Yielding transition
A dynamical perspective
(energy landscape is not everything)

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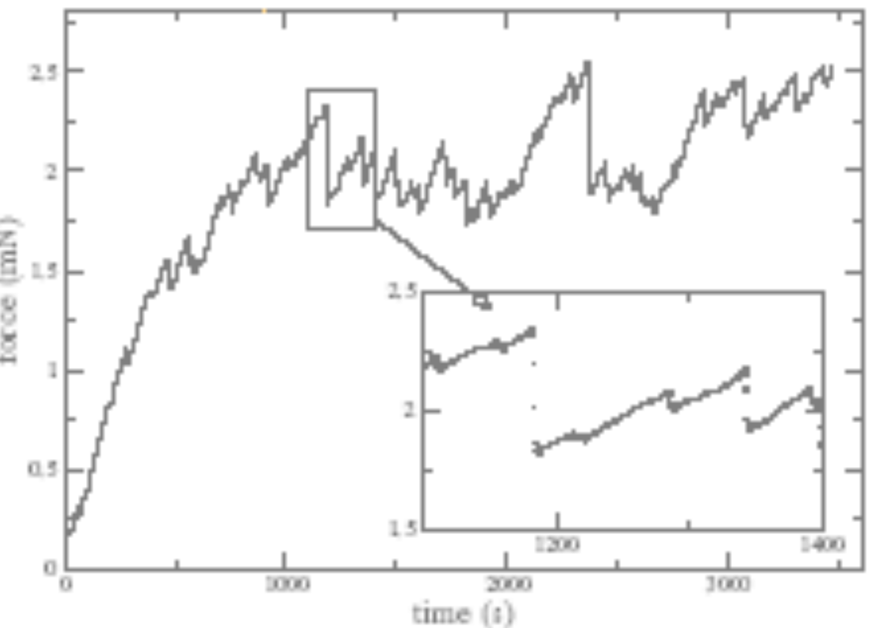
Outline

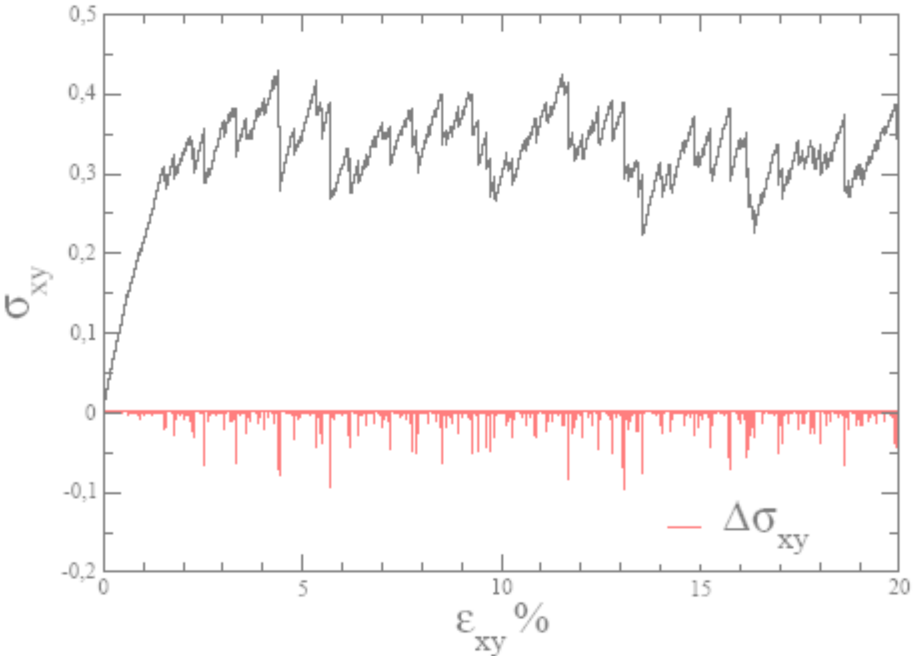
- Elastoplastic models
- Mean field treatments: Hébraud Lequeux, SGR
- Strain localisation vs continuous transitions
- Strain localisation in inertial systems
- Strain localisation in granular systems
- Creep

Deformation of amorphous systems at low T proceeds through well identified *plastic events* or *shear transformations* (Argon and Kuo, 1976)

Stress-strain curve at low strain rate, low temperature, small systems



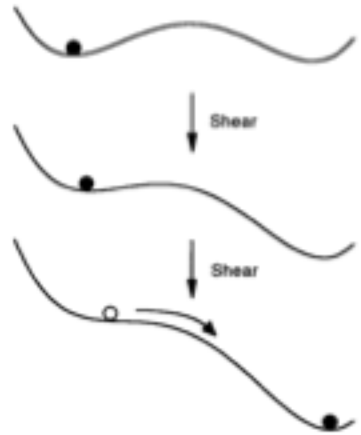
Plastic response of a foam (I. Cantat, O. Pitois, Phys. of fluids 2006)



Plastic response of a simulated Lennard-Jones glass (Tanguy, Leonforte, JLB, EPJ E 2006)

Events are **shear transformations** of Eshelby type

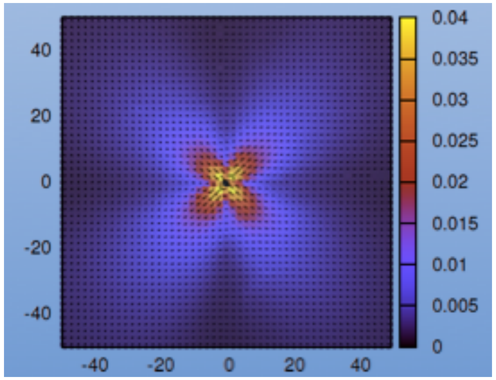
•Plastic instability in a very local region of the medium (irreversible) under the influence of the local stress.



Malandro, Lacks, PRL 1998

•Instability involves typically a few tens of particles and small shear strains (1 to 10%)

•Surroundings respond essentially as an homogeneous elastic medium (incompressible). Quadrupolar symmetry of the response.



Puosi, Rottler, JLB, PRE 2014

Events are **shear transformations** of Eshelby type

Proc. R. Soc. Lond. A 1957 **241**, 376-396

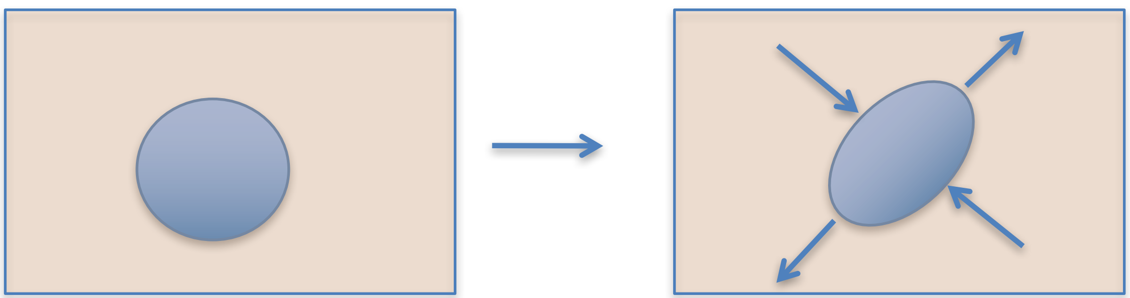
The determination of the elastic field of an ellipsoidal inclusion, and related problems

By J. D. Eshelby

Department of Physical Metallurgy, University of Birmingham

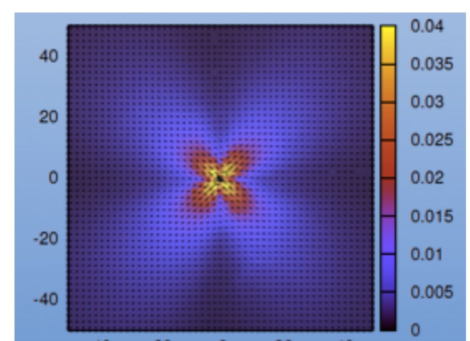
(*Communicated by R. E. Peierls, F.R.S.—Received* 1 *March* 1957)

Eshelby transformation: an inclusion within an elastic material undergoes a spontaneous change of shape (eigenstrain): circular to elliptical.



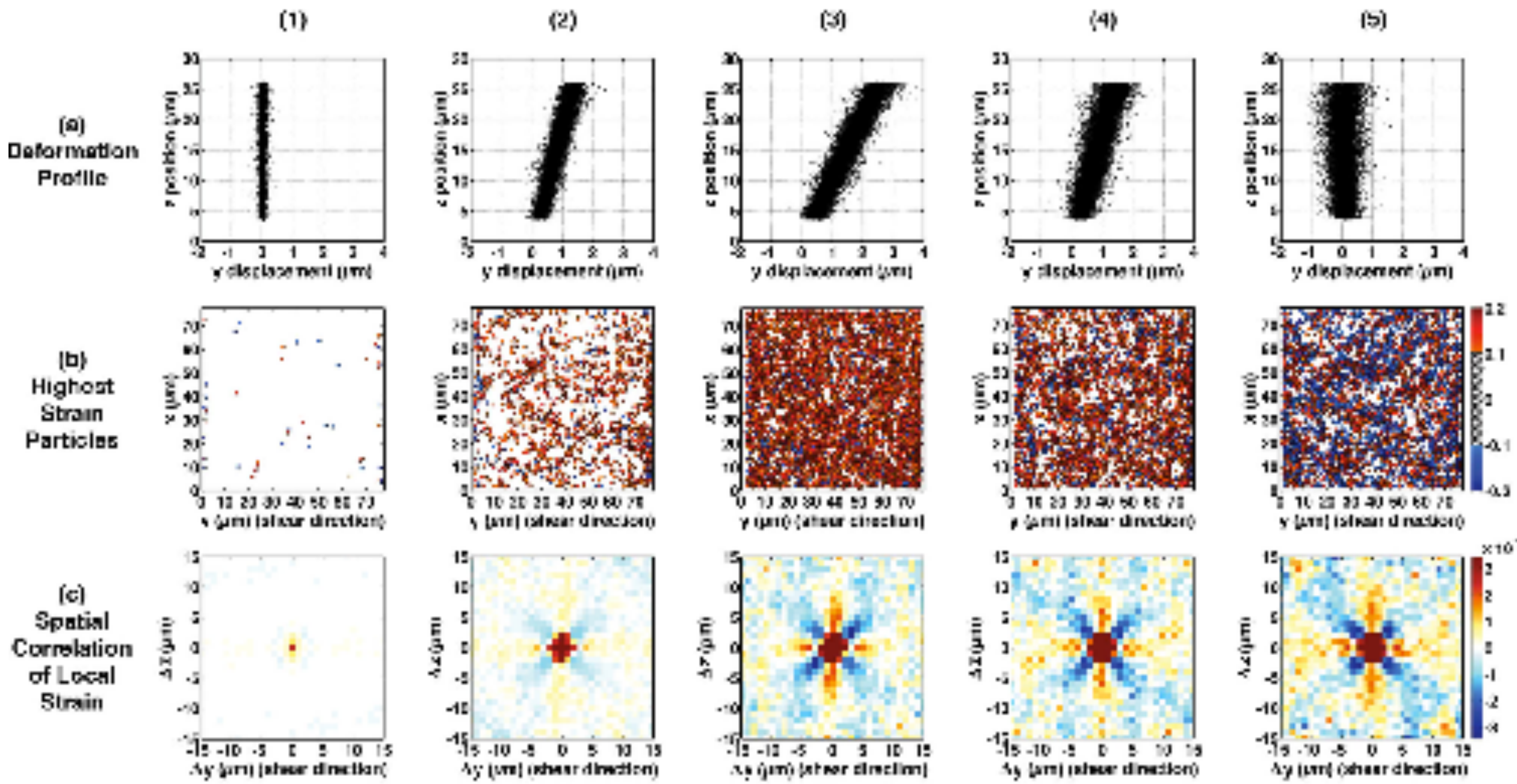
In an homeogeneous, linear elastic solid, the Induced shear stress outside the inclusion is proportional to the inclusion transformation strain and to the Eshelby propagator (response to two force dipoles):

$$G(r,\theta) = \frac{1}{\pi r^2} cos(4\theta)$$



Events are shear transformations of Eshelby type

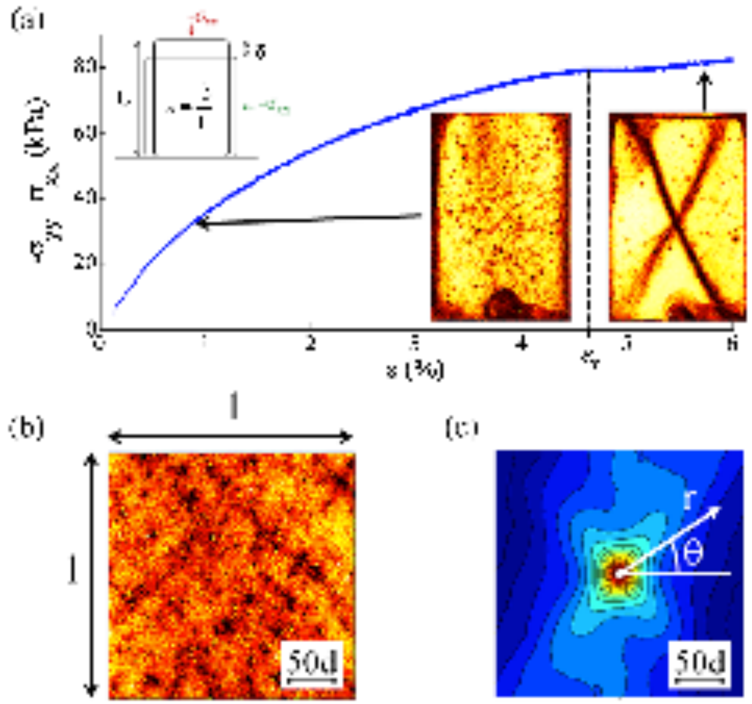
Best seen in experiments trough correlation patterns



