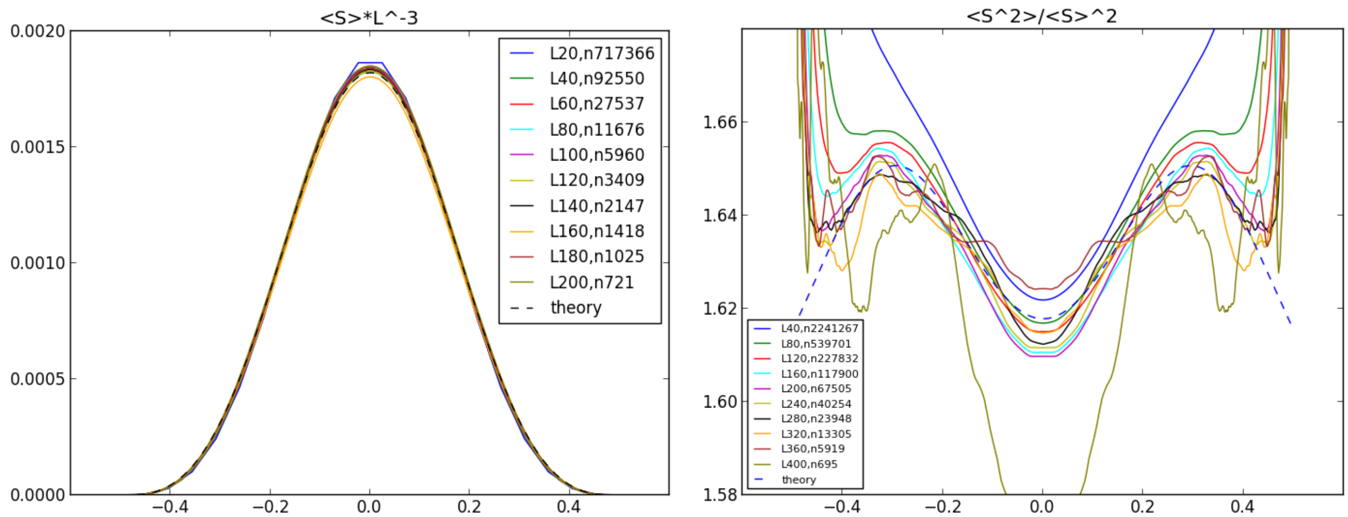


Figure 6: Left: The shape $\left\langle S(x/\ell)\right\rangle_\ell / \ell^3$ for ℓ between 40 and 400. To reduce statistical errors, we have symmetrized this function. Right: The symmetrized ratio $\left\langle S_\ell^2(x)\right\rangle_{\ell=1} / \left\langle S_\ell(x)\right\rangle_{\ell=1}^2$.

backward motion, which prevents a full analytical solution.

While inertia induces memory (the manifold not only remembers its position, but also its velocity, i.e. how it arrived), more realistic models with memory for magnetic systems typically contain apart from the interface velocity $\dot{u}(t)$ a "pressure term", modeling the instantaneous induced current, which decays by Ohmic dissipation [28]. Astonishingly, this model can still be solved analytically [KW84]. While in the standard ABBM model an avalanche stops if the instantaneous velocity $\dot{u}$ goes to zero, here, due to the pressure term, the avalanche can "revive", leading to a considerable change in the shape of an avalanche: Instead of a finite well-defined stopping time, velocities typically decay exponentially in time.

2.8 Spatial shape and extension of avalanches, aftershocks

Another important problem is the spatial shape and extension of an avalanche. Let us start with the spatial shape. For the Brownian Force Model, this problem can be tackled analytically, and one can write down the full probability distribution $\mathcal{P}[S(x)]$ for the shape $S(x)$ (advance in an avalanche at point x) [KW91]. Since this is a functional, already the evaluation of the mean shape $\langle S(x)\rangle$ is difficult, but can be achieved in a saddle-point analysis for large aspect ratio $S\ell^4$, where S is the size and ℓ the spatial extension of an avalanche [KW91].

I further considered the joint distribution of avalanche sizes and extensions in the BFM model [KW95], as well as several size distributions and driving protocols, i.e. whether one drives a single point or the whole interface, whether one restricts the ensemble of avalanches to those of fixed global size, etc. Driving dependence also exists for short-range correlated disorder, see [KW93], where we considered tip-driven motion.

Finally, these efforts were crowned by an analytic result for the shape $\langle S(x)\rangle$ [KW104] of an avalanche of given extension ℓ, as well as of its fluctuations encoded in the second moment $\langle S(x)^2\rangle$. While the analytic expressions are cumbersome (they involve the Weierstrass $\mathcal{P}$ and ζ functions, their numerical verification including all amplitudes is quite precise, see figure 6.

An important question is whether avalanches are correlated, or independent. In earthquakes the experimentally observed Omori law states that the probability to find an earthquake of a certain size decays algebraically with the time spent since the main earthquake. On the other hand, in mean-field models, as the ABBM model, avalanches are *completely uncorrelated*. For short-range correlated disorder, avalanches are anti-correlated, which is intuitive: After a big avalanche, the system has much relaxed and one expects smaller once to come next. [KW98]. This implies that laws such as the Omori law cited above need additional physical ingredients. Indeed, most attempts of earthquake modeling indeed introduce an additional, slowly relaxing dynamic variable, see e.g. [29, 30].

2.9 An efficient algorithm to simulate disordered elastic manifolds, and their equivalence to stochastic Manna sandpiles

Sampling an elastic manifold in a disordered environment usually demands to store the random force seen by each degree of freedom over a certain distance, in a discretized form. This is computationally intensive; a more important drawback is that when the manifold is moving slowly, the disorder force does not change during long times, inciting the programmer to go to finer grids in space.

These problems can be avoided by considering the coupled equation of motion for the velocity, and force, where the latter evolves according to [KW84]

$$\partial_t \mathcal{F}(x,t) = -\dot u(x,t)\mathcal{F}(x,t) + \eta(x,t)\sqrt{\dot u(x,t)} \tag{5}$$

with $\eta(x,t)$ a white noise, uncorrelated in space and time. One indeed shows that this is equivalent to a disorder correlator decaying exponentially in u [KW84][31]. Since the force is now a dynamic variable, the two problems mentioned above disappear, and simulations become much more efficient.

The remaining problem, namely that the noise-term $\eta(x,t)\sqrt{\dot u(x,t)}$ only decreases as $\sqrt{\delta t}$ with the time-discretization step δt, can be circumvented by solving exactly the stochastic equation of motion for a finite time-interval [32] [KW84]. We have already used this process with good success [KW84,KW104][31], and are employing it again in collaboration with Alejandro Kolton.

It had been conjectured for a long time that disordered elastic manifolds and Manna sandpiles are equivalent, but only circumstantial evidence had been given [33, 34, 35, 36, 37]. To prove the first part of this connection, namely the equivalence between disordered elastic manifolds and the so-called C-DP field theory, I found [KW89] that one simply has to identify the velocity $\dot u$ of disordered elastic manifolds with the activity ρ in Manna-sandpiles $\rho \equiv \dot u$, and the difference between activity and the number of grains $n = \phi$ per site (the conserved field) with the force $\mathcal{F}(x,t) = n(x,t) - \rho(x,t) + \text{const}$, to arrive at the field theory for conserved-directed percolation (C-DP), which is believed to be the *effective field theory* of the Manna model [36, 38, 39, 40, 35, 37, 41, 42, 43, 44]. Remained to show the equivalence between the Manna-model itself, and the C-DP field theory. I succeeded to do this in [KW90]; this will be discussed in more detail in section 9.2.

2.10 Other applications of FRG, and surprises

Together with my PhD student Alexander Dobrinevski [KW72], we considered a particle in one dimension submitted to amplitude and phase disorder. It can be mapped onto the complex Burgers equation, and provides a toy model for problems with interplay of interferences and disorder, such as the NSS model of hopping conductivity in disordered insulators and the Chalker-Coddington model for the (spin) quantum Hall effect. The model has three distinct phases.

A surprising application of FRG deals with loop-erased random walks (LERW, Lawler 1980), i.e. random walks in which loops are erased as they form; LERWs have applications in combinatorics, self-organized criticality, conformal field theory and SLE. We found [KW61] that the Functional RG developed for periodic systems provides a field theory for LERWs, where the dynamic exponent z encodes the fractal dimension of LERWs. The predicted subleading logarithmic corrections in $d = 4$ were recently tested numerically by P. Grassberger [45] and David Wilson [46]. Our recent insights are discussed in more detail in section 9.1.

Finally, to go beyond elastic descriptions and include plastic effects, we studied [KW59] two elastic layers coupled viscously. To one-loop FRG we found hysteresis in the velocity-force characteristics, i.e. two dynamically meta-stable branches, a situation forbidden in elastic depinning (Middleton's theorem). The 2-loop extension strongly hints at new phenomena and suggests that more work is needed to conclude.

2.11 Random-field $O(N)$ model, and LR interactions

By contrast to the random-field Ising model, the random-field $O(N)$ non-linear sigma model (hard-spin version) is amenable to studies via FRG at zero temperature [47]. For $N = 2$ there is a fixed point to one loop in $d = 4 - \epsilon$ describing a quasi-ordered *phase* while for $N = 3, 4, \ldots$ the fixed point is in $d = 4 + \epsilon$ [48, 49, 50, 51], describing the ferromagnetic-paramagnetic *transition*. Hence something must happen for $2 < N < 3$, with a lowering of the lower critical dimension. This was elucidated in a two-loop FRG calculation for this model [KW44]. Lowering N below $N_c = 2.83$, the transition fixed point in $4 + \epsilon$ dimensions plunges below $d = 4$, and becomes stable. Combining exact RG with our functional RG, which can only be done numerically, Tarjus and Tissier [52, 53], finally succeeded to see the RF fixed point for the Ising model.

Another interesting observation made in [54] is that for $N > 18$ the RF model recovers dimensional reduction, hence its large-N limit must be trivial. We studied the random anisotropy version of this model [KW54] and found there a non-trivial large-N limit with a non-analytic fixed point for any N. Thus the recovery of "dimensional reduction" seems to be rather specific for the random-field universality class. It also does not exist for disordered elastic manifolds discussed above.

Long-range correlated disorder occurs in some systems such as superconductors subjected to ion irradiation or liquids in aerogels. We therefore applied with my former postdoc A. Fedorenko the FRG to manifolds in long-range correlated disorder in [KW50] to one loop. A. Fedorenko later generalized this to 2 loops [55], establishing an interesting weak Bose glas phase, and to spin systems [56].

2.12 Replica Symmetry Breaking: Confrontations FRG-RSB, and avalanches in the SK model

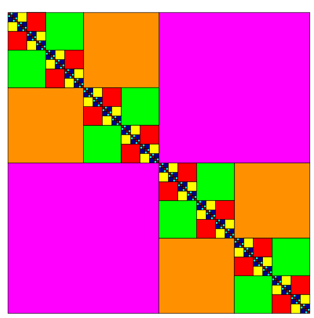


Figure 7: Example of a replica-symmetry broken matrix.

In some cases other analytical methods are available to study disordered elastic systems. It is thus important to confront and, if possible, reconcile the FRG with these methods. Studying the problem for a large number of components N [KW23, KW36, KW38, KW57], we have been able to make contact with the replica symmetry breaking (RSB) method, whose validity is restricted to the large-N limit. We have been able to show that the two methods calculate almost the same 2-point function at zero momentum. The difference can be traced to the different ways of preparing the system, allowing for spontaneous RSB in the case of the RSB-method, and for explicit RSB in the functional RG calculation. (The difference is quite similar to a pure magnet cooled down below the Curie-temperature, which can either be prepared in zero

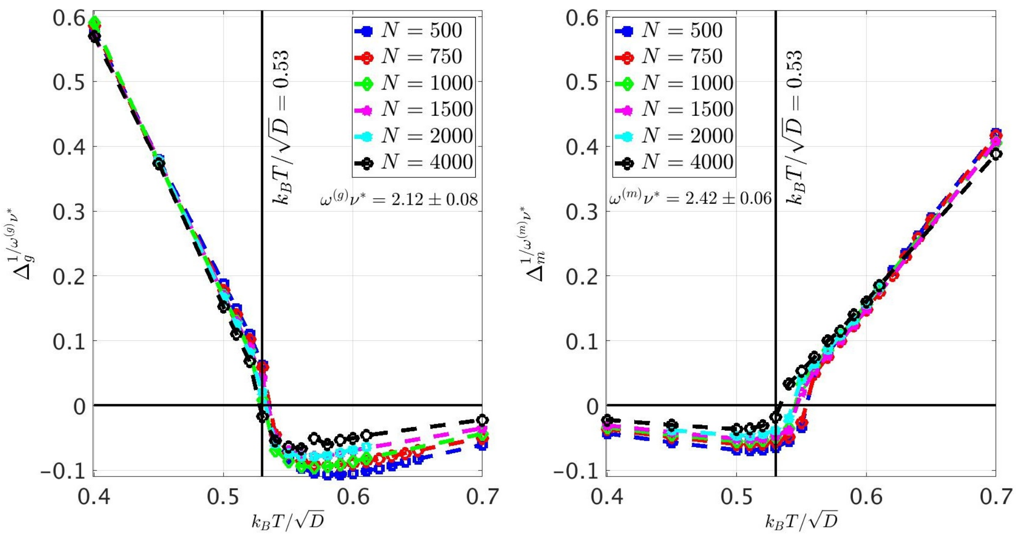


Figure 8: The two order parameters defined in Eq. (6) both go to zero at the transition temperature $T = T_c$..

magnetic field, leading to spontaneous symmetry breaking in the magnetization, or in an infinitesimaly small magnetic field, with which the magnetization will align. The RSB-method compares to the first case, and FRG to the latter.) Recently, we have been able to make this connection precise [KW57]: For manifolds embedded in a N-dimensional space, the Mézard-Parisi mean-field theory, based on Parisi's scheme of replica symmetry breaking (RSB) should become "exact" in a sense to be determined for $N \to \infty$. With Markus Müller [KW57] we understood this point and found that in presence of RSB there are *two* distinct scaling regions in the large-N limit, one which reproduces the full Mézard-Parisi result with $u^2 = O(1)$ and the other, $u^2 = O(N)$, which yields the previous FRG equation at large N. They are related to the appearance either of the first shock in the system as a response to an external field or of a thermodynamic number of shocks (one per independent volume element) in the two respective regimes.

A further direction of research has been the application of our formalism to avalanches in mean-field spin glasses [KW70, KW75], where full replica-symmetry breaking prevails, and the avalanche exponent τ decreases from the value $\tau = 3/2$ for the Brownian force model to $\tau = 1$, which we were able to extract from a full RSB solution.

3 RNA-folding

Together with Michael Lässig and François David, I have developed a field theoretical treatment for the RNA glass transition [KW45, KW49, KW68]. This project heavily relies on the techniques developed for self-avoiding membranes discussed in the next section.

RNA molecules (mostly) fold in planar configurations (in the sense of field theory). We are interested in the pairing statistics for large molecules. The RNA glass transition appears upon decrease of the temperature, and is due to sequence disorder. It corresponds to an increase in the roughness of the height-field from the random-walk exponent $1/2$ for homopolymers to about $2/3$ for the glass phase, equivalent to a decrease in the exponent ρ for the paring probability as a function of distance from $3/2$ to $4/3$. This has numerically be seen in [57].

Our field theory uses an analytical continuation of the homopolymer problem, by modifying the pairing probability of two monomers at a distance s, from $s^{-3/2}$ to $s^{-\rho_0}$, represented by polymers imbedded into $d = 2\rho_0$-dimensional space. Choosing $\rho_0 = 1 + \epsilon/2$, we construct a systematic ϵ-expansion, by treating sequence disorder in a perturbative expansion. As in the $2 + \epsilon$ expansions of the non-linear sigma model or KPZ, our RG-treatment finds a continuous phase transition (i.e. an IR-repulsive fixed point), and allows us to calculate the critical exponents at this transition. As a side-effect we find that already at the transition the critical exponents are given by those in the low-temperature phase ("locking scenario").

While together with R. Bundschuh and W. Baez, we have finally localized the transition, the problem remains difficult due to enormous finite-size effects. The key idea is to define order parameters Δ_g in the low-temperature (glass) phase, and Δ_m in the high-temperature (molten) phase,

$$\Delta_g \simeq \lim_{\ell\to\infty} \frac{\langle p(\ell)^2\rangle}{\langle p(\ell)\rangle}\ , \qquad \Delta_m \simeq \lim_{\ell\to\infty} \frac{\langle p(\ell)\rangle^2}{\langle p(\ell)^2\rangle}\ . \tag{6}$$

eq:lowTemp

These ratios are constructed to both yield 1 at $T = 0$ and $T \to \infty$, due to the locking at $T = 0$, and the decorrelation of copies at $T \to \infty$. As can be seen on Fig. 8, they both disappear with a characteristic power-law (with known exponent) at the critical temperature T_c.

With François David [KW49] we have been able to construct a field theory and an action for the RNA-folding problem, a long outstanding problem. We have also *proven* that the theory is renormalizable to all orders. Using this breakthrough, we have been able to calculate the exponents ρ to 2-loop order [KW49]. A detailed account (93 pages) of these calculations was published recently [KW68].

Together with PhD-student Christian Hagendorf, we have also included the effect of an external applied force [KW55].

We have further studied a toy-model, similar in spirit to a greedy algorithm in computer science [KW56], where the energetically most favorable bond is made first, then considering the remaining possible pairings. This model can be solved analytically; e.g. the roughness exponent is $\zeta = (\sqrt{17} - 3)/2 \approx 0.561$, and we have obtained higher correlation functions as well. Interestingly, our model can be reformulated as a dual tree growth process, opening the door to a completely different community. I hope that this insight will spur further progress for growth models. A first example was indeed proposed in [58].

A representation of RNA folding in the Poincaré plan has been the starting point of an outreach project with Jean-Pierre Hébert, artist in residence at the KITP, UCSB, Santa Barbara, USA. Our work, of which an example is displayed on the cover of this report, has been shown at SciArt 2011.

4 Self-avoiding membranes

Polymers and membranes have played an essential role in my research of the first ten years [KW3–KW9, KW11, KW13-KW15, KW17-KW19, KW21, KW22, KW24, KW29, KW37, KW39]. Two major classes of membranes can be distinguished: Fluid membranes, and polymerized (tethered) membranes. The last class is characterized by a fixed internal connectivity, i.e. the underlying lattice is regular. Nature uses these membranes in so different systems as cell walls, or graphene. More precisely, cell walls contain actin-filaments which can be isolated and studied in experiments. More abstractly, polymerized membranes are lattices made out of beads, connected by springs. They thus constitute a generalization of polymers, which can be viewed as a long chain of beads. As for polymers there are a couple of interesting questions: free movement, incorporation of hydrodynamic interactions, frozen or dynamic disorder, and more.

Polymerized membranes can be treated by field-theoretic methods. This is theoretically novel: The usual local field theories have to be generalized to multi-local ones. Besides a better understanding of membranes, it also leads to a deeper insight into field theory itself.

The subject was started in 1987 by Kardar and Nelson, and by Aronowitz and Lubensky [59, 60], who introduced D-dimensional polymerized membranes embedded into d dimensions. Self-avoidance can then be treated in an ϵ-expansion in $\epsilon = 2D - (2 - D)d/2$. It took some time until methods were developed to treat the model more systematically. They resulted in a proof of perturbative renormalizability by David, Duplantier and Guitter [61, 62, 63, 64]. During my PhD-thesis [KW7], I developed methods to perform perturbative calculations. Already self-avoidance at the 3-critical point and at leading order demands for its treatment to rely on numerical techniques for the evaluation of some of the integrals. However, I found out that expanding about a membrane with inner dimension $D = 4/3$ allows for a complete analytical treatment, as a result of an operator crossover between 3-point self-avoidance and modified 2-point self-avoidance (a second derivative of the delta-interaction) [KW3].

That the theory of tethered membranes is rather rich was shown in [KW5].

Experimentally relevant is the question whether polymerized tethered membranes will be seen flat or crumpled. Experimental realizations are graphene, or the spectrin network extracted from red blood cells. In a tour de force calculation [KW4, KW6], I succeeded to show that contrary to earlier studies [65], 2-loop results are quite reliable, and lead to a predicted Hausdorff-dimension of about 2.5. This result still awaits experimental and numerical confirmation, due to important finite-size problems in the latter.

When I went for my first postdoc position in Essen, both Lothar Schäfer and Hans-Werner Diehl were very interested in the dynamics of self-avoiding polymers. They suggested that I look at the dynamics of self-avoiding membranes as a generalization. For polymers, De Gennes had given in 1976 [66, 67] scaling arguments for the dynamical exponents, which however had never been rigorously been proven to be correct, even for polymers. I was able to show, both for selfavoiding polymers and for the generalization to polymerized membranes, that there is no proper dynamic renormalization *to all orders in perturbation theory*. This allowed me to relate the dynamic exponent to the static one, and thus to prove De Gennes' scaling analysis of 1976 to all orders in perturbation theory [KW8, KW9].

