

# Yielding

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Yielding of amorphous solids – Paris, October 28, 2017

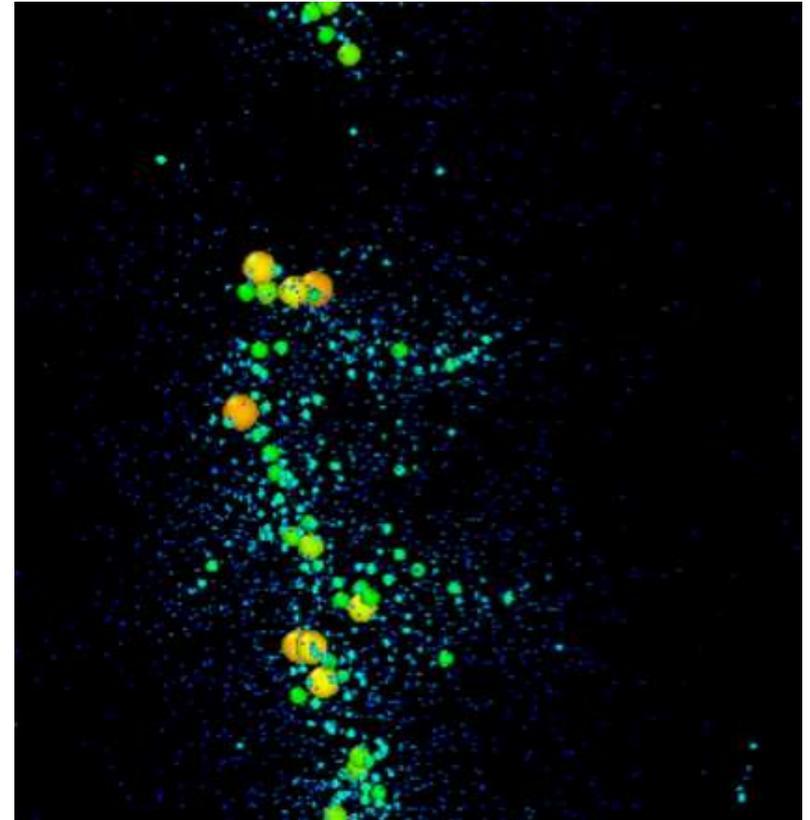


# Coworkers

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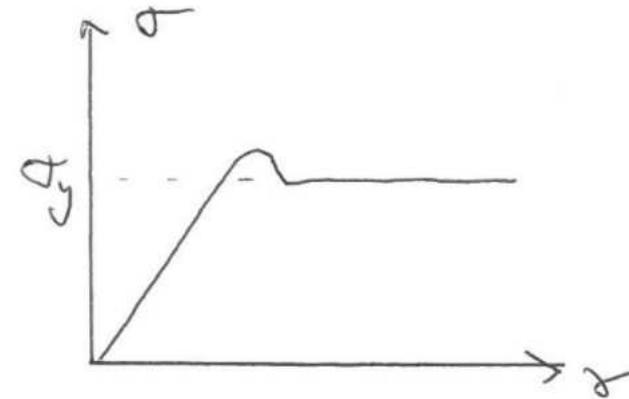
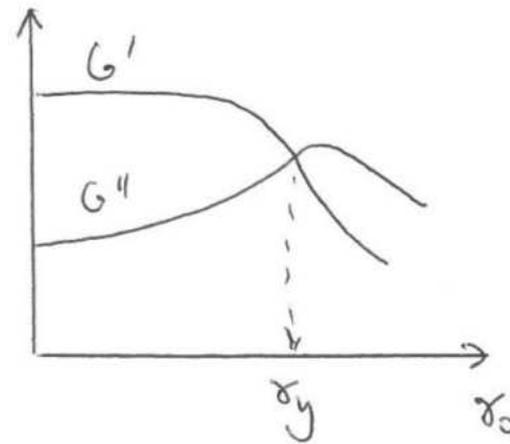
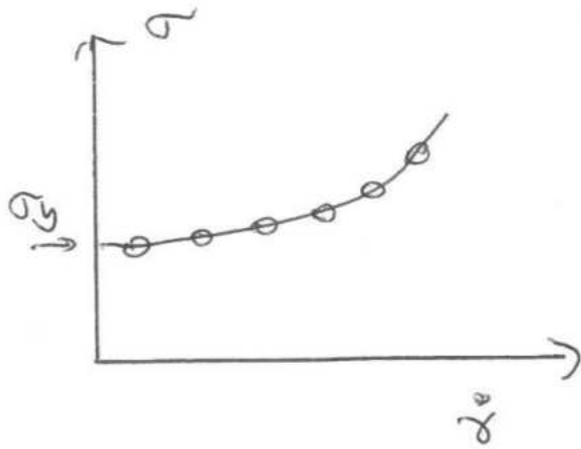
- On-going work with:

