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Abstract – For a driven elastic object near depinning, we derive from first principles the distribution of instantaneous velocities in an avalanche. We prove that above the upper critical dimension, $d \geq d_{uc}$, the n -times distribution of the center-of-mass velocity is equivalent to the prediction from the ABBM stochastic equation. Our method allows to compute space and time dependence from an instanton equation. We extend the calculation beyond mean field, to lowest order in $\epsilon = d_{uc} - d$.



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Obtaining a quantitative description of the dynamics during an avalanche is of great importance for systems whose dynamics is governed by jumps, such as magnets, superconductors, earthquakes, the contact line of fluids, or fracture [1–6]. In particular the motion of domain walls (DW) in magnets is important for many applications, such as magnetic recording. It can be measured from the Barkhausen (magnetization) noise [7,8], which is a complicated time-dependent signal.

A major step forward was accomplished by Alessandro, Beatrice, Bertotti and Montorsi (ABBM) [9] who introduced, on a phenomenological basis, a stochastic equation approximating the DW motion by a single degree of freedom submitted to a random-force landscape with long-range (Brownian) correlations. Although a crude description, this model has been used extensively to compare with experiments on magnets, with success in some “mean-field-like” cases, and failure in others [10,11]. Since microscopic disorder is usually short ranged, it can only hold as an effective “mean-field” model, and until now its validity was only established in the infinite-range limit [11,12]. No microscopic foundation for the validity of this model for an interface in a realistic disorder exists.

On the other hand, a sophisticated field theory was developed for systems with quenched disorder. In particular, for elastic interfaces, relevant for DW motion, functional RG methods (FRG) [1,13–15] recently allowed to derive the distribution of quasi-static avalanche sizes [16,17]. Until now, no description of the dynamics during an avalanche was available. In fact, since it involves

much faster motion than the average driving velocity, it led to difficulties in the early FRG approaches [14].

The aim of this letter is to compute from first principles the distribution of instantaneous velocities in an avalanche. We study a single elastic interface, of internal dimension d (total space dimension is $D = d + 1$) at zero temperature, near the depinning threshold. We expand around the upper critical dimension d_{uc} , with $d_{uc} = 4$ for standard elasticity, and $d_{uc} = 2$ in the presence of long-range elasticity, *e.g.* arising from dipolar forces. Remarkably, we find that for $d = d_{uc}$ (and above) and in the scaling limit, the n -time probability distribution (with n arbitrary) of the center of mass of the interface is equivalent to that of the ABBM stochastic equation, in terms of renormalized parameters which in some cases can be estimated. The two methods are rather different in spirit, and the identification non-trivial. Our result establishes the universality of the ABBM model for $d \geq d_{uc}$. In addition it allows to resolve the spatial structure, and gives the corrections to ABBM for $d < d_{uc}$.

Here we sketch a very simple derivation, for details and various subtleties involved we refer to [18]. Consider the equation of motion, in the comoving frame, for the local velocity of an interface driven at velocity v :

$$(\eta_0 \partial_t - \nabla_x^2) \dot{u}_{xt} = \partial_t F(vt + u_{xt}, x) - m^2 \dot{u}_{xt}. \quad (1)$$

It is obtained by time derivation (noted indifferently \dot{u} or $\partial_t u$) of the standard overdamped equation of motion. x is the d -dimensional internal coordinate, $vt + u_{xt}$ the space- and time-dependent displacement field and η_0 the friction coefficient. $F(u, x)$ is the quenched random pinning force from the impurities, with, *e.g.*, Gaussian distribution and

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variance $\overline{F(u, x)F(u', x')} = \delta^d(x - x')\Delta_0(u - u')$, where overbars denote disorder averages. m^2 is the strength of the restoring force $-m^2(u_{xt} - vt)$ (*i.e.*, the mass, or spring constant), which flattens the interface beyond a scale $L_m \sim 1/m$. In the small- m , large- L_m , limit, studied here, the interface has the roughness exponent ζ of the depinning transition, with $u \sim x^\zeta$ for $x \lesssim L_m$ and $u \sim L_m^\zeta$ for $L > L_m$. For simplicity we chose standard elasticity $\sim \nabla_x^2$, but it can be replaced by an arbitrary elastic kernel as needed in applications [2,4,10].

