Bose-Einstein condensation in Quantum Glasses

Giuseppe Carleo, Marco Tarzia, and Francesco Zamponi*
Phys. Rev. Lett. 103, 215302 (2009)

Collaborators: Florent Krzakala, Laura Foini, Alberto Rosso, Guilhem Semerjian

*Laboratoire de Physique Théorique, Ecole Normale Supérieure, 24 Rue Lhomond, 75231 Paris Cedex 05

March 29, 2010

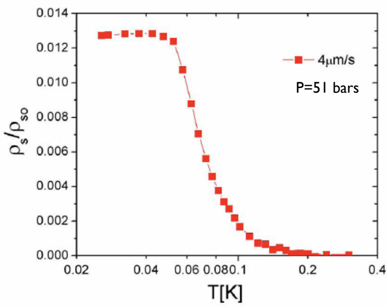
Outline

Outline

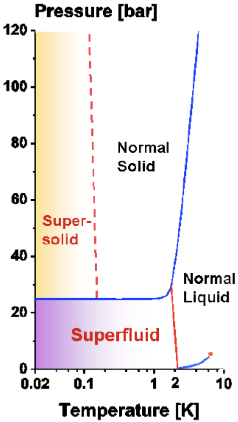
1. Motivations
   - Supersolidity of He^4
   - Helium 4: Monte Carlo results
2. The quantum cavity method
   - Regular lattices, Bethe lattices and random graphs
   - Recursion relations
   - Bose-Hubbard models on the Bethe lattice
3. A lattice model for the superglass
   - Extended Hubbard model on a random graph
   - Results
   - A variational argument
4. Discussion
   - Disordered Bose-Hubbard model: the Bose glass
   - 3D spin glass model with quenched disorder
   - Quantum Biroli-Mézard model: a lattice glass model
5. Conclusions

Motivations: supersolidity of He^4

Non-classical rotational inertia observed in solid He^4 (Kim and Chan)



Possible interpretation: supersolidity

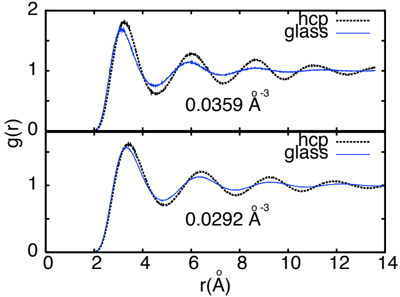


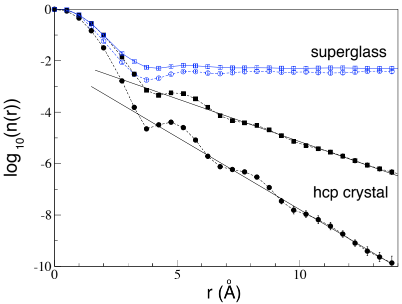
- Supersolidity excluded in perfect He^4 crystals (Boninsegni, Ceperley et al.)
- Supersolidity strongly enhanced by fast quenches (Rittner and Reppy)
- History dependent response and some evidence for aging (Davis et al.)

Are BEC and superfluidity possible in disordered solids?

Helium 4: Monte Carlo results

Quantum Monte Carlo simulation of He^4 at high pressure $P > 32$ bar Quench from the liquid phase down in the solid phase





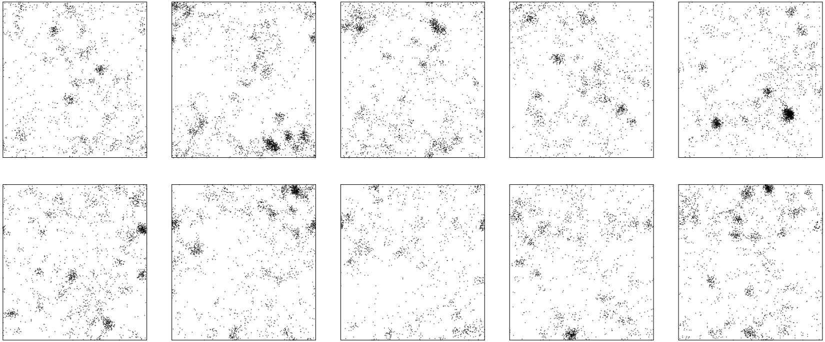
Density-density correlations similar to the liquid (large Lindemann ratio)

BONINSEGNI ET AL., PRL 96, 105301 (2006)

ODLRO observed in the one-particle density matrix → BEC, superfluidity
At $P = 32$ bar, $n_0 = 0.5\%$ and $\rho_s/\rho = 0.6$

Helium 4: Monte Carlo results

Amorphous condensate wavefunction: $n(r - r') \sim n_0 \phi(r)\phi(r')$



Plot of $\phi(x, y, z)$ on slices at fixed z

Boninsegni et al., PRL 96, 105301 (2006)

Many open problems

Difficulties of QMC

- Strong ergodicity problems in the glass phase
- No access to real time dynamics: is this phase really metastable?
- Difficulty in determining the phase boundary: what is the phase diagram?

