A new quantum glass phase: the superglass

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Outline

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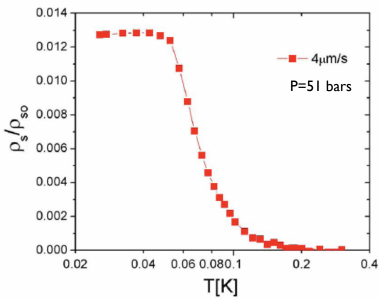
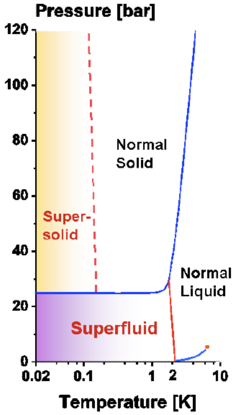
Motivations: supersolidity of He^4

Non-classical rotational inertia observed in solid He^4 (Kim and Chan)





Possible interpretation: supersolidity

- Supersolidity excluded in perfect He^4 crystals (Boninsegni, Ceperley et al.)
- Supersolidity strongly enhanced by fast quenches (Rittner and Reppy)
- History dependent response and some evidence for aging (Davis et al.)

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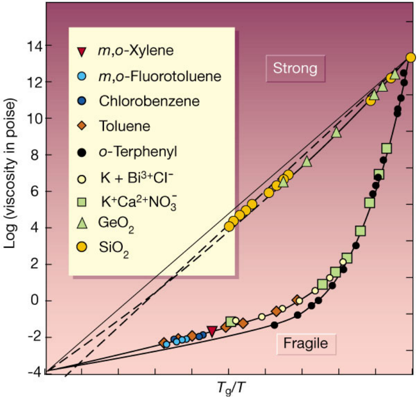
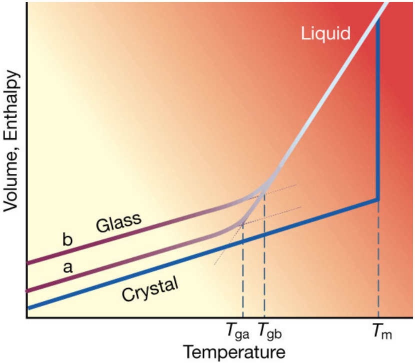
Phenomenology

Classical particle system (e.g. Lennard-Jones like potential)
No external disorder

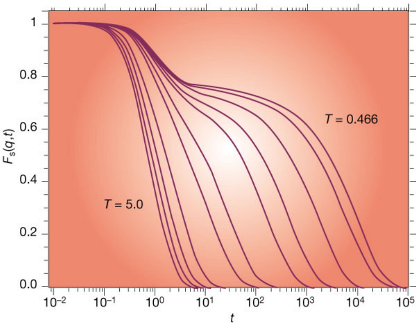




Huge increase of the viscosity (or density relaxation time) in a small range of temperature

Second order phase transition: jump in compressibility

Phenomenology



Two steps relaxation:

1. Intra-cage vibrational motion (τ_β)
2. Structural relaxation (τ_α)

First six decades of dynamic slowing down is well described by Mode-Coupling Theory (MCT)

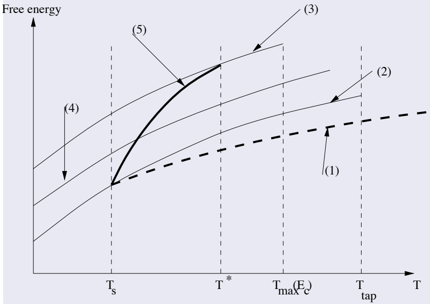
- MCT predicts power-law divergence, $\tau \sim (T - T_c)^{-\gamma}$, with too large T_c
- The divergence is "activated" $\tau \sim \exp(A/(T - T_0))$ instead
- Activation is neglected in MCT (mean field theory)

Mean field spin glass models

A mean field model for the glass transition: the *p-spin model*:

$$H = \sum_{i<j<k} J_{ijk} S_i S_j S_k$$

S_i Ising spins
J_{ijk} independent Gaussian random variables with zero average



- Liquid phase: dynamics is described by MCT-like equations
- "Activated" liquid phase: $e^{N\Sigma}$ states are populated
- Glass phase: "condensation", finite number of ground states