Together with Pierre Le Doussal, we looked at the dynamics in presence of quenched disorder. We were able to establish an RG framework, and found a new fixed point for a polymer or membrane living in a static random-force field, which is not purely the derivative of a random potential. Contrary to the latter case, there is a perturbatively accessible fixed point, with subleading velocity-force characteristics, leading to (weak) trapping and a dynamical exponent z only slightly larger than 2 [KW11, KW17]. (As a side remark, this means that numerical simulations are well feasible; they would be more than welcome to check the predictions.)

One of the most beautiful tricks in the treatment of polymers goes back to De Gennes [68]: Partition function and 2-point function for self-avoiding polymers are equivalent to analogous quantities in a N-component ϕ^4 theory, in which the limit $N \to 0$ has been taken. Stated differently, the ϕ^4-theory is a generalization of selfavoiding polymers. A different generalization has been discussed above, the generalization to polymerized membranes with inner dimension $D \neq 1$. The question arises, whether a common generalization exists. Together with Mehran Kardar, I succeeded to construct such a generalization [KW13, KW14]. Our theory of N-colored membranes allows for an intriguing conjecture for the nature of droplets dominating the 3d-Ising model at criticality, which is satisfied by our numerical results. Generalizing ϕ^4 theory with "cubic anisotropy" we find an *inverse* Coleman-Weinberg phenomenon.

Self-avoiding polymerized membranes represent a new class of field theories. It is important to understand them beyond perturbation theory. While this goal is still elusive, together with François David we have succeeded to properly understand the behavior at large orders in perturbation theory [KW15, KW39]. Interestingly, the large-order behavior is as in ϕ^4-theory given by an instanton; however the latter lives in an auxiliary space. It can be viewed as the critical potential necessary to collapse the membrane. We have been able to solve the theory at large embedding dimension d, and give an expansion in $1/d$. The calculations are a *tour de force*, and at this stage it is not clear how much they can improve our existing 2-loop results.

For the case of a manifold interacting with a single point, which was the first model studied by David, Duplantier and Guitter [61, 62], I have been able to make much progress with my PhD-student Henryk Pinnow. Starting from the observation that diagrams can be calculated analytically in the limit of inner dimension $D \to 2$, we have been able to resum the perturbation theory in the limit of $D \to 2$ to all orders, and to construct an expansion about $D = 2$ to fourth order [KW24, KW29, KW37]. While we have not been able to fully understand the crossover function necessary to extrapolate down to $D = 1$ (a case against which we can check), our all-order results and using the simplest crossover function were quite promising, and let us hope that one day one will be able to solve the self-avoiding case in the physically relevant limit of $D = 2$.

This work (up to 1999) has been reviewed in [KW18], published as volume 19 in the series "Domb-Lebowitz", Phase Transitions and Critical Phenomena. It is subject of my habilitation, granted in 1999.

5 Multifractality, and Conformal field theory

5.1 Multifractality

Multifractality is an intriguing property of system which have a dimensionless field. It is a common feature, and tool of analysis, in localization in electronic systems [69, 70, 71, 72]. These are quantum systems, and the question arizes whether the same phenomenology appears in classical *disordered* systems. While the *intuitive* answer is in the affirmative, we did not know of any analytical result. For this reason, I decided to study this problem, both for the 2d sine-Gordon model using methods of conformal perturbation theory, as for a charge-density wave, using FRG.

To this aim, in [KW83] we consider the random-phase sine-Gordon model in two dimensions. We calculate all higher cumulants and show that they grow with an amplitude containing the Riemann zeta function. This should be visible in Bragg scattering experiments of the dual fermion model.

For charge-density waves in $d = 4 - \epsilon$ dimensions, we develop a method based on functional determinants and the Gel'fand-Yaglom method [73], equivalent to summing an infinite set of diagrams. We obtain, in dimension $d = 4 - \epsilon$, the even As a corollary, we obtain an analytic expression for a class of n-loop integrals in $d = 4$, which appear in the perturbative determination of Konishi amplitudes, also accessible via AdS/CFT using integrability.

5.2 CFTs with current-current interactions

An intriguing way to arrive at disordered systems is to perturb a conformal field theory. The idea of this approach is that the starting model is highly constraining and so one hopes to be able to go further when perturbing it. Together with Andreas Ludwig, I have analyzed 2D conformal field theories with a "Kac-Moody current algebra" symmetry, which are perturbed by a left-right bilinear in the Noether-currents. An interesting conjecture has recently been advanced by Gerganov, LeClair and Moriconi [74], who consider a general current-algebra conformal field theory at level k, perturbed by right-left current bilinears, and proposed a β-function supposed to be exact to all orders in perturbation theory (in a specific regularization scheme). We have checked this proposal with a direct calculation at 4-loop order. We find the conjecture to be incompatible with our calculation in all possible regularization schemes [KW30].

5.3 Interfaces in the Random-Bond Potts Model and Relation to SLE

Here I consider interfaces in the random-bond Potts model. Formally, the random-bond Potts model is a perturbed conformal field theory [75, 76]. Using methods of conformal perturbation theory, I calculated the anomalous dimension of curve-creating operators, and described by Stochastic Löwner Evolution (SLE). We obtained [KW63] the fractal dimension of FK clusters in the random-Potts system analytically, and compared them favorably to numerics. Numerically also the putative SLE duality $\kappa_{\rm spin}\kappa_{\rm FK} = 16$ seems to hold for the disordered model.

5.4 Roughness in the sine-Gordon model

Using the renormalization group to two-loop order [KW79, KW80] we obtained analytically the roughness in the disorder-induced glass phase of the two-dimensional XY model with quenched random symmetry-breaking fields and without vortices. These results compare favorably to numerical simulations [KW79]. This result differs at two-loop order, i.e., $\mathcal{O}(\tau^3)$, from the prediction based on results from the "nearly conformal" field theory of a related fermion model [77].

6 Fractional Brownian motion and its extreme value statistics

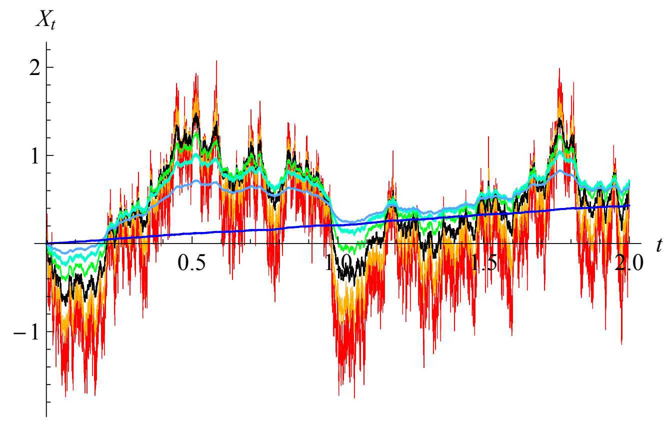


Figure 9: *Left*: The fractional Brownian motion discussed in the main text, for different values of H, using the algorithm of [78], from $H = 0.25$ (red) to $H = 1$ (blue, almost straight line).

6.1 Introduction

For a diffusing particle the mean-sqaured distance of its position X_t grows linearly in time, $\langle X_t^2 \rangle \sim t$, and one knows many things about its extreme value statistics: Its survival probability, maximal excursion, the time at the maximal excursion (the famous arc-sine law), the time the walk is above its initial position, and many more. On the other hand, not much is known for the anomalous diffusion of more complex objects. E.g. what is the effective theory of a single monomer which is part of a polymer? How does an RNA-molecule translocate through a nano-pore? How does a colloid diffuse in interaction with other colloids? Their motion is *non-Markovian*, i.e. has *memory*, a feature which is also seemingly important for the analysis of financial markets. Demanding that the process be Gaussian, translational-invariant both in time and space, and self-affine defines a class of processes called fractional Brownian motion (fBm). More precisely,

$$\langle X_t X_s \rangle = s^{2H} + t^{2H} - |t-s|^{2H} \ . \tag{7}$$

fBm

An example, constructed with the same random numbers in a Fourier decomposition [78] is given on the left of figure 9. All higher cumulants vanish (Gaussian process). For $H < 1/2$ the process is sub-diffusive, while for $H > 1/2$ it is super-diffusive. In the former case its velocity is anti-correlated, whereas in the latter case it is correlated, as can be seen from $\langle \partial_t X_t \, \partial_s X_s \rangle = 2H(2H-1)|s-t|^{2(H-1)}$. In this project, we are interested in the extremal behavior of fBm, which is often more relevant than the mere knowledge of expecations.

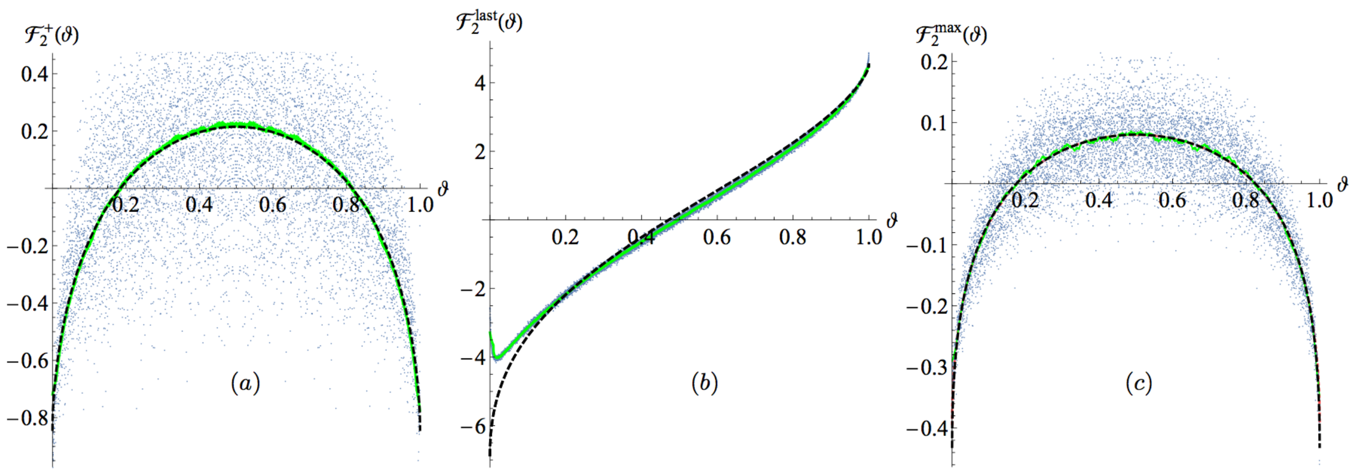


Figure 10: A comparison for the three $\mathcal{F}_2(\vartheta)$ obtained analytically (black dashed lines) and their measurement using formula (5) with $\epsilon = \pm\frac{1}{6}$. From left to right: (a) positive time, (b) time of the last visit to the origin, and (c) time for the maximum. The scattered dots are the raw data from trajectories of $N = 2^{13}$ time steps, averaged over 5×10^9 samples, which are coarse grained by a factor of 100 to give the green curve. Approximations of our analytical results are given in the supplementary material.

The theory I developed is based on an expansion around Brownian motion, setting

$$H = \frac{1}{2} + \epsilon \,. \tag{8}$$

The idea behind is that for Brownian motion one knows the propagator with absorbing boundaries, which allows to impose bounds on the motion. For a Brownian, it is also possible to keep track of other observables, as the time a process remains positive. The parameter ϵ then quantifies the "non-Markovianity". After some initial fruitless attempts, my current technology is well developed: It works with the Laplace-transform in time, which allows to do all space-integrations, and avoids integration over intermediate time points. The reader interested in the technical details can consult [KW97].

6.2 Main results

Let us now sketch the main results. A classic object of extreme value statistics is the probability that a fractional Brownian motion which starts close to the origin, remains positive up to time t, at which time it arrives at position x [KW71].

Levy's arcsine laws say that for a Brownian starting at the origin, three observables have the same probability distribution: the time the process is positive, the time it achieves its maximum (or minimum), and the time it last visits the origin.

In our formalism, the probability for the time to reach the maximum at time θ at total time $T = 1$ is given by

$$p_+(\vartheta) = \frac{\mathcal{N}_+}{[\vartheta(1-\vartheta)]^H}\, \mathrm{e}^{\epsilon \mathcal{F}_1^+(\vartheta) + \epsilon^2 \mathcal{F}_2^+(\vartheta) + \mathcal{O}(\epsilon^3)} \,. \tag{9}$$

In a first step, we obtained $\mathcal{F}_1^+(\vartheta)$, as well as the joined distribution of maximum and time when it is achieved [KW92, KW97]. Repeating similar calculations for the other two observables, we realized that only the last visit to the origin has a different law at leading order in ϵ, which prompted us to go to second order in ϵ [KW101]. This was a tour-de-force calculation. The results obtained and plotted on figure 10 compare favorably to the numerical verification, itself difficult since we have to extract a sub-subleading correction in the ϵ-expansion [KW101].

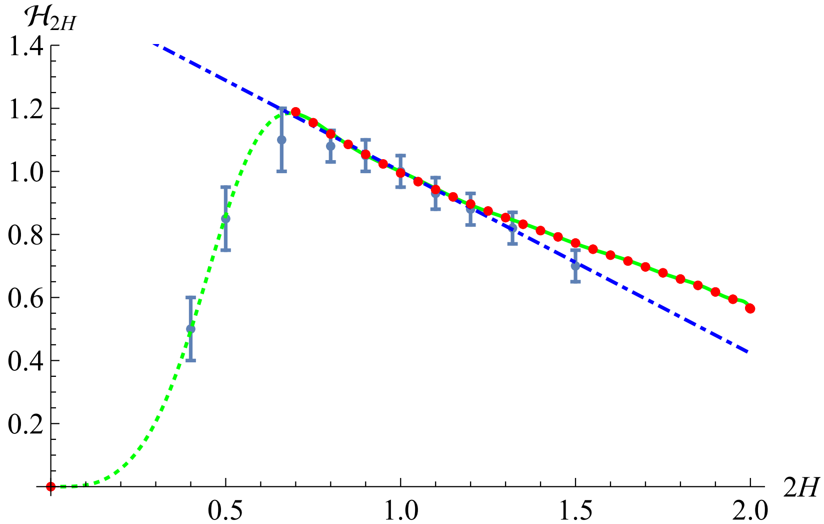


Figure 11: Comparison of numerical data of [85] (red dots), interpolation (green solid line where data quality is good, and dashed elsewhere) and our order-ϵ result (blue, dot-dashed). The remaining points are an estimation of Pickands constant using the distribution of the maximum of a fBm bridge.

We performed a similar analysis for fBm bridges, which are defined as processes which start and end at zero. While the last visit to the origin is always at the end, the two other probabilities, for the time when the maximum is achieved, as well as for the positive time, are distinct at order ϵ [KW99].

For generic Gaussian Random Processes, the tail of the distribution for large values of the maximum has attracted a lot of attention following the seminal work by Pickands [79], followed by Piterbarg [80, 81], and others [82, 83, 84]. It is now well proven that the tails of the probability contain a Gaussian factor, a subleading power-law, and a universal amplitude, termed the Pickands constant (or at least simply related to it) $\mathcal{H}_{2H}$. Curiously, this constant is analytically known only for $H = 1/2$ (Brownian motion), and $H = 1$ (a straight line). Using the techniques presented above, I was able to show that [KW100]

$$\mathcal{H}_{2H} = 1 - \gamma_{\rm E}(2H-1) + \mathcal{O}(2H-1)^2 \,, \tag{10}$$

where $\gamma_{\rm E}$ is Euler's constant. This is in agreement with the most precise estimations of Pickands' constant presented on figure 11. We have also been able to show rigorously that the fractional dimension of the record set of fBm with Hurst-exponent H is H [KW102].

Our last work deals with the 2-sided exit problem of fBm [KW107]. We were able to evaluate analytically the probability that an fBm starting at $x \in [0, 1]$ exists either at the upper end 1, or the lower end 0, and what expectation and variance of its exit time are. We confirmed our analytical findings with extensive numerical simulations (27 CPU years).

7 Graphene and Carbone Nanotubes

7.1 Carbon Nano-Tubes

Consider transport properties of a single-walled metallic carbon nano-tube with nearly perfect contacts to the electrodes. Experiments have shown that such systems can transport electrons ballistically, with only weak back-scattering, which mostly takes place at the two contact-points with the wires. The measured conductance as function of applied voltage and gate-potential shows a pattern of quantum interference due to resonant tunneling, which we calculate analytically using a 2-channel Luttinger liquid with backscattering in the contacts, using the Keldysh-formalism to capture non-equilibrium effects due to the finite applied

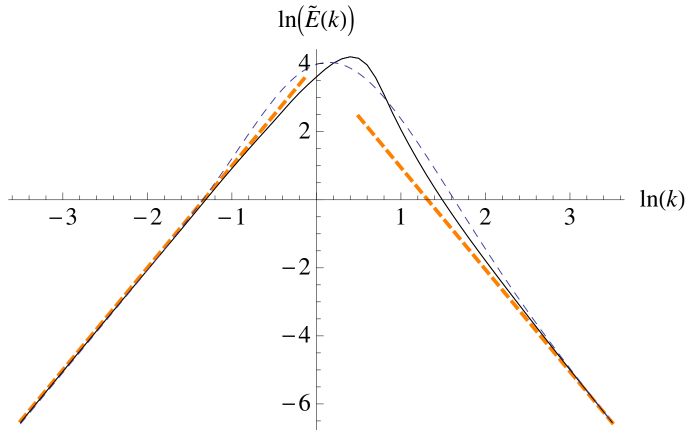


Figure 12: Double logarithmic plot for the rescaled energy $\tilde{E}(k) := k\tilde{\Delta}(k)$ as a function of k. The solid black line is our solution of the FRG fixed-point equation; the thick dashed lines show the asymptotic slopes ± 3. The thin grey-blue dashed line is the guessed function from which the numerical solution starts.

voltage. A variety of experimentally relevant predictions are made, including scattering at magnetic impurities and in presence of an external magnetic field [KW34].

7.2 Rippling transition in graphene

Together with Paco Guinea, we consider [KW86] a model of Dirac fermions coupled to flexural phonons to describe a graphene sheet fluctuating in dimension $2 + d$. We derive the self-consistent screening equations for the quantum problem, exact in the limit of large d. We first treat the membrane alone, and work out the quantum to classical, and harmonic to anharmonic crossover. For the coupled electron-membrane problem we calculate the dressed two-particle propagators of the elastic and electron interactions and find that it exhibits a collective mode which becomes unstable at some wave-vector q_c for large enough coupling g. The saddle point analysis, exact at large d, indicates that this instability corresponds to spontaneous and simultaneous appearance of gaussian curvature and electron puddles. The relevance to ripples in graphene is discussed.

8 Turbulence, and turbulent Advection

8.1 Passive Advection: Passive Scalar and Passive Polymer

In the context of turbulence, passive advection is an important testing ground for ideas, since it has been shown that higher moments of the density-density correlation function show multiscaling even in an advecting medium with correlations which are Gaussian and short-ranged in time. I have been able to generalize this “passive scalar” model to extended elastic objects like polymers and membranes (“passive polymer”). With – surprisingly – the same rigor as for particles, properties for polymers and membranes can be calculated, leading to some significant differences [KW19].

8.2 Fully developed turbulence

To understand fully developed turbulence with the methods of RG is challenging, not only due to the intrinsic unsolved problems of turbulence, but also due to its calculational complexity. We applied the functional renormalization-group (FRG) approach to decaying Navier-Stokes and Surface-Quasi-Geostrophic turbulence [KW81]. The method is based on a renormalized small-time expansion, equivalent to a loop expansion, and naturally produces a dissipative anomaly and a cascade after a finite time. A breakdown of energy conservation due to shocks and the appearance of a direct energy cascade corresponds to the failure of dimensional reduction in disordered systems. For Navier-Stokes in three dimensions, the velocity-velocity correlation function acquires a linear dependence on the distance, $\zeta_2 = 1$, in the inertial range, instead of Kolmogorov's $\zeta_2 = 2/3$; however the possibility remains for corrections at two- or higher-loop order. In two dimensions, we obtain a numerical solution which conserves energy and exhibits an inverse cascade, with explicit analytical results both for large and small distances, in agreement with the scaling proposed by Batchelor (see figure 12). In large dimensions, the one-loop FRG equation for Navier-Stokes converges to that of Burgers.