D. Coslovich,  
T. Kawasaki,  
A. Ninarello,  
M. Ozawa.



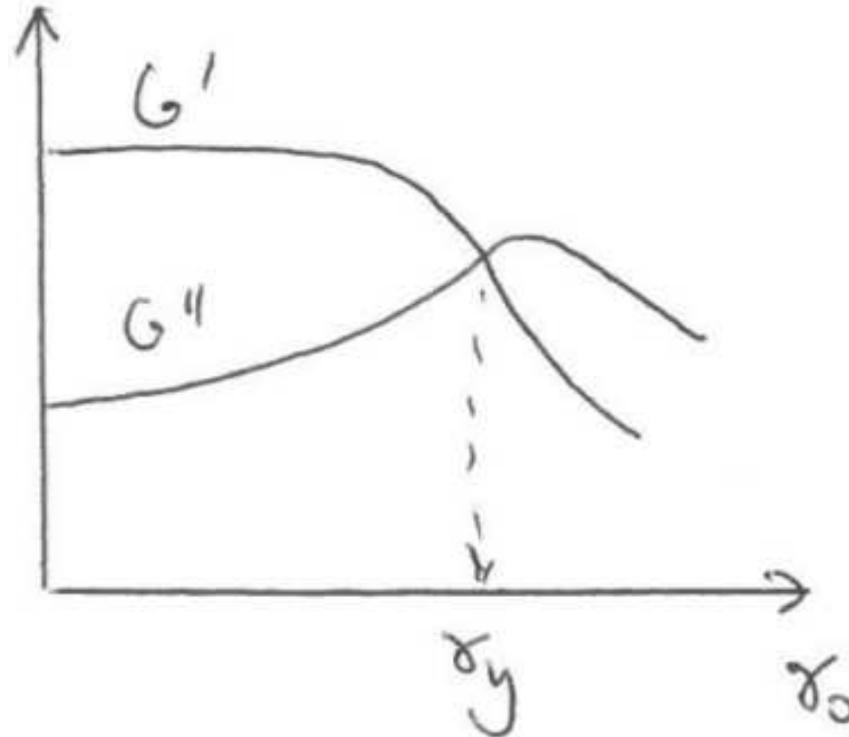
# Yielding(s): Experimental review

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# A critical yielding transition?

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- Ideas from irreversibility transition and absorbing states.

[Nagamanasa *et al.* PRE '14, Regev *et al.* PRE '13, Fiocco *et al.* PRE '13]

- Discontinuous transition observed in confocal microscopy.

