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Why one needs a functional renormalization group to survive in a disordered world

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Abstract. In this paper, we discuss why functional renormalization is an essential tool to treat strongly disordered systems. More specifically, we treat elastic manifolds in a disordered environment. These are governed by a disorder distribution, which after a finite renormalization becomes non-analytic, thus overcoming the predictions of the seemingly exact dimensional reduction. We discuss how a renormalizable field theory can be constructed even beyond 2-loop order. We then consider an elastic manifold embedded in N dimensions, and give the exact solution for $N \to \infty$. This is compared to predictions of the Gaussian replica variational ansatz, using replica symmetry breaking. Finally, the effective action at order $1/N$ is reported.

Keywords. Disordered systems; functional renormalization.

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1. Introduction

In this paper we consider an elastic manifold in a random potential, as a prototype for strongly disordered systems. Since for all these systems temperature is irrelevant, we will consider temperature as zero. The systems we have in mind are domain walls in dirty magnets, contact lines, charge density waves, vortex lattices, to just mention a few. These results were obtained in collaboration with Pierre Le Doussal [1–11]. For lack of space we restrict our discussion to equilibrium. Complementary material, especially for the depinning, can be found in an earlier review [12].

2. Physical realizations, model and observables

The simplest experimental realization is an Ising magnet. Imposing boundary conditions with all spins up at the upper boundary and all spins down at the lower boundary (see figure 1), at low temperatures, a domain wall separates a region with

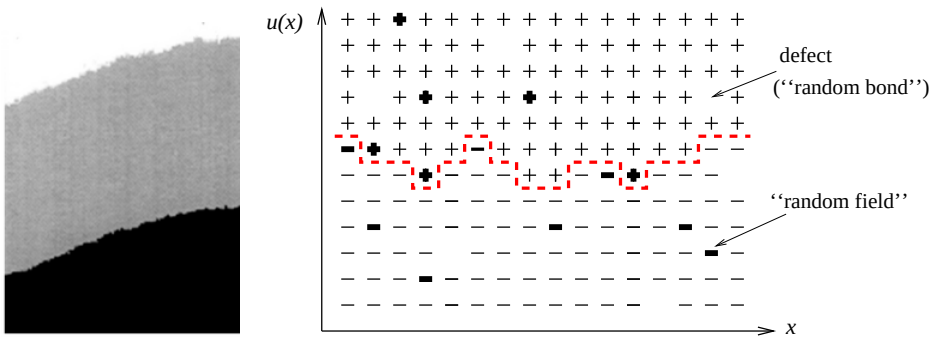


Figure 1. An Ising magnet at low temperatures forms a domain wall described by a function $u(x)$ (right). An experiment on a thin cobalt film (left) [13]; with kind permission of the authors.

spin up from a region with spin down. In a pure system at temperature $T = 0$, this domain wall is completely flat. Disorder can deform the domain wall, making it eventually rough again. Figure 1 shows how the domain wall is described by a displacement field $u(x)$. Another example is the contact line of water (or liquid helium) wetting a rough substrate. A realization with a 2-parameter displacement field $\vec{u}(\vec{x})$ is the deformation of a vortex lattice: the position of each vortex is deformed from $\vec{x}$ to $\vec{x} + \vec{u}(\vec{x})$. A three-dimensional example is the charge density wave.

All these models have in common that they are described by a displacement field $x \in \mathbb{R}^d \longrightarrow \vec{u}(x) \in \mathbb{R}^N$. For simplicity, we set $N = 1$, if not explicitly stated otherwise. After some initial coarse-graining, the energy $\mathcal{H} = \mathcal{H}_{\rm el} + \mathcal{H}_{\rm DO}$ consists of two parts: the elastic energy $\mathcal{H}_{\rm el}$ and the disorder energy $\mathcal{H}_{\rm DO}$

$$\mathcal{H}_{\rm el}[u] = \int \mathrm{d}^d x \, \frac{1}{2} \left(\nabla u(x)\right)^2 \ , \qquad \mathcal{H}_{\rm DO}[u] = \int \mathrm{d}^d x \, V(x, u(x)). \tag{1}$$