Open questions

- What are the physical ingredients (disorder, frustration)?
- What is the nature of the transition?
- Is it accompanied by slow dynamics in the liquid phase?
- Where does superfluidity come from?
- Strong interaction and disorder: need for a nonperturbative analysis

Our result – G.Carleo, M.Tarzia, FZ, PRL 103, 215302 (2009)

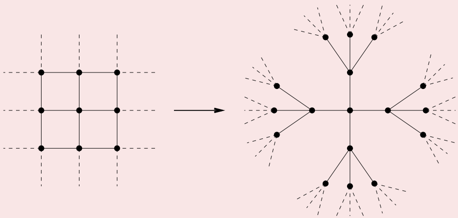
- A solvable model displaying a thermodynamic superglass phase
- Disorder is self-induced by frustration *in absence of external potentials*
- Realistic mechanism for He^4

Outline

Regular lattices, Bethe lattices and random graphs

Bethe approximation

Discard the small loops of the lattice, graph becomes a tree:

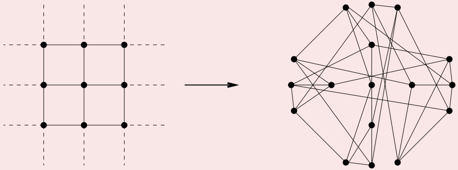


The tree allows for a simple recursive solution but the frustration is lost

Regular lattices, Bethe lattices and random graphs

Cavity approximation

Replace the lattice by a random graph of the same connectivity:

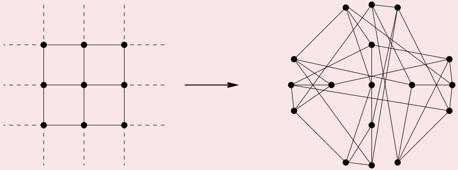


The exact solution on the random graph can still be obtained (cavity method)
Key property: loops have size $\log L$, locally tree-like

Regular lattices, Bethe lattices and random graphs

Cavity approximation

Replace the lattice by a random graph of the same connectivity:



The exact solution on the random graph can still be obtained (cavity method)
Key property: loops have size $\log L$, locally tree-like

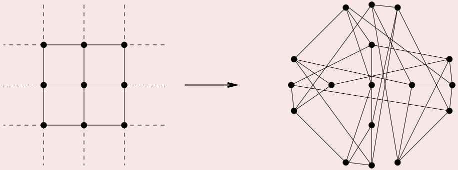
Unfrustrated phase (RS cavity method)

- Correlations decay fast enough
- Long loops can be neglected $\longrightarrow$ back to a tree
- Cavity approximation is equivalent to Bethe approximation

Regular lattices, Bethe lattices and random graphs

Cavity approximation

Replace the lattice by a random graph of the same connectivity:



The exact solution on the random graph can still be obtained (cavity method)
Key property: loops have size $\log L$, locally tree-like

Frustrated phase (1RSB cavity method)

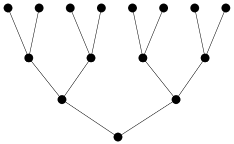
- Correlations do not decay fast enough
- The recursion relation on a tree is initiated from a given boundary condition
- For each boundary condition, a different fixed point is obtained
- One has to sum over boundary conditions in a consistent way
- Cavity approximation describes the glassy phase

Classical ferromagnet on the Bethe lattice

$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j$

$Z_{g+1}(\sigma) = \sum_{\sigma_1,\dots,\sigma_{z-1}} Z_g(\sigma_1) \dots Z_g(\sigma_{z-1})\, e^{\beta J \sigma(\sigma_1 + \dots + \sigma_{z-1})}$

Normalized probability : $\eta_g(\sigma) = \frac{Z_g(\sigma)}{Z_g(+)+Z_g(-)} = \frac{e^{\beta h_g \sigma}}{2\cosh(\beta h_g)}$



Recursion on the effective magnetic field :

$$h_{g+1} = \frac{z-1}{\beta} \mathrm{atanh}\left(\tanh(\beta J)\tanh(\beta h_g)\right)$$

Fixed point when $g \to \infty$: (infinitesimal field to break the symmetry)

- $h = 0$ at high temperature
- $h \neq 0$ at low temperature

True magnetic field can be recovered with z instead of $z-1$ neighbors on the root