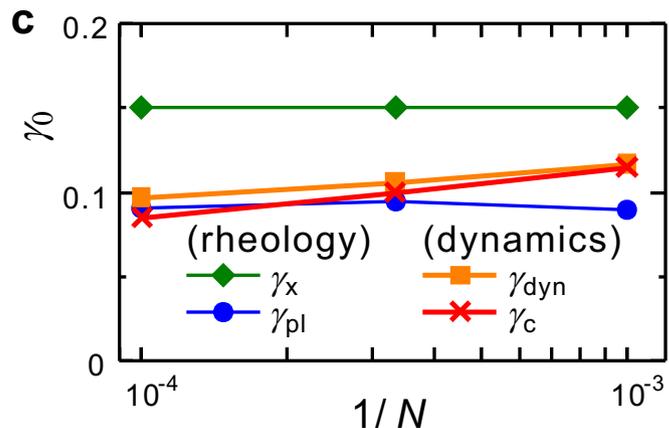
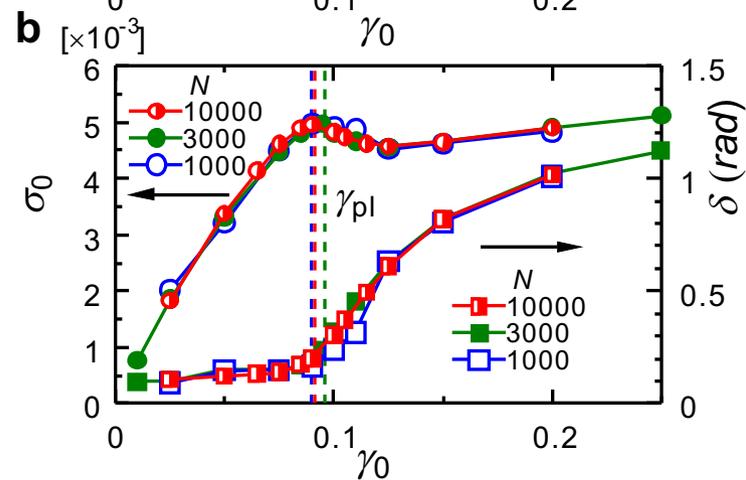
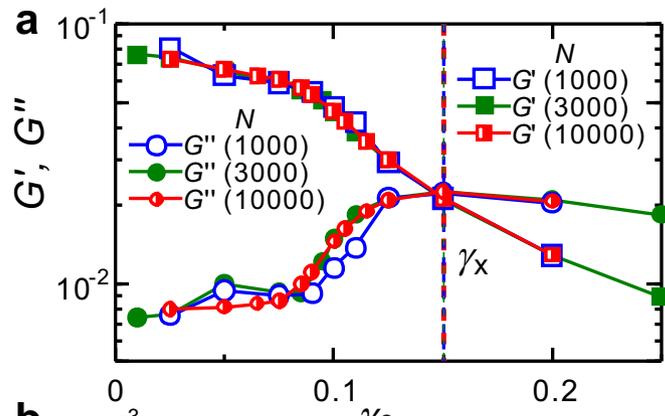
[Knowlton *et al.*, Soft Matter '14]

# Oscillatory shear: Setup

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- T. Kawasaki and L. Berthier, *Macroscopic yielding in jammed solids is accompanied by a non-equilibrium first-order transition in particle trajectories*, PRE 94, 022615 (2016).
- Overdamped athermal simulations of soft harmonic spheres above jamming: Shearing a simple jammed solid / glass at  $T = 0$ .
- Periodic boundary conditions: **spatially homogeneous** flow.
- Apply periodic deformation:  $\gamma(t) = \gamma_0(1 - \cos(\omega t))$ , with  $\omega \ll \epsilon/(a^2\xi)$ .
- Measure the **steady state** shear stress:  $\sigma(t) = \sigma_0 \cos(\omega t + \delta)$ . Extract  $(\sigma_0, \delta)$  to measure 'linear' response - even beyond linear regime.
- **Equivalently**:  $G'(\omega) + iG''(\omega) = \sigma_0/\gamma_0 e^{i\delta}$  are the well-known experimental measurements of the linear response. Extract  $(G', G'')$ .
- Access also **microscopic structure** (e.g.  $g(r)$ ,  $S(q)$ ) and **dynamics** (e.g. mean-squared displacement).

# Macroscopic rheology



- **Crossing** of  $G'$  and  $G''$  defines  $\gamma_x \approx 0.15$ . Standard location of 'yielding' transition under oscillatory shear.