We choose the disorder at the microscopic scale to be Gaussian, with correlations

$$\overline{V(x,u)V(x',u')} := \delta^d(x - x')R(u - u') \ . \tag{2}$$

The most interesting observable is the roughness exponent ζ, obtained from the behavior of the correlation function

$$\overline{\left[u(x) - u(y)\right]^2} \sim |x - y|^{2\zeta} \ . \tag{3}$$

Other observables are higher-order correlation functions or the free energy.

3. Dimensional reduction

There is a beautiful and rather mind-boggling theorem relating disordered systems to pure systems (i.e. without disorder), which applies to a large class of systems,

e.g. random field systems and elastic manifolds in disorder. It is called dimensional reduction and reads as follows [14]:

Theorem. *A d-dimensional disordered system at zero temperature is equivalent to all orders in perturbation theory to a pure system in $d-2$ dimensions at finite temperature.*

Let me give an example: The thermal expectation value for the 2-point function scales as $\langle [u(x)-u(y)]^2 \rangle \sim |x|^{2-d}$. Making the dimensional shift implied by dimensional reduction implies that the disorder-averaged 2-point function at zero temperature is

$$\overline{[u(x)-u(0)]^2} \sim x^{4-d} \equiv x^{2\zeta}, \qquad \text{i.e.} \quad \zeta = \frac{4-d}{2} \; . \tag{4}$$

We will see later that this is not true; but remains an important benchmark due to the fact that the 'theorem' is correct to all orders in the disorder strength and its moments (i.e. when expanding in $R''(0)$, $R''''(0)$, and so on).

4. The Larkin-length

To understand the failure of dimensional reduction, let us turn to an interesting argument given by Larkin [15]. He considers a piece of an elastic manifold of size L. If the disorder has correlation length r, and characteristic potential energy $\bar{\epsilon}$, this piece will typically see a potential energy of strength $E_{\rm DO} = \bar{\epsilon}(L/r)^{d/2}$. On the other hand, there is an elastic energy, which scales like $E_{\rm el} = c\,L^{d-2}$. These energies are balanced at the Larkin-length $L = L_c$ with $L_c = (\frac{c^2}{\bar{\epsilon}^2} r^d)^{1/(4-d)}$. More important than this value is the observation that in all physically interesting dimensions $d < 4$, and at scales $L > L_c$, the membrane is pinned by disorder; whereas on small scales elastic energy dominates. This means that $d = 4$ is the upper critical dimension.

5. The functional renormalization group (FRG)

Let us now discuss a way out of the dilemma, posed by dimensional reduction: We would like to make an $\epsilon = 4-d$ expansion. On the other hand, dimensional reduction tells us that the roughness is $\zeta = (4-d)/2$ (see eq. (4)). Even though this is systematically wrong below four dimensions, it tells us correctly that at the critical

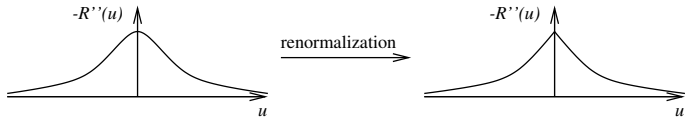


Figure 2. Change of $-R''(u)$ under renormalization and formation of the cusp.