Colloidal paste under simple shear
(Jensen, Weitz, Spaepen, PRE 2014)

Events are shear transformations of Eshelby type

Best seen in experiments trough correlation patterns

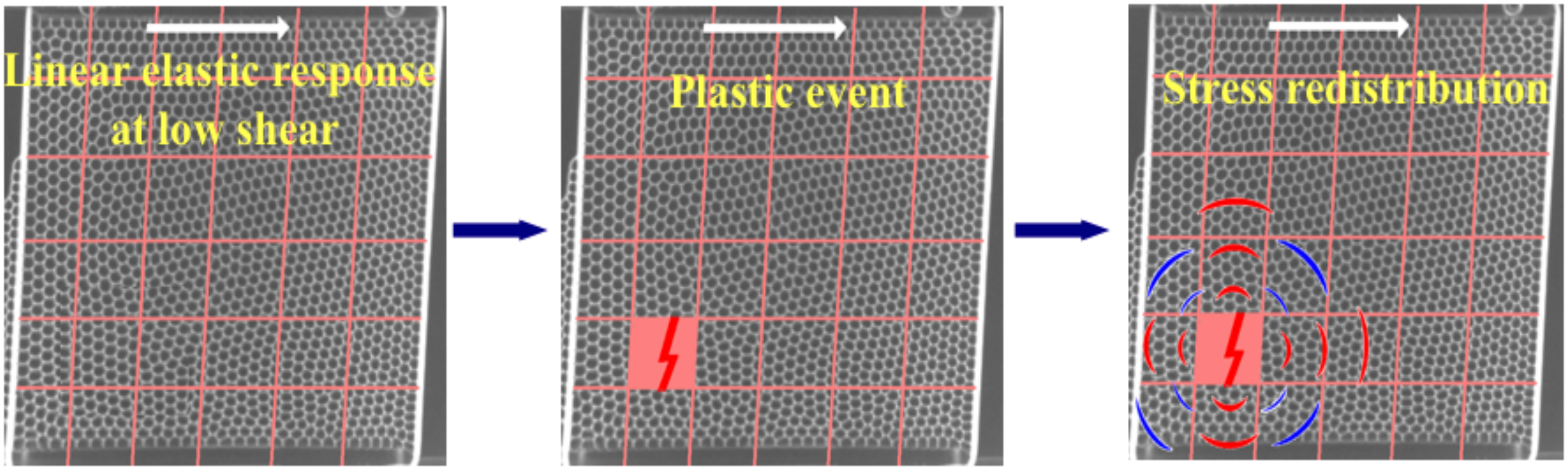


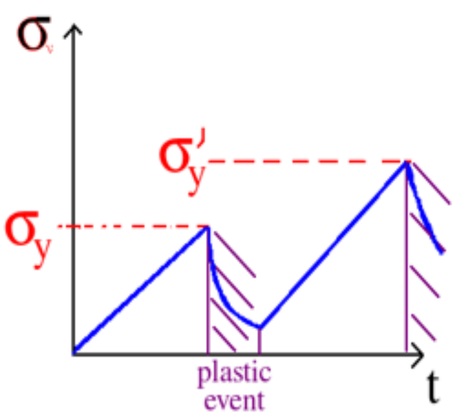
Granular medium under uniaxial deformation
(Le Bouil, Amon, Crassous, PRL 2014)

Three levels of modelling

- Microscopic : Particle based, molecular dynamics or athermal quasistatic deformations. Detailed information, limited sizes /times.
- Mesoscopic : Coarse grain and use the « shear transformations » as elementary events, with elastic interactions between them.
- Continuum : Stress, strain rate, and other state variables (« effective temperature ») treated as continuum fields.

Mesoscopic description

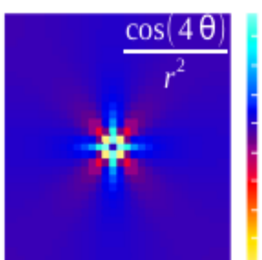




Time evolution of the local stress

$$\partial \boldsymbol{\sigma}(\boldsymbol{r},t) = \mu\dot{\gamma} + \int 2\mu \mathcal{G}(\boldsymbol{r},\boldsymbol{r}')\dot{\boldsymbol{\epsilon}}^{\mathrm{pl}}(\boldsymbol{r}',t)d^2\boldsymbol{r}'$$

$$\text{where } \dot{\boldsymbol{\epsilon}}^{\mathrm{pl}}(\boldsymbol{r}',t) \equiv \begin{cases} \frac{\boldsymbol{\sigma}(\boldsymbol{r}',t)}{2\mu\tau} & \text{if plastic} \\ \mathbf{0} & \text{otherwise} \end{cases}$$



$$\mathcal{G}\left(\boldsymbol{r},\boldsymbol{r}'\right) \propto \frac{\cos\left(4\theta\right)}{\left\|\boldsymbol{r}-\boldsymbol{r}'\right\|^2}$$

Mesoscopic description- an old idea

Self-organized criticality in a crack-propagation model of earthquakes

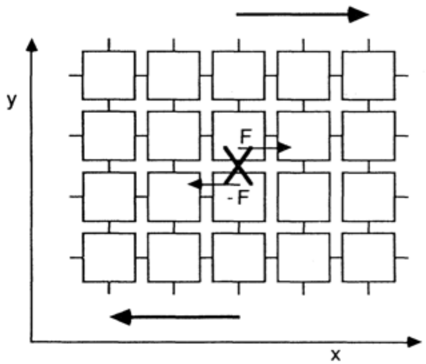
Kan Chen and Per Bak
Brookhaven National Laboratory, Upton, New York 11973

S. P. Obukhov
Landau Institute for Theoretical Physics, The U.S.S.R. Academy of Sciences, Moscow, U.S.S.R.
and Brookhaven National Laboratory, Upton, New York 11973

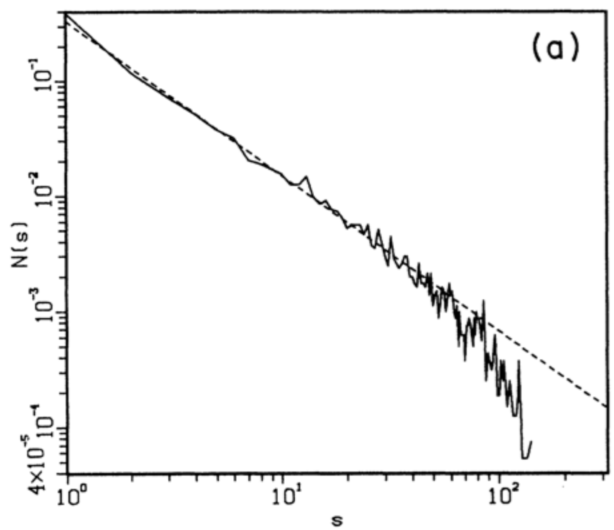
(Received 14 August 1990)

Phys Rev A, 1991

Spring network with threshold in force



external stress field. When the stress somewhere exceeds a critical value (which is must be eventually since the stress is ever increasing), the shear stress is released while the medium undergoes a local shear deformation (rupture). This causes a very anisotropic redistribution of elastic forces, falling off roughly as $1/r^d$ with the distance from the instability:[18] Somewhere the shear force in-



Slope -1.4 in 2D

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Two proposals for describing this scenario in a mean field manner

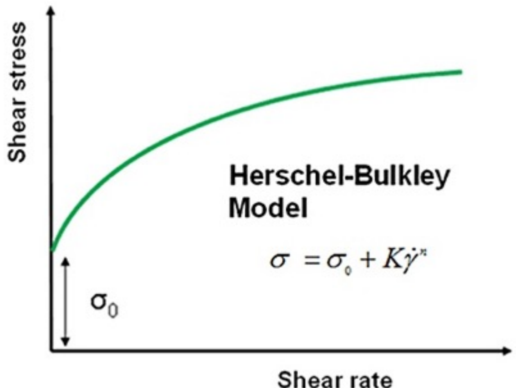
Rheology of soft glassy materials (**SGR**)
By: Sollich, P; Lequeux, F; Hébraud, P; Cates ME
PHYSICAL REVIEW LETTERS Volume: 78 Pages: 4657-4660 Published: JUN 16 1997

Very popular, based on Bouchaud's trap model

Mode-coupling theory for the pasty rheology of soft glassy materials (**HL**)
By: Hébraud, P; Lequeux, F
PHYSICAL REVIEW LETTERS Volume: 81 Pages: 2934-2937 Published: OCT 5 1998

Less popular, probably much more realistic

Both models predict flow curve of the Herschel Bulkley form



$$\sigma(\dot{\gamma}) = \sigma_Y + A\dot{\gamma}^{\alpha}$$

The trap model (J-P. Bouchaud)

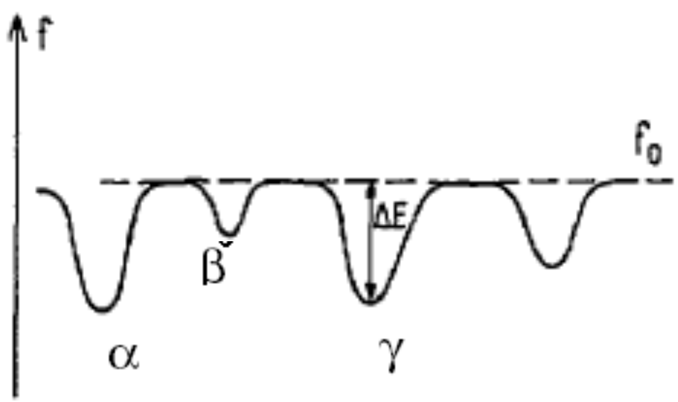
J. Phys. I France **2** (1992) 1705-1713 SEPTEMBER **1992**, PAGE 1705

Classification
Physics Abstracts
75.40 — 05.40 — 64.70

Short Communication

Weak ergodicity breaking and aging in disordered systems

J. P. Bouchaud



Escape time from trap α

$$\tau_\alpha = \exp(+|E_\alpha|/k_B T)$$

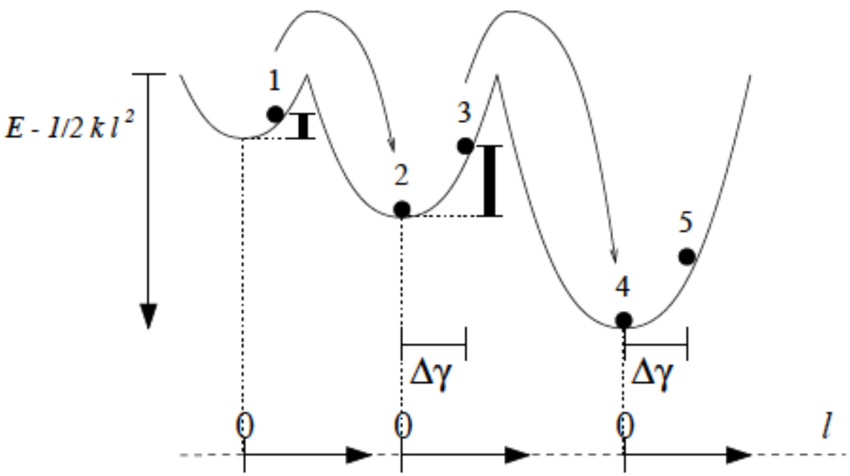
The distribution of trap depths is given by

$$\rho(E) = \exp(-|E|/k_B T_c)$$

Distribution of trapping times $P(\tau) \sim \left(\frac{\tau_0}{\tau}\right)^{(1+T/T_c)}$
$\rightarrow < \tau >$ is infinite $T < T_c$

A very popular model: Soft Glassy Rheology
(Sollich, Lequeux, Hébraud,Sollich, Fielding)

Sollich P., Lequeux, F., Hebraud P. and Cates M. E., "Rheology of Soft Glassy Materials", Phys. Rev. Lett. 78 (1987) 2020–2023.



•Exponential distribution of energy barriers (-> glass transition)

• *l* strain variable, increases linearly with time

$$E \to E - kl^2/2$$

P(L,E,t) distribution of systems in different « traps » and at different strains L.