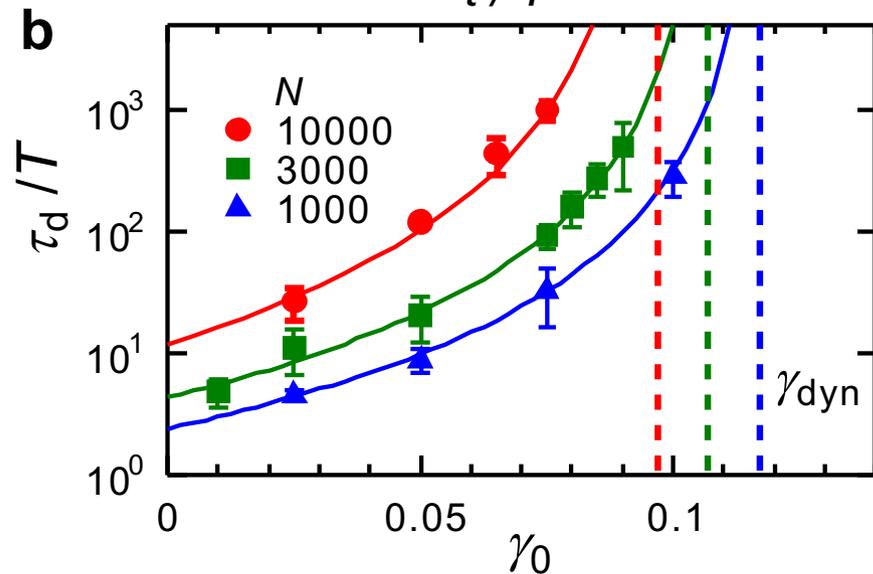
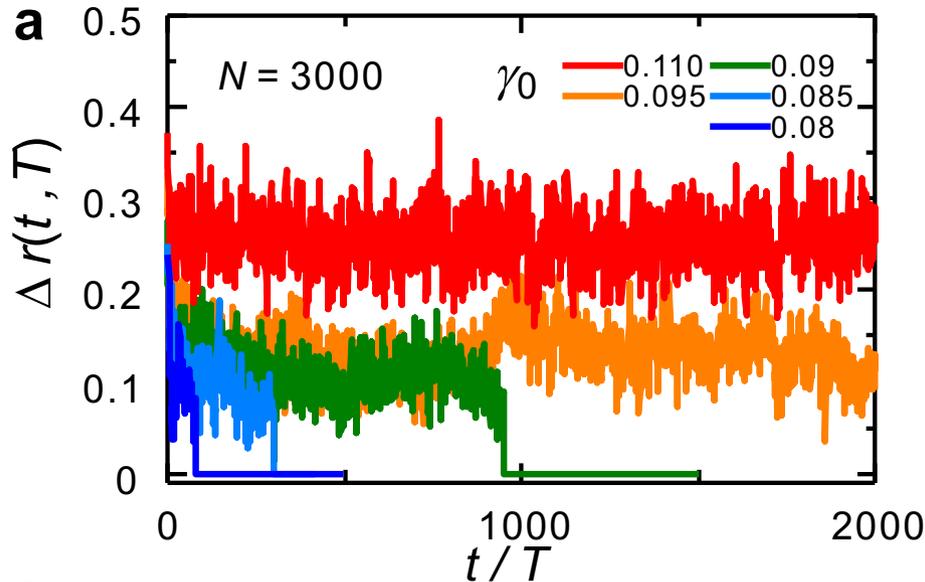
- **Maximum** of  $\sigma_0$  defines  $\gamma_{pl} \approx 0.1$  ( $\gamma_x$  is invisible here).

- $\gamma_{pl}$  also corresponds to **onset of dissipation**, when  $\delta > 0$ .

- We thus have at least 2 good definitions of 'the' yielding transition, which is ok if it's not a transition and **just a crossover**.

- Finite-size effects are small, which is ok.