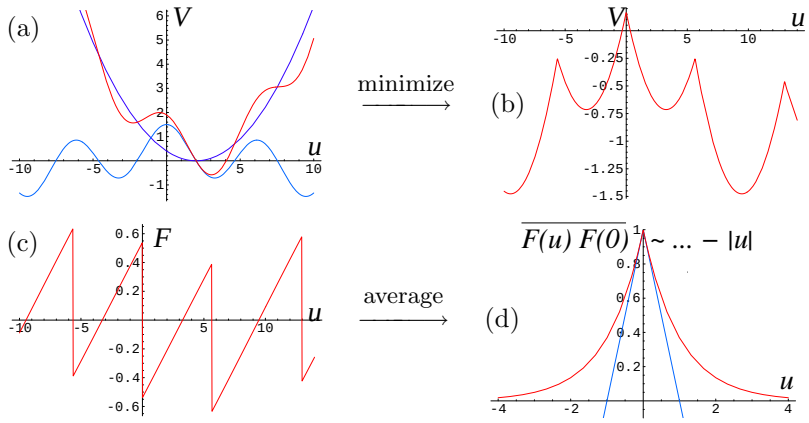


Figure 3. Generation of the cusp, as explained in the main text.

dimension $d = 4$, where disorder is marginally relevant, the field u is dimensionless. This means that having identified any relevant or marginal perturbation (as the disorder), we find immediately another such perturbation by adding more powers of the field. We can not thus restrict ourselves to keeping solely the first moments of the disorder, but have to keep the whole disorder-distribution function $R(u)$. Thus we need a functional renormalization group (FRG) treatment. Functional renormalization is an old idea going back to the seventies, and can e.g. be found in Wegner and Houghton [16]. For disordered systems, it was first proposed in 1986 by Fisher [17]. Performing an infinitesimal renormalization, i.e. integrating over a momentum shell *à la* Wilson, leads to the flow $\partial_\ell R(u)$, with $(\epsilon = 4 - d)$

$$\partial_\ell R(u) = (\epsilon - 4\zeta)\, R(u) + \zeta u R'(u) + \frac{1}{2} R''(u)^2 - R''(u) R''(0)\ . \tag{5}$$

The first two terms come from the rescaling of R and u respectively. The last two terms are the result of the 1-loop calculations, see e.g. [4].

More important than the form of this equation is its actual solution, sketched in figure 2. After some finite renormalization, the second derivative of the disorder $R''(u)$ acquires a cusp at $u = 0$; the length at which this happens is the Larkin-length. How does this overcome dimensional reduction? To understand this, it is interesting to study the flow of the second and fourth moments. Taking derivatives of (5) with respect to u and setting u to 0, we obtain

$$\partial_\ell R''(0) = (\epsilon - 2\zeta)\, R''(0) + R'''(0)^2 \ \longrightarrow\ (\epsilon - 2\zeta)\, R''(0); \tag{6}$$

$$\partial_\ell R''''(0) = \epsilon R''''(0) + 3R''''(0)^2 + 4R'''(0) R'''''(0) \ \longrightarrow\ \epsilon R''''(0) + 3R''''(0)^2. \tag{7}$$

Since $R(u)$ is an even function, $R'''(0)$ and $R'''''(0)$ are 0 as indicated in eqs (6) and (7). The above equations for $R''(0)$ and $R''''(0)$ are in fact closed. Equation

(6) tells us that the flow of $R''(0)$ is trivial and that $\zeta = \epsilon/2 \equiv \frac{4-d}{2}$. This is exactly the result predicted by dimensional reduction. The appearance of the cusp can be inferred from eq. (7). Its solution is $R''''(0)|_\ell = \frac{c\, e^{\epsilon\ell}}{1-3\,c(e^{\epsilon\ell}-1)/\epsilon}$, with $c := R''''(0)|_{\ell=0}$. Thus after a finite renormalization $R''''(0)$ becomes infinite, the cusp appears. By analyzing the solution of the flow equation (5), one also finds that beyond the Larkin-length $R''(0)$ is no longer given by (6) with $R'''(0)^2 = 0$, but $R'''(0)^2 \to R'''(0^+)^2 \equiv \lim_{u\to 0} R'''(u)^2$, which is non-zero after the cusp. Renormalization of the whole function thus overcomes dimensional reduction. The appearance of the cusp also explains why dimensional reduction breaks down. The simplest way to see this is by redoing the proof for elastic manifolds in disorder, which in the absence of disorder is a simple Gaussian theory. Terms contributing to the 2-point function involve $R''(0)$, $TR''''(0)$ and higher derivatives of $R(u)$ at $u = 0$, which all come with higher powers of T. To obtain the limit of $T \to 0$, one sets $T = 0$, and only $R''(0)$ remains. This is the dimensional reduction result. However, we just saw that $R''''(0)$ becomes infinite. Thus $R''''(0)T$ may also contribute, and the proof fails.