In a suitable limit (infinite number of spin in each interaction) reduces to the Random Energy Model (REM): 2^N levels E_i, i.i.d. Gaussian variables

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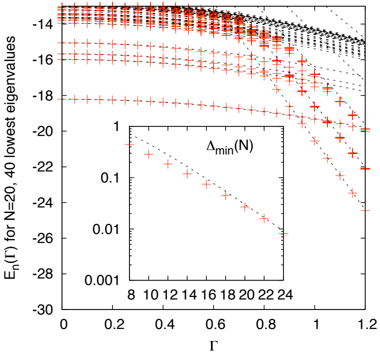
Quantum p-spin and QREM

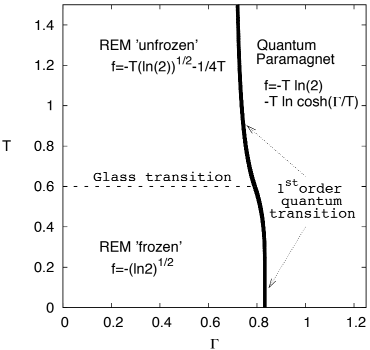
Quantum p-spin in a transverse field: (Goldschimdt; Cugliandolo et al.; Jorg et al.)

$$H = \sum_{i<j<k} J_{ijk} S_i^z S_j^z S_k^z - \Gamma \sum_i S_i^x$$

For infinite-body interaction: quantum REM, full spectrum

First order quantum phase transition (paramagnet $\rightarrow$ glass) at $T = 0$

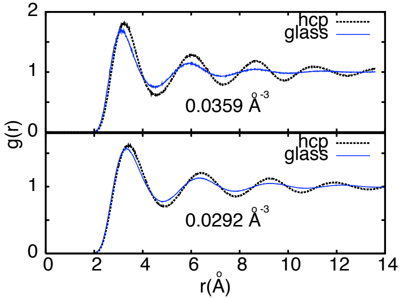




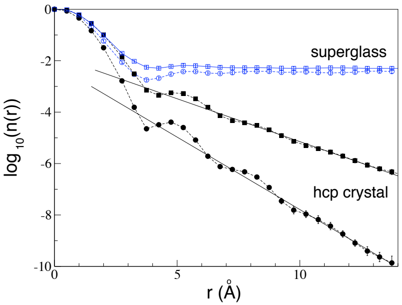
At $T = 0$, slow dynamics in the glass but not in the paramagnet; no slowing down observed on approaching Γ_c from above.

Helium 4: Monte Carlo results

Quantum Monte Carlo simulation of He^4 at high pressure $P > 32$ bar
Quench from the liquid phase down in the solid phase (Boninsegni et al.)



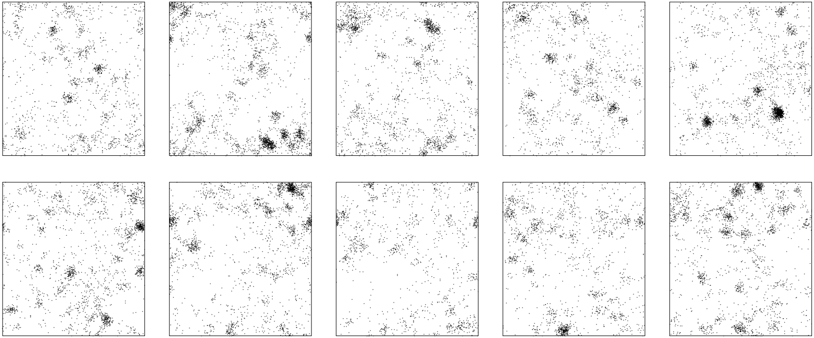
Density-density correlations similar to the liquid (large Lindemann ratio)



ODLRO observed in the one-particle density matrix → BEC, superfluidity
At $P = 32$ bar, $n_0 = 0.5\%$ and $\rho_s/\rho = 0.6$

Helium 4: Monte Carlo results

Amorphous condensate wavefunction: $n(r - r') \sim n_0 \phi(r) \phi(r')$



Plot of $\phi(x, y, z)$ on slices at fixed z

Many open problems

What is the nature of the transition?
Is it accompanied by slow dynamics in the liquid phase?
Where does superfluidity come from?