# Microscopic dynamics: Transition?



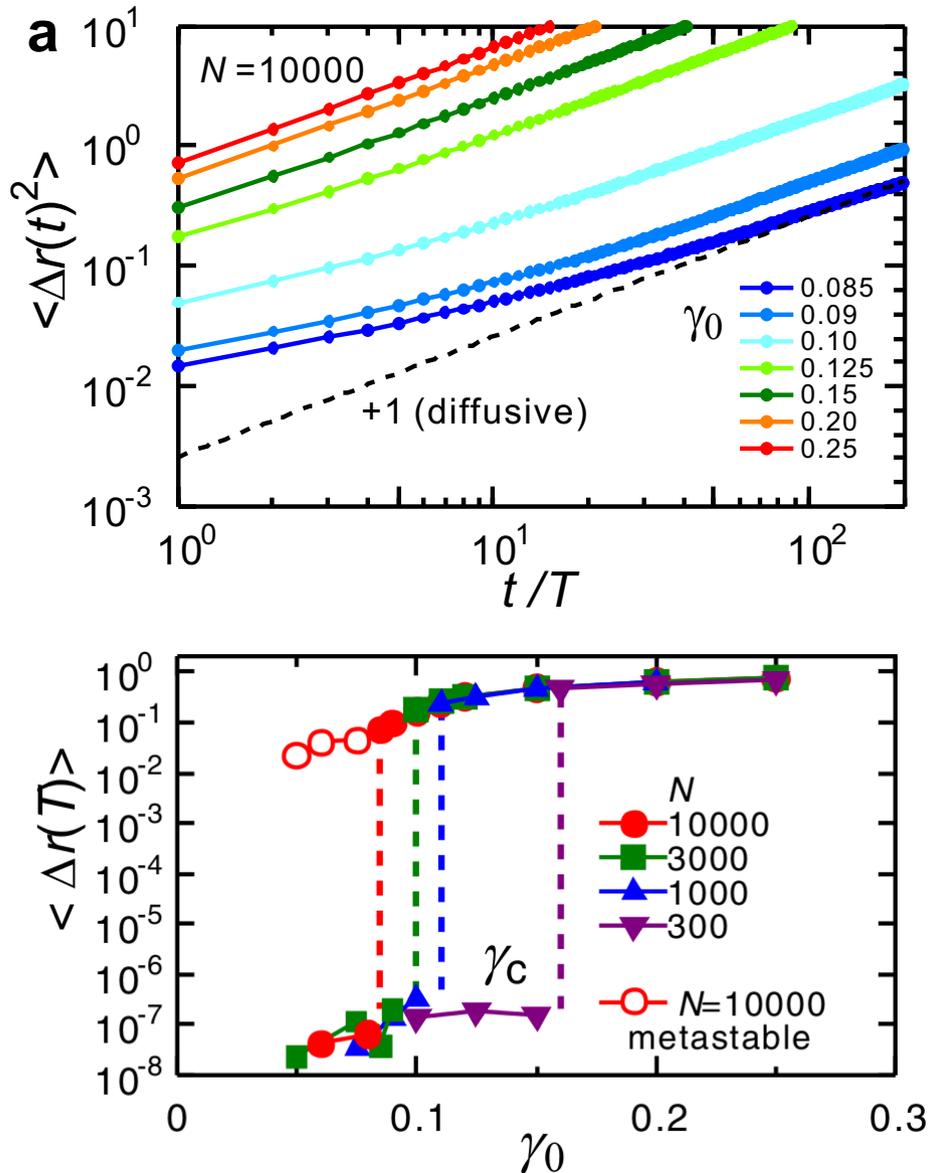
- Mean particle **displacement** after one cycle:  $\Delta r(t, T = 2\pi/\omega)$ .

- Starting from a **random configuration** at  $t = 0$ ,  $\Delta r(t, T)$  either **vanishes** when  $\gamma_0 < \gamma_{\text{dyn}}$  or **fluctuates** around well-defined average value otherwise.

- Timescale to reach (nearly) reversible state **diverges** as  $\gamma_0 \rightarrow \gamma_{\text{dyn}}^-$ : power law with **pronounced finite size effects**. [Seen by Foffi, Sastry, Reichhardt...]