6. The cusp and shocks

Let us give a simple argument as to why a cusp is a physical necessity, and not an artifact. The argument is quite old and appeared probably first in the treatment of correlation functions by shocks in Burgers turbulence. It became popular in [18]. Suppose we want to integrate out a single degree of freedom, whose average position due to the elastic energy connecting it to its neighbors is u. This harmonic potential and the disorder term are represented by the parabola and the lowest curve in figure 3a, respectively; their sum is the remaining curve. For a given disorder realization, the minimum of the potential as a function of u is reported in figure 3b. Note that it has non-analytic points, which mark the transition from one minimum to another. Taking the derivative of the potential leads to the force in figure 3c. It is characterized by almost linear pieces and shocks (i.e. jumps). Calculating the force–force correlator, the dominant contribution in its decay for small distances is due to the presence of shocks. Their contribution is proportional to their probability, itself proportional to the distance between the two observable points. This leads to $\overline{F(u)F(0)} = \overline{F(0)^2} - c|u|$, with some numerical coefficient c.

7. Beyond 1-loop?

Functional renormalization has successfully been applied to a bunch of problems at 1-loop order. From a field theory, we however demand more. Namely that it allows for systematic corrections beyond 1-loop order; be renormalizable; and thus allow us to make universal predictions. However, this has been a puzzle since 1986, and it has even been suggested that the theory is not renormalizable due to the appearance of terms of order $\epsilon^{3/2}$ [19]. Why is the next order so complicated? The

reason is that it involves terms proportional to $R'''(0)$. A look at figure 3 explains the puzzle. Shall we use the symmetry of $R(u)$ to conclude that $R'''(0)$ is 0? Or shall we take the left-hand or right-hand derivatives, related by

$$R'''(0^+) := \lim_{\substack{u>0\\u\to 0}} R'''(u) = -\lim_{\substack{u<0\\u\to 0}} R'''(u) =: -R'''(0^-)? \tag{8}$$

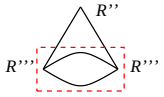
In the following, I will present the solution of this puzzle, at 2-loop order and large N. The latter approach allows for another independent control parameter, and sheds further light on the cusp formation.

8. Results at 2-loop order

For the flow equation at 2-loop order, the result is [1,4,20,21]

$$\begin{aligned}\partial_\ell R(u) = (\epsilon - 4\zeta)\, R(u) + \zeta u R'(u) + \frac{1}{2}R''(u)^2 - R''(u)R''(0)\\ + \frac{1}{2}\left(R''(u) - R''(0)\right) R'''(u)^2 - \frac{1}{2}R'''(0^+)^2 R''(u)\ .\end{aligned} \tag{9}$$

The first line is the result at 1-loop order, already given in (5). The second line is new. The most interesting term is the last one, which involves $R'''(0^+)^2$ and which we therefore call *anomalous*. The hard task is to fix the prefactor $(-\frac{1}{2})$. We have found five different prescriptions to calculate it: The sloop algorithm, recursive construction, reparametrization invariance, renormalizability, and potentiality [1,22]. For lack of space, we restrain our discussion to the last two. At 2-loop order the following diagram appears leading to the anomalous term:



$$\longrightarrow\ \frac{1}{2}\left(R''(u) - R''(0)\right) R'''(u)^2 - \frac{1}{2}R''(u)R'''(0^+)^2. \tag{10}$$

The integral (not written here) contains a subdivergence, which is indicated by the box. Renormalizability demands that its leading divergence (which is of order $1/\epsilon^2$) be canceled by a 1-loop counter-term. The latter is unique thus fixing the prefactor of the anomalous term.