Quantum lattice models

$$H = -J\sum_{\langle i,j\rangle}(a_i^\dagger a_j + a_j^\dagger a_i) + V(\underline{n}) = K + V(\underline{n})$$

$$Z = \mathrm{Tr}\, e^{-\beta H} \qquad |\underline{n}\rangle = |n_1, \dots, n_N\rangle \qquad \mathbb{1} = \sum_{\underline{n}} |\underline{n}\rangle\langle\underline{n}|$$

Quantum model ⇔ Classical model with one additional dimension (imaginary time)

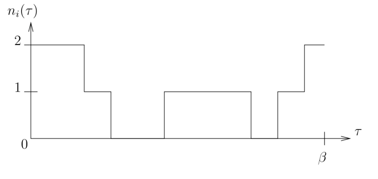
$$Z = \lim_{M\to\infty} \sum_{\underline{n}^1,\dots,\underline{n}^M} \exp\left[-\frac{\beta}{M}\sum_{\alpha=1}^{M} V(\underline{n}^\alpha)\right] \prod_{\alpha=1}^{M} \left\langle \underline{n}^\alpha | e^{-\frac{\beta}{M}K} | \underline{n}^{\alpha+1} \right\rangle$$

becomes a path integral :

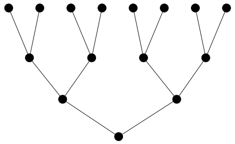
$$Z = \int \prod_{i=1}^{N} Dn_i(\tau) \exp\left[-\int_0^\beta d\tau V(\underline{n}(\tau))\right] \mathcal{W}[\underline{n}(\tau)]$$

where the $n_i(\tau)$ are piecewise constant integer functions (periodic)

- For a hopping h at time τ_h, $n_i \to n_i + 1$, and one of the neighbors j has $n_j \to n_j - 1$
- $\mathcal{W}_h = J\sqrt{n_i(\tau_h)+1}\sqrt{n_j(\tau_h)}$.
- $\mathcal{W}[\underline{n}(\tau)] = \prod_h \mathcal{W}_h$



Quantum lattice model on the Bethe lattice



- Path integral construction valid for any graph
- Classical degree of freedom = trajectories $n_i(\tau)$
- Recursive computation on trees, formally similar, but $n(\tau)$ (function) instead of $\sigma \in \{+1,-1\}$

$$Z_{g+1}(\sigma) = \sum_{\sigma_1,\ldots,\sigma_{z-1}} Z_g(\sigma_1)\ldots Z_g(\sigma_{z-1})\, e^{\beta J\sigma(\sigma_1+\cdots+\sigma_{z-1})}$$

becomes

$$Z_{g+1}[n(\tau)] = \int Dn_1(\tau) Z_g[n_1(\tau)]\ldots Dn_{z-1}(\tau) Z_g[n_{z-1}(\tau)]\; w[n,n_1,\ldots,n_{z-1}]$$

The quantum cavity method

- Functional recurrence equations for the local action $Z_g[n(\tau)] = \exp\{-S[n(\tau)]\}$
- A quadratic action gives back DMFT (correct for $z\to\infty$)
  Laumann, Scardicchio, Sondhi (2008), Byczuk, Vollhardt (2008)

Numerical resolution for bosons and spins – Krzakala, Rosso, Semerjian, FZ (2008)

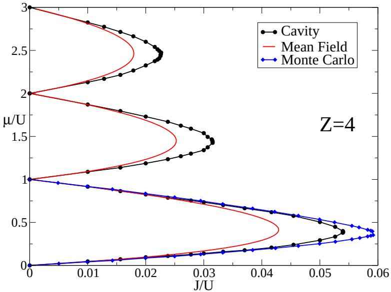
- Represent $Z[n(\tau)]$ by a weighted sample of trajectories
- Use the iteration equation to construct a new sample
- Works only if $Z[n(\tau)]$ is a probability
- Numerical method similar to QMC or GFMC, but here $L=\infty$ (no FSS)

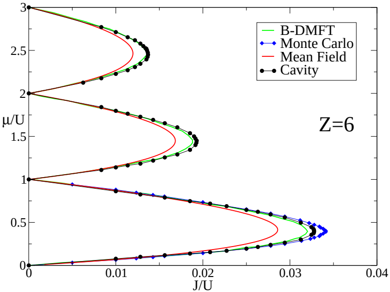
Bose-Hubbard models on the Bethe lattice

Test: study of the Mott transition in the Bose-Hubbard model

G. SEMERJIAN, M. TARZIA, FZ, PRB 80, 014524 (2009)

$$H = -J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i) + \frac{U}{2} \sum_i n_i(n_i - 1) - \sum_i \mu n_i$$