Fixed strain rate evolution $l = \dot{\gamma} t \quad \sigma = k\langle l \rangle$

Activated escape from traps due to « mechanical noise » x

Dynamical equation for the strain distribution function P(E,l,t) on a typical site:

$$\frac{\partial}{\partial t}P = -\dot{\gamma}\frac{\partial}{\partial l}P - \Gamma_0 e^{-(E-\frac{1}{2}kl^2)/x}P + \Gamma(t)\rho(E)\delta(l)$$

External drive

Activated yield events

x = mechanical noise temperature

Reset strain and energy after yield. Γ is the total plastic activity.

- Very successful model, describes many features of the flow o glassy systems + ageing
- glass transition at $x=x_g=1$; power law fluid $1<x<2$; Newtonian above
- for $x< x_g$: aging, yield stress σ_Y, $\sigma=\sigma_Y+A\,\gamma^{1-x}$

But..

- **mechanical temperature x is not defined self consistently, adjustable parameter**
- **does it correspond to anything physical ?**

The challenger: Hébraud Lequeux model: Stress diffusion due to mechanical noise + self consistency

P(s,t) probability distribution of stress on a typical site (no disorder, single local yield stress)

$$\partial_t \mathcal{P}(\sigma,t) = -G_0\dot{\gamma}(t)\,\partial_\sigma \mathcal{P} + D_{\mathrm{HL}}(t)\,\partial_\sigma^2 \mathcal{P} - \nu_{\mathrm{HL}}(\sigma,\sigma_c)\,\mathcal{P} + \Gamma(t)\,\delta(\sigma)$$

External drive

Stress diffusion

Yield if $\sigma > \sigma_c$

Reset to zero after yield

Yield rule and plastic activity

$$\nu_{\mathrm{HL}}(\sigma,\sigma_c) \equiv \frac{1}{\tau}\theta(\sigma-\sigma_c)$$

$$\Gamma(t) = \frac{1}{\tau}\int_{\sigma'>\sigma_c} d\sigma'\,\mathcal{P}(\sigma',t)$$

Non linear feedback

$$D_{\mathrm{HL}}(t) = \alpha\,\Gamma(t)$$

The challenger: Hébraud Lequeux model: Stress diffusion due to mechanical noise + self consistency

- Solve for a fixed value of D (linear equation, $P(\sigma, D, \dot{\gamma})$ is piecewise exponential).

- Obtain $\Gamma(D, \dot{\gamma})$ and enforce self consistency condition $D = \alpha\Gamma(D, \dot{\gamma}) \Rightarrow D(\dot{\gamma})$

- Obtain $<\sigma>= \int d\sigma \sigma P(\sigma, D(\dot{\gamma}), \dot{\gamma})$

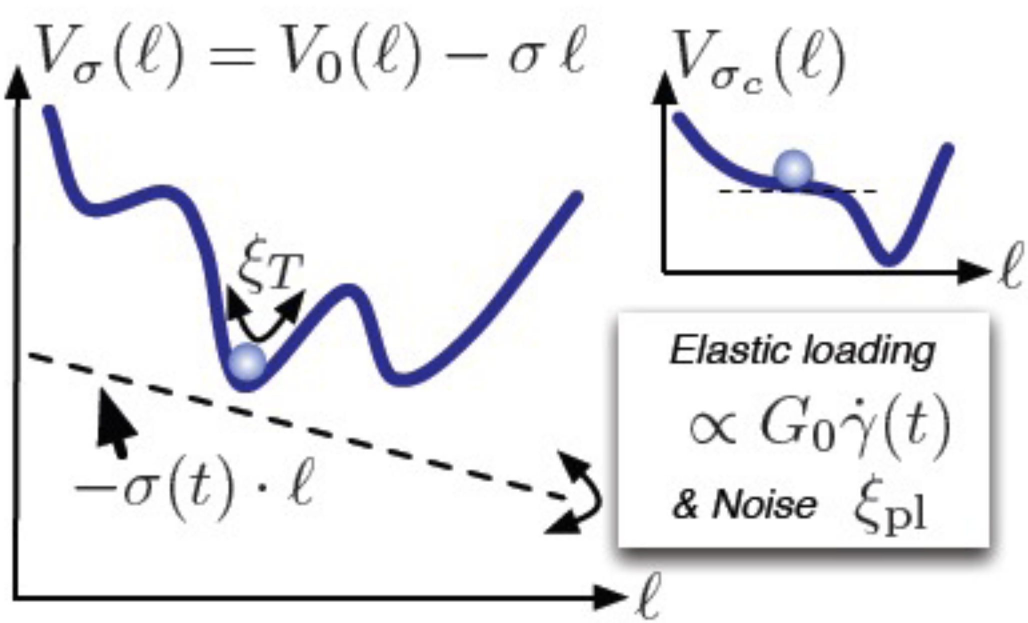
  - $\alpha > \alpha_c = 2$ Newtonian behaviour
    $\sigma \sim \dot{\gamma}$

  - $\alpha < \alpha_c = 2$ Herschel Bulkley law with exponent $1/2$:
    $\sigma = \sigma_Y + A\dot{\gamma}^{1/2}$

Main difference between the two models: description of the random process that triggers the yield event.

Mechanical noise is different from thermal noise!

Potential Energy Landscape
Picture for a small region (STZ):



- Thermal noise acts on strain variable l in a fixed landscape biased by the stress
- Mechanical noise acts a diffusive process on the stress bias itself

=> Very different escape times (Arrhenius vs diffusive)

A. Nicolas, K. Martens, JLB, EPL 2014
E. Agoritsas et al, EPJ E 2015

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Nature of the «yield» (arrested->flow) transition ?

$$\dot{\gamma} \propto (\sigma - \sigma_{\mathrm{yield}})^{\beta}$$

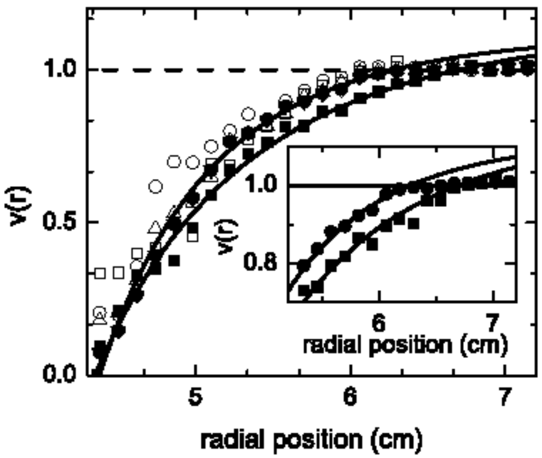
"Second order" critical behaviour, monotonous flow curve. Avalanche behavior at vanishing strain rates, analogies and differences with depinning problems.

Coexistence of flowing and nonflowing regions at the same value of the stress is also commonly observed = > possibility of "first order" transition, known as "strain localisation" or "shear banding".
"Spinodal" instability upon increasing strain-> Procaccia et al.
Here focus on stationary state, beyond yield.

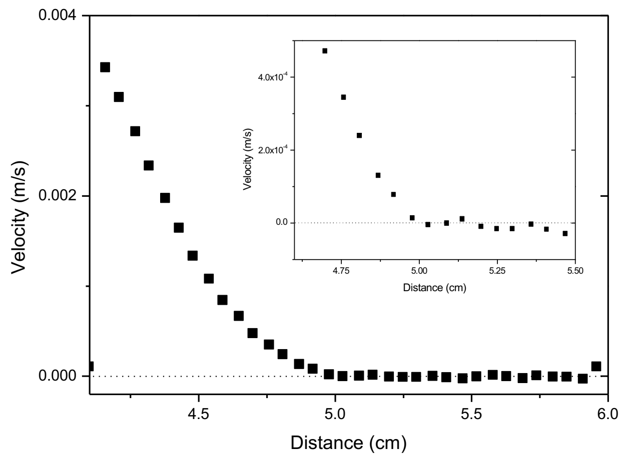
Strain localisation/ Shear banding

Coexistence of flowing regions and solid regions at the same value of the stress

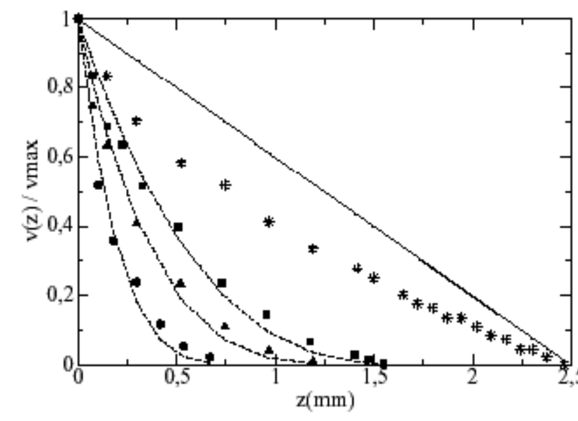
Bubble Rafts
(Dennin *et al.*, 2004)



Chocolate
(Coussot *et al.*)

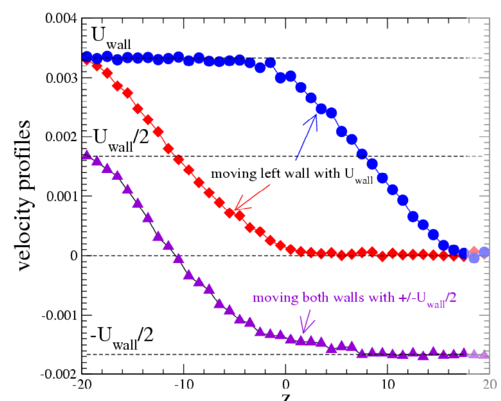


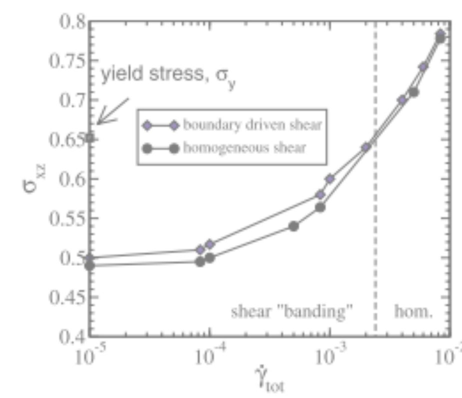
Granular pastes
(Barentin et al., 2003)



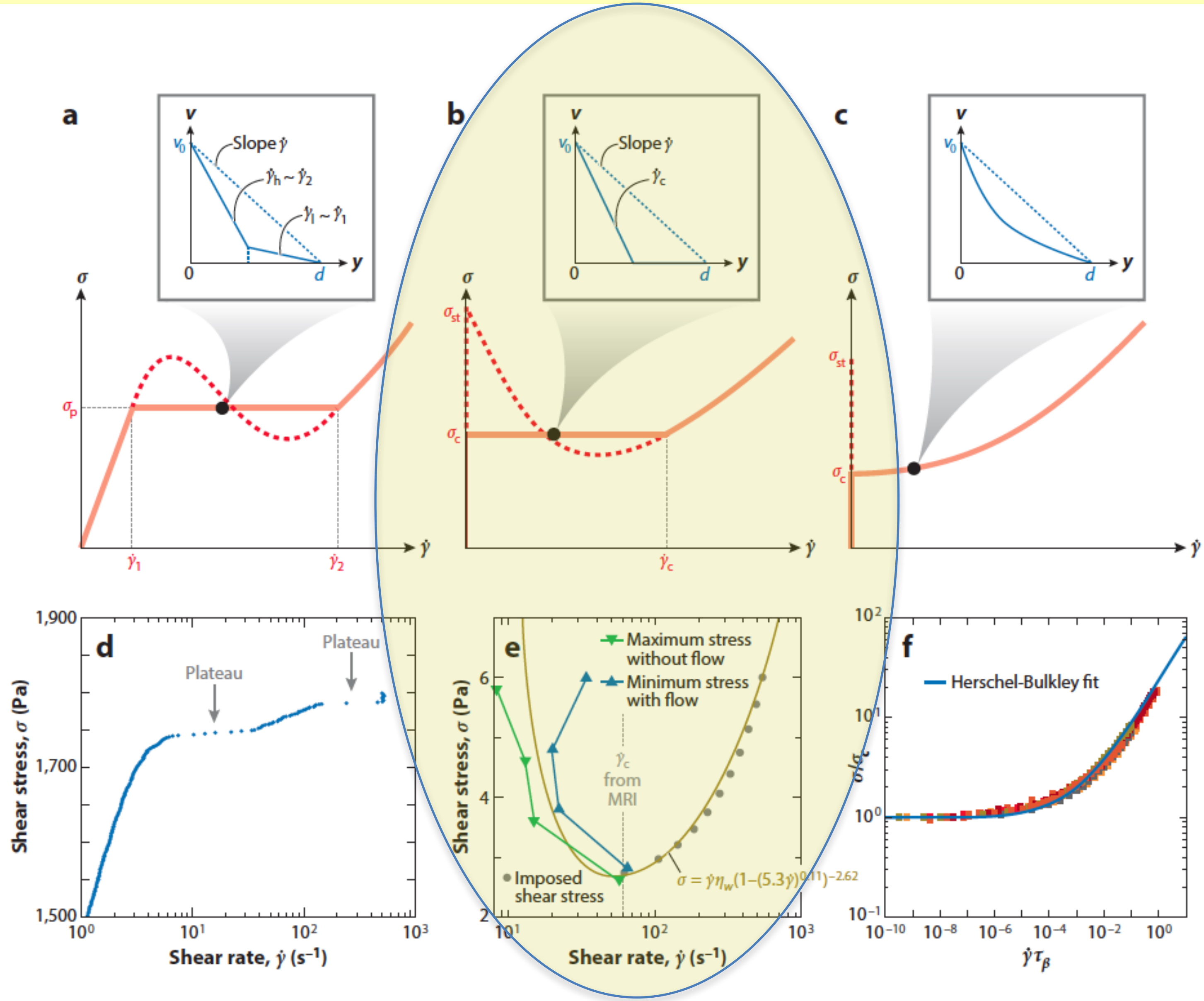
Lennard-Jones glass
(Simulation, Varnik, Bocquet, JLB, 2004)