Another very physical demand is that the problem remains potential, i.e. that forces still derive from a potential. The force–force correlation function being $-R''(u)$, this means that the flow of $R'(0)$ has to be strictly 0. From (8) one can check that this does not remain true if one changes the prefactor of the last term in (8), thus fixing it.

Let us give some results for random-bond disorder (short-ranged potential–potential correlation function). For this, we have to solve (9) numerically, with the result $\zeta = 0.20829804\epsilon + 0.006858\epsilon^2$. This compares well with numerical simulations (see table 1).

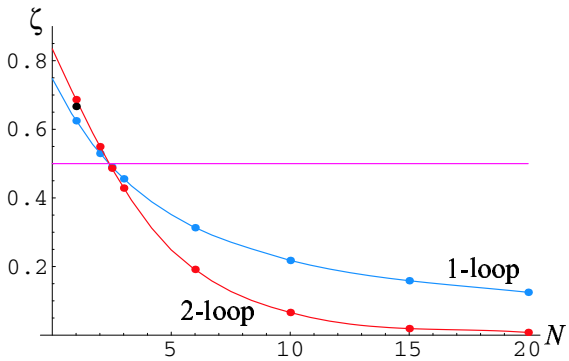


Figure 4. Results for the roughness ζ at 1- and 2-loop orders, as a function of the number of components N.

Table 1. Results for ζ in the random bond case.

ζ	One loop	Two loop	Estimate	Simulation and exact
$d = 3$	0.208	0.215	0.215 ± 0.01	0.22 ± 0.01 [23]
$d = 2$	0.417	0.444	0.42 ± 0.02	0.41 ± 0.01 [23]
$d = 1$	0.625	0.687	0.67 ± 0.02	2/3

9. Finite *N*

Up to now, we have studied the functional RG for one component $N = 1$. The general case of finite N is more difficult to handle, since derivatives of the renormalized disorder now depend on the direction, in which these derivatives are taken. Define amplitude $u := |\vec{u}|$ and direction $\hat{u} := \vec{u}/|\vec{u}|$ of the field. Then deriving the latter variable leads to terms proportional to $1/u$, which are diverging in the limit of $u \to 0$. This poses additional problems in the calculation, beyond the case $N = 1$. At 1-loop order everything is well-defined [19]. We have found a consistent RG equation at 2-loop order (see [12] and [24]).

The fixed point equation has to be integrated numerically, order by order in ϵ. The result, specialized to directed polymers, i.e. $\epsilon = 3$ is plotted in figure 4. We see that the 2-loop corrections are rather big at large N, and so some doubt on the applicability of the latter down to $\epsilon = 3$ is advised. However, both 1- and 2-loop results reproduce well the two known points on the curve: $\zeta = 2/3$ for $N = 1$ and $\zeta = 0$ for $N = \infty$. The latter result will be given in §10 Via the equivalence [25] of the directed polymer problem in N dimensions treated here and the KPZ-equation of nonlinear surface growth in N dimensions, we conclude that $d \approx 2.4$ is the upper critical dimension of KPZ.