We can compute many observables

- Local density $\langle n_i \rangle$ and condensate wavefunction $\langle a_i \rangle$
- Local green function $\left\langle a_i^\dagger(\tau) a_i(0) \right\rangle$ and correlation $\langle n_i(\tau) n_i(0) \rangle$
- Spatial correlations $\left\langle a_i^\dagger a_j \right\rangle$

Outline

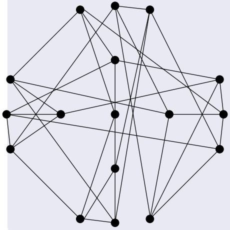
A lattice model for the superglass

Extended Hubbard model on a regular random graph at half-filling and $U = \infty$:

$$H = -J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i) + V \sum_{\langle i,j \rangle} n_i n_j - \sum_i \mu n_i$$

G. Carleo, M. Tarzia, FZ, PRL 103, 215302 (2009)

We study the model on a regular random graph of L sites and connectivity $z = 3$

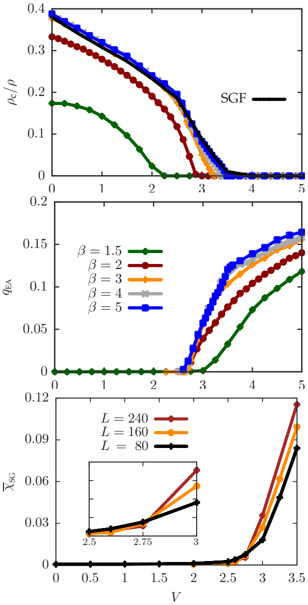


- No disorder in the interactions
- Frustration: loops of even and odd size
- Large loops, locally tree-like graph: *Bethe lattice without boundary*
- Solution for $L \to \infty$ possible via the cavity method
- Classical model ($J = 0$): spin glass transition (like Sherrington-Kirkpatrick model)
- Glass phase is thermodynamically stable
- RSB, many degenerate glassy states, slow dynamics

Methods

- Quantum cavity method: solution for $L \to \infty$
- Canonical Worm Monte Carlo: limited to $L < 240$ by ergodicity problems
- Variational calculation + Green Function Monte Carlo (for illustration)

Results at half-filling and $J = 1$



Condensate fraction:

$$\rho_c = \lim_{|i-j|\to\infty} \left\langle a_i^+ a_j \right\rangle = |\langle a \rangle|^2$$

Edwards-Anderson order parameter:

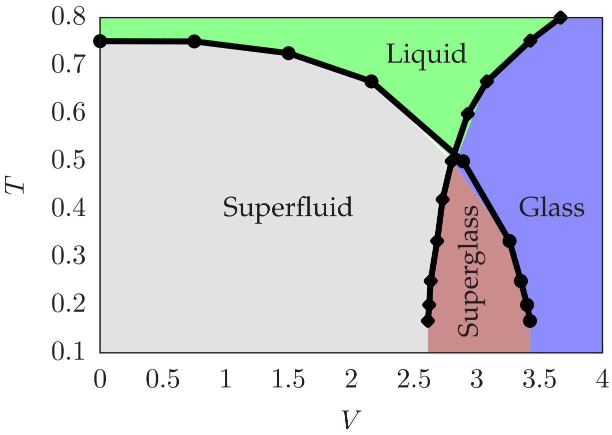
$$q_{EA} = \frac{1}{L} \sum_i \left\langle (\delta n_i)^2 \right\rangle \qquad \delta n_i = n_i - \langle n_i \rangle$$

Spin-Glass susceptibility:

$$\chi_{SG} = \frac{1}{L} \int_0^\beta d\tau \sum_{i,j} \left\langle \delta n_i(\tau) \delta n_j(0) \right\rangle^2$$

Phase diagram

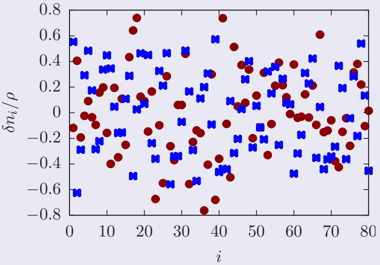
Phase diagram at half-filling and $J = 1$



A variational argument

Many spin glass states

- Each spin glass state breaks translation invariance
- Simplest variational wavefunction: $\langle \underline{n} | \Psi \rangle = \exp(\sum_i \alpha_i n_i)$
- A different set of parameters for each spin glass state
- Optimization of the parameters α_i depends on the initial condition