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Mapping on classical diffusive dynamics

- General mapping: Quantum Hamiltonian $\Leftarrow$ Fokker-Planck operator
- Diffusive dynamics (Brownian motion, Langevin equation):

$$\gamma_i \frac{d\mathbf{x}_i}{dt} = -\frac{\partial}{\partial \mathbf{x}_i} U_N(\mathbf{x}_1, \dots, \mathbf{x}_N) + \boldsymbol{\eta}_i(t) \ , \qquad i = 1, ..., N \ ,$$

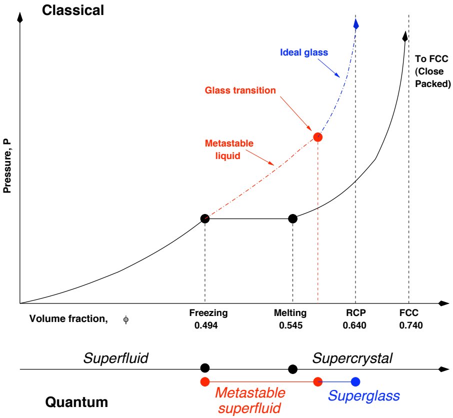
- Evolution of probability $P(\mathbf{x}_i; t)$: Fokker-Planck eq. $\partial_t P = -H_{FP} P$
- Equilibrium distribution $P_G = \exp(-\beta U_N)/Z$, $H_{FP} P_G = 0$
  All other eigenvectors $H_{FP} P_E = E\, P_E$ such that $E > 0$
- Associated quantum (Hermitian) Hamiltonian: $H = P_G^{-1/2}\, H_{FP}\, P_G^{1/2}$
- Ground state $\Psi_G(\mathbf{x}_i) = \sqrt{P_G(\mathbf{x}_i)}$ is a *Jastrow wavefunction*
  Full spectrum of H equal to spectrum of H_{FP} $\Rightarrow$ access to real time quantum dynamics

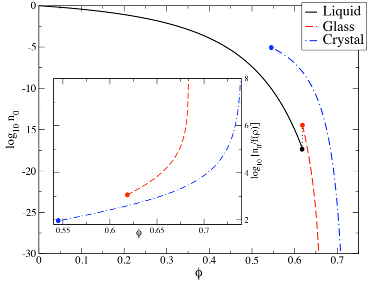
Remarks:

◇ H has special properties! No inverse mapping in general...
◇ Jastrow wavefunctions are good variational ground states for He^4

The phase diagram

We choose $U_N(\mathbf{x}_i) = \sum_{i<j} V_{HS}(\mathbf{x}_i - \mathbf{x}_j)$ (classical Hard Spheres)
Quantum potential: sticky Hard Sphere + sticky three-body interactions
Glass transition on increasing density





Solid phases are "classical": small Lindemann ratio
Finite n_0 but very small in both crystal and glass phases

Slow dynamics approaching the glass phase

Density-density correlation function:

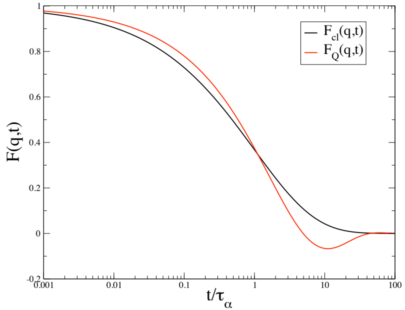
- $F_{cl}(q,t) = \langle \rho_q(t)\, \rho_{-q}(0) \rangle = \int_0^\infty \frac{d\omega}{2\pi}\, \rho_q(\omega)\, e^{-\omega t}$
- $F_Q(q,t) = \langle 0 | \{ \rho_q(it), \rho_q(0) \} | 0 \rangle = \int_0^\infty \frac{d\omega}{2\pi}\, \rho_q(\omega) \cos(\omega t)$