« Explained » by static vs dynamic yield stress





Strain localisation/ Shear banding



Flow profile (cylindrical Couette)

Flow curve

Example system

Divoux, Fardin, Manneville, Lerouge, *Annual review fluid mechanics* 2016

Strain localisation/ Shear banding

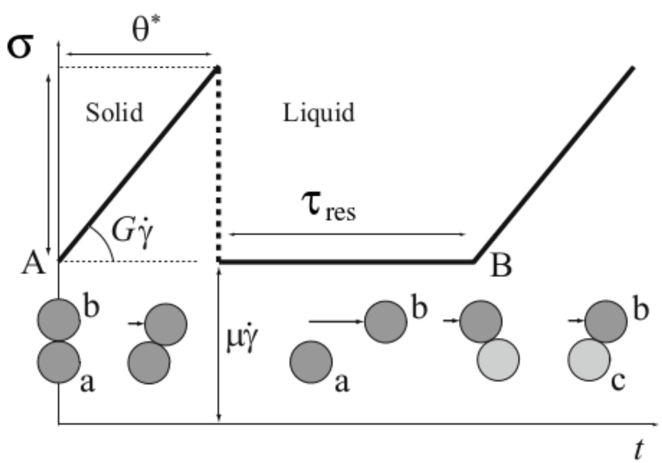
Many different possible microscopic mechanisms can lead to permanent localisation of deformation…

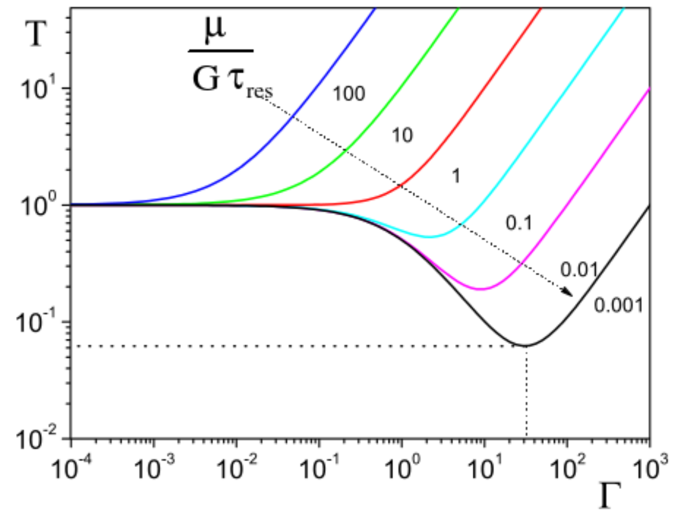
Three examples here: long recovery time (transient damage), inertia, friction

Strain localisation/ Shear banding

A possible mechanism from a mesoscopic viewpoint (Coussot and Ovarlez, Martens et al): long plastic events (large "healing time")

Coussot and Ovarlez mean field analysis (EPJE 2010)





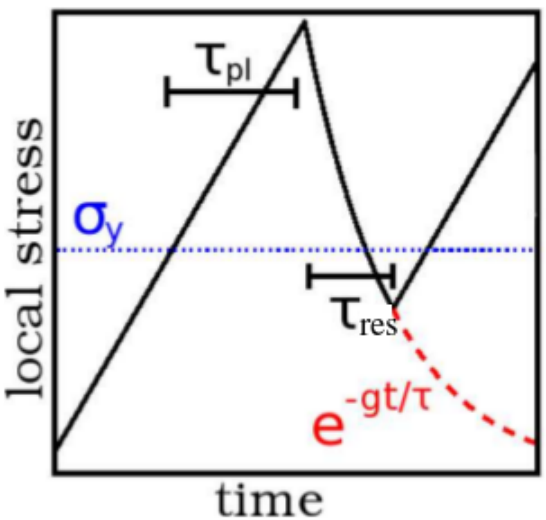
$$< \sigma >= \eta\dot{\gamma} + \frac{\sigma_c}{1 + \dot{\gamma}\tau_{res}/\gamma_c}$$

Constitutive curve becomes non monotonic at large τ_{res}

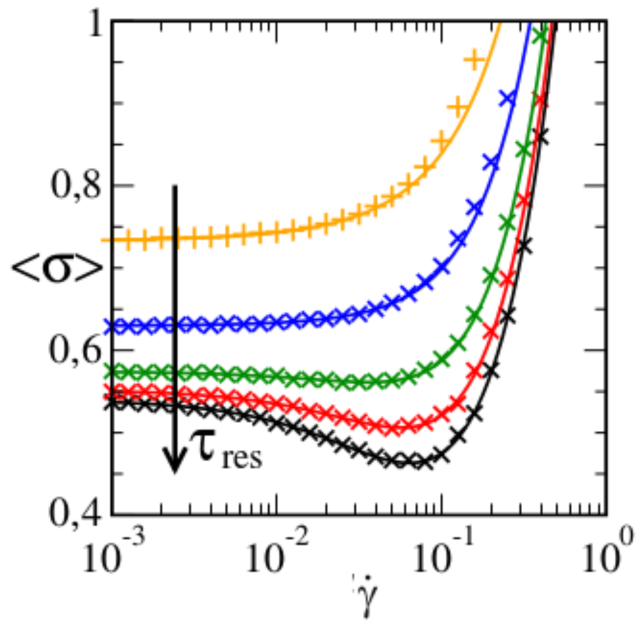
Strain localisation/ Shear banding

A possible mechanism from a mesoscopic viewpoint (Coussot and Ovarlez, Martens et al): long plastic events (large "healing time")

Assembly of elastoplastic blocks interacting via elastic propagator.
Healing time τ_{res} before elastic recovery varies.



Life cycle of a single block



Flow curves

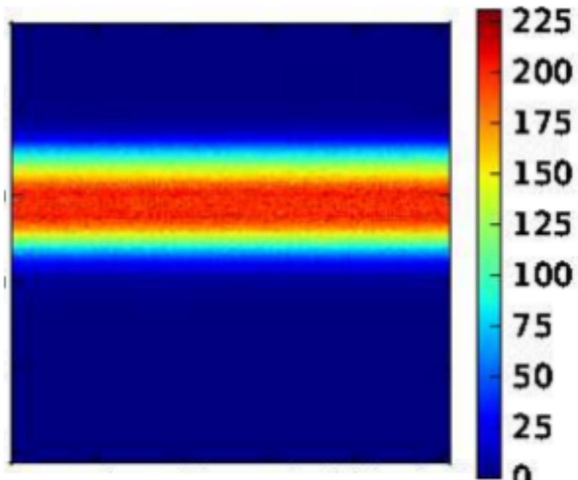
K Martens, L. Bocquet, JLB, Soft Matter 2012

Strain localisation/ Shear banding

A possible mechanism from a mesoscopic viewpoint (Coussot and Ovarlez, Martens et al): long plastic events (large “healing time”)

Picard's spatially resolved model :

assembly of elastoplastic blocks interacting via elastic propagator G

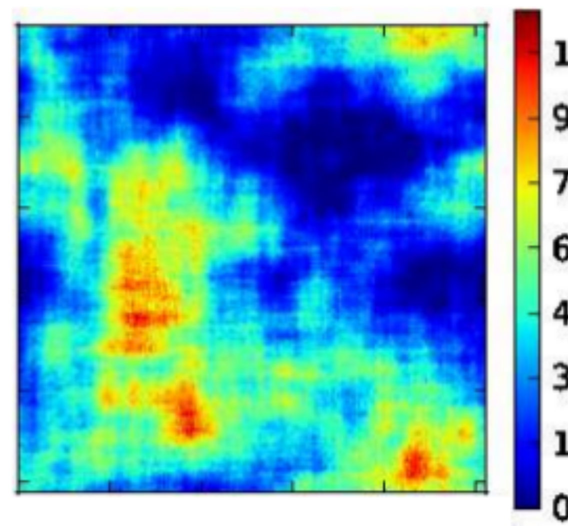


Cumulated plastic activity

Why linear structure ?

$\widetilde{\sigma}_{el} = \widetilde{G}.\widetilde{\varepsilon}_p$ =0

Outside an homogeneous plastic band
(soft mode of the elastic propagator)



Elastic propagator replaced by short range interaction

Martens, . Bocquet, JLB, Soft Matter 2012
Tyukodi, Patinet, Roux, Vandembroucq 2016 “soft modes in the depinning transition”

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Strain localisation in inertial systems

(Salerno and Robbins 2014, Nicolas Rottler Barrat PRL 2015, Karimi Barrat 2016)

Back to a microscopic model

Lennard-Jones particles, 2d system

Damping ζ, mass m, stress scale $\Sigma_0 = \epsilon/\sigma^2$ (in 2d).

Quality factor: $Q = \tau_{damp}/\tau_{vib}$

Overdamped: $Q \ll 1$ underdamped $Q \gg 1$

$$m\dot{v}_i = -\zeta v_i + F(x_i) + \theta_i(t)$$

$$\tau_{damp} = m/\zeta \quad ; \quad \tau_{vib} = \sqrt{m/\Sigma_0}$$

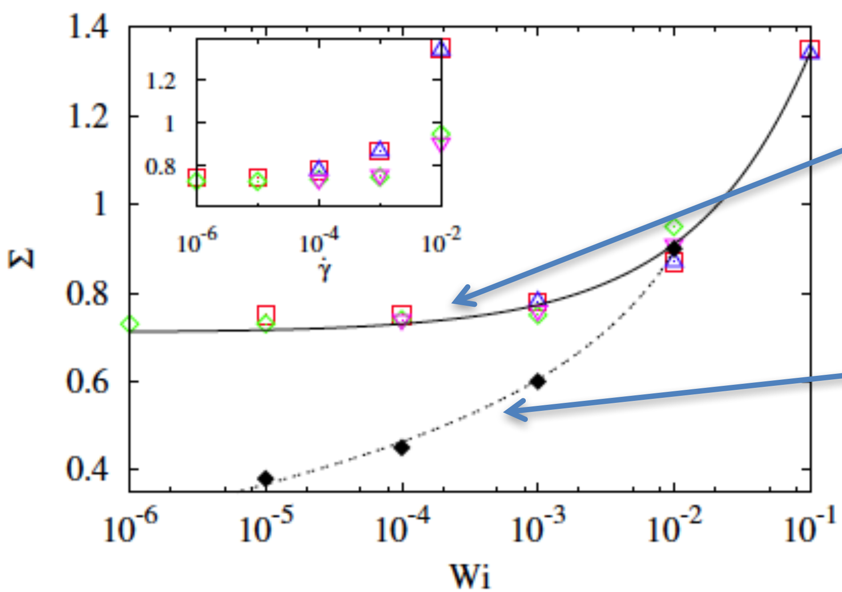
Strain localisation in inertial systems

(Salerno and Robbins 2014, Nicolas Rottler Barrat PRL 2015, Karimi Barrat 2016)

Overdamped system, zero temperature:

$$\Sigma(\dot{\gamma}, T=0) = \Sigma_0(0.72 + 2\sqrt{W})$$

with $W = \zeta\dot{\gamma}/\Sigma_0$



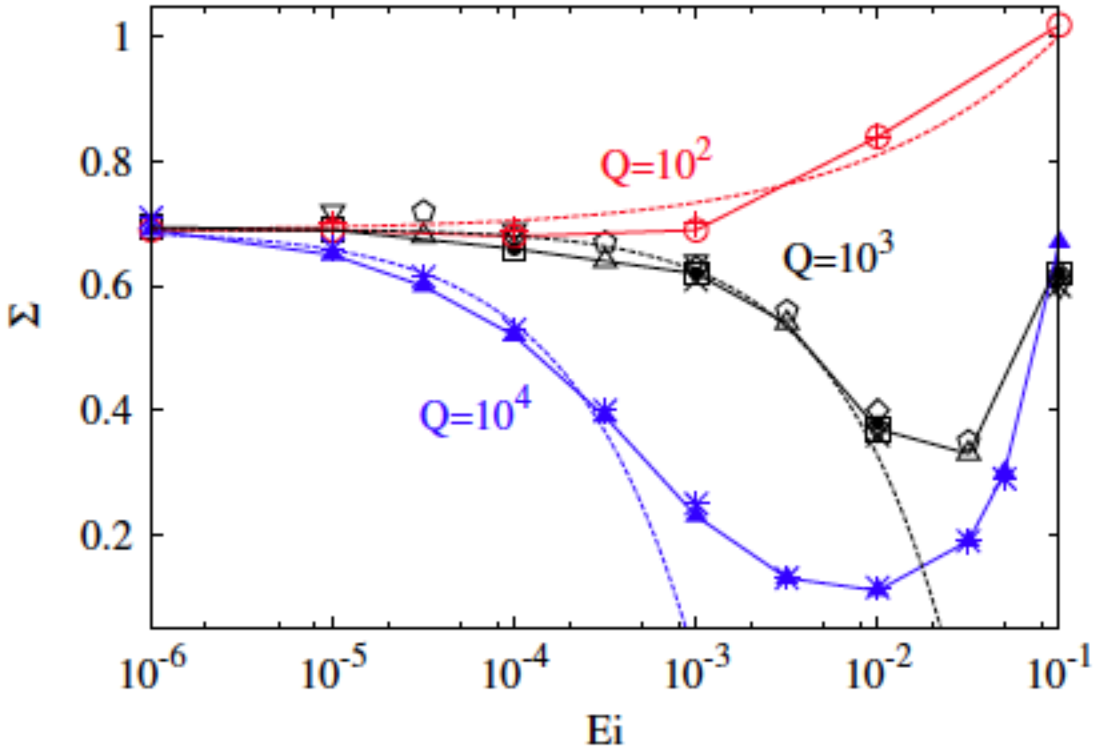
Zero temperature

Higher temperature (T=0.2)
Stress decreases

Strain localisation in inertial systems

(Salerno and Robbins 2014, Nicolas Rottler Barrat PRL 2015, Karimi Barrat 2016)

Underdamped systems, zero temperature: nonmonotonic flow curves!