Near the depinning transition, *i.e.* at small v , the interface proceeds via avalanches. This is easiest seen in the center-of-mass position $u_t = L^{-d} \int_x u_{xt}$. There is a well-defined quasi-static limit $v = 0^+$ where $u_t = u(w)$, with $w = vt$ the well position. The process $u(w)$ jumps at discrete locations w_i , *i.e.*, $u(w) = L^{-d} \sum_i S_i \theta(w - w_i)$, with S_i the avalanche sizes. Their statistics was predicted via FRG, and checked numerically [16,17,19]. There, the bare disorder correlator $\Delta_0(u)$ flows, under coarse graining, to the renormalized one $\Delta(u)$. The latter has an interpretation as a physical observable, proportional to the fluctuations of the restoring force, or equivalently center-of-mass position. It was calculated numerically in [20,21], and measured in experiments [4]. At the depinning transition it exhibits a linear cusp $-\Delta'(0^+) > 0$. This cusp is directly related to the moments of the normalized avalanche size distribution $P(S)$, via [17]

$$S_m := \frac{\langle S^2 \rangle}{2\langle S \rangle} = \frac{|\Delta'(0^+)|}{m^4}. \quad (2)$$

The confining well suppresses large avalanches, and sets the large-scale cutoff of $P(S)$ to be $S_m \sim m^{-(d+\zeta)}$. Here we study the dynamics *inside* these avalanches, which occur for small v on a time scale $\tau_m \sim L_m^z \ll \Delta w/v$, where Δw is the typical separation of avalanches in the same space region, and z the dynamical exponent of the depinning transition. Hence we consider small v so that avalanches remain separated, a condition equivalent to $L_m \ll \xi_v$, where ξ_v is the correlation length induced by driving with velocity v [13,14] near depinning (for $m = 0$). This is illustrated in fig. 1.

The information about the dynamics in an avalanche is contained in the n -times cumulants $C_n = \overline{\dot{u}_{t_1} \dots \dot{u}_{t_n}^c}$, $n \geq 2$ (with $\overline{\dot{u}_t} = 0$). In the limit $v \rightarrow 0^+$ the product $\dot{u}_{t_1} \dots \dot{u}_{t_n}$ vanishes unless all times are inside the same avalanche. The probability that exactly one avalanche occurs in a time interval $T < \Delta w/v$ is $\rho_0 v T$, with $\rho_0 = L^d/\langle S \rangle$ the avalanche density per unit w . Since the movement is non-smooth, C_n is $O(v)$, rather than $O(v^n)$. As the total displacement is by definition the avalanche size, C_n satisfies the sum rule $L^{nd} \int_{[-T/2, T/2]^n} dt_1 \dots dt_n \overline{\dot{u}_{t_1} \dots \dot{u}_{t_n}} = \rho_0 v T \langle S^n \rangle + O(v^2)$. It can be computed perturbatively in the (renormalized) disorder. For $n = 2$ and to lowest order

$$\overline{\dot{u}_{t_1} \dot{u}_{t_2}^c} = -L^{-d} \Delta'(0^+) \frac{v}{m^2 \eta} e^{-\frac{m^2}{\eta} |t_1 - t_2|}, \quad (3)$$

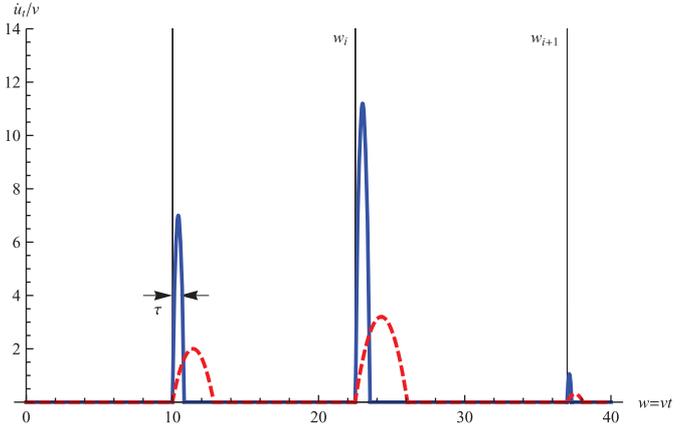


Fig. 1: (Colour on-line) Schematic plot of the instantaneous velocity (divided by v) as a function of vt for different v . The area under the curve is the avalanche size, hence is constant as $v \rightarrow 0^+$. The quasi-static avalanche positions w_i are indicated.

where η is the renormalized friction¹. Integrating over time one recovers (2).