10. Large *N*

In the last section, we have discussed renormalization in a loop expansion, i.e. expansion in ϵ. In order to independently check consistency it is good to have a

non-perturbative approach. This is achieved by the large-N limit, which can be solved analytically and to which we turn now. We start from the disorder-averaged energy with disorder correlator $B(\vec{u}^2) \equiv R(|\vec{u}|)$ where we use an N-component field $\vec{u}$. We then calculate the free energy in the presence of a source j, and finally the effective action $\Gamma(\vec{u})$ via a Legendré transform. For large N the saddle-point equation reads [2]

$$\tilde{B}'(u_{ab}^2) = B'(\chi_{ab}), \quad \chi_{ab} = u_{ab}^2 + 2TI_1 + 4I_2[\tilde{B}'(u_{ab}^2) - \tilde{B}'(0)]. \tag{11}$$

This equation gives the derivative of the effective (renormalized) disorder $\tilde{B}$ as a function of the (constant) background field $u_{ab}^2 = (u_a - u_b)^2$ in terms of the derivative of the microscopic (bare) disorder B, the temperature T and the integrals $I_n := \int_k \frac{1}{(k^2+m^2)^n}$.

The saddle-point equation can be turned into a closed functional renormalization group equation for $\tilde{B}$ by taking the derivative with respect to m (restricting ourselves to $T = 0$):

$$\partial_l \tilde{B}(x) \equiv -\frac{m\partial}{\partial m}\tilde{B}(x) = (\epsilon - 4\zeta)\,\tilde{B}(x) + 2\zeta x \tilde{B}'(x) + \frac{1}{2}\tilde{B}'(x)^2 - \tilde{B}'(x)\tilde{B}'(0). \tag{12}$$

This is a complicated nonlinear partial differential equation. It is therefore surprising that one can find an analytic solution. (The trick is to write down the flow equation for the inverse function of $\tilde{B}'(x)$, which is linear.) Let us only give the results of this analytic solution: First of all, for long-range correlated disorder of the form $\tilde{B}'(x) \sim x^{-\gamma}$, the exponent ζ can be calculated analytically as $\zeta = \frac{\epsilon}{2(1+\gamma)}$. It agrees with the replica treatment in [26] and the 1-loop treatment in [19]. For short-range correlated disorder, $\zeta = 0$. Secondly, it demonstrates that before the Larkin length, $\tilde{B}(x)$ is analytic and thus dimensional reduction holds. Beyond the Larkin length, $\tilde{B}''(0) = \infty$, a cusp appears and dimensional reduction is incorrect. This shows again that the cusp is not an artifact of the perturbative expansion, but an important property even of the exact solution of the problem (here, in the limit of large N).

11. Relation to replica symmetry breaking (RSB)

There is another treatment of the limit of large N given by Mézard and Parisi [26]. They make a Gaussian variational ansatz of the form

$$\mathcal{H}_{\rm g}[\vec{u}] = \frac{1}{2T}\sum_{a=1}^{n}\int_x \vec{u}_a(x)\left(-\nabla^2 + m^2\right)\vec{u}_a(x) - \frac{1}{2T^2}\sum_{a,b=1}^{n}\sigma_{ab}\vec{u}_a(x)\vec{u}_b(x), \tag{13}$$

which becomes exact for $N \to \infty$. The art is to make an appropriate ansatz for σ_{ab}. The simplest possibility, $\sigma_{ab} = \sigma$ for all $a \neq b$ reproduces the dimensional reduction result, which breaks down at the Larkin length. Beyond that scale,

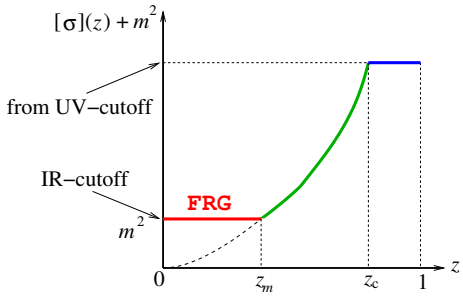
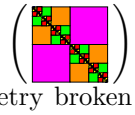


Figure 5. The function $[\sigma](u) + m^2$ as given in [26].

a replica symmetry broken (RSB) ansatz for σ_{ab} of the form $\sigma_{ab} =$  is necessary. Parisi has shown that this infinitely often replica symmetry broken matrix can be parametrized by a function $[\sigma](z)$ with $z \in [0,1]$ where $z = 0$ describes distant states, whereas $z = 1$ describes nearby states. The solution of the large-N saddle-point equations leads to the curve depicted in figure 5. Knowing it, the 2-point function is given by $\langle u_k u_{-k} \rangle = \frac{1}{k^2+m^2} \left(1 + \int_0^1 \frac{\mathrm{d}z}{z^2} \frac{[\sigma](z)}{k^2+[\sigma](z)+m^2}\right)$.