Grains in 3d

$$Q \simeq 0.1 a\sqrt{\rho\Sigma_0}/\eta$$

$$Ei = \dot{\gamma}\sqrt{m/\Sigma_0} = \dot{\gamma}\tau_{vib}$$

Strain localisation in inertial systems

(Salerno and Robbins 2014, Nicolas Rottler Barrat PRL 2015, Karimi Barrat 2016)

Interpretation: inertial vibrations at a "bath temperature" T=0 act as a finite temperature

=> Data at large Q can be obtained from data at smaller Q and higher temperature.

$$\Sigma(Ei, Q, T_0) = \Sigma\left(Ei, 1, T_K(Q, Ei, T_0)\right)$$

$$\text{Energy dissipation proportional to } \Sigma\dot{\gamma}$$

$$T_K \simeq C|\dot{\gamma}| + T_0$$

Rate weakening effect compensated at large strain rates by standard increase with strain rate

Nonmonotonic flow curve – Shear bands ?

Strain localisation in inertial systems

Nonmonotonic flow curve – Shear bands ?
=> Stability analysis of homogeneous flow (K. Martens, V. Venkatesh, work in progress). Assume monotonous constitutive relation :

$$\Sigma(\dot{\gamma}, T_K) = \Sigma_Y(T_K) + A(T_K)\dot{\gamma}^\alpha$$

Force Balance

$$\rho \frac{\partial v_x}{\partial t} = \frac{\partial \Sigma}{\partial z} = \frac{\partial \Sigma}{\partial T_K}\frac{\partial T_K}{\partial z} + \frac{\partial \Sigma}{\partial \dot{\gamma}}\frac{\partial \dot{\gamma}}{\partial z}$$

Temperature diffusion

$$C\left(\frac{\partial T_K}{\partial t} - \frac{T_K}{\tau}\right) = \lambda \frac{\partial^2 T_K}{\partial z^2} + \Sigma.\dot{\gamma}$$

Strain localisation in inertial systems

Homogeneous flow becomes linearly unstable if the system is larger than a critical size

$$\ell_c = 2\pi\sqrt{\lambda}\left(-\frac{\Sigma\partial\Sigma/\partial T}{\partial\Sigma/\partial\dot{\gamma}} - \frac{C}{\tau}\right)^{-1/2}$$

Below this length scale heat diffusion is too fast and the shear bands do not persist in time.

Confirmed quantitatively by large scale molecular dynamics simulations.

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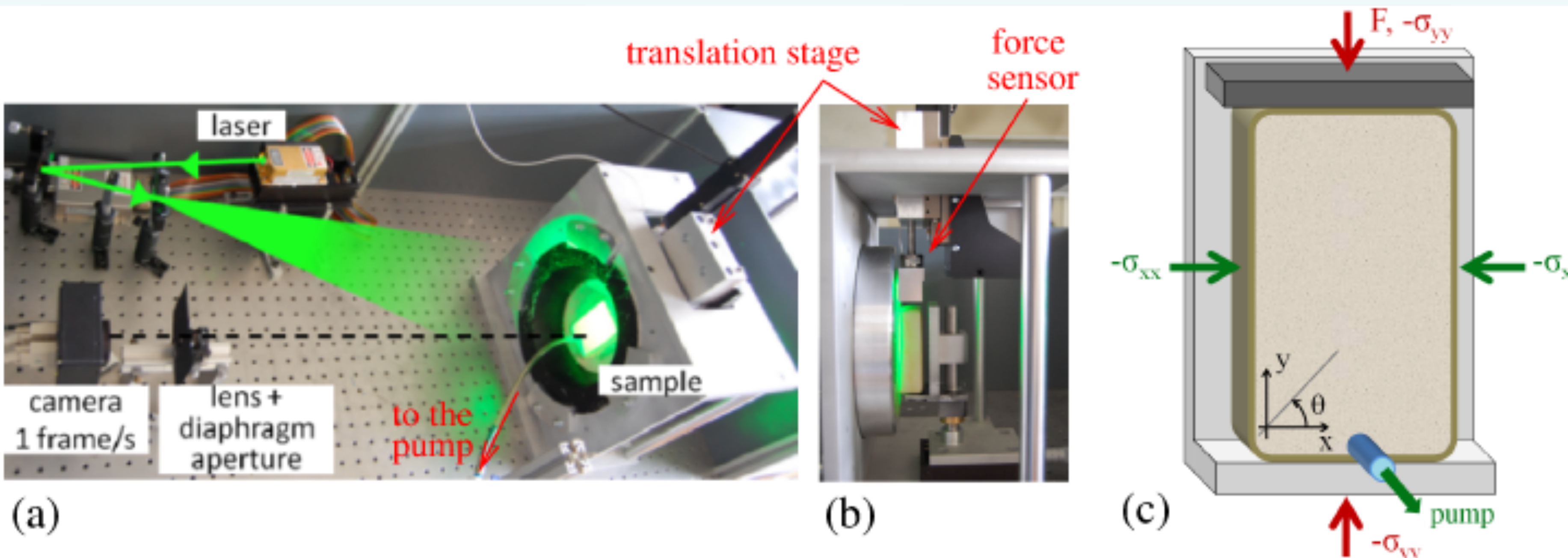
Experiments by Amon, Crassous et al

PRL 112, 246001 (2014) PHYSICAL REVIEW LETTERS week ending 20 JUNE 2014

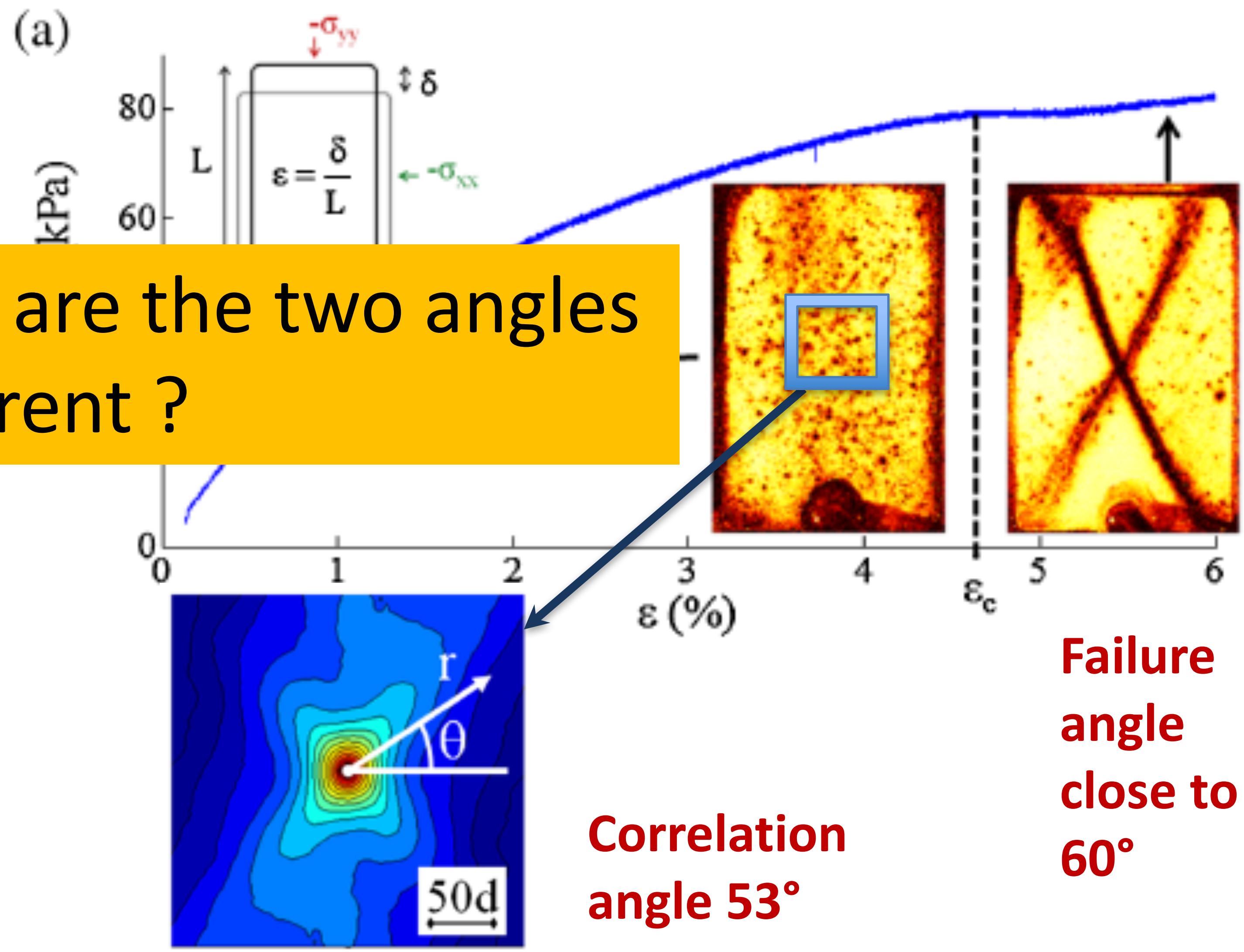
Emergence of Cooperativity in Plasticity of Soft Glassy Materials

Antoine Le Bouil, Axelle Amon, Sean McNamara, and Jérôme Crassous
Université de Rennes 1, Institut de Physique de Rennes (UMR UR1-CNRS 6251),
Bât. 11A, Campus de Beaulieu, F-35042 Rennes, France



A. Amon, Habilitation thesis

Experiments by Amon, Crassous et al



Why are the two angles different ?