To obtain all moments at once, as well as the velocity distribution, we compute the generating function

$$Z[\lambda] = L^{-d} \partial_v \overline{e^{\int_{xt} \lambda_{xt} (v + \dot{u}_{xt})}} \Big|_{v=0^+}. \quad (4)$$

The average over disorder (and initial conditions) is obtained from the dynamical action $S = S_0 + S_{\text{dis}}$ of (1):

$$S_0 = \int_{xt} \tilde{u}_{xt} (\eta \partial_t - \nabla_x^2 + m^2) \dot{u}_{xt}, \quad (5)$$

$$S_{\text{dis}} = -\frac{1}{2} \int_{xtt'} \tilde{u}_{xt} \tilde{u}_{xt'} \partial_t \partial_{t'} \Delta(v(t-t') + u_{xt} - u_{xt'}). \quad (6)$$

This yields

$$Z[\lambda] = L^{-d} \partial_v \int \mathcal{D}[\dot{u}] \mathcal{D}[\tilde{u}] e^{-S + \int_{xt} \lambda_{xt} (v + \dot{u}_{xt})} \Big|_{v=0^+} \quad (7)$$

with $Z[0] = 0$. We write

$$\begin{aligned} \partial_t \partial_{t'} \Delta(v(t-t') + u_{xt} - u_{xt'}) &= \\ (v + \dot{u}_{xt}) \partial_{t'} \Delta'(v(t-t') + u_{xt} - u_{xt'}) &= \\ (v + \dot{u}_{xt}) \Delta'(0^+) \partial_{t'} \text{sgn}(t-t') + \dots, \end{aligned} \quad (8)$$

where we have used that the interface is only moving forward (Middleton theorem [22]). We can thus rewrite the disorder term as $S = S_{\text{dis}}^{\text{tree}} + \dots$, where

$$S_{\text{dis}}^{\text{tree}} = \Delta'(0^+) \int_{xt} \tilde{u}_{xt} \tilde{u}_{xt} (v + \dot{u}_{xt}) \quad (9)$$

is the *tree level* or *mean-field* action (see footnote ¹). The terms neglected are $O(\Delta''(0^+))$ and higher derivatives,

¹We use the improved action, where $\eta_0 \rightarrow \eta$ and $\Delta_0 \rightarrow \Delta$ in order to obtain the correct result for $d \geq d_{\text{uc}}$, see [17]. Note that m^2 and v are not corrected.

and we have shown that they contribute only to order $O(\epsilon)$ to $Z[\lambda]$, hence can be neglected at tree level.

We now study the tree approximation for $Z[\lambda]$, *i.e.* (7) with S_{dis} replaced by (9). Thus, the highly non-linear action (6) has been reduced to a much simpler cubic theory! Even more remarkably, \dot{u}_{xt} appears only linearly in (9), and viewing \dot{u} as a response field, the tree level theory is *equivalent to the following non-linear equation*:

$$(\eta\partial_t + \nabla_x^2 - m^2)\tilde{u}_{xt} - \Delta'(0^+)\tilde{u}_{xt}^2 + \lambda_{xt} = 0. \quad (10)$$

We denote \tilde{u}_{xt}^λ the solution of this equation for a given source λ_{xt} . Performing the derivative with respect to v in (7) gives

$$\begin{aligned} Z[\lambda] &= L^{-d} \int_{xt} \lambda_{xt} - \Delta'(0^+)(\tilde{u}_{xt}^\lambda)^2 \\ &= L^{-d} \int_{xt} (-\eta\partial_t - \nabla_x^2 + m^2)\tilde{u}_{xt}^\lambda = m^2 L^{-d} \int_{xt} \tilde{u}_{xt}^\lambda, \end{aligned} \quad (11)$$

where we have used eq. (10) and, in the last equality, assumed that \tilde{u}_{xt}^λ vanishes at large t and x . To analyze the result, it is convenient to use dimensionless equations, replacing $x \rightarrow x/m$, $L \rightarrow L/m$, $t \rightarrow \tau_m t$, $v \rightarrow v v_m$, $\lambda \rightarrow \lambda/S_m$ and $\tilde{u}_{xt} \rightarrow \tilde{u}_{xt}/m^2 S_m$, where $v_m = S_m m^d / \tau_m$, and $\tau_m = \eta/m^2$. From now on we will use these units, and consider the center-of-mass velocity, thus choosing $\lambda_{xt} = \lambda_t$ uniform.