9 Recent excitements on various subjects

9.1 Field Theories for loop-erased Random Walks, and their equivalence to CDWs at depinning

Self-avoiding walks (SAWs) and loop-erased random walks (LERWs) are two ensembles of random paths with numerous applications in mathematics, statistical physics and quantum field theory. While SAWs are described by the $n \to 0$ limit of ϕ^4-theory with $O(n)$-symmetry, LERWs have no obvious field-theoretic description. I analysed [KW106] two candidates for a field theory of LERWs, and discovered a connection between the corresponding and a priori unrelated theories. The first such candidate is the $O(n)$-symmetric ϕ^4 theory at $n = -2$ whose link to LERWs was known in two dimensions due to conformal field theory. Here it is established via a perturbation expansion in the coupling constant in arbitrary dimension. The second candidate is a field theory for charge-density waves pinned by quenched disorder, whose relation to LERWs had been conjectured earlier using analogies with Abelian sandpiles. We explicitly show that both theories yield identical results to 4-loop order and give both a perturbative and a non-perturbative proof of their equivalence. For the fractal dimension of LERWs in $d = 3$ our theory gives at 5-loop order $z = 1.624 \pm 0.002$, in agreement with the estimate $z = 1.62400 \pm 0.00005$ of numerical simulations [46].

9.2 Coherent-state path integral, imaginary noise, reaction diffusion systems, and an effective field theory for the Manna model

The coherent-state path integral is one of the central methods used to study stochastic systems in a field-theoretic setting, i.e. to construct a dynamic action, (a.k.a. Martin-Siggia-Rose, MSR). There are, however, severe problems in the literature on its applications: Decoupling of quartic vertices is done with a real instead of an imaginary noise. What are occupation probabilities for complex coherent states? How can one derive effective stochastic equations of motion with real noise? Is there a Mean-Field limit, and how is it constructed? Since all these questions were not, or only badly discussed in the literature, I decided to find answer to these questions. The result are comprehensive lecture notes [KW90] which I plan on incorporating into the class I am going to teach next term. As a corollary, I was able to give the first microscopic derivation of the stochastic equations of motion for the Manna-model. The latter is a stochastic model for a sandpile: if on a given site there are two or more grains, then move them to randomly chosen neighbors. A mean-field approximation for this model consists in moving the grains not to nearest neighbors, but to any other site. The resulting equations for the i-times occupation probability then close, and the steady state can be found analytically. Starting from this solution, one has to add diffusion terms plus noise terms, which come from the fact that the number of grains changes by an integer, but we want to write effective equa-

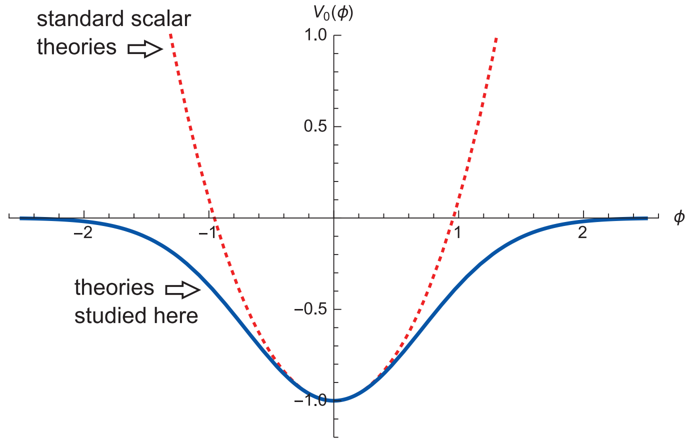


Figure 13: The function $\mathcal{V}_0(\phi)$, for ϕ^4 theory (top, red, dashed), and a bounded potential (bottom, blue, solid).

tions of motion for a continuous variable. The latter are fixed by demanding that are correctly reproduced both the *mean* amd the *fluctuations* of the i-times occupation probabilities. The latter being the relevant dynamic variables also allows us to understand why one needs more than one dynamical variable, contrary to the coherent-state path integral which only has one (complex) variable. It turns out that for the Manna model close to the transition one can get away with two variables. This allowed me to finally *derive*, with the same rigor as one derives the Mean-Field approximation for the Ising model, the effective stochastic description corresponding to the Manna model, known as the C-DP field theory.

9.3 Non-trivial fixed points of the Renormalization Group

Evaluating the partition function of a field theory in presence of a potential $\mathcal{V}_0(u)$, one typically gets a flow equation of the form [86, 2, 87, 88], (confusingly) also referred to as *exact RG*:

$$-m\partial_m \mathcal{V}(u) = -m\partial_m \int^\Lambda \frac{\mathrm{d}^d k}{(2\pi)^d} \ln\left(1 + \frac{\mathcal{V}_0''(u)}{k^2+m^2}\right) . \tag{11}$$

Keeping only the leading non-linear term [KW96] leads to the simple flow equation for the dimensionless potential $R(u) := -m^{4\zeta-\epsilon}\mathcal{V}(um^{-\zeta})$

$$-m\partial_m R(u) = (\epsilon - 4\zeta)R(u) + \zeta u R'(u) + \frac{1}{2}R''(u)^2 + \dots . \tag{12}$$

This equation is similar to the FRG flow equation (1) for disordered elastic manifolds. It reproduces the standard RG-equation for ϕ^4 theory, setting $R(u) = -gu^4$. Renormalization of the propagator can be avoided for $n = -2$ components (field theory for loop-erased random walks).

The question now arises what happens if one starts with a smooth bounded potential, as the one given on Fig. 13, where $R(u) > 0$? I showed that

(i) $R(u)$ develops a cusp at $u = 0$, and a cubic singularity at $u = u_c > 0$.

(ii) The flow equation (12) has an infinity of fixed points, indexed by $\zeta \in [\frac{\epsilon}{4}, \infty]$.

(iii) The solution chosen dynamically when starting from smooth initial conditions is $\zeta = \frac{\epsilon}{3}$. Its analytic expression for $0 \le u \le 1$ reads

$$R_{\zeta=\frac{\epsilon}{3}}(u) = \epsilon\left[\frac{1}{18}(1-u)^3 - \frac{1}{72}(1-u)^4\right] . \tag{13}$$

It vanishes for $u > 1$, and is continued symmetrically to $u < 0$.

This scenario is quite unusual: While there is a family of perturbatively accessible fixed points, only one of them is chosen dynamically. It is yet not clear to which physical system it applies. A possible candidate is wetting, but as discussed in the literature [89, 90, 91, 92, 93, 94], experiments can be described by a flow equation *linear* in $R(u)$.

10 Other topics

10.1 Casimir forces

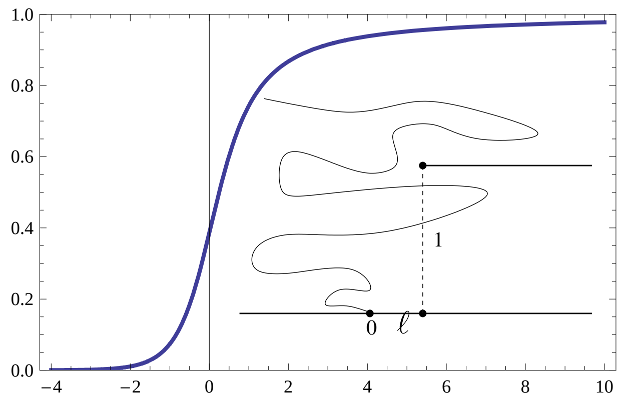


Figure 14: The probability to avoid a wall starting from $(\ell, 1)$ to $(\infty, 1)$. Inset: the geometry in question.

Due to new experiments, Casimir physics has seen a revival in recent years, both for the quantum mechanical as for the classical version, known as the critical Casimir effect. Stimulated by a workshop at the KITP in Santa Barbara, which I attended in summer 2009, I asked myself what the fluctuation-induced forces of a self-avoiding polymer would be [KW64]. Interestingly, recent results from Schramm-Löwner evolution (SLE) allow to calculate analytically the force exerted by one or several polymers on a small impenetrable disk, in various 2-dimensional domain geometries. Contrary to the quantum version, Casimir forces can be repulsive, and allow e.g. for a stable position of the disk, "trapped" between two polymers.

The method also allows to give a closed expression for the probability that a self-avoiding walk starting at the origin does not intersect a given line, see figure 14, a question often asked for directed random walks in the context of extremal value statistics. However the standard methods of extremal value statistics can not handle this situation, where the polymer can return towards the axis from which it started.

10.2 KPZ-equation

A fascinating phenomenon is the growth of surfaces. One of the most interesting models is the KPZ-equation, which describes non-linear surface growth. I have been able to clarify some controversial issues in the literature, pertaining to the 2-loop and all-order β-functions for the roughening transition in $d = 2+\epsilon$ dimensions [KW10, KW12].

10.3 KPZ and qKPZ for self-sustained reaction fronts

Self-sustained reaction fronts in a disordered flow display self-affine roughening, pinning and depinning transitions, as depicted on figure 15. In collaboration with my experimental colleagues Séverine Atis, Dominique Salin, and Laurent Talon, we measure spatial and temporal fluctuations which are consistent

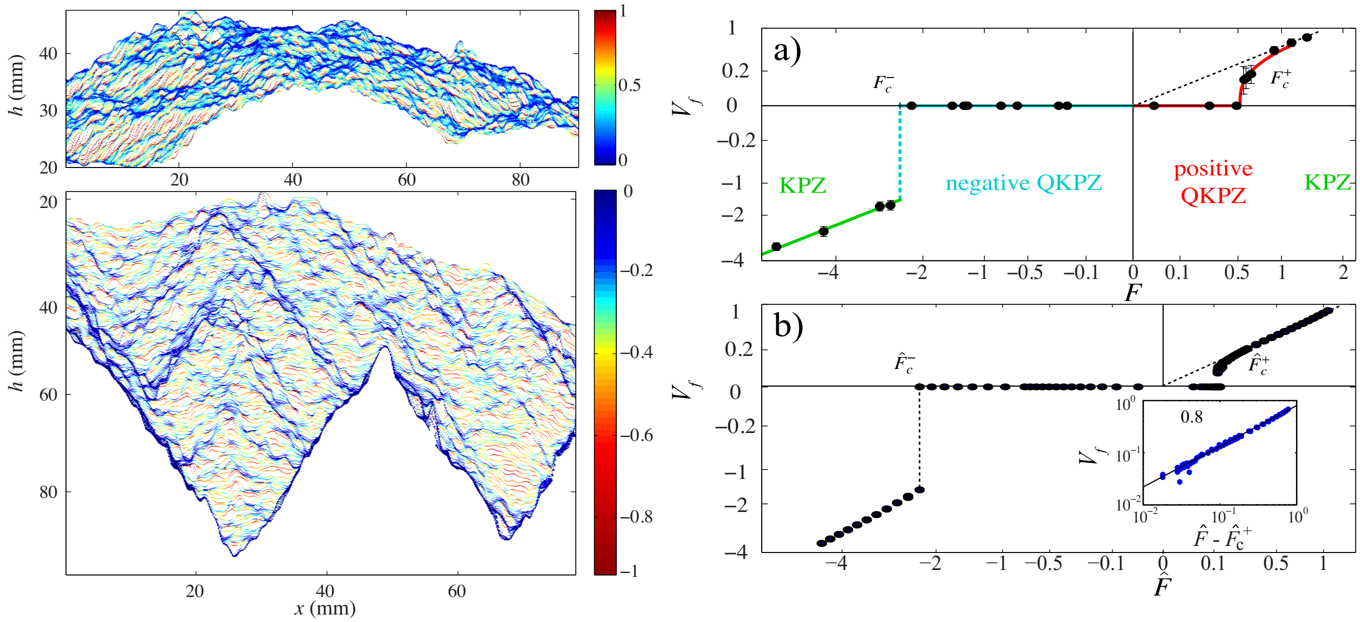


Figure 15: Left: Successive experimental fronts at constant time intervals. Color represents local front velocity. Top left: upward propagating front near F_c^+. Bottom left: backward propagating front near F_c^-.
Right: Front velocity V_{f} versus the applied force F, in adverse flow. a) experiments (black dots with error bars), b) numerics. Dashed lines are a linear extrapolation of the advancing branch. To put all data on one plot, axes are rescaled according to $F \to F/|F|^{1/2}$, $V_{\mathrm{f}} \to V_{\mathrm{f}}/|V_{\mathrm{f}}|^{1/2}$. Insert: log-log plot of front velocity versus $\hat{F} - \hat{F}_{c^+}$. The continuous line corresponds to $v(\hat{F}) \propto (\hat{F} - \hat{F}_{c^+})^{0.8 \pm 0.05}$.

with three distinct universality classes in $d = 1 + 1$, controlled by a single parameter, the mean (imposed) flow velocity [KW88]. The three distinct universality classes are

1. the Kardar-Parisi-Zhang (KPZ) class for fast advancing or receding fronts, with a roughness exponent of $\zeta \approx 0.5$.

2. the quenched Kardar-Parisi-Zhang class (positive-qKPZ) when the mean-flow velocity almost cancels the reaction rate. It has roughness $\zeta \approx 0.63$, in agreement with values from directed percolation [95, 96, 97]. A depinning transition with non-linear velocity-force characteristics, $v \sim |F - F_c|^{\alpha}$ is observed, see figure 15.

3. the negative-qKPZ equation for transitory receding fronts, close to the lower depinning threshold $\hat{F}_{c^-}$. One observes characteristic saw-tooth shapes, see Fig. 15, bottom left.

To my knowledge, this system is the first where all three KPZ universality classes have been observed in a single experiments.

10.4 Reaction-diffusion equation

Together with Sanjay Puri [KW28], we have developed an efficient perturbative expansion to obtain solutions for the initial-value problem of the Fisher equation, and the time-dependent Ginzburg-Landau equation. The starting point of our expansion is an improvement of the corresponding singular-perturbation solution, for which we give a 5-line derivation (as compared to the original derivation, resumming series of diagrams and making 3 steps of approximation.) This approach transforms the solution of non-linear reaction-diffusion equations into the solution of a hierarchy of linear equations. Our numerical results show that this hierarchy rapidly converges to the exact solution. (I currently use these techniques for different avalanche observables for which a closed solution cannot be given.)

10.5 Supersymmetric non-linear sigma model

In my diploma thesis, I showed that for supersymmetric non-linear sigma-models there are no corrections at 3-loop order to the dimension of relevant operators [KW1, KW2], contrary to bosonic non-linear sigma-models.

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12 Publications by Kay J. Wiese

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Links to pdf-files of the articles are provided as pdf. Hyperlinks to the journals are given when available.

[KW1] K.J. Wiese, *Die Zeta-Funktion für ein supersymmetrisches nichtlineares Sigma-Modell*, Diplomarbeit, Uni Heidelberg, 1992.

Die Renormierungsgruppen ζ-Funktion für supersymmetrischeh nichtlineare Sigma-Modelle wird berechnet bis zur 3-loop Ordnung. For eine große Klasse von Modellen, eingeschlossen das N-Vector Modell und Matrix-Modelle, kann das Resultat wie folgt zusammengefasst werden: Wenn die ζ-Funktion für das bosonische Modell $\zeta_{\rm Bos}(t_R) = a t_R + O(t_R^2)$ ist, dann besitzt die ζ-Funktion für das supersymmetrische Modell die Form $\zeta_{\rm Susy} = a t_R + O(t_R^4)$. Dieses Resultat ist gültig für beliebige harmonische Polynome des Feldes (sog. "Weiche Operatoren"). pdf

[KW2] K.J. Wiese, *Anomalous dimensions of soft operators in supersymmetric nonlinear sigma-models*, Mod. Phys. Lett. A **8** (1993) 3845–3852.

The renormalization ζ-function for supersymmetric nonlinear sigma-models is calculated up to three-loop order. For a wide class of models, which includes the N-vector model and matrix models, the result can be summarized as follows: If the ζ-function for the bosonic model is $\zeta_{\rm Bos}(t_R) = at_R + O(t_R^2)$, then the ζ-function for the supersymmetric model takes the form $\zeta_{\rm SUSY}(t_R) = at_R + O(t_R^4)$. This is the case for arbitrary harmonic polynomials of the field variables (so called "soft operators").

[KW3] K.J. Wiese and F. David, *Self-avoiding tethered membranes at the tricritical point*, Nucl. Phys. B **450** (1995) 495–557, cond-mat/9503126.

The scaling properties of self-avoiding tethered membranes at the tricritical point (theta-point) are studied by perturbative renormalization group methods. To treat the 3-body repulsive interaction (known to be relevant for polymers), new analytical and numerical tools are developped and applied to 1-loop calculations. These technics are a prerequisite to higher order calculations for self-avoiding membranes. The cross-over between the 3-body interaction and the modified 2-body interaction, attractive at long range, is studied through a new double epsilon-expansion. It is shown that the latter interaction is relevant for 2-dimensional membranes at the theta-point.

[KW4] K.J. Wiese, *Classification of perturbations for membranes with bending rigidity*, Phys. Lett. **B 387** (1996) 57–63, cond-mat/9607192.

A complete classification of the renormalization-group flow is given for impurity-like marginal operators of membranes whose elastic stress scales like $(\Delta r)^2$ around the external critical dimension $d_c = 2$. These operators are classified by characteristic functions on $\mathbb{R}^2 \times \mathbb{R}^2$.

[KW5] F. David and K.J. Wiese, *Scaling of self-avoiding tethered membranes: 2-loop renormalization group results*, Phys. Rev. Lett. **76** (1996) 4564, cond-mat/9602125.

The scaling properties of selfavoiding polymerized membranes are studied using renormalization group methods. The scaling exponent ν is calculated for the first time at two loop order. ν is found to agree with the Gaussian variational estimate for large space dimension d and to be close to the Flory estimate for $d = 3$.

[KW6] K.J. Wiese and F. David, *New renormalization group results for scaling of self-avoiding tethered membranes*, Nucl. Phys. **B 487** (1997) 529–632, cond-mat/9608022.

The scaling properties of self-avoiding polymerized 2-dimensional membranes are studied via renormalization group methods based on a multilocal operator product expansion. The renormalization group functions are calculated to second order. This yields the scaling exponent ν to order ϵ^2. Our extrapolations for ν agree with the Gaussian variational estimate for large space dimension d and are close to the Flory estimate for $d = 3$. The interplay between self-avoidance and rigidity at small d is briefly discussed.

[KW7] K.J. Wiese, *Membranes polymérisées auto-évitantes*, PhD thesis, Université d'Orsay, July 1996.

Cette thèse fait le point sur le comportement d'échelle des membranes auto-évitantes. Dans un premier temps, leurs caractéristiques et une comparaison avec les membranes fluides sont présentées. Des résultats connus concernant des expériences, des simulations numériques et des approches théoriques sont rappelés. Dans un deuxième temps, des nouveaux outils techniques développés, aussi bien analytiques que numériques sont présentés. Pour la première fois, l'exposant du comportement en échelle ν est calculé à l'ordre de deux boucles. Pour une grande dimension d'espace, ν est égal à sa valeur dans l'estimation de la méthode variationnelle gaussienne et pour $d = 3$ proche de la valeur dans l'approximation de Flory. Des membranes au point tri-critique (point Θ) sont regardées dans la suite. Au point Θ, deux opérateurs sont en concurrence : une interaction à trois points et une interaction à deux points, attractive à grande distance mais répulsive à courte distance. Leur "cross-over" est étudié à l'aide d'un nouveau double développement en ε. En conclusion : tandis que le point Θ des polymères est dominé par l'interaction à trois corps, c'est l'interaction à deux corps modifiée qui est pertinente pour décrire des membranes. Dans la suite la classification des opérateurs marginaux pour une membrane rigide est discutée. Le comportement en grands ordres en théorie de perturbation des membranes polymérisées auto-évitantes est discuté. Dans le dernier chapitre, une approche du groupe de renormalisation fonctionnel est développé.

[KW8] K.J. Wiese, *Dynamics of selfavoiding tethered membranes I, model A dynamics (Rouse model)*, Eur. Phys. J. **B 1** (1998) 269–272, cond-mat/9702020.
The dynamical scaling properties of selfavoiding polymerized membranes with internal dimension D are studied using model A dynamics. It is shown that the theory is renormalizable to all orders in perturbation theory and that the dynamical scaling exponent z is given by $z = 2 + D/\nu$. This result applies especially to membranes ($D = 2$) but also to polymers ($D = 1$), for which this scaling relation had been suggested but not proven.

[KW9] K.J. Wiese, *Dynamics of selfavoiding tethered membranes II, inclusion of hydrodynamic interactions (Zimm model)*, Eur. Phys. J. **B 1** (1998) 273–276, cond-mat/9702023.
The dynamical scaling properties of selfavoiding polymerized membranes with internal dimension D embedded into d dimensions are studied including hydrodynamical interactions. It is shown that the theory is renormalizable to all orders in perturbation theory and that the dynamical scaling exponent z is given by $z = d$. The crossover to the region, where the membrane is crumpled swollen but the hydrodynamic interaction irrelevant is discussed. The results apply as well to polymers ($D = 1$) as to membranes ($D = 2$).