What is the relation between the two approaches, both of which pretend to calculate the same 2-point function? Comparing the analytical solutions, we find that the 2-point function given by FRG is the same as that of RSB, if in the latter expression we only take into account the contribution from the most distant states, i.e. those for z between 0 and z_m (see figure 5). To understand why this is so, we have to remember that the two calculations were done under quite different assumptions. In contrast to the RSB calculation, the FRG approach calculated the partition function in the presence of an external field j, which was then used to give the effective action via a Legendre transformation. Even if the field j is finally turned to 0, the system will remember its preparation, as is the case for a magnet.

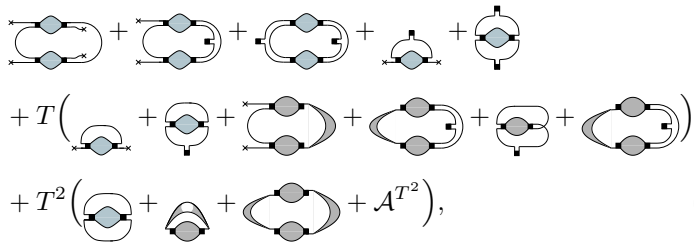
By explicitly breaking the replica symmetry through an applied field, all replicas will settle in distant states, and the close states from the Parisi function $[\sigma](z)+m^2$ (which describes *spontaneous* RSB) will not contribute. However, we found that the full RSB result can be reconstructed by remarking that the part of the curve between z_m and z_c is independent of the infra-red cut-off m, and then integrating over m [2] (m_c is the mass corresponding to z_c):

$$\langle u_k u_{-k} \rangle \Big|_{k=0}^{\mathrm{RSB}} = \frac{\tilde{R}'_m(0)}{m^4} + \int_m^{m_c} \frac{\mathrm{d}\tilde{R}'_\mu(0)}{\mu^4} + \frac{1}{m_c^2} - \frac{1}{m^2} \ . \tag{14}$$

We also note that a similar effective action has been proposed in [18]. While it agrees qualitatively, it does not reproduce the correct FRG 2-point function, as it should.

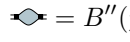
12. Corrections at order $1/N$

In a graphical notation, we find [11]

$\delta B^{(1)} =$ 

$$+ T\Big($$ $$\Big)$$

$$+ T^2\Big($$ $$+ \mathcal{A}^{T^2}\Big), \qquad (15)$$

 $= B''(\chi_{ab})\,(1 - 4A_d I_2(p) B''(\chi_{ab}))^{-1}$,  $= B(\chi_{ab}), \qquad (16)$

where the explicit expressions are given in [11]. By varying the IR-regulator, one can derive a β-function at order $1/N$, see [11]. At $T = 0$, it is UV-convergent, and should allow us to find a fixed point. We have been able to do this at order ϵ, showing consistency with the 1-loop result, see §9. Other dimensions are more complicated.

A β-function can also be defined at finite T. However, since temperature is an irrelevant variable, it makes the theory non-renormalizable, i.e. in order to define it, one must keep an explicit infra-red cut-off. These problems will be treated in a forthcoming publication.

13. Perspectives

Other interesting problems have been treated by the above methods, especially dynamic problems (see [12] for a review); and many more are now in reach. Some open points have already been raised in these notes, others are the strong disorder phase of random field problems, or whether FRG can also be applied to spin-glasses. We have to leave these problems for future research and as a challenge for the reader to plunge deeper into the mysteries of functional renormalization.

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