Stability of the glass state

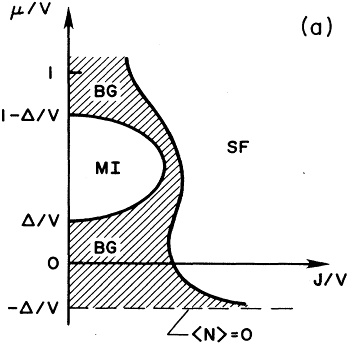
- Green Function Monte Carlo: $|\Psi(\tau)\rangle = e^{-\tau H}|\Psi\rangle$
- The time τ needed to escape from the initial state $|\Psi\rangle$ increases with system size

Outline

Disordered Bose-Hubbard model: the Bose glass

$$H = -J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i) + \frac{U}{2} \sum_i n_i(n_i - 1) - \sum_i (\mu + \varepsilon_i) n_i$$

$\varepsilon_i \in [-\Delta, \Delta]$ *quenched external disorder*



- Mott insulator: one particle/site
  Strong localization $\Rightarrow$ no BEC, $\rho_c = 0$
  Zero compressibility
- Bose glass: additional defects
  Anderson localization, $\rho_c = 0$
  Finite compressibility

No frustration, no RSB
No slow dynamics

Fisher, Weichman, Grinstein, Fisher, PRB 40, 546 (1989)

Open question

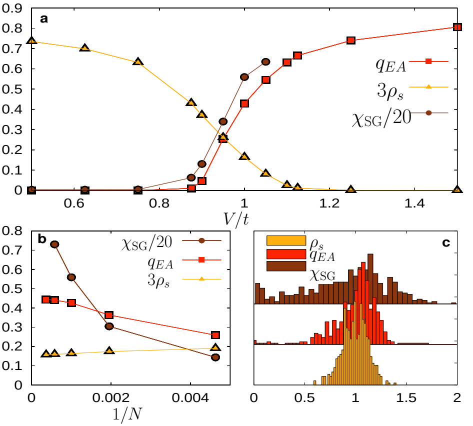
Does the Bose Glass phase exist on the Bethe lattice? Ioffe and Mézard, arXiv:0909.2263

3D spin glass model with quenched disorder

$$H = -J\sum_{\langle i,j\rangle}(a_i^\dagger a_j + a_j^\dagger a_i) + \sum_{\langle i,j\rangle} V_{ij}(n_i - 1/2)(n_j - 1/2)$$

Tam, Geraedts, Inglis, Gingras, Melko, arXiv:0909.1845

QMC for the 3D cubic lattice with couplings $V_{ij} = \pm V$



- Same phase diagram
- But frustration is induced by quenched disorder

Quantum Biroli-Mézard model: a lattice glass model

Towards a more realistic model of structural glasses: the Biroli-Mézard model

$$H = -J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i) + V \sum_i n_i q_i \theta(q_i) - \sum_i \mu n_i$$
$$q_i = \sum_{j \in \partial i} n_j - \ell$$

Classical model ($J = 0$): glass transition similarly to Hard Spheres

- Random First Order Transition: discontinuous q_{EA}, 2nd order phase transition
- Glassy phase both on 3D cubic and Bethe lattices (quantitatively similar)
- Self-generated disorder and RSB
- Very slow dynamics (divergence stronger than power-law)
- Believed (by some) to be in the same universality class of particle systems (e.g. Lennard-Jones, Hard Spheres): *RFOT theory of the glass transition*

Add quantum fluctuations ($J \neq 0$)

- A quantum glass transition? Slow dynamics? Aging?
- Nature of the transition (first or second order)?
- Preliminary indications of a quite complex phase diagram
- Work in progress – L. Foini, G. Semerjian, FZ
- Many body interaction could be realized in experiment with cold molecules
  Buchler, Micheli, Zoller, Nature Physics 3, 726 (2007)

Outline

Conclusions

Our results

- Quantum cavity method: a general method to deal with frustrated boson and spin systems
- Exact solution for Bethe lattice models for $L \to \infty$
- Semi-realistic model for interacting bosons displays a frustration-induced superglass phase
- Second order spin glass like transition
- Thermodynamically stable glass phase

Related works

- Quantum Mode Coupling Theory – Reichmann, Miyazaki
- B-DMFT – Vollhardt, Hofstetter, et al.
- Monte Carlo simulations – Boninsegni, Prokof'ev, Svistunov, et al.
- Variational calculations – Biroli, Chamon, FZ

Perspectives

- Experiments: He^4 and cold molecules
- Lattice glass models for structural glasses – Foini, Semerjian, FZ
- Simulations of binary mixtures – Biroli, Carleo, Tarzia, FZ