[KW10] K.J. Wiese, *Critical discussion of the 2-loop calculations for the KPZ-equation*, Phys. Rev. **E 56** (1997) 5013–5017, cond-mat/9706009.
In this article, we perform a careful analysis of the renormalization procedure used in existing calculations to derive critical exponents for the KPZ-equation at 2-loop order. This analysis explains the discrepancies between the results of the different groups. The correct critical exponents in $d = 2 + \epsilon$ dimensions at the crossover between weak- and strong-coupling regime are $\zeta = O(\epsilon^3)$ and $z = 2 + O(\epsilon^3)$. No strong-coupling fixed point exists at 2-loop order.

[KW11] P. Le Doussal and K.J. Wiese, *Glassy trapping of elastic manifolds in nonpotential static random flows*, Phys. Rev. Lett. **80** (1998) 2362, cond-mat/9708112.
We study the dynamics of polymers and elastic manifolds in non potential static random flows. We find that barriers are generated from combined effects of elasticity, disorder and thermal fluctuations. This leads to glassy trapping even in pure barrier-free divergenceless flows $v \stackrel{f\to 0}{\sim} f^{\phi}$ ($\phi > 1$). The physics is described by a new RG fixed point at finite temperature. We compute the anomalous roughness $R \sim L^{\zeta}$ and dynamical $t \sim L^{z}$ exponents for directed and isotropic manifolds.

[KW12] K.J. Wiese, *On the perturbation expansion of the KPZ-equation*, J. Stat. Phys. **93** (1998) 143–154, cond-mat/9802068.
We present a simple argument to show that the beta-function of the d-dimensional KPZ-equation ($d \geq 2$) is to all orders in perturbation theory given by $\beta(g) = (d-2)g - 2(8\pi)^{-d/2}\Gamma(2-d/2)g^2$. Neither the dynamical exponent z nor the roughness-exponent ζ have any correction in any order of perturbation theory. This shows that standard perturbation theory cannot attain the strong-coupling regime and in addition breaks down at $d = 4$. We also calculate a class of correlation-functions exactly.

[KW13] K.J. Wiese and M. Kardar, *Generalizing the $O(N)$-field theory to N-colored manifolds of arbitrary internal dimension D*, Nucl. Phys. **B 528** (1998) 469–522, cond-mat/9803389.
We introduce a geometric generalization of the $O(N)$-field theory that describes N-colored membranes with arbitrary dimension D. As the $O(N)$-model reduces in the limit $N \to 0$ to self-avoiding polymers, the N-colored manifold model leads to self-avoiding tethered membranes. In the other limit, for inner dimension $D \to 1$, the manifold model reduces to the $O(N)$-field theory. We analyze the scaling properties of the model at criticality by a one-loop perturbative renormalization group analysis around an upper critical line. The freedom to optimize with respect to the expansion point on this line allows us to obtain the exponent ν of standard field theory to much better precision that the usual 1-loop calculations. Some other field theoretical techniques, such as the large N limit and Hartree approximation, can also be

applied to this model. By comparison of low and high temperature expansions, we arrive at a conjecture for the nature of droplets dominating the 3d-Ising model at criticality, which is satisfied by our numerical results. We can also construct an appropriate generalization that describes cubic anisotropy, by adding an interaction between manifolds of the same color. The two parameter space includes a variety of new phases and fixed points, some with Ising criticality, enabling us to extract a remarkably precise value of 0.6315 for the exponent ν in $d = 3$. A particular limit of the model with cubic anisotropy corresponds to the random bond Ising problem; unlike the field theory formulation, we find a fixed point describing this system at 1-loop order. pdf

[KW14] K.J. Wiese and M. Kardar, *A geometric generalization of field theory to manifolds of arbitrary dimension*, Eur. Phys. J. **B 7** (1998) 187–190, cond-mat/9803279.
We introduce a generalization of the $O(N)$ field theory to N-colored membranes of arbitrary inner dimension D. The $O(N)$ model is obtained for $D \to 1$, while $N \to 0$ leads to self-avoiding tethered membranes (as the $O(N)$ model reduces to self-avoiding polymers). The model is studied perturbatively by a 1-loop renormalization group analysis, and exactly as $N \to \infty$. Freedom to choose the expansion point D, leads to precise estimates of critical exponents of the $O(N)$ model. Insights gained from this generalization include a conjecture on the nature of droplets dominating the 3d-Ising model at criticality; and the fixed point governing the random bond Ising model. pdf

[KW15] F. David and K.J. Wiese, *Large orders for self-avoiding membranes*, Nucl. Phys. **B 535** (1998) 555–595, cond-mat/9807160.
We derive the large order behavior of the perturbative expansion for the continuous model of tethered self-avoiding membranes. It is controlled by a classical configuration for an effective potential in bulk space, which is the analog of the Lipatov instanton, solution of a highly non-local equation. The n-th order is shown to have factorial growth as $(-\text{cst})^n (n!)^{1-\epsilon/D}$, where D is the "internal" dimension of the membrane and ϵ the engineering dimension of the coupling constant for self-avoidance. The instanton is calculated within a variational approximation, which is shown to become exact in the limit of large dimension d of bulk space. This is the starting point of a systematic $1/d$ expansion. As a consequence, the epsilon-expansion of self-avoiding membranes has a factorial growth, like the epsilon-expansion of polymers and standard critical phenomena, suggesting Borel summability. Consequences for the applicability of the 2-loop calculations are examined. pdf

[KW16] S. Kehrein, C. Münkel and K.J. Wiese, *The finite one–dimensional wire problem*, physics/9808038 (1998).
We discuss an elementary problem in electrostatics: What does the charge distribution look like for a free charge on a strictly one-dimensional wire of finite length? To the best of our knowledge this question has so far not been discussed anywhere. One notices that a solution of this problem is not as simple as it might appear at first sight. pdf

[KW17] K.J. Wiese and P. Le Doussal, *Polymers and manifolds in static random flows: a RG study*, Nucl. Phys. **B 552** (1999) 529–598, cond-mat/9808330.
We study the dynamics of a polymer or a D-dimensional elastic manifold diffusing and convected in a non-potential static random flow (the "randomly driven polymer model"). We find that short-range (SR) disorder is relevant for $d < 4$ for directed polymers (each monomer sees a different flow) and for $d < 6$ for isotropic polymers (each monomer sees the same flow) and more generally for $d < d_c(D)$ in the case of a manifold. This leads to new large scale behavior, which we analyze using field theoretical methods. We show that all divergences can be absorbed in multilocal counter-terms which we compute to one loop order. We obtain the non trivial roughness ζ, dynamical z and transport exponents ϕ in a dimensional expansion. For directed polymers we find ζ about 0.63 ($d = 3$), ζ about 0.8 ($d = 2$) and for isotropic polymers ζ about 0.8 ($d = 3$). In all cases $z > 2$ and the velocity versus applied force characteristics is sublinear, i.e. at small forces $v(f) \sim f^{\phi}$ with $\phi > 1$. It indicates that this new state is glassy, with dynamically generated barriers leading to trapping, even by a divergenceless (transversal) flow. For random flows with long-range (LR) correlations, we find continuously varying exponents with the ratio g_L/g_T of potential to transversal disorder, and interesting crossover phenomena between LR and SR behavior. For isotropic polymers new effects (e.g. a sign change of $\zeta - \zeta_0$) result from the

competition between localization and stretching by the flow. In contrast to purely potential disorder, where the dynamics gets frozen, here the dynamical exponent z is not much larger than 2, making it easily accessible by simulations. The phenomenon of pinning by transversal disorder is further demonstrated using a two monomer "dumbbell" toy model.

[KW18] K.J. Wiese, *Polymerized membranes, a review*. Volume *19* of *Phase Transitions and Critical Phenomena*, Acadamic Press, London, 1999.
Membranes are of great technological and biological as well as theoretical interest. Two main classes of membranes can be distinguished: Fluid membranes and polymerized, tethered membranes. Here, we review progress in the theoretical understanding of polymerized membranes, i.e. membranes with a fixed internal connectivity. We start by collecting basic physical properties, clarifying the role of bending rigidity and disorder, theoretically and experimentally as well as numerically. We then give a thorough introduction into the theory of self-avoiding membranes, or more generally non-local field theories with δ-like interactions. A couple of tools is developed. Based on a proof of perturbative renormalizability for non-local field-theories, renormalization group calculations can be performed up to 2-loop order, which in 3 dimensions predict a crumpled phase with fractal dimension of about 2.4; this phase is however seemingly unstable towards the inclusion of bending rigidity. The tricritical behavior of membranes is discussed and shown to be quite different from that of polymers. Dynamical properties are studied in the same frame-work. Exact scaling relations, suggested but not demonstrated long time ago by De Gennes for polymers, are established. Along the same lines, disorder can be included leading to interesting applications. We also construct a generalization of the $O(N)$-model, which in the limit $N \to 0$ reduces to self-avoiding membranes in analogy with the $O(N)$-model, which in the limit $N \to 0$ reduces to self-avoiding polymers. Since perturbation theory is at the basis of the above approach, one has to ensure that the perturbation expansion is not divergent or at least Borel-summable. Using a suitable reformulation of the problem, we obtain the instanton governing the large-order behavior. This suggests that the perturbation expansion is indeed Borel-summable and the presented approach meaningful. Some technical details are relegated to the appendices. A final collection of various topics may also serve as exercises.

[KW19] K.J. Wiese, *The passive polymer problem*, J. Stat. Phys. **101** (2000) 843–891, chao-dyn/9911005.
In this article, we introduce a generalization of the diffusive motion of point-particles in a turbulent convective flow with given correlations to a polymer or membrane. In analogy to the passive scalar problem we call this the passive polymer or membrane problem. We shall focus on the expansion about the marginal limit of velocity-velocity correlations which are uncorrelated in time and grow with the distance x as $|x|^{\epsilon}$, and ϵ small. This relation gets modified for polymers and membranes (the marginal advecting flow has correlations which are shorter ranged.) The construction is done in three steps: First, we reconsider the treatment of the passive scalar problem using the most convenient treatment via field theory and renormalization group. We explicitly show why IR-divergences and thus the system-size appear in physical observables, which is rather unusual in the context of ordinary field-theories, like the ϕ^4-model. We also discuss, why the renormalization group can nevertheless be used to sum these divergences and leads to anomalous scaling of $2n$-point correlation functions as e.g. $S^{2n}(x) := \left\langle [\Theta(x,t) - \Theta(0,t)]^{2n} \right\rangle$. In a second step, we reformulate the problem in terms of a Langevin equation. This is interesting in its own, since it allows for a distinction between single-particle and multi-particle contributions, which is not obvious in the Focker-Planck treatment. It also gives an efficient algorithm to determine S^{2n} numerically, by measuring the diffusion of particles in a random velocity field. In a third and final step, we generalize the Langevin treatment of a particle to polymers and membranes, or more generally to an elastic object of inner dimension D with $0 \leq D \leq 2$. These objects can intersect each other. We also analyze what happens when self-intersections are no longer allowed.

[KW20] P. Chauve, P. Le Doussal and K.J. Wiese, *Renormalization of pinned elastic systems: How does it work beyond one loop?*, Phys. Rev. Lett. **86** (2001) 1785–1788, cond-mat/0006056.
We examine, beyond one loop, the candidate field theories for equilibrium and driven dynamics of elastic systems pinned by disorder. To escape dimensional reduction, the action is non-analytic at T=0. We show two-loop renormalizability of (quasi-static) depinning and compute roughness ζ and dynamical exponents z for periodic systems and interfaces. We prove that random field disorder attracts shorter range disorder and find a $\mathcal{O}(\epsilon^2)$ violation of the conjecture $\zeta = \epsilon/3$, in

agreement with simulations. We then discuss the issues arising in the statics. Depinning and static β-functions differ at two loop and contain novel anomalous terms due to the non-analytic nature of the theory. pdf

[KW21] K.J. Wiese, *Polymerisierte Membrane*. Jahrbuch der Göttinger Akademie der Wissenschaften, Vandenhoeck & Ruprecht in Göttingen, Göttingen, 2000. pdf

[KW22] K.J. Wiese, *Principles of non-local field theories and their application to polymerized membranes*, in W. Janke, A. Pelster, H.-J. Schmidt and M. Bachmann, editors, *Fluctuating paths and fields*, World Scientific, Singapore, 2001. cond-mat/0106361.
In these lecture notes, we give an overview about non-local field-theories and their application to polymerized membranes, i.e. membranes with a fixed internal connectivity. The main technical tool is the multi-local operator product expansion (MOPE), generalizing ideas from local field theories to the multi-local situation. Dedicated to Hagen Kleinert at the occasion of his 60th birthday. pdf

[KW23] P. Le Doussal and K.J. Wiese, *Functional renormalization group at large N for random manifolds*, Phys. Rev. Lett. **89** (2002) 125702, cond-mat/0109204.
We introduce a method, based on an exact calculation of the effective action at large N, which aims to bridge the gap between mean field theory and renormalization in complex systems. We apply it to a d-dimensional manifold in a random potential for large embedding space dimension N. This yields a functional renormalization group equation valid for any d, which contains both the $O(\epsilon)$ results of Balents-Fisher and some of the non-trivial results of the Mezard-Parisi solution thus shedding light on both. Corrections are computed at order $O(1/N)$. Applications to the problems of KPZ, random field and mode coupling in glasses are mentioned. pdf

[KW24] H.A. Pinnow and K.J. Wiese, *Interacting crumpled manifolds*, J. Phys. **A 35** (2002) 1195–1229, cond-mat/0110011.
In this article we study the effect of a δ-interaction on a polymerized membrane of arbitrary internal dimension D. Depending on the dimensionality of membrane and embedding space, different physical scenarios are observed. We emphasize on the difference of polymers from membranes. For the latter, non-trivial contributions appear at the 2-loop level. We also exploit a “massive scheme” inspired by calculations in fixed dimensions for scalar field theories. Despite the fact that these calculations are only amenable numerically, we found that in the limit of $D \to 2$ each diagram can be evaluated *analytically*. This property extends in fact to any order in perturbation theory, allowing for a summation of all orders. This is a novel and quite surprising result. Finally, an attempt to go beyond $D = 2$ is presented. Applications to the case of self-avoiding membranes are mentioned. pdf

[KW25] P. Le Doussal, K.J. Wiese and P. Chauve, *2-loop functional renormalization group analysis of the depinning transition*, Phys. Rev. B **66** (2002) 174201, cond-mat/0205108.
We construct the field theory which describes the universal properties of the quasi-static isotropic depinning transition for interfaces and elastic periodic systems at zero temperature, taking properly into account the non-analytic form of the dynamical action. This cures the inability of the 1-loop flow-equations to distinguish between statics and quasi-static depinning, and thus to account for the irreversibility of the latter. We prove two-loop renormalizability, obtain the 2-loop β-function and show the generation of “irreversible” anomalous terms, originating from the non-analytic nature of the theory, which cause the statics and driven dynamics to differ at 2-loop order. We obtain the roughness exponent ζ and dynamical exponent z to order ϵ^2. This allows to test several previous conjectures made on the basis of the 1-loop result. First it demonstrates that random-field disorder does indeed attract all disorder of shorter range. It also shows that the conjecture $\zeta = \epsilon/3$ is incorrect, and allows to compute the violations, as $\zeta = \frac{\epsilon}{3}(1 + 0.14331\epsilon)$, $\epsilon = 4 - d$. This solves a longstanding discrepancy with simulations. For long-range elasticity it yields $\zeta = \frac{\epsilon}{3}(1 + 0.39735\epsilon)$, $\epsilon = 2 - d$ (vs. the standard prediction $\zeta = 1/3$ for $d = 1$), in reasonable agreement with the most recent simulations. The high value of $\zeta \approx 0.5$ found in experiments both on the contact line depinning of liquid Helium and on slow crack fronts is discussed. pdf

[KW26] K.J. Wiese, *Disordered systems and the functional renormalization group: A pedagogical introduction*, Acta Physica Slovaca **52** (2002) 341, cond-mat/0205116.
In this article, we review basic facts about disordered systems, especially the existence of many metastable states and and the resulting failure of dimensional reduction. Besides techniques based on the Gaussian variational method and replica-symmetry breaking (RSB), the functional renormalization group (FRG) is the only general method capable of attacking strongly disordered systems. We explain the basic ideas of the latter method and why it is difficult to implement. We finally review current progress for elastic manifolds in disorder.

[KW27] P. Le Doussal and K.J. Wiese, *Functional renormalization group for anisotropic depinning and relation to branching processes*, Phys. Rev. E **67** (2003) 016121, cond-mat/0208204.
Using the functional renormalization group, we study the depinning of elastic objects in presence of anisotropy. We explicitly demonstrate how the KPZ-term is always generated, even in the limit of vanishing velocity, except where excluded by symmetry. This mechanism has two steps: First a non-analytic disorder-distribution is generated under renormalization beyond the Larkin-length. This non-analyticity then generates the KPZ-term. We compute the β-function to one loop taking properly into account the non-analyticity. This gives rise to additional terms, missed in earlier studies. A crucial question is whether the non-renormalization of the KPZ-coupling found at 1-loop order extends beyond the leading one. Using a Cole-Hopf-transformed theory we argue that it is indeed uncorrected to all orders. The resulting flow-equations describe a variety of physical situations: We study manifolds in periodic disorder, relevant for charge density waves, as well as in non-periodic disorder. Further the elasticity of the manifold can either be short-range (SR) or long-range (LR). A careful analysis of the flow yields several non-trivial fixed points. All these fixed points are transient since they possess one unstable direction towards a runaway flow, which leaves open the question of the upper critical dimension. The runaway flow is dominated by a Landau-ghost-mode. For LR elasticity, relevant for contact line depinning, we show that there are two phases depending on the strength of the KPZ coupling. For SR elasticity, using the Cole-Hopf transformed theory we identify a non-trivial 3-dimensional subspace which is invariant to all orders and contains all above fixed points as well as the Landau-mode. It belongs to a class of theories which describe branching and reaction-diffusion processes, of which some have been mapped onto directed percolation.

[KW28] SS. Puri and K.J. Wiese, *Perturbative linearization of reaction-diffusion equations*, J. Phys. A **36** (2003) 2043–2054, cond-mat/0209524.
We develop perturbative expansions to obtain solutions for the initial-value problems of two important reaction-diffusion systems, viz., the Fisher equation and the time-dependent Ginzburg-Landau (TDGL) equation. The starting point of our expansion is the corresponding singular-perturbation solution. This approach transforms the solution of nonlinear reaction-diffusion equations into the solution of a hierarchy of linear equations. Our numerical results demonstrate that this hierarchy rapidly converges to the exact solution.

[KW29] H.A. Pinnow and K.J. Wiese, *Interacting crumpled manifolds: Exact results to all orders of perturbation theory*, Europhys. Lett. **64** (2003) 371–377, cond-mat/0210007.
In this letter, we report progress on the field theory of polymerized tethered membranes. For the toy-model of a manifold repelled by a single point, we are able to sum the perturbation expansion in the strength g_0 of the interaction exactly in the limit of internal dimension $D \to 2$. This exact solution is the starting point for an expansion in $2-D$, which aims at connecting to the well studied case of polymers ($D=1$). We here give results to order $(2-D)^4$, where again all orders in g_0 are resummed. This is a first step towards a more complete solution of the self-avoiding manifold problem, which might also prove valuable for polymers.

[KW30] A.W.W. Ludwig and K.J. Wiese, *The 4-loop β-function in the 2D non-abelian Thirring model, and comparison with its conjectured exact form*, Nucl. Phys. B **661** (2003) 577–607, cond-mat/0211531.

[KW31] A. Rosso, W. Krauth, P. Le Doussal, J. Vannimenus and K.J. Wiese, *Universal interface width distributions at the depinning threshold*, Phys. Rev. E **68** (2003) 036128, cond-mat/0301464.

We compute the probability distribution of the interface width at the depinning threshold, using recent powerful algorithms. It confirms the universality classes found previously. In all cases, the distribution is surprisingly well approximated by a generalized Gaussian theory of independant modes which decay with a characteristic propagator $G(q) = 1/q^{d+2\zeta}$; ζ, the roughness exponent, is computed independently. A functional renormalization analysis explains this result and allows to compute the small deviations, i.e. a universal kurtosis ratio, in agreement with numerics. We stress the importance of the Gaussian theory to interpret numerical data and experiments. pdf

[KW32] P. Le Doussal and K.J. Wiese, *Higher correlations, universal distributions and finite size scaling in the field theory of depinning*, Phys. Rev. E **68** (2003) 046118, cond-mat/0301465.