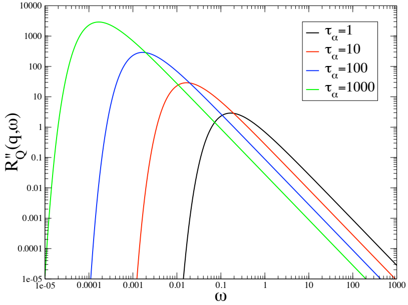
Separation of time scales: $\rho_q(\omega) = \rho_\beta(\omega \tau_\beta) + \rho_\alpha(\omega \tau_\alpha)$ with $\tau_\beta \ll \tau_\alpha$

For $\tau_\beta \ll t \ll \tau_\alpha$:

- the contribution of $\rho_\beta(\omega\tau_\beta)$ decays to zero
- the contribution of $\rho_\alpha(\omega\tau_\alpha)$ is the same since $e^{-\omega t} \sim \cos(\omega t) \sim 1$

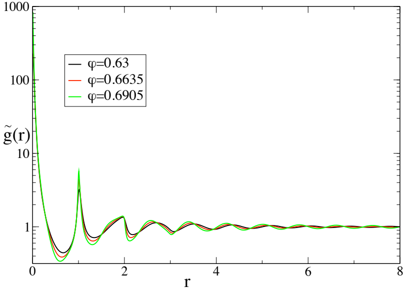
hence $F_{cl}(q,t) \sim F_Q(q,t) \sim \int_0^\infty \frac{d\omega}{2\pi}\, \rho_\alpha(\omega\tau_\alpha) \Rightarrow$ same plateau!



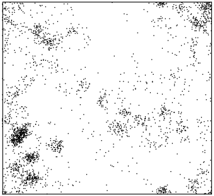


Condensate fluctuation in the glass

In the glass state $\tau_\alpha = \infty \rightarrow$; liquid freezes in many possible states
Amorphous density profile $\rho_\alpha(r)$ and condensate profile $\phi_\alpha(r)$



$g_\phi(r - r') \propto \sum_\alpha p_\alpha \phi_\alpha(r)\phi_\alpha(r')$
correlation function of condensate fluctuations



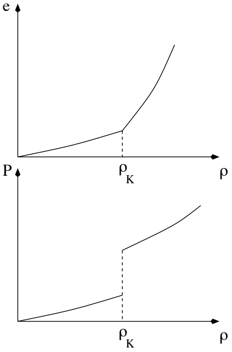
Superfluid properties

Superfluidity requires a linear spectrum ("phonons"): $v_c \leq \min_k[\epsilon(k)/k]$

In our model $e(\rho) \equiv 0 \Rightarrow$ sound velocity $c = \frac{d}{d\rho}\rho^2 \frac{de}{d\rho} = 0 \Rightarrow v_c = 0$

(follows from a special symmetry that allows to map H into a Fokker-Planck operator)

Introduce a perturbation $\delta v(r)$; then $\delta e(\rho) = \frac{\rho}{2} \int dr\, g(r)\, \delta v(r)$



- sound velocity $c \neq 0 \Rightarrow \rho_s \neq 0$
- first order transition at ρ_K
  [very weak jump in $e'(\rho) = P/\rho^2$]

Perspectives

Weak points in the theory:

- "Classical"-like solids, small Lindemann ratio and superfluid fraction
- "Ad hoc" inclusion of phonons
- New quantum phase transition: first order with slow dynamics.
  How general?
- Quantitative computation for He^4, cold atoms...
  [ρ_K for He^4 is 10 times larger than the one of Boninsegni et al.]
- What happens at finite temperature?