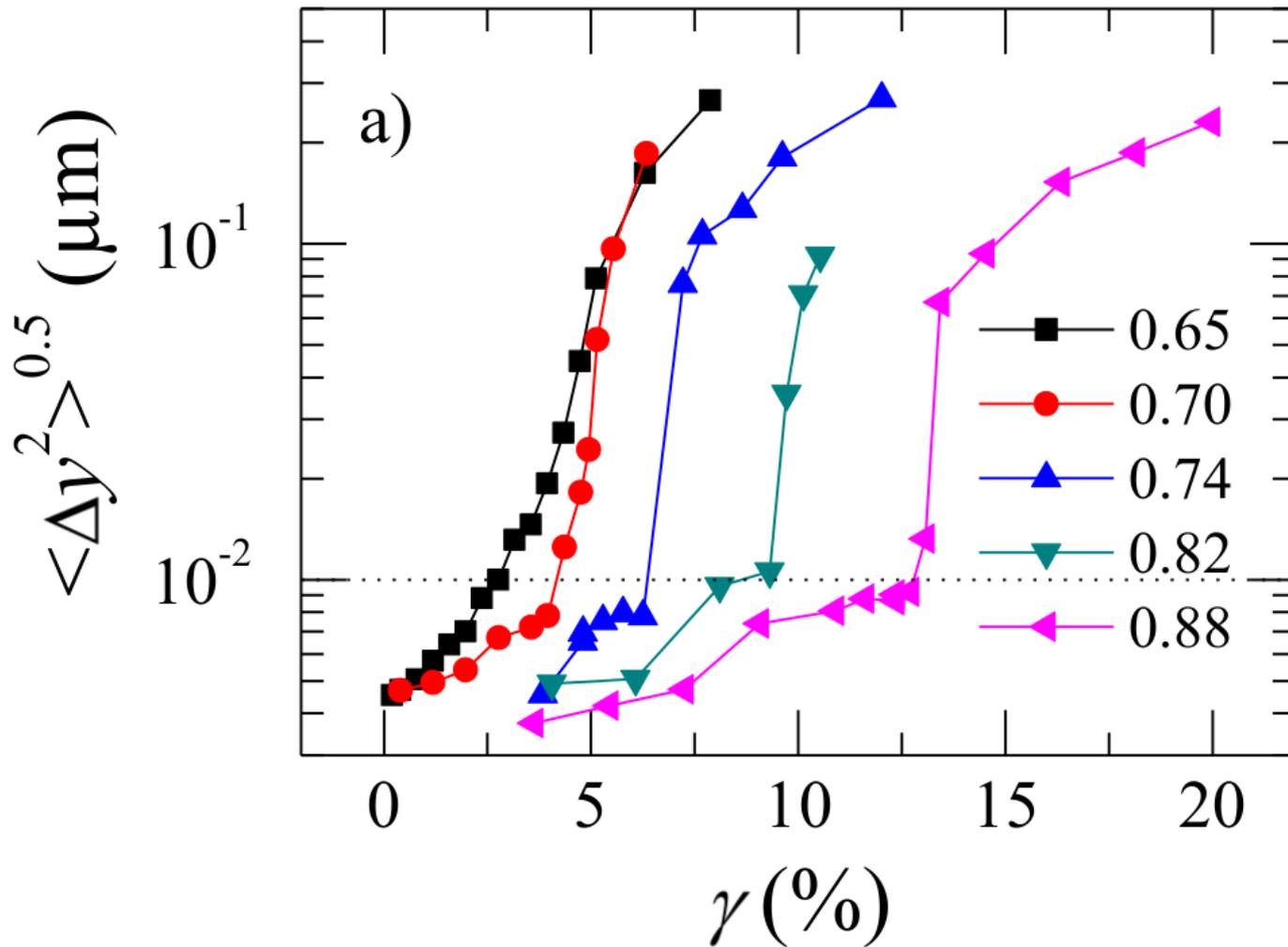
- Empirically, we find that  $\gamma_{\text{dyn}} \approx 0.1$  when  $N$  gets larger: **Critical point?**