Recently we constructed a renormalizable field theory up to two loops for the quasi-static depinning of elastic manifolds in a disordered environment. Here we explore further properties of the theory. We show how higher correlation functions of the displacement field can be computed. Drastic simplifications occur, unveiling much simpler diagrammatic rules than anticipated. This is applied to the universal scaled width-distribution. The expansion in $d = 4 - \epsilon$ predicts that the scaled distribution coincides to the lowest orders with the one for a Gaussian theory with propagator $G(q) = 1/q^{d+2\zeta}$, ζ being the roughness exponent. The deviations from this Gaussian result are small and involve higher correlation functions, which are computed here for different boundary conditions. Other universal quantities are defined and evaluated: We perform a general analysis of the stability of the fixed point. We find that the correction-to-scaling exponent is $\omega = -\epsilon$ and not $-\epsilon/3$ as used in the analysis of some simulations. A more detailed study of the upper critical dimension is given, where the roughness of interfaces grows as a power of a logarithm instead of a pure power. pdf

[KW33] K.J. Wiese, *The functional renormalization group treatment of disordered systems: a review*, Ann. Henri Poincaré **4** (2003) 473–496, cond-mat/0302322.

We review current progress in the functional renormalization group treatment of disordered systems. After an elementary introduction into the phenomenology, we show why in the context of disordered systems a functional renormalization group treatment is necessary, contrary to pure systems, where renormalization of a single coupling constant is sufficient. This leads to a disorder distribution, which after a finite renomalization becomes non-analytic, thus overcoming the predictions of the seemingly exact dimensional reduction. We discuss, how a renormalizable field theory can be constructed, even beyond 1-loop order. We then discuss an elastic manifold imbedded in N dimensions, and give the exact solution for $N \to \infty$. This is compared to predictions of the Gaussian replica variational ansatz, using replica symmetry breaking. We finally discuss depinning, both isotropic and anisotropic, and the scaling function for the width distribution of an interface. pdf

[KW34] S. Peca, L. Balents and K.J. Wiese, *Fabry-Perot interference and spin filtering in carbon nanotubes*, Phys. Rev. B **68** (2003) 205423, cond-mat/0304496.

We study the two-terminal transport properties of a metallic single-walled carbon nanotube with good contacts to electrodes, which have recently been shown [W. Liang et al, Nature 441, 665-669 (2001)] to conduct ballistically with weak backscattering occurring mainly at the two contacts. The measured conductance, as a function of bias and gate voltages, shows an oscillating pattern of quantum interference. We show how such patterns can be understood and calculated, taking into account Luttinger liquid effects resulting from strong Coulomb interactions in the nanotube. We treat back-scattering in the contacts perturbatively and use the Keldysh formalism to treat non-equilibrium effects due to the non-zero bias voltage. Going beyond current experiments, we include the effects of possible ferromagnetic polarization of the leads to describe spin transport in carbon nanotubes. We thereby describe both incoherent spin injection and coherent resonant spin transport between the two leads. Spin currents can be produced in both ways, but only the latter allow this spin current to be controlled using an external gate. In all cases, the spin currents, charge currents, and magnetization of the nanotube exhibit components varying quasiperiodically with bias voltage, approximately as a superposition of periodic interference oscillations of spin- and charge-carrying "quasiparticles" in the nanotube, each with its own period. The amplitude of the higher-period signal is largest in single-mode quantum wires, and is somewhat suppressed in metallic nanotubes due to their sub-band degeneracy. pdf

[KW35] P. Le Doussal, K.J. Wiese and P. Chauve, *Functional renormalization group and the field theory of disordered elastic systems*, Phys. Rev. E **69** (2004) 026112, cond-mat/0304614.

We study elastic systems such as interfaces or lattices, pinned by quenched disorder. To escape triviality as a result of "dimensional reduction", we use the functional renormalization group. Difficulties arise in the calculation of the renormalization group functions beyond 1-loop order. Even worse, observables such as the 2-point correlation function exhibit the same problem already at 1-loop order. These difficulties are due to the non-analyticity of the renormalized disorder correlator at zero temperature, which is inherent to the physics beyond the Larkin length, characterized by many metastable states. As a result, 2-loop diagrams, which involve derivatives of the disorder correlator at the non-analytic point, are naively "ambiguous". We examine several routes out of this dilemma, which lead to a unique renormalizable field-theory at 2-loop order. It is also the only theory consistent with the potentiality of the problem. The β-function differs from previous work and the one at depinning by novel "anomalous terms". For interfaces and random bond disorder we find a roughness exponent $\zeta = 0.20829804\epsilon + 0.006858\epsilon^2$, $\epsilon = 4 - d$. For random field disorder we find $\zeta = \epsilon/3$ and compute universal amplitudes to order $O(\epsilon^2)$. For periodic systems we evaluate the universal amplitude of the 2-point function. We also clarify the dependence of universal amplitudes on the boundary conditions at large scale. All predictions are in good agreement with numerical and exact results, and an improvement over one loop. Finally we calculate higher correlation functions, which turn out to be equivalent to those at depinning to leading order in ϵ. pdf

[KW36] P. Le Doussal and K.J. Wiese, *Functional renormalization group at large N for disordered elastic systems, and relation to replica symmetry breaking*, Phys. Rev. B **68** (2003) 174202, cond-mat/0305634.

We study the replica field theory which describes the pinning of elastic manifolds of arbitrary internal dimension d in a random potential, with the aim of bridging the gap between mean field and renormalization theory. The full effective action is computed exactly in the limit of large embedding space dimension N. The second cumulant of the renormalized disorder obeys a closed self-consistent equation. It is used to derive a Functional Renormalization Group (FRG) equation valid in any dimension d, which correctly matches the Balents Fisher result to first order in $\epsilon = 4 - d$. We analyze in detail the solutions of the large-N FRG for both long-range and short-range disorder, at zero and finite temperature. We find consistent agreement with the results of Mezard Parisi (MP) from the Gaussian variational method (GVM) in the case where full replica symmetry breaking (RSB) holds there. We prove that the cusplike non-analyticity in the large N FRG appears at a finite scale, corresponding to the instability of the replica symmetric solution of MP. We show that the FRG exactly reproduces, for any disorder correlator and with no need to invoke Parisi's spontaneous RSB, the non-trivial result of the GVM for small overlap. A formula is found yielding the complete RSB solution for all overlaps. Since our saddle-point equations for the effective action contain both the MP equations and the FRG, it can be used to describe the crossover from FRG to RSB. A qualitative analysis of this crossover is given, as well as a comparison with previous attempts to relate FRG to GVM. Finally, we discuss applications to other problems and new perspectives. pdf

[KW37] H.A. Pinnow and K.J. Wiese, *Scaling behavior of tethered crumpled manifolds with inner dimension close to d=2: Resumming the perturbation theory*, Nucl. Phys. B **711** (2005) 530–564, cond-mat/0403734.

The field theory of self-avoiding tethered membranes still poses major challenges. In this article, we report progress on the toy-model of a manifold repelled by a single point. Our approach allows to sum the perturbation expansion in the strength g_0 of the interaction exactly in the limit of internal dimension $D \to 2$, yielding an analytic solution for the strong-coupling limit. This analytic solution is the starting point for an expansion in $2 - D$, which aims at connecting to the well studied case of polymers ($D = 1$). We give results to fourth order in $2 - D$, where the dependence on g_0 is again summed exactly. As an application, we discuss plaquette density functions, and propose a Monte-Carlo experiment to test our results. These methods should also allow to shed light on the more complex problem of self-avoiding manifolds. pdf

[KW38] P. Le Doussal and K.J. Wiese, *Derivation of the functional renormalization group β-function at order $1/N$ for manifolds pinned by disorder*, Nucl. Phys. B **701** (2004) 409–480, cond-mat/0406297.

In an earlier publication, we have introduced a method to obtain, at large N, the effective action for d-dimensional manifolds in a N-dimensional disordered environment. This allowed to obtain the Functional Renormalization Group (FRG) equation for $N = \infty$ and was shown to reproduce, with no need for ultrametric replica symmetry breaking, the predictions of the Mézard-Parisi solution. Here we compute the corrections at order $1/N$. We introduce two novel complementary methods, a diagrammatic and an algebraic one, to perform the complicated resummation of an infinite number of loops, and derive the β-function of the theory to order $1/N$. We present both the effective action and the corresponding functional renormalization group equations. The aim is to explain the conceptual basis and give a detailed account of the novel aspects of such calculations. The analysis of the FRG flow, comparison with other studies, and applications, e.g. to the strong-coupling phase of the Kardar-Parisi-Zhang equation are examined in a subsequent publication. pdf

[KW39] F. David and K.J. Wiese, *Instanton calculus for the self-avoiding manifold model*, J. Stat. Phys. **120** (2005) 875–1035, cond-mat/0409765.
We compute the normalisation factor for the large order asymptotics of perturbation theory for the self-avoiding manifold (SAM) model describing flexible tethered (D-dimensional) membranes in d-dimensional space, and the ϵ-expansion for this problem. For that purpose, we develop the methods inspired from instanton calculus, that we introduced in a previous publication (Nucl. Phys. B 534 (1998) 555), and we compute the functional determinant of the fluctuations around the instanton configuration. This determinant has UV divergences and we show that the renormalized action used to make perturbation theory finite also renders the contribution of the instanton UV-finite. To compute this determinant, we develop a systematic large-d expansion. For the renormalized theory, we point out problems in the interplay between the limits $\epsilon \to 0$ and $d \to \infty$, as well as IR divergences when $\epsilon = 0$. We show that many cancellations between IR divergences occur, and argue that the remaining IR-singular term is associated to amenable non-analytic contributions in the large-d limit when $\epsilon = 0$. The consistency with the standard instanton-calculus results for the self-avoiding walk is checked for $D = 1$. pdf

[KW40] P. Le Doussal, K.J. Wiese, E. Raphael and Ramin Golestanian, *Can non-linear elasticity explain contact-line roughness at depinning?*, Phys. Rev. Lett. **96** (2006) 015702, cond-mat/0411652.
We examine whether cubic non-linearities, allowed by symmetry in the elastic energy of a contact line, may result in a different universality class at depinning. Standard linear elasticity predicts a roughness exponent $\zeta = 1/3$ (one loop), $\zeta = 0.388 \pm 0.002$ (numerics) while experiments give $\zeta \approx 0.5$. Within functional RG we find that a non-local KPZ-type term is generated at depinning and grows under coarse graining. A fixed point with $\zeta \approx 0.45$ (one loop) is identified, showing that large enough cubic terms increase the roughness. This fixed point is unstable, revealing a rough strong-coupling phase. Experimental study of contact angles θ near $\pi/2$, where cubic terms in the energy vanish, is suggested. pdf

[KW41] K.J. Wiese, *Supersymmetry breaking in disordered systems and relation to functional renormalization and replica-symmetry breaking*, J. Phys.: Condens. Matter. **17** (2005) S1889–S1898, cond-mat/0411656.
In this article, we study an elastic manifold in quenched disorder in the limit of zero temperature. Naively it is equivalent to a free theory with elasticity in Fourier-space proportional to k^4 instead of k^2, i.e. a model without disorder in two space-dimensions less. This phenomenon, called dimensional reduction, is most elegantly obtained using supersymmetry. However, scaling arguments suggest, and functional renormalization shows that dimensional reduction breaks down beyond the Larkin length. Thus one equivalently expects a break-down of supersymmetry. Using methods of functional renormalization, we show how supersymmetry is broken. We also discuss the relation to replica-symmetry breaking, and how our formulation can be put into work to lift apparent ambiguities in standard functional renormalization group calculations. Dedicated to Lothar Schäfer at the occasion of his 60th birthday. pdf

[KW42] K.J. Wiese, *Why one needs a functional renormalization group to survive in a disordered world*, Pramana **64** (2005) 817–827, cond-mat/0511529.
In these proceedings, we discuss why functional renormalization is an essential tool to treat strongly disordered systems.

More specifically, we treat elastic manifolds in a disordered environment. These are governed by a disorder distribution, which after a finite renormalization becomes non-analytic, thus overcoming the predictions of the seemingly exact dimensional reduction. We discuss how a renormalizable field theory can be constructed even beyond 2-loop order. We then consider an elastic manifold embedded in N dimensions, and give the exact solution for $N \to \infty$. This is compared to predictions of the Gaussian replica variational ansatz, using replica symmetry breaking. Finally, the effective action at order $1/N$ is reported. pdf

[KW43] P. Le Doussal and K.J. Wiese, *2-loop functional renormalization for elastic manifolds pinned by disorder in N dimensions*, Phys. Rev. E **72** (2005) 035101 (R), cond-mat/0501315.
We study elastic manifolds in a N-dimensional random potential using functional RG. We extend to $N > 1$ our previous construction of a field theory renormalizable to two loops. For isotropic disorder with $O(N)$ symmetry we obtain the fixed point and roughness exponent to next order in $\epsilon = 4 - d$, where d is the internal dimension of the manifold. Extrapolation to the directed polymer limit $d = 1$ allows some handle on the strong coupling phase of the equivalent N-dimensional KPZ growth equation, and eventually suggests an upper critical dimension $d_{\rm u} \approx 2.5$. pdf

[KW44] P. Le Doussal and K.J. Wiese, *Random field spin models beyond one loop: a mechanism for decreasing the lower critical dimension*, Phys. Rev. Lett. **96** (2006) 197202, cond-mat/0510344.
The functional RG for the random field and random anisotropy O(N) sigma-models is studied to two loop. The ferromagnetic/disordered (F/D) transition fixed point is found to next order in $d = 4 + \epsilon$ for $N > N_c$ ($N_c = 2.8347408$ for random field, $N_c = 9.44121$ for random anisotropy). For $N < N_c$ the lower critical dimension $d = d_{\rm lc}$ plunges below $d_{\rm lc} = 4$: we find two fixed points, one describing the quasi-ordered phase, the other is novel and describes the F/D transition. $d_{\rm lc}$ can be obtained in an $(N_c - N)$-expansion. The theory is also analyzed at large N and a glassy regime is found. pdf

[KW45] M. Lässig and K.J. Wiese, *The freezing of random RNA*, Phys. Rev. Lett. **96** (2006) 228101, arXiv:q-bio/0511032.
We study secondary structures of random RNA molecules by means of a renormalized field theory based on an expansion in the sequence disorder. We show that there is a continuous phase transition from a molten phase at higher temperatures to a low-temperature glass phase. The primary freezing occurs above the critical temperature, with local islands of stable folds forming within the molten phase. The size of these islands defines the correlation length of the transition. Our results include critical exponents at the transition and in the glass phase. pdf

[KW46] A.A. Middleton, P. Le Doussal and K.J. Wiese, *Measuring functional renormalization group fixed-point functions for pinned manifolds*, Phys. Rev. Lett. **98** (2007) 155701, cond-mat/0606160.
Exact numerical minimization of interface energies is used to test the functional renormalization group (FRG) analysis for interfaces pinned by quenched disorder. The fixed-point function $R(u)$ (the correlator of the coarse-grained disorder) is computed. In dimensions $D = d + 1$, a linear cusp in $R''(u)$ is confirmed for random bond ($d = 1, 2, 3$), random field ($d = 0, 2, 3$), and periodic ($d = 2, 3$) disorders. The functional shocks that lead to this cusp are seen. Small, but significant, deviations from 1-loop FRG results are compared to 2-loop corrections. The cross-correlation for two copies of disorder is compared with a recent FRG study of chaos. pdf

[KW47] C. Bachas, P. Le Doussal and K.J. Wiese, *Wetting and minimal surfaces*, Phys. Rev. E **75** (2007) 031601, hep-th/0606247.
We study minimal surfaces which arise in wetting and capillarity phenomena. Using conformal coordinates, we reduce the problem to a set of coupled boundary equations for the contact line of the fluid surface, and then derive simple diagrammatic rules to calculate the non-linear corrections to the Joanny-de Gennes energy. We argue that perturbation theory is quasi-local, i.e. that all geometric length scales of the fluid container decouple from the short-wavelength deformations of the contact line. This is illustrated by a calculation of the linearized interaction between contact lines on two opposite parallel walls. We present a simple algorithm to compute the minimal surface and its energy based on these

ideas. We also point out the intriguing singularities that arise in the Legendre transformation from the pure Dirichlet to the mixed Dirichlet-Neumann problem.

[KW48] A. Fedorenko, P. Le Doussal and K.J. Wiese, *Universal distribution of threshold forces at the depinning transition*, Phys. Rev. E **74** (2006) 041110, cond-mat/0607229.
We study the distribution of threshold forces at the depinning transition for an elastic system of finite size, driven by an external force in a disordered medium at zero temperature. Using the functional renormalization group (FRG) technique, we compute the distribution of pinning forces in the quasi-static limit. This distribution is universal up to two parameters, the average critical force, and its width. We discuss possible definitions for threshold forces in finite-size samples. We show how our results compare to the distribution of the latter computed recently within a numerical simulation of the so-called critical configuration.

[KW49] F. David and K.J. Wiese, *Systematic field theory of the RNA glass transition*, Phys. Rev. Lett. **98** (2007) 128102, q-bio.BM/0607044.
We prove that the Laessig-Wiese (LW) field theory for the freezing transition of the secondary structure of random RNA is renormalizable to all orders in perturbation theory. The proof relies on a formulation of the model in terms of random walks and on the use of the multilocal operator product expansion. Renormalizability allows us to work in the simpler scheme of open polymers, and to obtain the critical exponents at 2-loop order. It also allows to prove some exact exponent identities, conjectured in LW.

[KW50] A. Fedorenko, P. Le Doussal and K.J. Wiese, *Statics and dynamics of elastic manifolds in media with long-range correlated disorder*, Phys. Rev. E **74** (2006) 061109, cond-mat/0609234.
We study the statics and dynamics of an elastic manifold in a disordered medium with quenched defects correlated as $\sim r^{-a}$ for large separation r. We derive the functional renormalization group equations to one-loop order which allow to describe the universal properties of the system in equilibrium and at the depinning transition. Using a double $\varepsilon = 4-d$ and $\delta = 4-a$ expansion we compute the fixed points characterizing different universality classes and analyze their regions of stability. The long-range disorder-correlator remains analytic but generates short-range disorder whose correlator exhibits the usual cusp. The critical exponents and universal amplitudes are computed to first order in ε and δ at the fixed points. At depinning a velocity-versus-force exponent β larger than unity can occur. We discuss possible realizations using extended defects.

[KW51] P. Le Doussal and K.J. Wiese, *How to measure Functional RG fixed-point functions for dynamics and at depinning*, EPL **77** (2007) 66001, cond-mat/0610525.
We show how the renormalized force correlator $\Delta(u)$, the function computed in the functional RG (FRG) field theory, can be measured directly in numerics and experiments on the dynamics of elastic manifolds in presence of pinning disorder. For equilibrium dynamics we recover the relation obtained recently in the statics between $\Delta(u)$ and a physical observable. Its extension to depinning reveals interesting relations to stick-slip models of avalanches used in dry friction and earthquake dynamics. The particle limit ($d = 0$) is solved for illustration: $\Delta(u)$ exhibits a cusp and differs from the statics. We propose that the FRG functions be measured in wetting and magnetic interfaces experiments.

[KW52] A. Rosso, P. Le Doussal and K.J. Wiese, *Numerical calculation of the functional renormalization group fixed-point functions at the depinning transition*, Phys. Rev. B **75** (2007) 220201, cond-mat/0610821.
We compute numerically the sequence of successive pinned configurations of an elastic line pulled quasistatically by a spring in a random bond (RB) and random field (RF) potential. Measuring the fluctuations of the center of mass of the line allows one to obtain the functional renormalization group (FRG) functions at the depinning transition. Following this procedure we are able to directly test the main predictions of FRG calculations. In particular, the universal form of the second cumulant $\Delta(u)$ is found to have a linear cusp at the origin, to be identical for RBs and RFs, different from the statics and in good agreement with two-loop FRG calculations. The cusp is due to avalanches, which we visualize. Avalanches also produce a cusp in the third cumulant, whose universal form is obtained, as predicted by FRG functions.

[KW53] K.J. Wiese and P. Le Doussal, *Functional renormalization for disordered systems: Basic recipes and gourmet dishes*, Markov Processes Relat. Fields **13** (2007) 777–818, cond-mat/0611346.

We give a pedagogical introduction into the functional renormalization group treatment of disordered systems. After a review of its phenomenology, we show why in the context of disordered systems a functional renormalization group treatment is necessary, contrary to pure systems, where renormalization of a single coupling constant is sufficient. This leads to a disorder distribution, which after a finite renormalization becomes non-analytic, thus overcoming the predictions of the seemingly exact dimensional reduction. We discuss, how the non-analyticity can be measured in a simulation or experiment. We then construct a renormalizable field theory beyond leading order. We discuss an elastic manifold embedded in N dimensions, and give the exact solution for N to infinity. This is compared to predictions of the Gaussian replica variational ansatz, using replica symmetry breaking. We further consider random field magnets, and supersymmetry. We finally discuss depinning, both isotropic and anisotropic, and universal scaling function.