Correlation angle 53°

Failure angle close to 60°

Experiments by Amon, Crassous et al

- Biaxial test of granular medium
- Decorrelation of speckle pattern gives access to local plastic activity (near the surface).
- Correlation maps of plastic activity reported during deformation

Stress redistribution+ Failure criterion

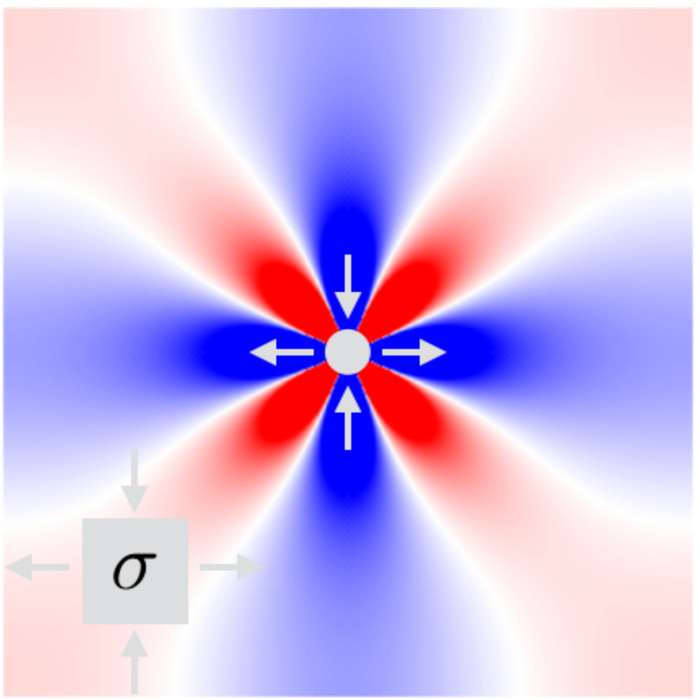
Stress tensor in 2d

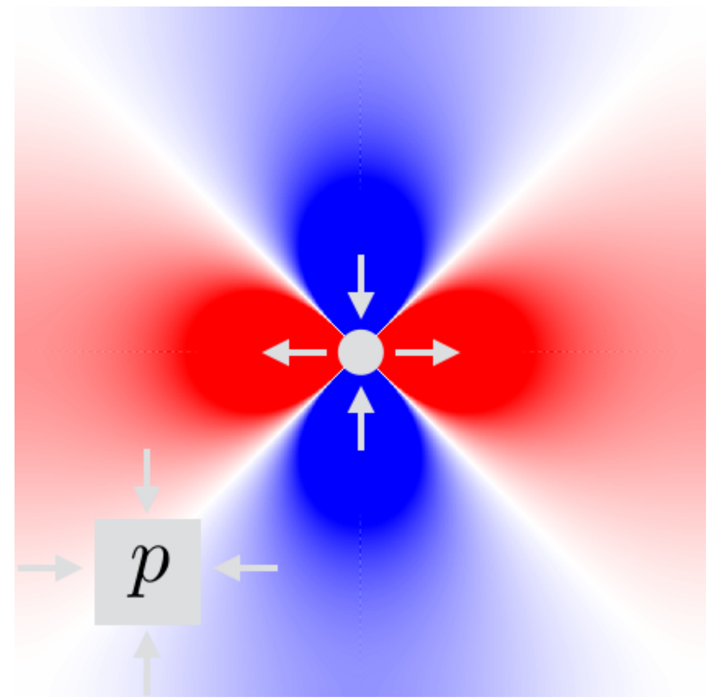
$$\sigma_{\alpha\beta} = -p\delta_{\alpha\beta} + \sigma(\delta_{\alpha x}\delta_{\beta x} - \delta_{\alpha y}\delta_{\beta y}) + \sigma_{xy}(\delta_{\alpha x}\delta_{\beta y} + \delta_{\alpha y}\delta_{\beta x})$$

Shear transformation aligned with x, y axis and located at the origin generates changes in the three stress components:

$$\begin{aligned}
\delta p &= \frac{\Delta\sigma}{1+\frac{\mu}{K}}\frac{1}{r^2}\cos 2\theta \\
\delta\sigma &= -\frac{\Delta\sigma}{1+\frac{\mu}{K}}\frac{1}{r^2}\cos 4\theta \\
\delta\sigma_{xy} &= -\frac{\Delta\sigma}{1+\frac{\mu}{K}}\frac{1}{r^2}\sin 4\theta
\end{aligned}$$

Stress redistribution





Failure criterion

Assume Mohr-Coulomb criterion with a local friction angle ϕ. Failure if

$$|\tau_m| \geq (p \sin \phi + c \cos \phi)$$

with

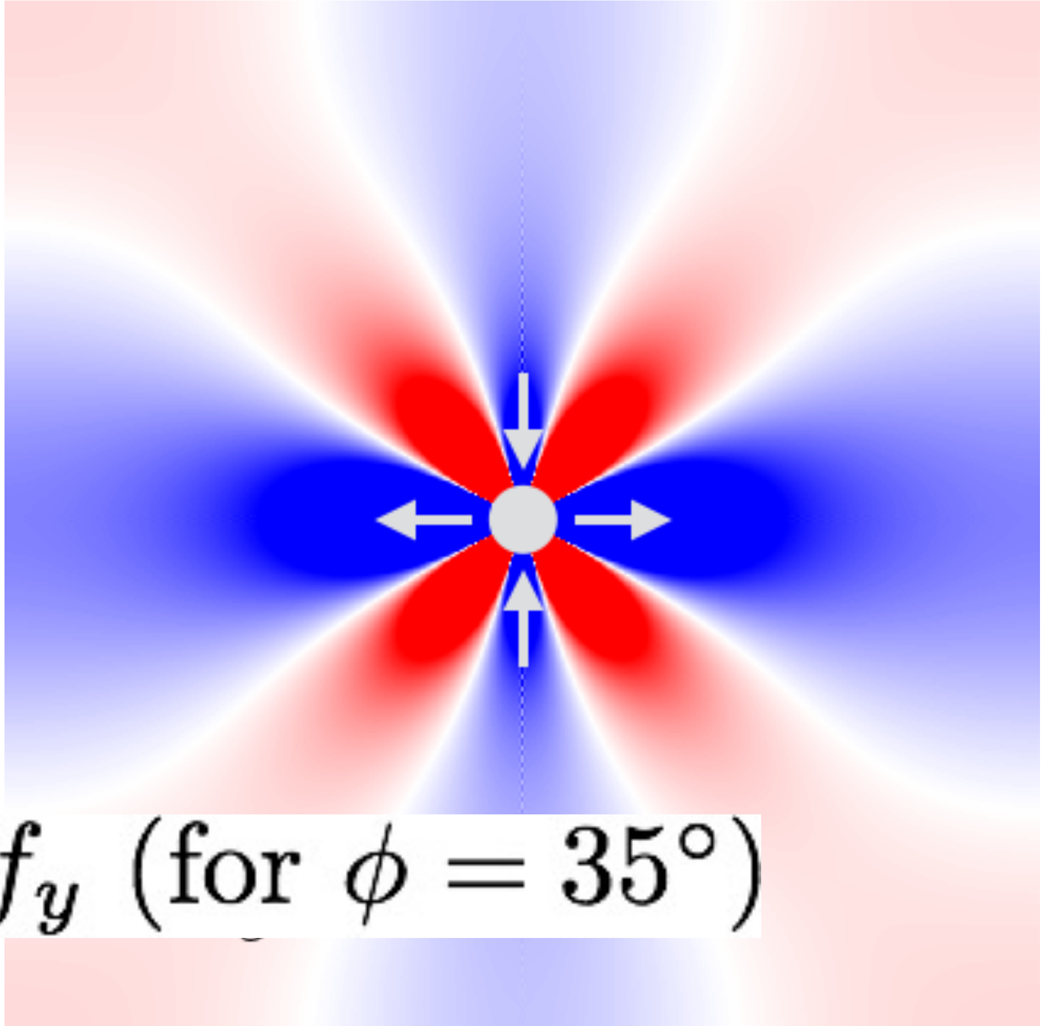
$$|\tau_m| = (\sigma^2 + \sigma_{xy}^2)^{\frac{1}{2}}$$

$c \geq 0$ cohesion strength.

Define yield function

$$f_y = |\tau_m| - (p \sin \phi + c \cos \phi)$$

Change in yield function in response to shear transformation



$$-\delta f_y \text{ (for } \phi = 35^\circ\text{)}$$

Correlations

Strong correlations between events are expected for directions where $-\delta f_y$ is large. Maximisation leads to

$$\theta_{\max} = \frac{1}{2}\cos^{-1}(-\frac{1}{4}\sin\,\phi)$$

For $\phi = 0$ (corresponding to von Mises criterion) the usual value $45°$ is recovered.

Extension to the case where the local transformation involves a dilation stress Δp leads to

$$\theta_{\max} = \frac{1}{2}\cos^{-1}[-\frac{1}{4}(\frac{\mu}{K}\frac{\Delta p}{\Delta\sigma} + \sin\,\phi)].$$

Shear band ?

- Plastic activity localized inside a linea region
- Picture band as a linear array of shear transformations
- If activity homogeneous on the line, stress redistribution is zero everywhere outside
- Proposed criterion for selecting the band orientation: maximise $-\delta f_y$ inside the band itself

Shear band orientation

Sum of shear transformations $\epsilon_{\alpha\beta}(\vec{r}) = \epsilon^*_{\alpha\beta}\, a^d \sum_i \delta(\vec{r} - \vec{r}_i)$, with $\vec{r}_i$ along a line at angle α

$$\delta p(\vec{r}) \;=\; 2\epsilon^* a^d \frac{\mu}{1+\frac{\mu}{K}} \cos 2\alpha \sum_i \delta(\vec{r} - \vec{r}_i)$$

$$\delta\sigma(\vec{r}) \;=\; 2\epsilon^* a^d [\mu - \frac{1}{2}\frac{\mu}{1+\frac{\mu}{K}}(1+\cos 4\alpha)] \sum_i \delta(\vec{r} - \vec{r}_i).$$

Maximize change in yield function w.r.t. α:

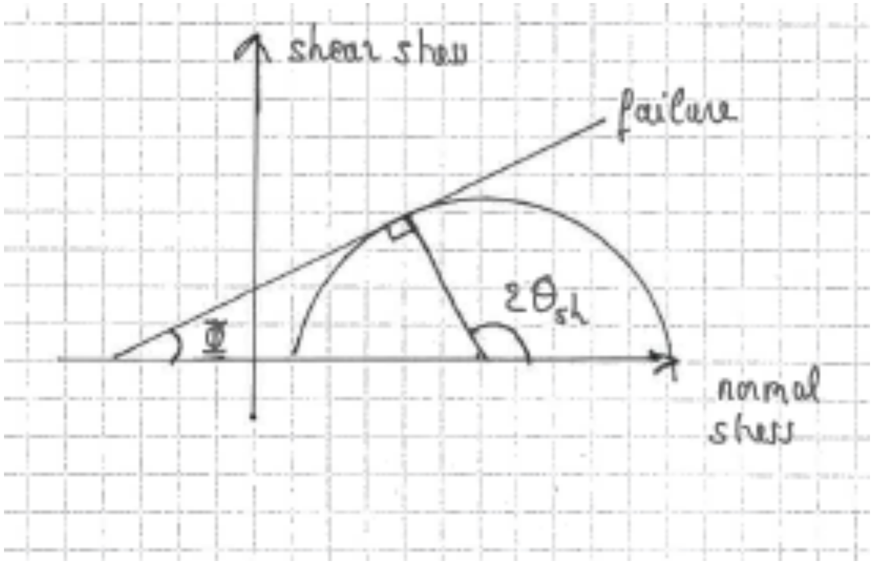
$$\frac{\partial}{\partial\alpha}\delta f_y|_{\alpha=\theta_{\rm sh}} = 0 : \theta_{\rm sh} = \tfrac{1}{2}\cos^{-1}(-\tfrac{1}{2}\sin\,\phi).$$

Different from correlation angle unless friction angle is zero!

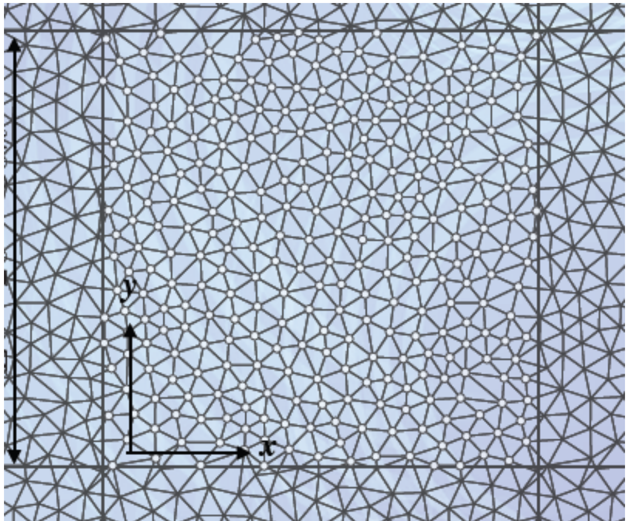
Macroscopic friction

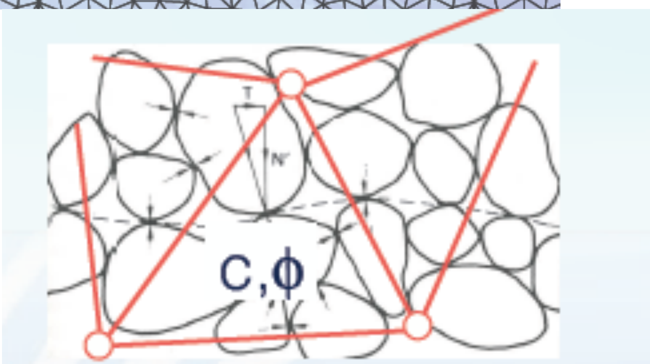
θ_{sh} can be used to define a macroscopic friction angle Φ using the Mohr Coulomb relation for the failure direction:

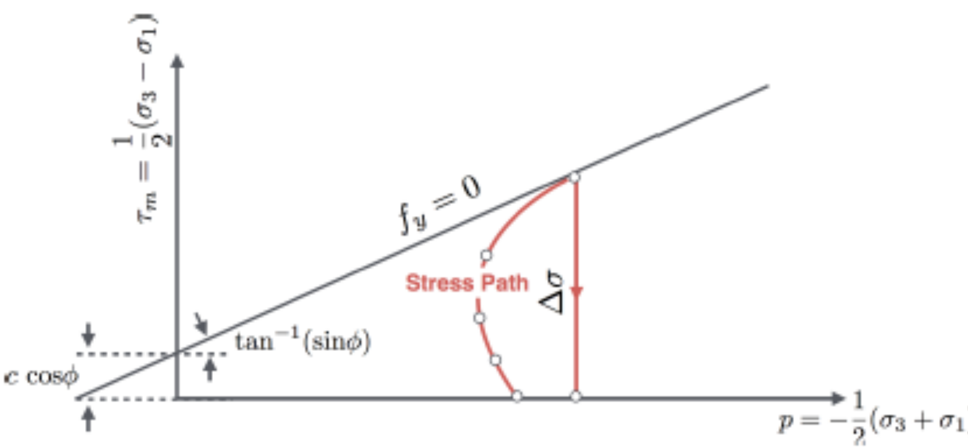
$$\theta_{\mathrm{sh}} = \frac{\pi}{4} + \frac{\Phi}{2}$$



Numerical test



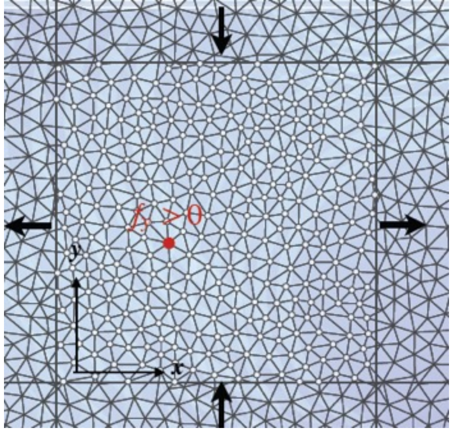




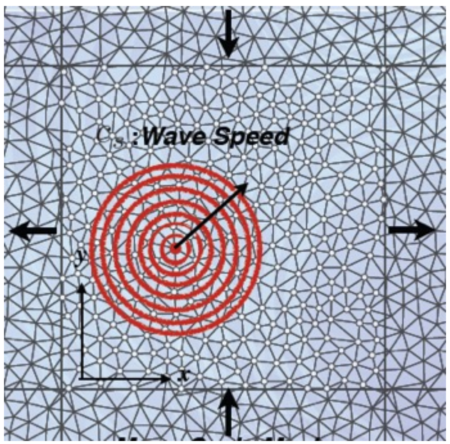
Finite element grid. Each element has elastoviscoplastic behavior,.