The 1-time probability at time $t=0$ is given by $\lambda_t = \lambda\delta(t)$ through its Laplace transform

$$\tilde{Z}(\lambda) = L^{-d} \partial_v \overline{e^{L^d \lambda(v+\dot{u})}} \Big|_{v=0^+}. \quad (12)$$

$\dot{u} = \dot{u}_{t=0}$ and the notation \tilde{Z} reminds us that we use dimensionless units. $\tilde{u}_{xt} = \tilde{u}_t$ and we need to solve

$$(\partial_t - 1)\tilde{u}_t + \tilde{u}_t^2 = -\lambda\delta(t) \quad (13)$$

with $\tilde{u}_t \rightarrow 0$ at $t = \pm\infty$. The solution is

$$\tilde{u}_t = \frac{\lambda}{\lambda + (1-\lambda)e^{-t}} \theta(-t). \quad (14)$$

Inserting into (12) gives

$$\tilde{Z}(\lambda) = \int_t \tilde{u}_t = -\ln(1-\lambda). \quad (15)$$

Calling τ_i , $i = 1, \dots, N$, the duration of the i -th avalanche out of N , and defining $\langle\tau\rangle := \frac{1}{N} \sum_{i=1}^N \tau_i$ the mean duration, the probability p_a that $t=0$ belongs to an avalanche is $p_a = \rho_0 v \langle\tau\rangle$. Hence the total 1-time velocity probability is

$$P(\dot{u}) = (1-p_a)\delta(v+\dot{u}) + p_a \tilde{P}(\dot{u}), \quad (16)$$

where $\tilde{P}(\dot{u})$ is the probability given that $t=0$ belongs to an avalanche. Both \tilde{P} and P are normalized to unity.

One notes the two (always) exact relations $\langle\dot{u}\rangle_P = 0$, $p_a \langle\dot{u} + v\rangle_{\tilde{P}} = v$. Hence for $v = 0^+$ one has $\rho_0 \langle\tau\rangle \langle\dot{u}\rangle_{\tilde{P}} = 1$ and, in dimensionful units

$$\begin{aligned} Z(\lambda) &= \frac{1}{m^d v_m} \tilde{Z}(m^d v_m \lambda) \\ &= L^{-d} \rho_0 \langle\tau\rangle \int d\dot{u} \tilde{P}(\dot{u}) (e^{L^d \lambda \dot{u}} - 1). \end{aligned} \quad (17)$$

We obtain, in the slow-driving limit, the instantaneous velocity distribution in the range $v_0 \ll \dot{u} \sim \tilde{v}_m$ (v_0 is a small-velocity cutoff):

$$\tilde{P}(\dot{u}) = \frac{1}{\rho_0 \langle\tau\rangle \tilde{v}_m^2} p\left(\frac{\dot{u}}{\tilde{v}_m}\right), \quad p(x) = \frac{1}{x} e^{-x}. \quad (18)$$

We defined $\tilde{v}_m = (mL)^{-d} v_m = L^{-d} S_m / \tau_m$. Hence $\langle\dot{u}\rangle_{\tilde{P}} \approx \tilde{v}_m / \ln(\frac{\tilde{v}_m}{v_0})$. Note that $p(x)$ is not a probability, but is normalized by $\int dx x p(x) = 1$.