Possible strategies:

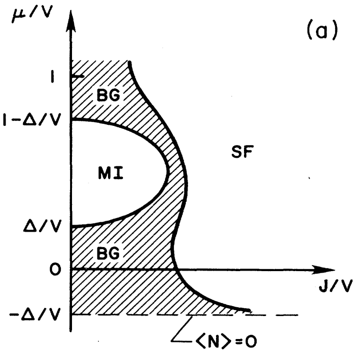
- Better variational wavefunctions: Shadow and Jastrow with three body interactions; should give larger Lindemann ratio and ρ_s
- Quantum Mode Coupling Theory (Reichmann and Miyazaki)
- Replica computation at finite temperature
- Leggett bound: relation between $\rho(r)$ and ρ_s, apply to superglass
  It seems that disorder does not help superfluidity

Outline

Disordered Bose-Hubbard model: the Bose glass

$$H = -J \sum_{<i,j>} (a_i^\dagger a_j + a_j^\dagger a_i) + \frac{U}{2} \sum_i n_i(n_i - 1) - \sum_i (\mu + \varepsilon_i) n_i$$

$\varepsilon_i \in [-\Delta, \Delta]$ *quenched external disorder*



- Mott insulator: one particle/site
  Strong localization $\Rightarrow$ no BEC, $\rho_s = 0$
  Zero compressibility
- Bose glass: additional defects
  Anderson localization
  Finite compressibility

No frustration, no RSB
No slow dynamics

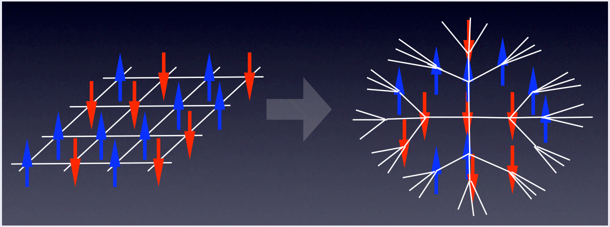
Quantum Biroli-Mézard model: a superglass?

$$H = -J \sum_{<i,j>} (a_i^\dagger a_j + a_j^\dagger a_i) + \sum_{<i_1,\cdots,i_k>} V(n_{i_1},\cdots,n_{i_k}) - \sum_i \mu n_i$$

Classical model ($J = 0$): glass transition similarly to Hard Spheres
Self-generated disorder, RSB, slow dynamics

Add quantum fluctuations ($J \neq 0$)

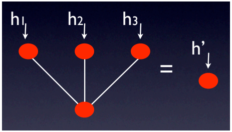
A quantum glass transition? Slow dynamics? Aging?
Nature of the transition (first or second order)?

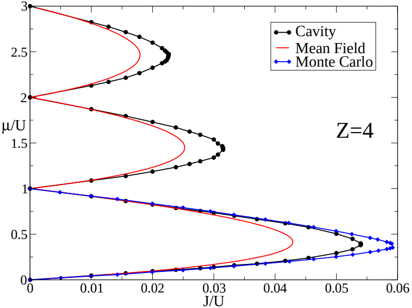


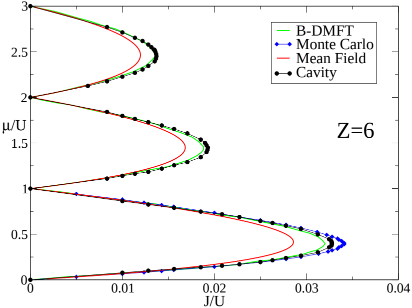
Strategy: solve the model on the Bethe lattice

Solution of Bose-Hubbard models on the Bethe lattice

- Solution of functional recurrence equations for the local action
- Gives back DMFT for $Z \to \infty$
- Successfully tested on the ordered Bose-Hubbard







Work in progress... (with G. Semerjian and M. Tarzia)

Conclusions

Our results:

- A semi-realistic model for interacting Bosons displays a superglass phase
- First order quantum glass transition with real time slow dynamics
- Variational calculation for more realistic potentials
- Possibility of exact solution for Bethe lattice models

Related works:

- Quantum Mode Coupling Theory (Reichmann, Miyazaki)
- B-DMFT (Vollhardt, Hofstetter, et al.)
- Monte Carlo simulations (Boninsegni, Prokof'ev, Svistunov, et al.)