Local failure criterion with Mohr Coulomb criterion. Local cohesion c drawn from an exponential distribution, uniform local friction angle ϕ.

Numerical test

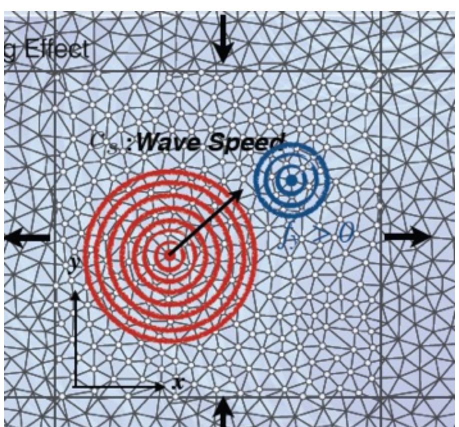


Simple shear loading triggers first yield event



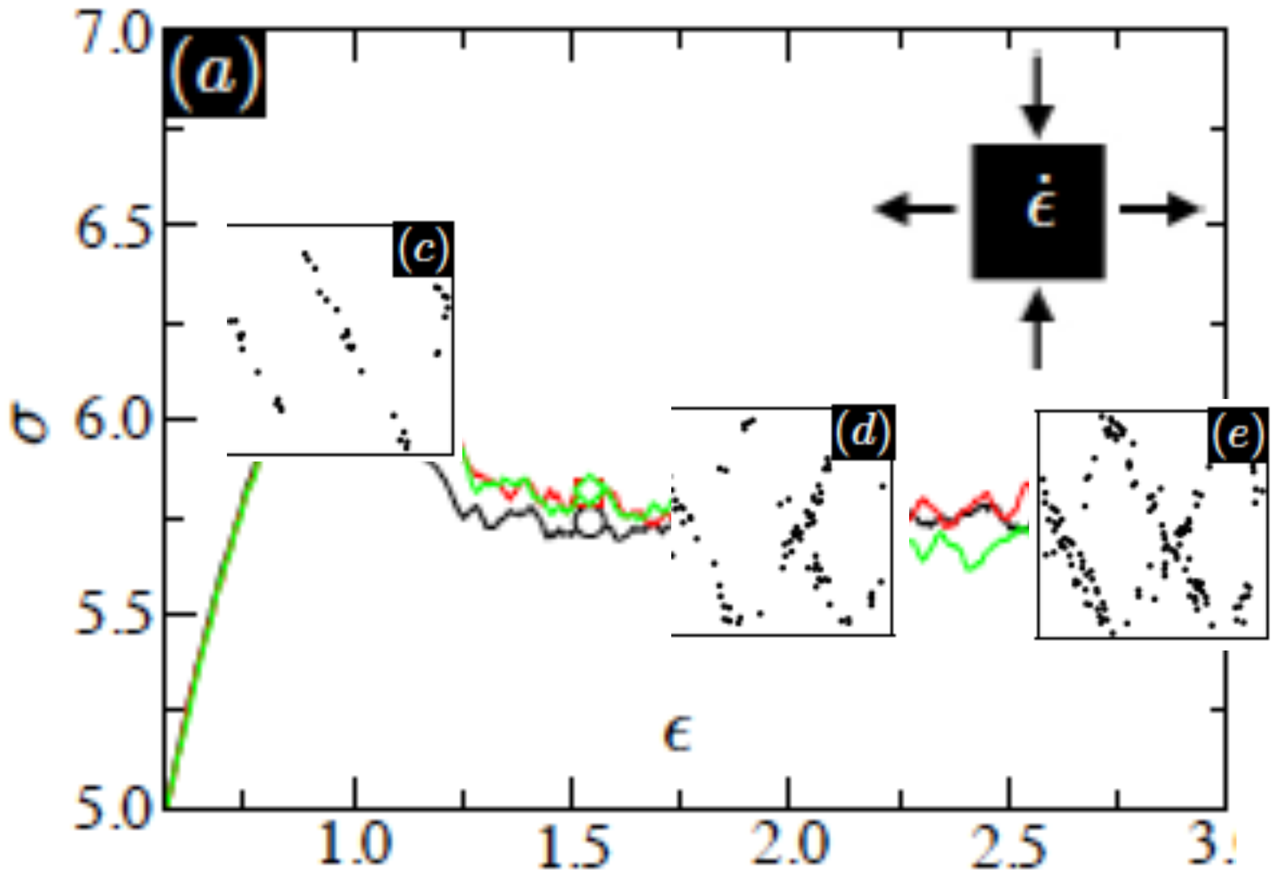
Solve for local displacement and stresses (overdamped propagation)

$$\partial_t^2 u = c_s^2 \nabla^2 u + \nu \nabla^2 (\partial_t u)$$



Trigger new events if local failure criterion is reached

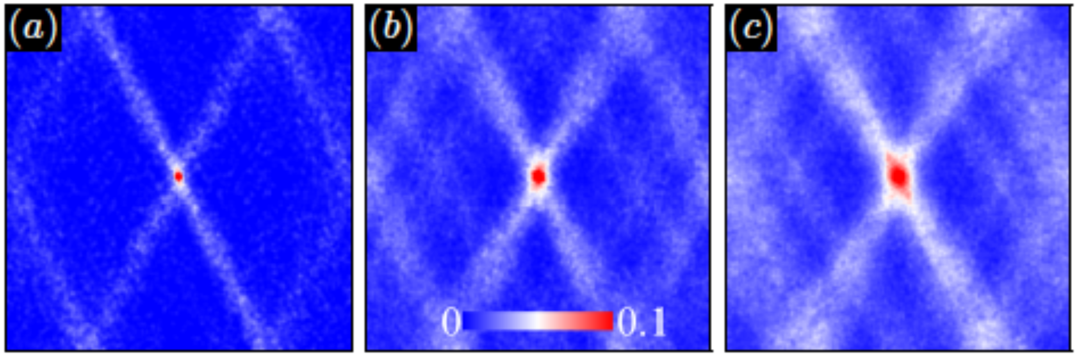
Loading curve



Small strains: Transient fluctuations

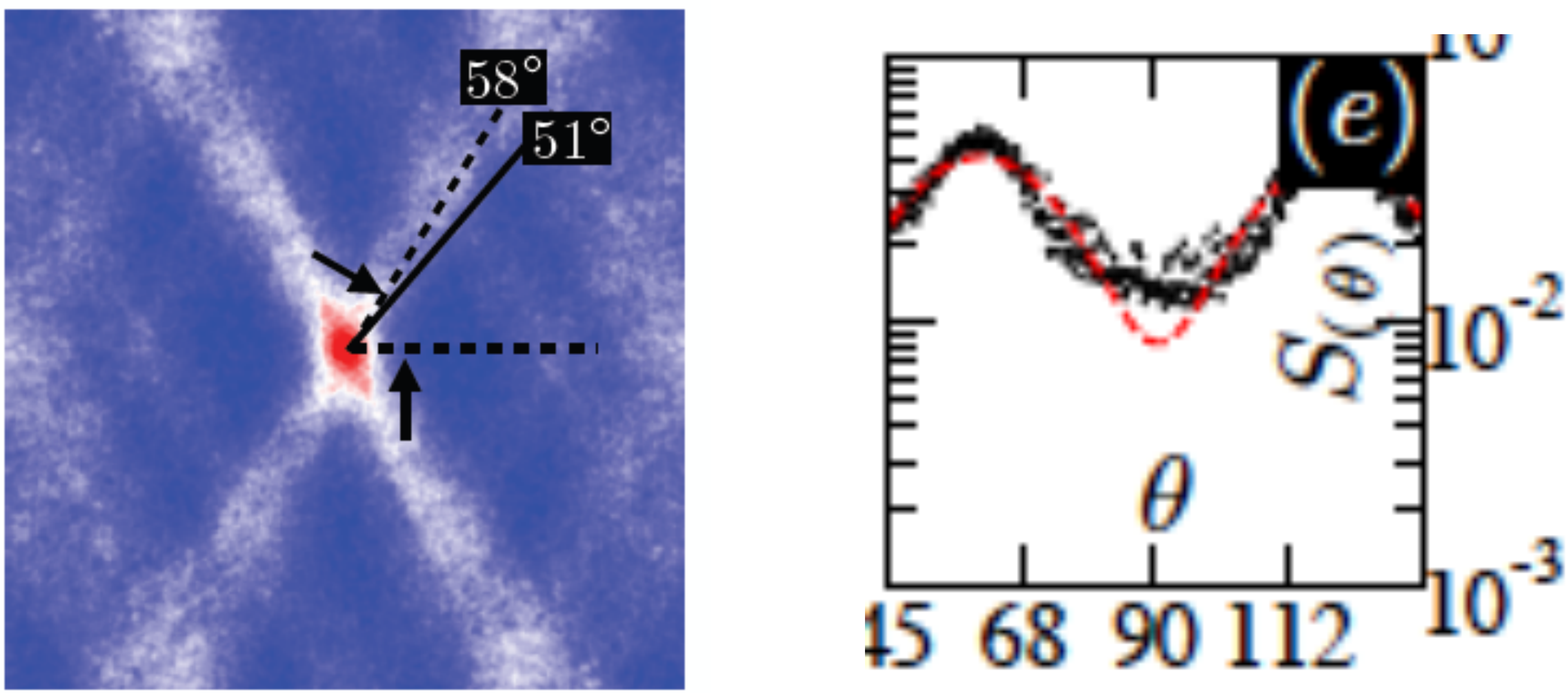
Large strains:stable bands

Correlations in plastic activity

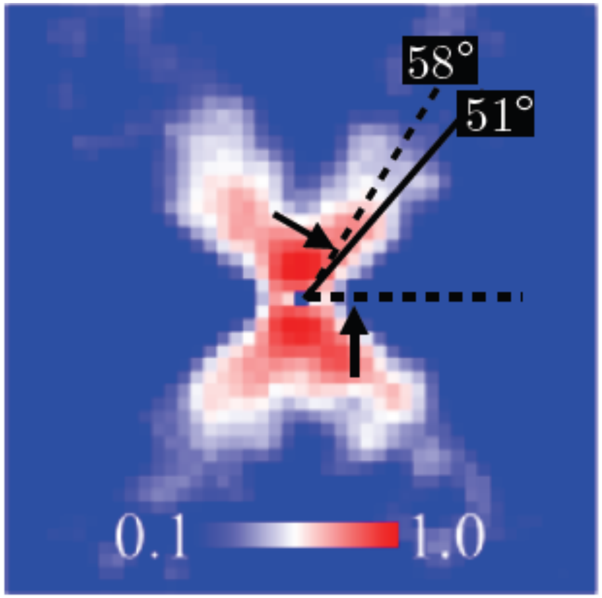


Predicted angles:
Correlation $\theta_{max} = 51^o$
Shear bands $\theta_{sh} = 58^o$