Similarly one obtains the n -time distribution of the center-of-mass velocity solving (13) with $\lambda_t = \sum_{j=1}^n \lambda_j \delta(t - t_j)$, noting $z_{ij} := 1 - e^{-|t_i - t_j|/\tau_m}$,

$$\tilde{Z}_n(\lambda_1, \dots, \lambda_n) = -\ln \left(\sum_{\Lambda \subset \{1, \dots, n\}} \prod_{i \in \Lambda} [-\lambda_i] \prod_{\{i, j\} \subset \Lambda, i < j} z_{ij} \right). \quad (19)$$

For $n=2$ one finds $\tilde{Z}_2 = -\ln(1 - \lambda_1 - \lambda_2 + \lambda_1 \lambda_2 z)$ with $z = 1 - e^{-|t_2 - t_1|/\tau_m}$. From this we obtain the probability $q_{12} = v q'_{12}$ that both t_1 and t_2 belong to the same avalanche, and the velocity distribution \tilde{P} conditioned to this event:

$$q'_{12} \tilde{P}(\dot{u}_1, \dot{u}_2) = \frac{1}{\tilde{v}_m^3} p\left(\frac{\dot{u}_1}{\tilde{v}_m}, \frac{\dot{u}_2}{\tilde{v}_m}\right), \quad (20)$$

$$p(v_1, v_2) = \frac{e^{-\frac{t}{2} - \frac{v_1 + v_2}{1 - e^{-t}}}}{(1 - e^{-t}) \sqrt{v_1 v_2}} I_1\left(\frac{2e^{-t/2} \sqrt{v_1 v_2}}{1 - e^{-t}}\right) \quad (21)$$

with $t = |t_2 - t_1|/\tau_m$, $q'_{12} \tilde{v}_m = \ln(1/z)$, and $I_1(x)$ is the Bessel- I function of the first kind. The probability that t_1 but not t_2 belongs to an avalanche is

$$q'_1 \tilde{P}_1(\dot{u}_1) = \frac{1}{\tilde{v}_m^2} p\left(\frac{\dot{u}_1}{\tilde{v}_m}\right), \quad p(\dot{u}_1) = \frac{e^{-\dot{u}_1/z}}{\dot{u}_1} \quad (22)$$

with $p'_a = q'_1 + q'_{12}$. Since the probability that there exists an avalanche starting in $[t_1, t_1 + dt_1]$ and ending in $[t_2, t_2 + dt_2]$ is $-dt_1 dt_2 \partial_{t_1} \partial_{t_2} q_{12}$, we obtain the distribution of durations τ as

$$P(\tau) = \frac{1}{\rho_0 \tilde{v}_m \tau_m^2} \frac{e^{-\tau/\tau_m}}{(1 - e^{-\tau/\tau_m})^2}. \quad (23)$$

For small durations $\tau \ll \tau_m$, $P(\tau) \approx \frac{1}{\rho_0 \tilde{v}_m \tau_m^2}$, for $\tau > \tau_0$, where we denote τ_0 the lower time cutoff not to be confused with τ_i above. This gives $\langle\tau\rangle = \frac{1}{\rho_0 \tilde{v}_m} \ln(\frac{\tau_m}{\tau_0})$ in good agreement with the above, using $\ln(\frac{\tau_m}{\tau_0}) \approx \ln(\frac{\tilde{v}_m}{v_0})$.

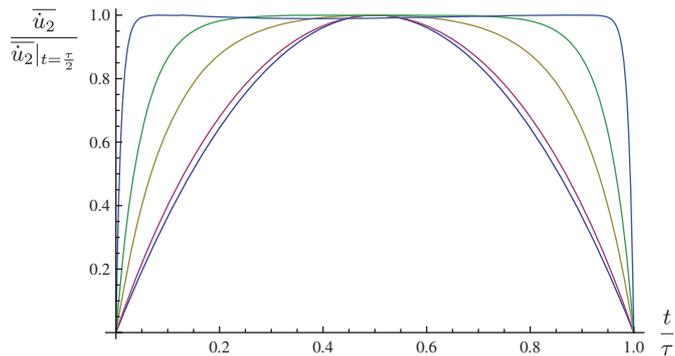


Fig. 2: (Colour on-line) Pulse shape: the normalized velocity at time t in an avalanche of duration τ for $\tau \ll \tau_m$ (lower curve) to $\tau \gg \tau_m$ (upper curve).

Note that $q'_{12} \tilde{P}(0^+, 0^+)$ is proportional to the probability that an avalanche starts at t_1 and ends at t_2 .