[KW54] P. Le Doussal and K.J. Wiese, *Stability of random-field and random-anisotropy fixed points at large N*, Phys. Rev. Lett. **98** (2007) 269704, cond-mat/0612310.

In this note, we clarify the stability of the large-N functional RG fixed points of the order/disorder transition in the random-field (RF) and random-anisotropy (RA) O(N) models. We carefully distinguish between infinite N, and large but finite N. For infinite N, the Schwarz-Soffer inequality does not give a useful bound, and all fixed points found in cond-mat/0510344 (Phys. Rev. Lett. 96, 197202 (2006)) correspond to physical disorder. For large but finite N (i.e. to first order in $1/N$) the non-analytic RF fixed point becomes unstable, and the disorder flows to an analytic fixed point characterized by dimensional reduction. However, for random anisotropy the fixed point remains non-analytic (i.e. exhibits a cusp) and is stable in the $1/N$ expansion, while the corresponding dimensional-reduction fixed point is unstable. In this case the Schwarz-Soffer inequality does not constrain the 2-point spin correlation. We compute the critical exponents of this new fixed point in a series in $1/N$ and to 2-loop order.

[KW55] F. David, C. Hagendorf and K.J. Wiese, *Random RNA under tension*, EPL **78** (2007) 68003, q-bio.BM/0701049.

The Lässig-Wiese (LW) field theory for the freezing transition of random RNA secondary structures is generalized to the situation of an external force. We find a second-order phase transition at a critical applied force $f = f_c$. For $f < f_c$ forces are irrelevant. For $f > f_c$, the extension $\mathcal{L}$ as a function of pulling force f scales as $\mathcal{L}(f) \sim (f - f_c)^{1/\gamma - 1}$. The exponent γ is calculated in an ϵ-expansion: At 1-loop order $\gamma = 1/2$, equivalent to the disorder-free case. At 2-loop order $\gamma = 0.6$. Using a locking argument, we speculate that this result extends to the strong-disorder phase.

[KW56] F. David, C. Hagendorf and K.J. Wiese, *A growth model for RNA secondary structures*, J. Stat. Mech. (2007) P04008, arXiv:0711.3421.

A hierarchical model for the growth of planar arch structures for RNA secondary structures is presented, and shown to be equivalent to a tree-growth model. Both models can be solved analytically, giving access to scaling functions for large molecules, and corrections to scaling, checked by numerical simulations of up to 6500 bases. The equivalence of both models should be helpful in understanding more general tree-growth processes.

[KW57] P. Le Doussal, M. Müller and K.J. Wiese, *Cusps and shocks in the renormalized potential of glassy random manifolds: How functional renormalization group and replica symmetry breaking fit together*, Phys. Rev. B **77** (2007) 064203, arXiv:0711.3929.

We compute the Functional Renormalization Group (FRG) disorder-correlator function $R(v)$ for d-dimensional elastic manifolds pinned by a random potential in the limit of infinite embedding space dimension N. It measures the equilibrium response of the manifold in a quadratic potential well as the center of the well is varied from 0 to v. We find two distinct scaling regimes: (i) a "single shock" regime, $v^2 \sim L^{-d}$ where L^d is the system volume and (ii) a "thermodynamic" regime, $v^2 \sim N$. In regime (i) all the equivalent replica symmetry breaking (RSB) saddle points within the Gaussian variational approximation contribute, while in regime (ii) the effect of RSB enters only through a single anomaly. When the RSB is continuous (e.g., for short-range disorder, in dimension $2 \leq d \leq 4$), we prove that regime (ii)

yields the large-N FRG function obtained previously. In that case, the disorder correlator exhibits a cusp in both regimes, though with different amplitudes and of different physical origin. When the RSB solution is 1-step and non-marginal (e.g., $d < 2$ for SR disorder), the correlator $R(v)$ in regime (ii) is considerably reduced, and exhibits no cusp. Solutions of the FRG flow corresponding to non-equilibrium states are discussed as well. In all cases the regime (i) exhibits a cusp non-analyticity at $T = 0$, whose form and thermal rounding at finite T is obtained exactly and interpreted in terms of shocks. The results are compared with previous work, and consequences for manifolds at finite N, as well as extensions to spin glasses and related models are discussed.

[KW58] K.J. Wiese and P. Le Doussal, *How to measure the effective action for disordered systems*, in Wolfhard Janke and Axel Pelster, editors, *Path Integrals - New Trends and Perspectives*, World Scientific, 2008, arXiv:0712.4286.

In contrast to standard critical phenomena, disordered systems need to be treated via the Functional Renormalization Group. The latter leads to a coarse grained disorder landscape, which after a finite renormalization becomes non-analytic, thus overcoming the predictions of the seemingly exact dimensional reduction. We review recent progress on how the nonanalytic effective action can be measured both in simulations and experiments, and confront theory with numerical work.

[KW59] P. Le Doussal, M.C. Marchetti and K.J. Wiese, *Depinning in a two-layer model of plastic flow*, Phys. Rev. B **78** (2008) 224201, arXiv:0801.0137.

We study a model of two layers, each consisting of a d-dimensional elastic object driven over a random substrate, and mutually interacting through a viscous coupling. For this model, the mean-field theory (i.e. a fully connected model) predicts a transition from elastic depinning to hysteretic plastic depinning as disorder or viscous coupling is increased. A functional RG analysis shows that any small inter-layer viscous coupling destablizes the standard (decoupled) elastic depinning FRG fixed point for $d \leq 4$, while for $d > 4$ most aspects of the mean-field theory are recovered. A one-loop study at non-zero velocity indicates, for $d < 4$, coexistence of a moving state and a pinned state below the elastic depinning threshold, with hysteretic plastic depinning for periodic and non-periodic driven layers. A 2-loop analysis of quasi-statics unveils the possibility of more subtle effects, including a new universality class for non-periodic objects. We also study the model in $d = 0$, i.e. two coupled particles, and show that hysteresis does not always exist as the periodic steady state with coupled layers can be dynamically unstable. It is also proved that stable pinned configurations remain dynamically stable in presence of a viscous coupling in any dimension d. Moreover, the layer model for periodic objects is stable to an infinitesimal commensurate density coupling. Our work shows that a careful study of attractors in phase space and their basin of attraction is necessary to obtain a firm conclusion for dimensions $d = 1, 2, 3$.

[KW60] P. Le Doussal, A.A. Middleton and K.J. Wiese, *Statistics of static avalanches in a random pinning landscape*, Phys. Rev. E **79** (2009) 050101 (R), arXiv:0803.1142.

We study the minimum-energy configuration of a d-dimensional elastic interface in a random potential tied to a harmonic spring. As a function of the spring position, the center of mass of the interface changes in discrete jumps, also called shocks or "static avalanches". We obtain analytically the distribution of avalanche sizes and its cumulants within an $\epsilon = 4 - d$ expansion from a tree and 1-loop resummation, using functional renormalization. This is compared with exact numerical minimizations of interface energies for random field disorder in $d = 2, 3$. Connections to the Burgers equation and to dynamic avalanches are discussed.

[KW61] A.A. Fedorenko, P. Le Doussal and K.J. Wiese, *Field theory conjecture for loop-erased random walks*, J. Stat. Phys. **133** (2008) 805–812, arXiv:0803.2357.

We give evidence that the functional renormalization group (FRG), developed to study disordered systems, may provide a field theoretic description for the loop-erased random walk (LERW), allowing to compute its fractal dimension in a systematic expansion in $\epsilon = 4 - d$. Up to two loop, the FRG agrees with rigorous bounds, correctly reproduces the leading logarithmic corrections at the upper critical dimension $d = 4$, and compares well with numerical studies. We obtain the universal subleading logarithmic correction in $d = 4$, which can be used as a further test of the conjecture.

[KW62] P. Le Doussal and K.J. Wiese, *Driven particle in a random landscape: disorder correlator, avalanche distribution and extreme value statistics of records*, Phys. Rev. E **79** (2009) 051105, arXiv:0808.3217.
We review how the renormalized force correlator $\Delta(u)$, the function computed in the functional RG field theory, can be measured directly in numerics and experiments on the *dynamics* of elastic manifolds in presence of pinning disorder. We show how this function can be computed analytically for a particle dragged through a 1-dimensional random-force landscape. The limit of small velocity allows to access the critical behavior at the depinning transition. For uncorrelated forces one finds three universality classes, corresponding to the three extreme value statistics, Gumbel, Weibull, and Fréchet. For each class we obtain analytically the universal function $\Delta(u)$, the corrections to the critical force, and the joint probability distribution of avalanche sizes s and waiting times w. We find $P(s) = P(w)$ for all three cases. All results are checked numerically. For a Brownian force landscape, known as the ABBM model, avalanche distributions and $\Delta(u)$ can be computed for any velocity. For 2-dimensional disorder, we perform large-scale numerical simulations to calculate the renormalized force correlator tensor $\Delta_{ij}(\vec{u})$, and to extract the anisotropic scaling exponents $\zeta_x > \zeta_y$. We also show how the Middleton theorem is violated. Our results are relevant for the record statistics of random sequences with linear trends, as encountered e.g. in some models of global warming. We give the joint distribution of the time s between two successive records and their difference in value w. pdf

[KW63] Jesper L. Jacobsen, Pierre Le Doussal, Marco Picco, Raoul Santachiara and Kay Jörg Wiese, *Critical interfaces in the random-bond Potts model*, Phys. Rev. Lett. **102** (2009) 070601, arXiv:0809.3985.
We study geometrical properties of interfaces in the random-temperature q-states Potts model as an example of a conformal eld theory weakly perturbed by quenched disorder. Using conformal perturbation theory in $q-2$ we compute the fractal dimension of Fortuin Kasteleyn domain walls. We also compute it numerically both via the Wolff cluster algorithm for $q = 3$ and via transfer-matrix evaluations. We also obtain numerical results for the fractal dimension of spin clusters interfaces for $q = 3$. These are found numerically consistent with the duality $\kappa_{\rm spin}\kappa_{\rm FK} = 16$ as expressed in putative SLE parameters. pdf

[KW64] P. Le Doussal and K.J. Wiese, *Fluctuation force exerted by a planar self-avoiding polymer*, EPL **86** (2009) 22001, arXiv:0812.1700.
Using results from Schramm Loewner evolution (SLE), we give the expression of the fluctuation-induced force exerted by a polymer on a small impenetrable disk, in various 2-dimensional domain geometries. We generalize to two polymers and examine whether the fluctuation force can trap the object into a stable equilibrium. We compute the force exerted on objects at the domain boundary, and the force mediated by the polymer between such objects. The results can straightforwardly be extended to any SLE interface, including Ising, percolation, and loop-erased random walks. Some are relevant for extremal value statistics. pdf

[KW65] P. Le Doussal and K.J. Wiese, *Size distributions of shocks and static avalanches from the functional renormalization group*, Phys. Rev. E **79** (2009) 051106, arXiv:0812.1893.
Interfaces pinned by quenched disorder are often used to model jerky self-organized critical motion. We study static avalanches, or shocks, defined here as jumps between distinct global minima upon changing an external field. We show how the full statistics of these jumps is encoded in the functional-renormalization-group fixed-point functions. This allows us to obtain the size distribution $P(S)$ of static avalanches in an expansion in the internal dimension d of the interface. Near and above $d = 4$ this yields the mean-field distribution $P(S) \sim S^{-3/2}\mathrm{e}^{-S/4S_m}$ where S_m is a large-scale cut-off, in some cases calculable. Resumming all 1-loop contributions, we find $P(S) \sim S^{-\tau}\exp\big(C(S/S_m)^{1/2} - \frac{B}{4}(S/S_m)^{\delta}\big)$ where B, C, δ, τ are obtained to first order in $\epsilon = 4 - d$. Our result is consistent to $O(\epsilon)$ with the relation $\tau = \tau_\zeta := 2 - \frac{2}{d+\zeta}$, where ζ is the static roughness exponent, often conjectured to hold at depinning. Our calculation applies to all static universality classes, including random-bond, random-field and random-periodic disorder. Extended to long-range elastic systems, it yields a different size distribution for the case of contact-line elasticity, with an exponent compatible with $\tau = 2 - \frac{1}{d+\zeta}$ to $O(\epsilon = 2 - d)$. We discuss consequences for avalanches at depinning and for sandpile models, relations to Burgers turbulence and the possibility that the relation $\tau = \tau_\zeta$ be violated to higher loop order. Finally, we show that the avalanche-size distribution on a hyper-plane of co-dimension one is in mean-field (valid close to and above

$d = 4$) given by $P(S) \sim K_{\frac{1}{3}}(S)/S$, where K is the Bessel-K function, thus $\tau_{\text{hyper plane}} = \frac{4}{3}$. pdf

[KW66] A. Rosso, P. Le Doussal and K.J. Wiese, *Avalanche-size distribution at the depinning transition: A numerical test of the theory*, Phys. Rev. B **80** (2009) 144204, arXiv:0904.1123.

We calculate numerically the sizes S of jumps (avalanches) between successively pinned configurations of an elastic line ($d = 1$) or interface ($d = 2$), pulled by a spring of (small) strength m^2 in a random-field landscape. We obtain strong evidence that the size distribution, away from the small-scale cutoff, takes the form $P(S) = \frac{\langle S \rangle}{S_m^2} p(S/S_m)$ where $S_m := \frac{\langle S^2 \rangle}{2\langle S \rangle} \sim m^{-d-\zeta}$ is the scale of avalanches, and ζ the roughness exponent at the depinning transition. Measurement of the scaling function $f(s) := s^\tau p(s)$ is compared with the predictions from a recent Functional RG (FRG) calculation, both at mean-field and one-loop level. The avalanche-size exponent τ is found in good agreement with the conjecture $\tau = 2 - 2/(d+\zeta)$, recently confirmed to one loop via the FRG. The function $f(s)$ exhibits a shoulder and a stretched exponential decay at large s, $\ln f(s) \sim -s^\delta$, with $\delta \approx 7/6$ in $d = 1$. The function $f(s)$, universal ratios of moments, and the generating function $\langle e^{\lambda s} \rangle$ are found in excellent agreement with the one-loop FRG predictions. The distribution of *local* avalanche sizes S_ϕ, i.e. of the jumps of a subspace of the manifold of dimension d_ϕ, is also computed and compared to our FRG predictions, and to the conjecture $\tau_\phi = 2 - 2/(d_\phi + \zeta)$. pdf

[KW67] P. Le Doussal, K.J. Wiese, S. Moulinet and E. Rolley, *Height fluctuations of a contact line: A direct measurement of the renormalized disorder correlator*, EPL **87** (2009) 56001, arXiv:0904.4156.

We have measured the center-of-mass fluctuations of the height of a contact line at depinning for two different systems: liquid hydrogen on a rough cesium substrate and isopropanol on a silicon wafer grafted with silanized patches. The contact line is subject to a confining quadratic well, provided by gravity. From the second cumulant of the height fluctuations, we measure the renormalized disorder correlator $\Delta(u)$, predicted by the Functional RG theory to attain a fixed point, as soon as the capillary length is large compared to the Larkin length set by the microscopic disorder. The experiments are consistent with the asymptotic form for $\Delta(u)$ predicted by Functional RG. The observed small deviations could be used as a probe of the underlying physical processes. The third moment, as well as avalanche-size distributions are measured and compared to predictions from Functional RG. pdf

[KW68] F. David and K.J. Wiese, *Field theory of the RNA freezing transition*, J. Stat. Mech. (2009) P10019, arXiv:0906.1472.

Folding of RNA is subject to a competition between entropy, relevant at high temperatures, and the random, or random looking, sequence, determining the low-temperature phase. It is known from numerical simulations that for random as well as biological sequences, high- and low-temperature phases are different, e.g. the exponent ρ describing the pairing probability between two bases is $\rho = \frac{3}{2}$ in the high-temperature phase, and $\rho \approx \frac{4}{3}$ in the low-temperature (glass) phase. Here, we present, for random sequences, a field theory of the phase transition separating high- and low-temperature phases. We establish the existence of the latter by showing that the underlying theory is renormalizable to all orders in perturbation theory. We test this result via an explicit 2-loop calculation, which yields $\rho \approx 1.36$ at the transition, as well as diverse other critical exponents, including the response to an applied external force (denaturation transition). pdf

[KW69] P. Le Doussal and K.J. Wiese, *Elasticity of a contact-line and avalanche-size distribution at depinning*, Phys. Rev. E **82** (2010) 011108, arXiv:0908.4001.

Motivated by recent experiments, we extend the Joanny-deGennes calculation of the elasticity of a contact line to an arbitrary contact angle and an arbitrary plate inclination in presence of gravity. This requires a diagonalization of the elastic modes around the non-linear equilibrium profile, which is carried out exactly. We then make detailed predictions for the avalanche-size distribution at quasi-static depinning: we study how the universal (i.e. short-scale independent) rescaled size distribution and the ratio of moments of local to global avalanches depend on the precise form of the elastic kernel. pdf

[KW70] P. Le Doussal, M. Müller and K.J. Wiese, *Avalanches in mean-field models and the Barkhausen noise in spin-glasses*, EPL **91** (2010) 57004, arXiv:1007.2069.

We obtain a general formula for the distribution of sizes of "static avalanches", or shocks, in generic mean-field glasses with replica-symmetry-breaking saddle points. For the Sherrington-Kirkpatrick (SK) spin-glass it yields the density $\rho(\Delta M)$ of the sizes of magnetization jumps ΔM along the equilibrium magnetization curve at zero temperature. Continuous replica-symmetry breaking allows for a power-law behavior $\rho(\Delta M) \sim 1/(\Delta M)^{\tau}$ with exponent $\tau = 1$ for SK, related to the criticality (marginal stability) of the spin-glass phase. All scales of the ultrametric phase space are implicated in jump events. Similar results are obtained for the sizes S of static jumps of pinned elastic systems, or of shocks in Burgers turbulence in large dimension. In all cases with a one-step solution, $\rho(S) \sim S e^{-AS^2}$. A simple interpretation relating droplets to shocks, and a scaling theory for the equilibrium analog of Barkhausen noise in finite-dimensional spin glasses are discussed. pdf

[KW71] K.J. Wiese, S.N. Majumdar and A. Rosso, *Perturbation theory for fractional Brownian motion in presence of absorbing boundaries*, Phys. Rev. E **83** (2011) 061141, arXiv:1011.4807.
Fractional Brownian motion is a Gaussian process $x(t)$ with zero mean and two-time correlations $\langle x(t_1)x(t_2)\rangle = D\left(t_1^{2H} + t_2^{2H} - |t_1 - t_2|^{2H}\right)$, where H, with $0 < H < 1$ is called the Hurst exponent. For $H = 1/2$, $x(t)$ is a Brownian motion, while for $H \neq 1/2$, $x(t)$ is a non-Markovian process. Here we study $x(t)$ in presence of an absorbing boundary at the origin and focus on the probability density $P_+(x,t)$ for the process to arrive at x at time t, starting near the origin at time 0, given that it has never crossed the origin. It has a scaling form $P_+(x,t) \sim t^{-H} R_+(x/t^H)$. Our objective is to compute the scaling function $R_+(y)$, which up to now was only known for the Markov case $H = 1/2$. We develop a systematic perturbation theory around this limit, setting $H = 1/2 + \epsilon$, to calculate the scaling function $R_+(y)$ to first order in ϵ. We find that $R_+(y)$ behaves as $R_+(y) \sim y^{\phi}$ as $y \to 0$ (near the absorbing boundary), while $R_+(y) \sim y^{\gamma} \exp(-y^2/2)$ as $y \to \infty$, with $\phi = 1 - 4\epsilon + O(\epsilon^2)$ and $\gamma = 1 - 2\epsilon + O(\epsilon^2)$. Our ϵ-expansion result confirms the scaling relation $\phi = (1 - H)/H$ proposed in PRL 102, 120602 (2009). We verify our findings via numerical simulations for $H = 2/3$. The tools developed here are versatile, powerful, and adaptable to different situations. pdf

[KW72] A. Dobrinevski, P. Le Doussal and K.J. Wiese, *Interference in disordered systems: A particle in a complex random landscape*, Phys. Rev. E **83** (2011) 061116, arXiv:1101.2411.
We consider a particle in one dimension submitted to amplitude and phase disorder. It can be mapped onto the complex Burgers equation, and provides a toy model for problems with interplay of interferences and disorder, such as the NSS model of hopping conductivity in disordered insulators and the Chalker-Coddington model for the (spin) quantum Hall effect. The model has three distinct phases: (I) a high-temperature or weak disorder phase, (II) a pinned phase for strong amplitude disorder, and (III) a diffusive phase for strong phase disorder, but weak amplitude disorder. We compute analytically the renormalized disorder correlator, equivalent to the Burgers velocity-velocity correlator at long times. In phase III, it assumes a universal form. For strong phase disorder, interference leads to a logarithmic singularity, related to zeroes of the partition sum, or poles of the complex Burgers velocity field. These results are valuable in the search for the adequate field theory for higher-dimensional systems. pdf

[KW73] P. Le Doussal and K.J. Wiese, *Distribution of velocities in an avalanche*, EPL **97** (2012) 46004, arXiv:1104.2629.
For a driven elastic object near depinning, we derive from first principles the distribution of instantaneous velocities in an avalanche. We prove that above the upper critical dimension, $d \geq d_{\rm uc}$, the n-times distribution of the center-of-mass velocity is equivalent to the prediction from the ABBM stochastic equation. mean field. Our method allows to compute space and time dependence from an instanton equation. We extend the calculation beyond mean field, to lowest order in $\epsilon = d_{\rm uc} - d$. pdf

[KW74] P. Le Doussal, A. Rosso and K.J. Wiese, *Shock statistics in higher-dimensional Burgers turbulence*, EPL **96** (2011) 14005, arXiv:1104.5048.
We conjecture the exact shock statistics in the inviscid decaying Burgers equation in $D > 1$ dimensions, with a special class of correlated initial velocities, which reduce to Brownian for $D = 1$. The prediction is based on a field-theory argument, and receives support from our numerical calculations. We find that, along any given direction, shocks sizes and locations are uncorrelated. pdf

[KW75] P. Le Doussal, M. Müller and K.J. Wiese, *Equilibrium avalanches in spin glasses*, Phys. Rev. B **85** (2012) 214402, arXiv:1110.2011.