Correlations in the shear banded regime

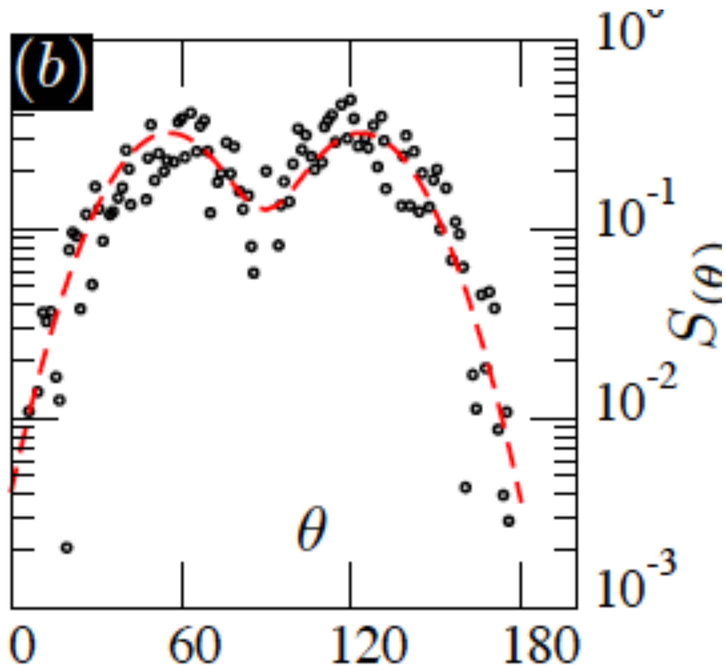


Correlations in plastic activity



Correlations at small strain

Sliding average in the strain interval 0 – yield strain



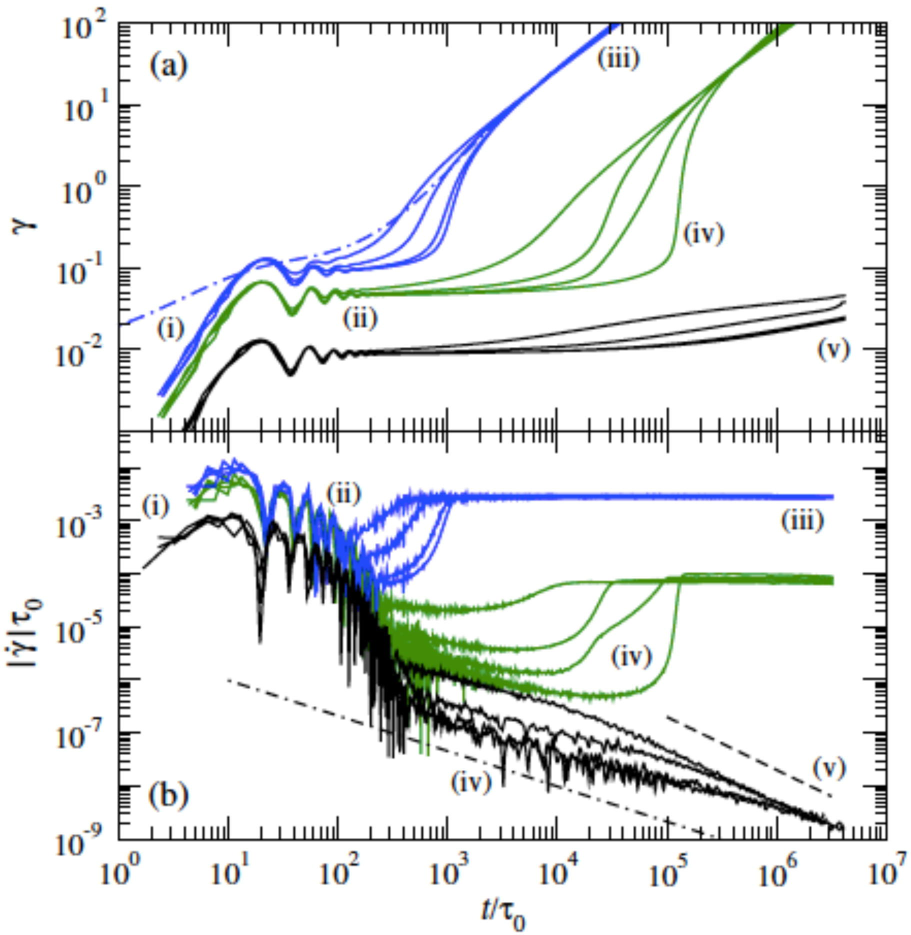
Outline

- Elastoplastic models
- Mean field treatments: Hébraud Lequeux, SGR
- Strain localisation vs continuous transitions
- Strain localisation in inertial systems
- Strain localisation in granular systems
- Creep

Creep: Apply a fixed stress σ and measure the strain γ (t)

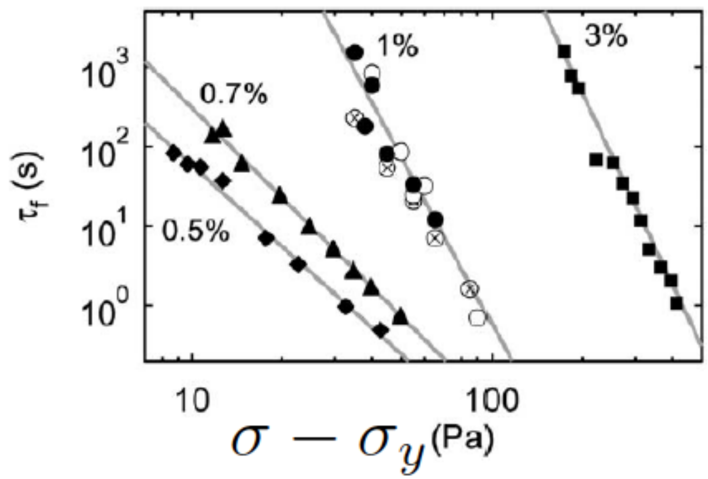
Siegenbürger et al, PRL 2012
Creep in a colloidal glass

Strain response: different stress levels, different waiting times



Divoux et al, Soft Matter 2011
Carbopol microgel

Fluidization time behaves as a power law of the distance to yield stress



Stress controlled version of elastoplastic models (mean field version)

Chen Liu, Kirsten Martens, JLB arxiv:1705.06912

$$\partial_t P(\sigma,t) = \underbrace{-G_o\dot{\gamma}(t)\partial_\sigma P}_{\text{Elastic Response To Shear}} + \underbrace{\alpha\Gamma(t)\partial_\sigma^2 P}_{\text{Mechanical Noise From Plastic Events}} \underbrace{- \frac{1}{\tau}\theta(|\sigma|-\sigma_c)P + \Gamma(t)\delta(\sigma)}_{\text{Plastic events \& Recovery}}$$

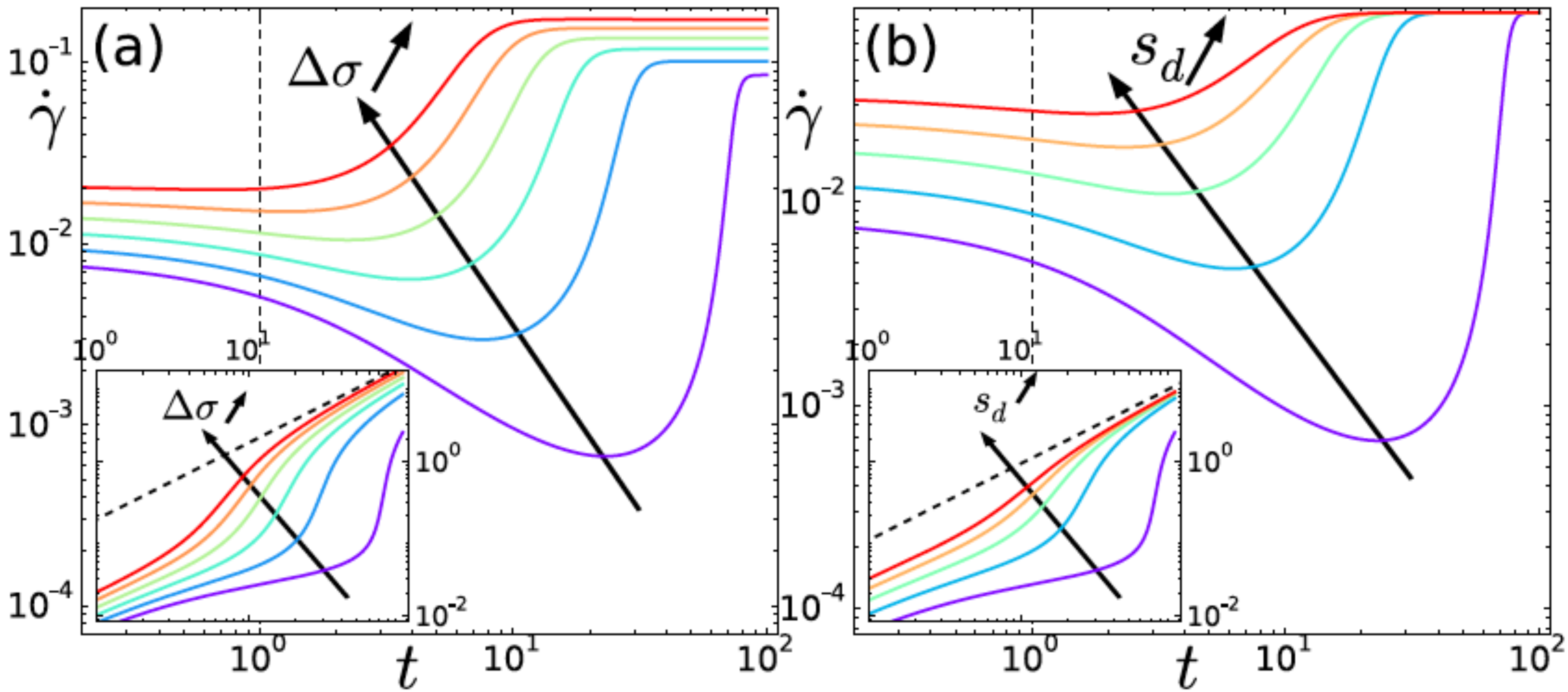
$$\Gamma(t) = \frac{1}{\tau}\int \theta(|\sigma|-\sigma_c)P(\sigma,t)d\sigma$$ Plastic Events Over All System

By imposing $\dot{\gamma}(t) = Cst \longrightarrow$ Steady State Shear Rheology

By imposing $\int P(\sigma)\sigma d\sigma = Cst$ Stress control protocol: Creep

$$\dot{\gamma}(t) = \frac{1}{\tau G_o}\int_{|\sigma|>\sigma_c} P(\sigma,t)\sigma d\sigma$$

Results qualitatively similar to experiments; very strong dependence on the initial condition for the probability distribution function

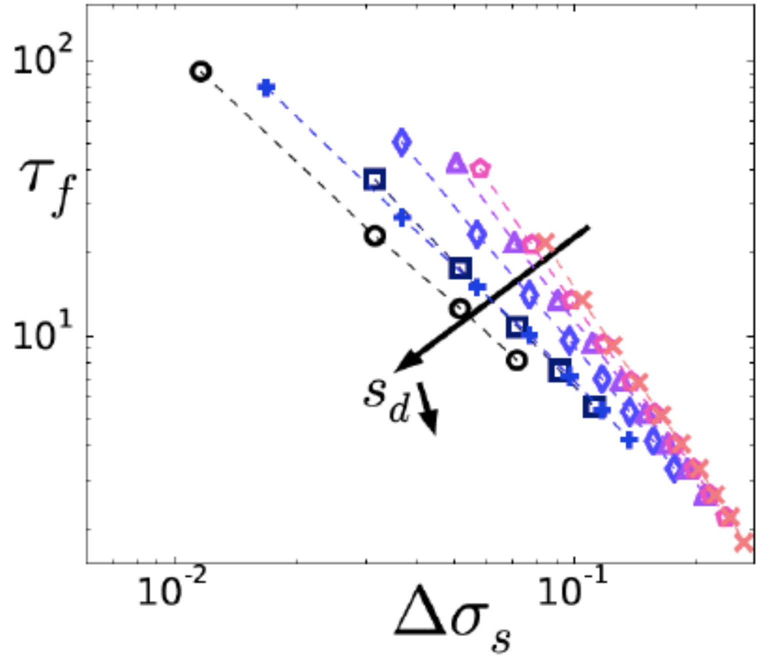


$$\Delta\sigma = \sigma^{imp} - \sigma_y(\alpha)$$

S_d: decreases when system ages

Fluidization time follows a power law with the static yield stress as a reference (can be identified with overshoot in stress strain curve).

Exponent is not universal, depends on system age.



$$\Delta\sigma_s = (\sigma - \sigma_y^S)$$

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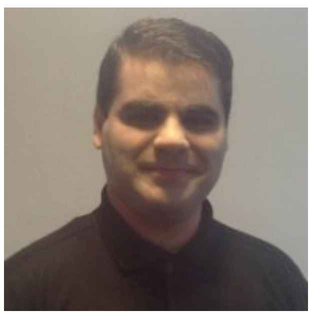

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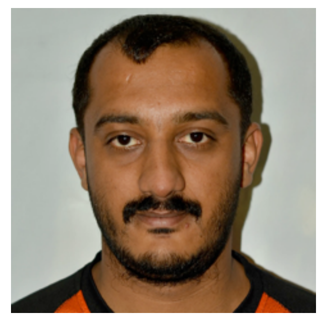

Vishwas Venkatesh

Chen Liu

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