The *shape* of an avalanche with duration τ can then be extracted from the probabilities at 3 times $(t_1, t_2, t_3) = (0, t, \tau)$ setting $\dot{u}_1 = \dot{u}_3 = 0^+$. From the generating function (19) for 3 times, the velocity probability distribution for the intermediate time is $P(\dot{u}_2) = b^2 \dot{u}_2 e^{-\dot{u}_2 b}$, with $\tilde{v}_m b := \frac{1}{z_{12}} + \frac{1}{z_{23}} - 1$ resulting in the average *shape*

$$\bar{u}_2 = \frac{2}{b} = \tilde{v}_m \frac{4 \sinh\left(\frac{t}{2\tau_m}\right) \sinh\left(\frac{\tau}{2\tau_m} \left[1 - \frac{t}{\tau}\right]\right)}{\sinh\left(\frac{\tau}{2\tau_m}\right)}. \quad (24)$$

This interpolates from a parabola for small $\tau \ll \tau_m$ to a flat shape for the longest avalanches (see fig. 2). For long avalanches, the velocity reaches a steady state, the plateau in fig. 2. This result holds for an interface at or above its upper critical dimension, which previously was used [8] on the basis of the ABBM model.

We now clarify the relation to the phenomenological ABBM theory [9]. The latter models the interface as a single point driven in a long-range-correlated random-force landscape, $F(u)$, with *Brownian* statistics. It amounts to suppressing the space dependence in (1), hence corresponds in our general model to the special case $d=0$ and $\Delta_0(0) - \Delta_0(u) = \sigma|u|$. The instantaneous velocity $\mathbf{v} = \dot{u}_t + v$ satisfies the stochastic equation $\eta d\mathbf{v} = m^2(v - \mathbf{v})dt + dF$ where $dF^2 = 2\sigma v dt$, with associated Fokker-Planck equation

$$\eta \partial_t Q = \partial_v \left[\frac{\sigma}{\eta} \partial_v (vQ) + m^2(v - v)Q \right] \quad (25)$$

for the velocity probability $Q \equiv Q(\mathbf{v}, t | \mathbf{v}_1, 0)$. For $v > 0$ it evolves to the stationary distribution $Q_0(\mathbf{v}) = v_m^{-v/v_m} v^{v/v_m - 1} e^{-v/v_m} / \Gamma(v/v_m)$ with $v_m = S_m / \tau_m$ and here $S_m = \sigma / m^4$ and $\tau_m = \eta / m^2$. For $v = 0^+$ one recovers (18), up to a normalization which entails a small-scale cutoff. Similarly for $v = 0^+$ one finds the propagator

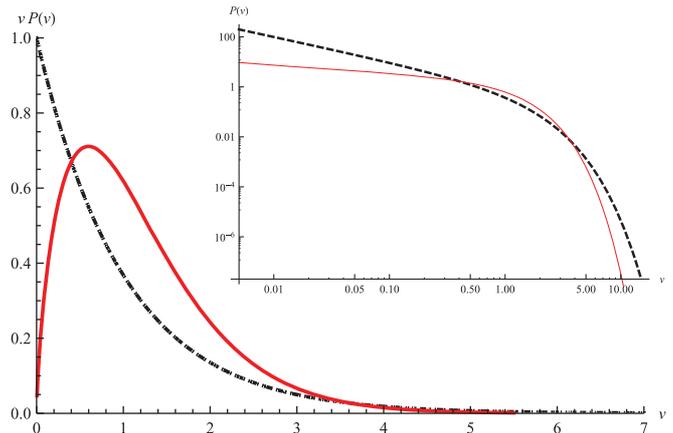


Fig. 3: (Colour on-line) Main plot: $vP(v)$ as a function of $v = \dot{u}$. The dashed line is the mean-field result (18). The red (solid) line is obtained from an $\epsilon = 4 - d$ expansion, extrapolated to $\epsilon = 3$. Inset: the same as a log-log plot.