We study the distribution of equilibrium avalanches (shocks) in Ising spin glasses which occur at zero temperature upon small changes in the magnetic field. For the infinite-range Sherrington-Kirkpatrick model we present a detailed derivation of the density $\rho(\Delta M)$ of the magnetization jumps ΔM. It is obtained by introducing a multi-component generalization of the Parisi-Duplantier equation, which allows us to compute all cumulants of the magnetization. We find that $\rho(\Delta M) \sim \Delta M^{-\tau}$ with an avalanche exponent $\tau = 1$ for the SK model, originating from the marginal stability (criticality) of the model. It holds for jumps of size $1 \ll \Delta M < N^{1/2}$ being provoked by changes of the external field by $\delta H = O(N^{-1/2})$ where N is the total number of spins. Our general formula also suggests that the density of overlap q between initial and final state in an avalanche is $\rho(q) \sim 1/(1-q)$. These results show interesting similarities with numerical simulations for the out-of-equilibrium dynamics of the SK model. For finite-range models, using droplet arguments, we obtain the prediction $\tau = (d_{\rm f} + \theta)/d_{\rm m}$ where $d_{\rm f}, d_{\rm m}$ and θ are the fractal dimension, magnetization exponent and energy exponent of a droplet, respectively. This formula is expected to apply to other glassy disordered systems, such as the random-field model and pinned interfaces. We make suggestions for further numerical investigations, as well as experimental studies of the Barkhausen noise in spin glasses. pdf

[KW76] P. Le Doussal and K.J. Wiese, *First-principle derivation of static avalanche-size distribution*, Phys. Rev. E **85** (2011) 061102, arXiv:1111.3172.

We study the energy minimization problem for an elastic interface in a random potential plus a quadratic well. As the position of the well is varied, the ground state undergoes jumps, called shocks or static avalanches. We introduce an efficient and systematic method to compute the statistics of avalanche sizes and manifold dis- placements. The tree-level calculation, i.e. mean-field limit, is obtained by solving a saddle-point equation. Graphically, it can be interpreted as a the sum of all tree graphs. The 1-loop corrections are computed using re- sults from the functional renormalization group. At the upper critical dimension the shock statistics is described by the Brownian Force model (BFM), the static version of the so-called ABBM model in the non-equilibrium context of depinning. This model can itself be treated exactly in any dimension and its shock statistics is that of a Levy process. Contact is made with classical results in probability theory on the Burgers equation with Brownian initial conditions. In particular we obtain a functional extension of an evolution equation introduced by Carraro and Duchon, which recursively constructs the tree diagrams in the field theory. pdf

[KW77] A. Dobrinevski, P. Le Doussal and K.J. Wiese, *Non-stationary dynamics of the Alessandro-Beatrice-Bertotti-Montorsi model*, Phys. Rev. E **85** (2012) 031105, arXiv:1112.6307.

We obtain an exact solution for the motion of a particle driven by a spring in a Brownian random-force landscape, the Alessandro-Beatrice-Bertotti-Montorsi (ABBM) model. Many experiments on quasi-static driving of elastic interfaces (Barkhausen noise in magnets, earthquake statistics, shear dynamics of granular matter) exhibit the same universal be- havior as this model. It also appears as a limit in the field theory of elastic manifolds. Here we discuss predictions of the ABBM model for monotonous, but otherwise arbitrary, time-dependent driving. Our main result is an explicit formula for the generating functional of particle velocities and positions. We apply this to derive the particle-velocity distribution following a quench in the driving velocity. We also obtain the joint avalanche size and duration distribution and the mean avalanche shape following a jump in the position of the confining spring. Such non-stationary driving is easy to realize in experiments, and provides a way to test the ABBM model beyond the stationary, quasi-static regime. We study extensions to two elastically coupled layers, and to an elastic interface of internal dimension d, in the Brownian force landscape. The effective action of the field theory is equal to the action, up to 1-loop corrections obtained exactly from a functional determinant. This provides a connection to renormalization-group methods. pdf

[KW78] P. Le Doussal, A. Petković and K.J. Wiese, *Distribution of velocities and acceleration for a particle in Brownian correlated disorder: Inertial case*, Phys. Rev. E **85** (2012) 061116, arXiv:1203.5620.

We study the motion of an elastic object driven in a disordered environment in presence of both dissipation and inertia. We consider random forces with the statistics of random walks and reduce the problem to a single degree of freedom. It is the extension of the mean field ABBM model in presence of an inertial mass m. While the ABBM model can be

solved exactly, its extension to inertia exhibits complicated history dependence due to oscillations and backward motion. The characteristic scales for avalanche motion are studied from numerics and qualitative arguments. To make analytical progress we consider two variants which coincide with the original model whenever the particle moves only forward. Using a combination of analytical and numerical methods together with simulations, we characterize the distributions of instantaneous acceleration and velocity, and compare them in these three models. We show that for large driving velocity, all three models share the same large-deviation function for positive velocities, which is obtained analytically for small and large m, as well as for $m = 6/25$. The effect of small additional thermal and quantum fluctuations can be treated within an approximate method.

[KW79] A. Perret, A. Ristivojevic, P. Le Doussal, Grégory Schehr and K. J. Wiese, *Super-rough glassy phase of the random field xy model in two dimensions*, Phys. Rev. Lett. **109** (2012) 157205, arXiv:1204.5685.
We study both analytically, using the renormalization group (RG) to two loop order, and numerically, using an exact polynomial algorithm, the disorder-induced glass phase of the two-dimensional XY model with quenched random symmetry-breaking fields and without vortices. In the super-rough glassy phase, i.e. below the critical temperature $T_{\rm c}$, the disorder and thermally averaged correlation function $B(r)$ of the phase field $\theta(\mathbf{x})$, $B(r) = \overline{\langle[\theta(\mathbf{x}) - \theta(\mathbf{x}+\mathbf{r})]^2\rangle}$ behaves, for $r \gg a$, as $B(r) \simeq A(\tau)\,\ln^2(r/a)$ where $r = |\mathbf{r}|$ and a is a microscopic length scale. We derive the RG equations up to cubic order in $\tau = (T_{\rm c} - T)/T_{\rm c}$ and predict the universal amplitude $A(\tau) = 2\tau^2 - 2\tau^3 + \mathcal{O}(\tau^4)$. The universality of $A(\tau)$ results from nontrivial cancellations between nonuniversal constants of RG equations. Using an exact polynomial algorithm on an equivalent dimer version of the model we compute $A(\tau)$ numerically and obtain a remarkable agreement with our analytical prediction, up to $\tau \approx 0.5$.

[KW80] Z. Ristivojevic, P. Le Doussal and K.J. Wiese, *Super-rough phase of the random-phase sine-Gordon model: Two-loop results*, Phys. Rev. B **86** (2012) 054201, arXiv:1204.6221.
We consider the two-dimensional random-phase sine-Gordon and study the vicinity of its glass transition temperature $T_{\rm c}$,, in an expansion in small $\tau = (T_{\rm c} - T)/T_{\rm c}$, where T denotes the temperature. We derive renormalization group equations in cubic order in the anharmonicity, and show that they contain two universal invariants. Using them we obtain that the correlation function in the super-rough phase for temperature $T < T_{\rm c}$ behaves at large distances as $\overline{\langle[\theta(x) - \theta(0)]^2\rangle} = \mathcal{A}\ln^2(|x|/a) + \mathcal{O}(\ln(x/a))$, where the amplitude $\mathcal{A}$ is a universal function of temperature $\mathcal{A} = 2\tau^2 - 2\tau^3 + \mathcal{O}(\tau^4)$. This result differs at two-loop order, i.e., $O(\tau^3)$, from the prediction based on results from the "nearly conformal" field theory of a related fermion model. We also obtain the correction-to-scaling exponent.

[KW81] A. Fedorenko, P. Le Doussal and K.J. Wiese, *Functional renormalization-group approach to decaying turbulence*, J. Stat. Mech. (2013) P04014, arXiv:1212.2117.
We reconsider the functional renormalization-group (FRG) approach to decaying Burgers turbulence, and extend it to decaying Navier-Stokes and Surface-Quasi-Geostrophic turbulence. The method is based on a renormalized small-time expansion, equivalent to a loop expansion, and naturally produces a dissipative anomaly and a cascade after a finite time. We explicitly calculate and analyze the one-loop FRG equations in the zero-viscosity limit as a function of the dimension. For Burgers they reproduce the FRG equation obtained in the context of random manifolds, extending previous results of one of us. Breakdown of energy conservation due to shocks and the appearance of a direct energy cascade corresponds to failure of dimensional reduction in the context of disordered systems. For Navier-Stokes in three dimensions, the velocity-velocity correlation function acquires a linear dependence on the distance, $\zeta_2 = 1$, in the inertial range, instead of Kolmogorov's $\zeta_2 = 2/3$; however the possibility remains for corrections at two- or higher-loop order. In two dimensions, we obtain a numerical solution which conserves energy and exhibits an inverse cascade, with explicit analytical results both for large and small distances, in agreement with the scaling proposed by Batchelor. In large dimensions, the one-loop FRG equation for Navier-Stokes converges to that of Burgers.

[KW82] P. Le Doussal and K.J. Wiese, *Avalanche dynamics of elastic interfaces*, Phys. Rev. E **88** (2013) 022106, arXiv:1302.4316.

Slowly driven elastic interfaces, such as domain walls in dirty magnets, contact lines wetting a non-homogenous substrate, or cracks in brittle disordered material proceed via intermittent motion, called avalanches. Here we develop a field-theoretic treatment to calculate, from first principles, the space-time statistics of instantaneous velocities within an avalanche. For elastic interfaces at (or above) their (internal) upper critical dimension $d \geq d_{\rm uc}$ ($d_{\rm uc} = 2, 4$ respectively for long-ranged and short-ranged elasticity) we show that the field theory for the *center of mass* reduces to the motion of a *point particle* in a random-force landscape, which is itself a random walk (ABBM model). Furthermore, the full *spatial* dependence of the velocity correlations is described by the Brownian-force model (BFM) where each point of the interface sees an independent Brownian-force landscape. Both ABBM and BFM can be solved exactly in any dimension d (for monotonous driving) by summing tree graphs, equivalent to solving a (non-linear) *instanton* equation. We focus on the limit of slow uniform driving. This tree approximation is the mean-field theory (MFT) for realistic interfaces in short-ranged disorder, up to the renormalization of two parameters at $d = d_{\rm uc}$. We calculate a number of observables of direct experimental interest: Both for the center of mass, and for a given Fourier mode q, we obtain various correlations and probability distribution functions (PDF's) of the velocity inside an avalanche, as well as the avalanche shape and its fluctuations (second shape). Within MFT we find that velocity correlations at non-zero q are asymmetric under time reversal. Next we calculate, beyond MFT, i.e. including loop corrections, the 1-time PDF of the center-of-mass velocity $\dot{u}$ for dimension $d < d_{\rm uc}$. The singularity at small velocity $\mathcal{P}(\dot{u}) \sim 1/\dot{u}^{\mathsf{a}}$ is substantially reduced from $\mathsf{a} = 1$ (MFT) to $\mathsf{a} = 1 - \frac{2}{9}(4-d) + ...$ (short-ranged elasticity) and $\mathsf{a} = 1 - \frac{4}{9}(2-d) + ...$ (long-ranged elasticity). We show how the dynamical theory recovers the avalanche-size distribution, and how the instanton relates to the response to an infinitesimal step in the force. pdf

[KW83] P. Le Doussal, Z. Ristivojevic and K.J. Wiese, *Exact form of the exponential correlation function in the glassy super-rough phase*, Phys. Rev. B **87** (2013) 214201, arXiv:1304.4612.
We consider the random-phase sine-Gordon model in two dimensions. It describes two-dimensional elastic systems with random-periodic disorder, such as pinned flux-line arrays, random-field XY models, and surfaces of disordered crystals. The model exhibits a super-rough glass phase at low temperature $T < T_c$ with relative displacements growing with distance r as $\overline{\langle[\theta(r) - \theta(0)]^2\rangle} \simeq A(\tau)\ln^2(r/a)$, where $A(\tau) = 2\tau^2 - 2\tau^3 + \mathcal{O}(\tau^4)$ near the transition and $\tau = 1 - T/T_c$. We calculate all higher cumulants and show that they grow as $\overline{\langle[\theta(r) - \theta(0)]^{2n}\rangle}_c \simeq [2(-1)^{n+1}(2n)!\zeta(2n-1)\tau^2 + \mathcal{O}(\tau^3)]\ln(r/a)$, $n \geq 2$, where ζ is the Riemann zeta function. By summation, we obtain the decay of the exponential correlation function as $\overline{\langle e^{iq[\theta(r)-\theta(0)]}\rangle} \simeq (a/r)^{\eta(q)} \exp\big(-\frac{1}{2}\mathcal{A}(q)\ln^2(r/a)\big)$ where $\eta(q)$ and $\mathcal{A}(q)$ are obtained for arbitrary $q \leq 1$ to leading order in τ. The anomalous exponent is $\eta(q) = cq^2 - \tau^2 q^2[2\gamma_{\rm E} + \psi(q) + \psi(-q)]$ in terms of the digamma function ψ, where c is non-universal and $\gamma_{\rm E}$ is the Euler constant. The correlation function shows a faster decay at $q = 1$, corresponding to fermion operators in the dual picture, which should be visible in Bragg scattering experiments. pdf

[KW84] A. Dobrinevski, P. Le Doussal and K.J. Wiese, *Statistics of avalanches with relaxation and Barkhausen noise: A solvable model*, Phys. Rev. E **88** (2013) 032106, arXiv:1304.7219.
We study a generalization of the Alessandro-Beatrice-Bertotti-Montorsi (ABBM) model of a particle in a Brownian force landscape, including retardation effects. We show that under monotonous driving the particle moves forward at all times, as it does in absence of retardation (Middleton's theorem). This remarkable property allows us to develop an analytical treatment. The model with an exponentially decaying memory kernel is realized in Barkhausen experiments with eddy-current relaxation, and has previously been shown numerically to account for the experimentally observed asymmetry of Barkhausen-pulse shapes. We elucidate another qualitatively new feature: the breakup of each avalanche of the standard ABBM model into a cluster of sub-avalanches, sharply delimited for slow relaxation under quasi-static driving. These conditions are typical for earthquake dynamics. With relaxation and aftershock clustering, the present model includes important ingredients for an effective description of earthquakes. We analyze quantitatively the limits of slow and fast relaxation for stationary driving with velocity $v > 0$. The v-dependent power-law exponent for small velocities, and the critical driving velocity at which the particle velocity never vanishes, are modified. We also analyze non-stationary avalanches following a step in the driving magnetic field. Analytically, we obtain the mean avalanche shape at fixed size, the duration distribution of the first sub-avalanche, and the time dependence of the mean velocity. We propose to study

these observables in experiments, allowing to directly measure the shape of the memory kernel, and to trace eddy current relaxation in Barkhausen noise.

[KW85] A.A. Fedorenko, P. Le Doussal and K.J. Wiese, *Non-Gaussian effects and multifractality in the Bragg glass*, EPL **105** (2014) 16002, arXiv:1309.6529.

We study, beyond the Gaussian approximation, the decay of the translational order correlation function for a d-dimensional scalar periodic elastic system in a disordered environment. We develop a method based on functional determinants, equivalent to summing an infinite set of diagrams. We obtain, in dimension $d = 4 - \epsilon$, the even n-th cumulant of relative displacements as $\overline{\langle [u(r) - u(0)]^n \rangle^c} \simeq \mathcal{A}_n \ln r$ with $\mathcal{A}_n = -(\varepsilon/3)^n \Gamma(n - \frac{1}{2}) \zeta(2n - 3)/\sqrt{\pi}$, multifractal dimension x_q of the exponential field $e^{qu(r)}$. As a corollary, we obtain an analytic expression for a class of n-loop integrals in $d = 4$, which appear in the perturbative determination of Konishi amplitudes, also accessible via AdS/CFT using integrability.

[KW86] F. Guinea, P. Le Doussal and K.J. Wiese, *Collective excitations in a large-d model for graphene*, Phys. Rev. B **89** (2014) 125428, arXiv:1312.2854.

We consider a model of Dirac fermions coupled to flexural phonons to describe a graphene sheet fluctuating in dimension $2 + d$. We derive the self-consistent screening equations for the quantum problem, exact in the limit of large d. We first treat the membrane alone, and work out the quantum to classical, and harmonic to anharmonic crossover. For the coupled electron-membrane problem we calculate the dressed two-particle propagators of the elastic and electron interactions and find that it exhibits a collective mode which becomes unstable at some wave-vector q_c for large enough coupling g. The saddle point analysis, exact at large d, indicates that this instability corresponds to spontaneous and simultaneous appearance of gaussian curvature and electron puddles. The relevance to ripples in graphene is discussed.

[KW87] A. Dobrinevski, P. Le Doussal and K.J. Wiese, *Avalanche shape and exponents beyond mean-field theory*, EPL **108** (2014) 66002, arXiv:1407.7353.

Elastic systems, such as magnetic domain walls, density waves, contact lines, and cracks, are all pinned by substrate disorder. When driven, they move via successive jumps called avalanches, with power-law distributions of size, duration and velocity. Their exponents, and the shape of an avalanche, defined as its mean velocity as function of time, have recently been studied. They are known approximatively from experiments and simulations, and were predicted from mean-field models, such as the Brownian force model (BFM), where each point of the elastic interface sees a force field which itself is a random walk. As we showed in EPL **97** (2012) 46004, the BFM is the starting point for an $\varepsilon = d_{\rm c} - d$ expansion around the upper critical dimension, with $d_{\rm c} = 4$ for short-ranged elasticity, and $d_{\rm c} = 2$ for long-ranged elasticity. Here we calculate analytically the $\mathcal{O}(\varepsilon)$, i.e. 1-loop, correction to the *avalanche shape* at fixed duration T, for both types of elasticity. The exact expression is well approximated by $\langle \dot{u}(t = xT) \rangle_T \simeq [Tx(1 - x)]^{\gamma - 1} \exp(\mathcal{A}[\frac{1}{2} - x])$, $0 \le x \le 1$. The asymmetry $\mathcal{A} \approx -0.336(1 - d/d_{\rm c})$ is negative for d close to $d_{\rm c}$, skewing the avalanche towards its end, as observed in numerical simulations in $d = 2$ and 3. The exponent $\gamma = (d + \zeta)/z$ is given by the two independent exponents at depinning, the roughness ζ and the dynamical exponent z. We propose a general procedure to predict other avalanche exponents in terms of ζ and z. We finally introduce and calculate the shape *at fixed avalanche size*, not yet measured in experiments or simulations.

[KW88] S. Atis, A.K. Dubey, D. Salin, L. Talon, P. Le Doussal and K.J. Wiese, *Experimental evidence for three universality classes for reaction fronts in disordered flows*, Phys. Rev. Lett. **114** (2015) 234502, arXiv:1410.1097.