$Q(\mathbf{v}, t | \mathbf{v}_1, 0) = v_m^{-1} \tilde{Q}\left(\frac{\mathbf{v}}{v_m}, \frac{t}{\tau_m} \middle| \frac{\mathbf{v}_1}{v_m}, 0\right)$ with

$$\tilde{Q}(v_2, t | v_1, 0) = v_1 e^{v_1} \left[p(v_1, v_2) + \frac{1}{v_1} e^{-\frac{v_1}{1 - e^{-t}}} \delta(v_2) \right], \quad (26)$$

and $p(v_1, v_2)$ given in eq. (21). $\tilde{Q}(v_2, t | v_1, 0)$ is solution of (25) with $Q(v_2, 0^+ | v_1, 0) = \delta(v_2 - v_1)$. The piece $\sim \delta(v_2)$ corresponds to avalanches which have already terminated at time t , and is necessary for Q to conserve probability. The *joint distribution* $\tilde{Q}(v_2, t | v_1, 0) \frac{1}{v_1} e^{-v_1}$ reproduces the 1-time and 2-times probabilities given in eqs. (20) and (22), up to a global normalization. More generally, since $\mathbf{v}(t)$ is a Markov-process, the n -time velocity probability obtained from (10) is $q'_{1p} \tilde{P}(\dot{u}_1, \dots, \dot{u}_n) = \frac{1}{\dot{u}_1} e^{-\dot{u}_1} \prod_{j=1}^{n-1} Q(\dot{u}_{j+1} t_{j+1} | \dot{u}_j t_j)$.

Several remarks are in order: Applying the dynamical-action method to the case where the force landscape is exactly Brownian, for an interface in any d or for a point (ABBM model), we find at $v = 0^+$ that *the tree approximation is exact*. In the field theory it means that there are no loop corrections. Hence $\Delta'(u) = \Delta'_0(u) = -\sigma \text{sgn}(u)$ is an exact FRG fixed point (with $\zeta = 4 - d$) as noted in [17]. This remarkable property is not valid for any other, *e.g.*, shorter-ranged, force landscape. In that sense, the model proposed by ABBM [9] appears very judicious.

Second, since a realistic interface in a short-ranged random force is described for $d \geq d_{uc}$ by the tree approximation, we proved that the temporal correlations of its center-of-mass velocity for $v \rightarrow 0$ are given by the ABBM model. Only two parameters enter, η and S_m , which at $d = d_c$ acquire a logarithmic dependence on m [17], that could be searched for in experiments.

Third, for the Brownian-force landscape the present method extends to $v > 0$ [23], and leads to the famous dependence of the exponent τ on v [9]. For realistic SR disorder, however, it is known that beyond the scale ξ_v the interface crosses over to the Edwards-Wilkinson regime.

It remains to be seen whether that can account for the data for $\tau(v)$ presented, *e.g.*, in [11].

Fourth, the present theory allows to go *beyond* the ABBM model in several ways: In $d \geq 4$, the non-linear equation (10) allows to study the full time and space dependence of velocity correlations, as in the statics [17]. Second, including loop corrections allows to compute corrections in a systematic expansion in $d = 4 - \epsilon$ [18]. Since the calculations are much more technical than those presented here, we restrict to some key results: The small- v behavior of the 1-time velocity distribution for $v_0 \ll v \ll \tilde{v}_m$ is to first order in ϵ

$$P(v) \sim 1/v^a, \quad a = 1 - \epsilon(1 - \zeta_1)/3 + O(\epsilon^2), \quad (27)$$

i.e., $a = 1 - \frac{2}{9}\epsilon$ for a non-periodic interface, and $a = 1 - \frac{\epsilon}{3}$ for a charge density wave (CDW). The scaling function also changes, see fig. 3. At small velocity, the divergence is smoothed, as the short-ranged nature of the disorder is more important in lower dimension. At large velocity, the decay becomes faster than the pure exponential of the ABBM model. The decay occurs at velocity scale \tilde{v}_m with $\eta_m \sim m^{2-z}$, determined by the dynamical exponent $z = 2 - \frac{2}{9}\epsilon$ of the depinning transition for non-periodic disorder and $z = 2 - \frac{\epsilon}{3}$ for CDW [13–15].

To conclude, we introduced a method to compute both spatial and temporal velocity correlations in an avalanche. Its tree approximation is exact at and above the upper critical dimension $d \geq d_{uc}$. There, the center-of-mass motion is equivalent to the phenomenological ABBM model, establishing the range of validity of the latter. For $d < d_{uc}$ corrections are calculated in a controlled expansion in $\epsilon = d_{uc} - d$. Other observables such as local velocities, measured in [3], should be computable.

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