Self-sustained reaction fronts in a disordered medium subject to an external flow display self-affine roughening, pinning and depinning transitions. We measure spatial and temporal fluctuations of the front in 1+1 dimensions, controlled by a single parameter, the mean flow velocity. Three distinct universality classes are observed, consistent with the Kardar-Parisi-Zhang (KPZ) class for fast advancing or receding fronts, the quenched KPZ class (positive-qKPZ) when the mean flow approximately cancels the reaction rate, and the negative-qKPZ class for slowly receding fronts. Both quenched KPZ classes exhibit distinct depinning transitions, in agreement with the theory.

[KW89] P. Le Doussal and K.J. Wiese, *An exact mapping of the stochastic field theory for Manna sandpiles to interfaces in random media*, Phys. Rev. Lett. **114** (2014) 110601, arXiv:1410.1930.

We show that the stochastic field theory for directed percolation in presence of an additional conservation law (the C-DP class) can be mapped exactly to the continuum theory for the depinning of an elastic interface in short-range correlated quenched disorder. On one line of parameters commonly studied, this mapping leads to the simplest overdamped dynamics. Away from this line, an additional memory term arises in the interface dynamics; we argue that it does not change the universality class. Since C-DP is believed to describe the Manna class of self-organized criticality, this shows that Manna stochastic sandpiles and disordered elastic interfaces (i.e. the quenched Edwards-Wilkinson model) share the same universal large-scale behavior. pdf

[KW90] K.J. Wiese, *Coherent-state path integral versus coarse-grained effective stochastic equation of motion: From reaction diffusion to stochastic sandpiles*, Phys. Rev. E **93** (2016) 042117, arXiv:1501.06514.

We derive and study two different formalisms used for non-equilibrium processes: The coherent-state path integral, and an effective, coarse-grained stochastic equation of motion. We first study the coherent-state path integral and the corresponding field theory, using the annihilation process $A + A \to A$ as an example. The field theory contains counter-intuitive quartic vertices. We show how they can be interpreted in terms of a first-passage problem. Reformulating the coherent-state path integral as a stochastic equation of motion, the noise generically becomes imaginary. This renders it not only difficult to interpret, but leads to convergence problems at finite times. We then show how alternatively an effective coarse-grained stochastic equation of motion with real noise can be constructed. The procedure is similar in spirit to the derivation of the mean-field approximation for the Ising model, and the ensuing construction of its effective field theory. We finally apply our findings to stochastic Manna sandpiles. We show that the coherent-state path integral is inappropriate, or at least inconvenient. As an alternative, we derive and solve its mean-field approximation, which we then use to construct a coarse-grained stochastic equation of motion with real noise. pdf

[KW91] T. Thiery, P. Le Doussal and K.J. Wiese, *Spatial shape of avalanches in the Brownian force model*, J. Stat. Mech. **2015** (2015) P08019, arXiv:1504.05342.

We study the Brownian force model (BFM), a solvable model of avalanche statistics for an interface, in a general discrete setting. The BFM describes the overdamped motion of elastically coupled particles driven by a parabolic well in independent Brownian force landscapes. Avalanches are defined as the collective jump of the particles in response to an arbitrary monotonous change in the well position (i.e. in the applied force). We derive an exact formula for the joint probability distribution of these jumps. From it we obtain the joint density of local avalanche sizes for stationary driving in the quasi-static limit near the depinning threshold. A saddle-point analysis predicts the spatial shape of avalanches in the limit of large aspect ratios for the continuum version of the model. We then study fluctuations around this saddle point, and obtain the leading corrections to the mean shape, the fluctuations around the mean shape and the shape asymmetry, for finite aspect ratios. Our results are finally confronted to numerical simulations. pdf

[KW92] M. Delorme and K.J. Wiese, *Maximum of a fractional Brownian motion: Analytic results from perturbation theory*, Phys. Rev. Lett. **115** (2015) 210601, arXiv:1507.06238.

Fractional Brownian motion is a non-Markovian Gaussian process X_t, indexed by the Hurst exponent H. It generalises standard Brownian motion (corresponding to $H = 1/2$). We study the probability distribution of the maximum m of the process and the time $t_{\rm max}$ at which the maximum is reached. They are encoded in a path integral, which we evaluate perturbatively around a Brownian, setting $H = 1/2 + \varepsilon$. This allows us to derive analytic results beyond the scaling exponents. Extensive numerical simulations for different values of H test these analytical predictions and show excellent agreement, even for large ε. pdf

[KW93] L.E. Aragon, A.B. Kolton, P. Le Doussal, K.J. Wiese and E. Jagla, *Avalanches in tip-driven interfaces in random media*, EPL **113** (2016) 10002, arXiv:1510.06795.

We analyse by numerical simulations and scaling arguments the avalanche statistics of 1-dimensional elastic interfaces

in random media driven at a single point. Both global and local avalanche sizes are power-law distributed, with universal exponents given by the depinning roughness exponent ζ and the interface dimension d, and distinct from their values in the uniformly driven case. A crossover appears between uniformly driven behaviour for small avalanches, and point-driven behaviour for large avalanches. The scale of the crossover is controlled by the ratio between the stiffness of the pulling spring and the elasticity of the interface; it is visible both in the global and local avalanche-size distributions, as in the average spatial avalanche shape. Our results are relevant to model experiments involving locally driven elastic manifolds at low temperatures, such as magnetic domain walls or vortex lines in superconductors.

[KW94] G. Durin, F. Bohn, M.A. Correa, R.L. Sommer, P. Le Doussal and K.J. Wiese, *Quantitative scaling of magnetic avalanches*, Phys. Rev. Lett. **117** (2016) 087201, arXiv:1601.01331.
We provide the first quantitative comparison between Barkhausen noise experiments and recent predictions from the theory of avalanches for pinned interfaces, both in and beyond mean-field. We study different classes of soft magnetic materials: polycrystals and amorphous samples, characterized by long-range and short-range elasticity, respectively; both for thick and thin samples, i.e. with and without eddy currents. The temporal avalanche shape at fixed size, and observables related to the joint distribution of sizes and durations, are analyzed in detail. Both long-range and short-range samples with no eddy currents are fitted extremely well by the theoretical predictions. In particular, the short-range samples provide the first reliable test of the theory beyond mean-field. The thick samples show systematic deviations from the scaling theory, providing unambiguous signatures for the presence of eddy currents.

[KW95] M. Delorme, P. Le Doussal and K.J. Wiese, *Distribution of joint local and total size and of extension for avalanches in the Brownian force model*, Phys. Rev. E **93** (2016) 052142, arXiv:1601.04940.
The Brownian force model (BFM) is a mean-field model for the local velocities during avalanches in elastic interfaces of internal space dimension d, driven in a random medium. It is exactly solvable via a non-linear differential equation. We study avalanches following a kick, i.e. a step in the driving force. We first recall the calculation of the distributions of the global size (total swept area) and of the local jump size for an arbitrary kick amplitude. We extend this calculation to the joint density of local and global sizes within a single avalanche, in the limit of an infinitesimal kick. When the interface is driven by a single point we find new exponents $\tau_0 = 5/3$ and $\tau = 7/4$, depending on whether the force or the displacement is imposed. We show that the extension of a *single avalanche* along one internal direction (i.e. the total length in $d = 1$) is finite and we calculate its distribution, following either a local or a global kick. In all cases it exhibits a divergence $P(\ell) \sim \ell^{-3}$ at small ℓ. Most of our results are tested in a numerical simulation in dimension $d = 1$.

[KW96] K.J. Wiese, *Dynamical selection of critical exponents*, Phys. Rev. E **93** (2016) 042105, arXiv:1602.00601.
In renormalized field theories there are in general one or few fixed points which are accessible by the renormalization-group flow. They can be identified from the fixed-point equations. Exceptionally, an infinite family of fixed points exists, parameterized by a scaling exponent ζ, itself function of a non-renormalizing parameter. Here we report a different scenario with an infinite family of fixed points of which seemingly only one is chosen by the renormalization-group flow. This dynamical selection takes place in systems with an attractive interaction $\mathcal{V}(\phi)$, as in standard ϕ^4 theory, but where the potential $\mathcal{V}$ at large ϕ goes to zero, as e.g. the attraction by a defect.

[KW97] M. Delorme and K.J. Wiese, *Perturbative expansion for the maximum of fractional Brownian motion*, Phys. Rev. E **94** (2016) 012134, arXiv:1603.00651.
Brownian motion is the only random process which is Gaussian, stationary and Markovian. Dropping the Markovian property, i.e. allowing for memory, one obtains a class of processes called fractional Brownian motion, indexed by the Hurst exponent H. For $H = 1/2$, Brownian motion is recovered. We develop a perturbative approach to treat the non-locality in time in an expansion in $\varepsilon = H - 1/2$. This allows us to derive analytic results beyond scaling exponents for various observables related to extreme value statistics: The maximum m of the process and the time $t_{\max}$ at which this maximum is reached, as well as their joint distribution. We test our analytical predictions with extensive numerical simulations for different values of H. They show excellent agreement, even for H far from $1/2$.

[KW98] T. Thiery, P. Le Doussal and K.J. Wiese, *Universal correlations between shocks in the ground state of elastic interfaces in disordered media*, Phys. Rev. E **94** (2016) 012110, arXiv:1604.05556.
The ground state of an elastic interface in a disordered medium undergoes collective jumps upon variation of external parameters. These mesoscopic jumps are called shocks, or static avalanches. Submitting the interface to a parabolic potential centered at w, we study the avalanches which occur as w is varied. We are interested in the correlations between the avalanche sizes S_1 and S_2 occurring at positions w_1 and w_2. Using the Functional Renormalization Group (FRG), we show that correlations exist for realistic interface models below their upper critical dimension. Notably, the connected moment $\langle S_1 S_2 \rangle^c$ is up to a prefactor exactly the renormalized disorder correlator, itself a function of $|w_2 - w_1|$. The latter is the universal function at the center of the FRG; hence correlations between shocks are universal as well. All moments and the full joint probability distribution are computed to first non-trivial order in an ϵ-expansion below the upper critical dimension. To quantify the local nature of the coupling between avalanches, we calculate the correlations of their local jumps. We finally test our predictions against simulations of a particle in random-bond and random-force disorder, with surprisingly good agreement. pdf

[KW99] M. Delorme and K.J. Wiese, *Extreme-value statistics of fractional Brownian motion bridges*, Phys. Rev. E **94** (2016) 052105, arXiv:1605.04132.
Fractional Brownian motion is a self-affine, non-Markovian and translationally invariant generalization of Brownian motion, depending on the Hurst exponent H. Here we investigate fractional Brownian motion where both the starting and the end point are zero, commonly referred to as bridge processes. Observables are the time t_+ the process is positive, the maximum m it achieves, and the time $t_{\max}$ when this maximum is taken. Using a perturbative expansion around Brownian motion ($H = \frac{1}{2}$), we give the first-order result for the probability distribution of these three variables, and the joint distribution of m and $t_{\max}$. Our analytical results are tested, and found in excellent agreement, with extensive numerical simulations, both for $H > \frac{1}{2}$ and $H < \frac{1}{2}$. This precision is achieved by sampling processes with a free endpoint, and then converting each realization to a bridge process, in generalization to what is usually done for Brownian motion. pdf

[KW100] M. Delorme, A. Rosso and K.J. Wiese, *Pickands' constant at first order in an expansion around Brownian motion*, J. Phys. A **50** (2017) 16LT04, arXiv:1609.07909.
In the theory of extreme values of Gaussian processes, many results are expressed in terms of the Pickands constant $\mathcal{H}_\alpha$. This constant depends on the local self-similarity exponent α of the process, i.e. locally it is a fractional Brownian motion (fBm) of Hurst index $H = \alpha/2$. Despite its importance, only two values of the Pickands constant are known: $\mathcal{H}_1 = 1$ and $\mathcal{H}_2 = 1/\sqrt{\pi}$. Here, we extend the recent perturbative approach to fBm to include drift terms. This allows us to investigate the Pickands constant $\mathcal{H}_\alpha$ around standard Brownian motion ($\alpha = 1$) and to derive the new exact result $\mathcal{H}_\alpha = 1 - (\alpha - 1)\gamma_{\rm E} + \mathcal{O}(\alpha - 1)^2$. pdf

[KW101] T. Sadhu, M. Delorme and K.J. Wiese, *Generalized arcsine laws for fractional Brownian motion*, Phys. Rev. Lett. **120** (2018) 040603, arXiv:1706.01675.
The *three arcsine laws* for Brownian motion are a cornerstone of extreme-value statistics. For a Brownian B_t starting from the origin, and evolving during time T, one considers the following three observables: (i) the duration t_+ the process is positive, (ii) the time $t_{\rm last}$ the process last visits the origin, and (iii) the time $t_{\max}$ when it achieves its maximum (or minimum). All three observables have the same cumulative probability distribution expressed as an arcsine function, thus the name of *arcsine laws*. We show how these laws change for fractional Brownian motion X_t, a non-Markovian Gaussian process indexed by the Hurst exponent H. It generalizes standard Brownian motion (*i.e.* $H = \frac{1}{2}$). We obtain the three probabilities using a perturbative expansion in $\epsilon = H - \frac{1}{2}$. While all three probabilities are different, this distinction can only be made at second order in ϵ. Our results are confirmed to high precision by extensive numerical simulations. pdf

[KW102] L. Benigni, C. Cosco, A. Shapira and K.J. Wiese, *Hausdorff dimension of the record set of a fractional Brownian motion*, Electron. Commun. Probab. **23** (2018) 1–8, arXiv:1706.09726.

We prove that the Hausdorff dimension of the record set of a fractional Brownian motion with Hurst parameter H equals H. pdf

[KW103] C. Husemann and K.J. Wiese, *Field theory of disordered elastic interfaces to 3-loop order: Results*, Nucl. Phys. B **932** (2018) 589–618, arXiv:1707.09802.
In disordered elastic systems, driven by displacing a parabolic confining potential adiabatically slowly, all advance of the system is in bursts, termed avalanches. Avalanches have a finite extension in time, which is much smaller than the waiting-time between them. Avalanches also have a finite extension ℓ in space, i.e. only a part of the interface of size ℓ moves during an avalanche. Here we study their spatial shape $\left<S(x)\right>_\ell$ given ℓ, as well as its fluctuations encoded in the second cumulant $\left<S^2(x)\right>_\ell^{\rm c}$. We establish scaling relations governing the behavior close to the boundary. We then give analytic results for the Brownian force model, in which the microscopic disorder for each degree of freedom is a random walk. Finally, we confirm these results with numerical simulations. To do this properly we elucidate the influence of discretization effects, which also confirms the assumptions entering into the scaling ansatz. This allows us to reach the scaling limit already for avalanches of moderate size. We find excellent agreement for the universal shape, its fluctuations, including all amplitudes. pdf

[KW104] Z. Zhu and K.J. Wiese, *The spatial shape of avalanches*, Phys. Rev. E **96** (2017) 062116, arXiv:1708.01078.
In disordered elastic systems, driven by displacing a parabolic confining potential adiabatically slowly, all advance of the system is in bursts, termed avalanches. Avalanches have a finite extension in time, which is much smaller than the waiting-time between them. Avalanches also have a finite extension ℓ in space, i.e. only a part of the interface of size ℓ moves during an avalanche. Here we study their spatial shape $\left<S(x)\right>_\ell$ given ℓ, as well as its fluctuations encoded in the second cumulant $\left<S^2(x)\right>_\ell^{\rm c}$. We establish scaling relations governing the behavior close to the boundary. We then give analytic results for the Brownian force model, in which the microscopic disorder for each degree of freedom is a random walk. Finally, we confirm these results with numerical simulations. To do this properly we elucidate the influence of discretization effects, which also confirms the assumptions entering into the scaling ansatz. This allows us to reach the scaling limit already for avalanches of moderate size. We find excellent agreement for the universal shape, its fluctuations, including all amplitudes. pdf

[KW105] K.J. Wiese, C. Husemann and P. Le Doussal, *Field theory of disordered elastic interfaces at 3-loop order: The β-function*, Nucl. Phys. B **932** (2018) 540–588, arXiv:1801.08483.
We calculate the effective action for disordered elastic manifolds in the ground state (equilibrium) up to 3-loop order. This yields the renormalziation-group β-function and the critical exponents to third order in $\epsilon = 4-d$, in an expansion in the dimension d around the upper critical dimension $d = 4$. The calculations are performed using exact RG, and several other techniques, which allow us to treat the problems associated with the cusp of the renormalized disorder. We also obtain the full 2-point function up to order ϵ^2, and the correction-to-scaling exponents. pdf

[KW106] K.J. Wiese and A.A. Fedorenko, *Field theories for loop-erased random walks*, submitted to PRX (2018), arXiv:1802.08830.
We analyze candidate field theories for loop-erased random walks (LERWs) in dimensions $2 \leq d \leq 4$. The first such candidate is ϕ^4-theory with $O(n)$-symmetry at $n = -2$. The link is established via a perturbation expansion in the coupling constant. The second candidate is a field theory for charge-density waves pinned by quenched disorder. Here the depinning transition is described by a non-analytic fixed point whose relation to the LERW had been conjectured earlier using analogies with Abelian sandpiles. We show diagrammatically order by order in the coupling constant that both theories yield identical results for key quantities such as the renormalization-group β-function, and the scaling dimensions of the observables which we identify with the fractal dimension of LERWs. While in ϕ^4 theory the latter is obtained from the crossover exponent encoded in the operator $\phi_1\phi_2$, in the charge-density-wave formulation it is given by the dynamical exponent z. The formal equivalence between the two theories is explicitly checked to 4-loop order. For the fractal dimension of LERWs in $d = 3$ it gives at 5-loop order $z = 1.624 \pm 0.002$, in agreement with the prediction

$z = 1.62400 \pm 0.00005$ of numerical simulations. We also show that a minimal description of LERWs can be formulated in terms of complex fermions. These three models constitute a hierarchy of field theories for LERWs. pdf

[KW107] K.J. Wiese, *First passage in an interval for fractional Brownian motion*, submitted to PRE (2018), arXiv:1807.08807.

Be X_t a random process starting at $x \in [0,1]$ with absorbing boundary conditions at both ends of the interval. Be $P_1(x)$ the probability to first exit at the upper boundary. For Brownian motion, $P_1(x) = x$, equivalent to $P_1'(x) = 1$. For fractional Brownian motion with Hurst exponent H, we establish that $P_1'(x) = \mathcal{N}[x(1-x)]^{\frac{1}{H}-2} \mathrm{e}^{\epsilon \mathcal{F}(x)+\mathcal{O}(\epsilon^2)}$, where $\epsilon = H - \frac{1}{2}$. The function $\mathcal{F}(x)$ is analytic, and well approximated by its Taylor expansion, $\mathcal{F}(x) \simeq 16(C-1)(x-1/2)^2 + \mathcal{O}(x-1/2)^4$, where $C = 0.915...$ is the Catalan-constant. A similar result holds for moments of the exit time starting at x. We then consider the span of X_t, i.e. the size of the (compact) domain it visited up to time t. For Brownian motion, we derive an analytic expression for the probability that the span reaches 1 for the first time, then generalized to fBm. Using large-scale numerical simulations with system sizes up to $N = 2^{24}$ and a broad range of H, we confirm our analytic results. There are important finite-size corrections which we quantify. They are most severe for small H, necessitating to go to the large systems mentioned above. pdf

[KW108] W.D. Baez, K.J. Wiese, R. Bundschuh, *On the behavior of random RNA secondary structures near the glass transition*, submitted to PRE (2018), arXiv:1808.02351.

RNA forms elaborate secondary structures through intramolecular base pairing. These structures perform critical biological functions within each cell. Due to the availability of a polynomial algorithm to calculate the partition function over these structures, they are also a suitable system for the statistical physics of disordered systems. In this model, below the denaturation temperature, random RNA secondary structures exist in one of two phases: a strongly disordered, low-temperature glass phase, and a weakly disordered, high-temperature molten phase. The probability of two bases to pair decays with their distance with an exponent 3/2 in the molten phase, and about 4/3 in the glass phase. Inspired by previous results from a renormalized field theory of the glass transition separating the two phases, we numerically study this transition. We introduce distinct order parameters for each phase, that both vanish at the critical point. We finally explore the driving mechanism behind this transition. pdf

[KW109] K.J. Wiese, *Span observables - "When is a foraging rabbit no longer hungry?"*, (2019), arXiv:1903.06036.

Be X_t a random walk. We study its span S, i.e. the size of the domain visited up to time t. We want to know the probability that S reaches 1 for the first time, as well as the density of the span given t. Analytical results are presented, and checked against numerical simulations. We then generalize this to include drift, and one or two reflecting boundaries. We also derive the joint probability of the maximum and minimum of a process. Our results are based on the diffusion propagator with reflecting or absorbing boundaries, for which a set of useful formulas is derived. pdf

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SUMMARY OF PAST RESEARCH