# First-order dynamic transition



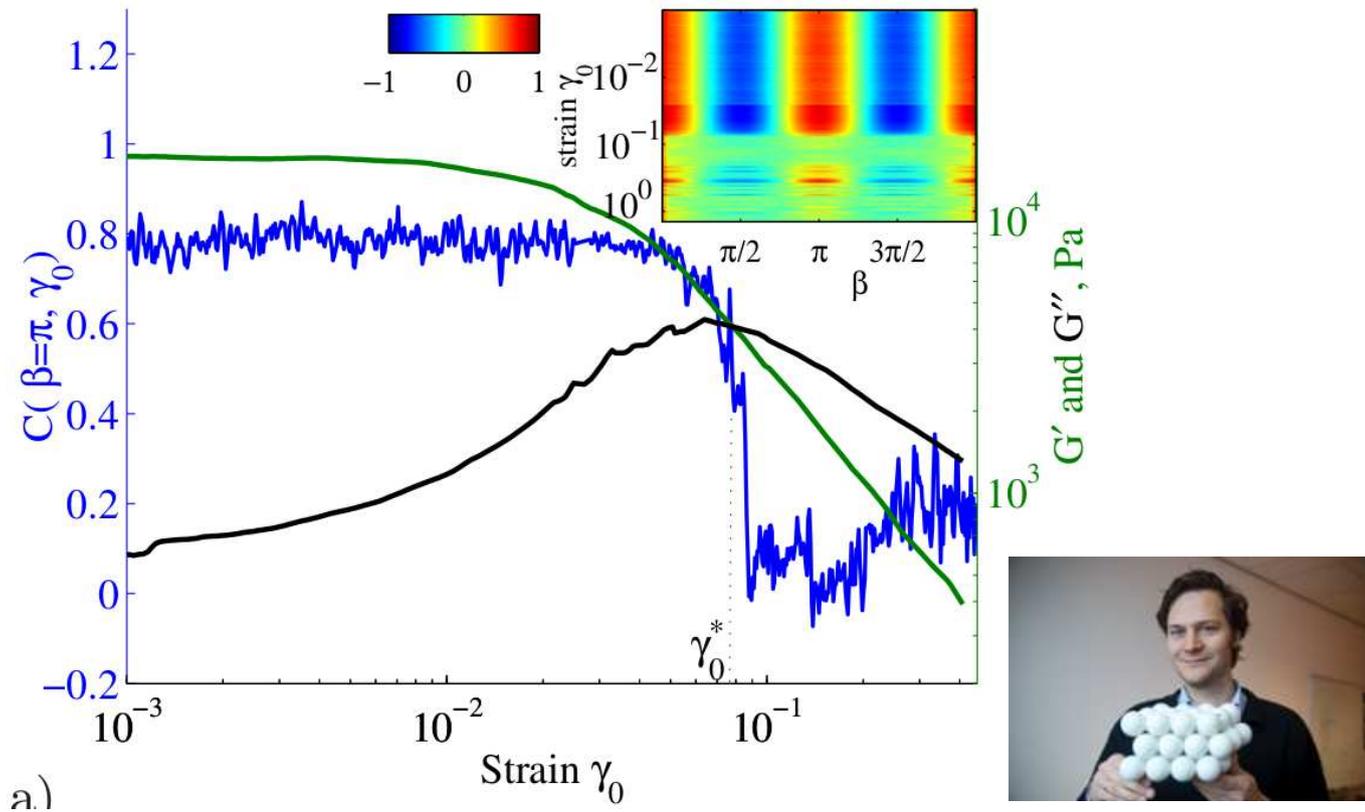
- Dynamics in steady-state regime for  $\gamma_0 > \gamma_c$  is **diffusive**.
- **Modest** decrease of diffusion constant as  $\gamma_0 \rightarrow \gamma_c^+$ .
- **Discontinuous** jump to zero at  $\gamma_c$ .
- Transition in microscopic dynamics appears **first-order**.
- Consistent with '**metastable**' dynamics seen in transient regime.
- Empirically, we find  $\gamma_c \approx \gamma_{\text{dyn}} \approx 0.1$  when  $N$  gets larger.

# 1 - Cipelletti's experiment



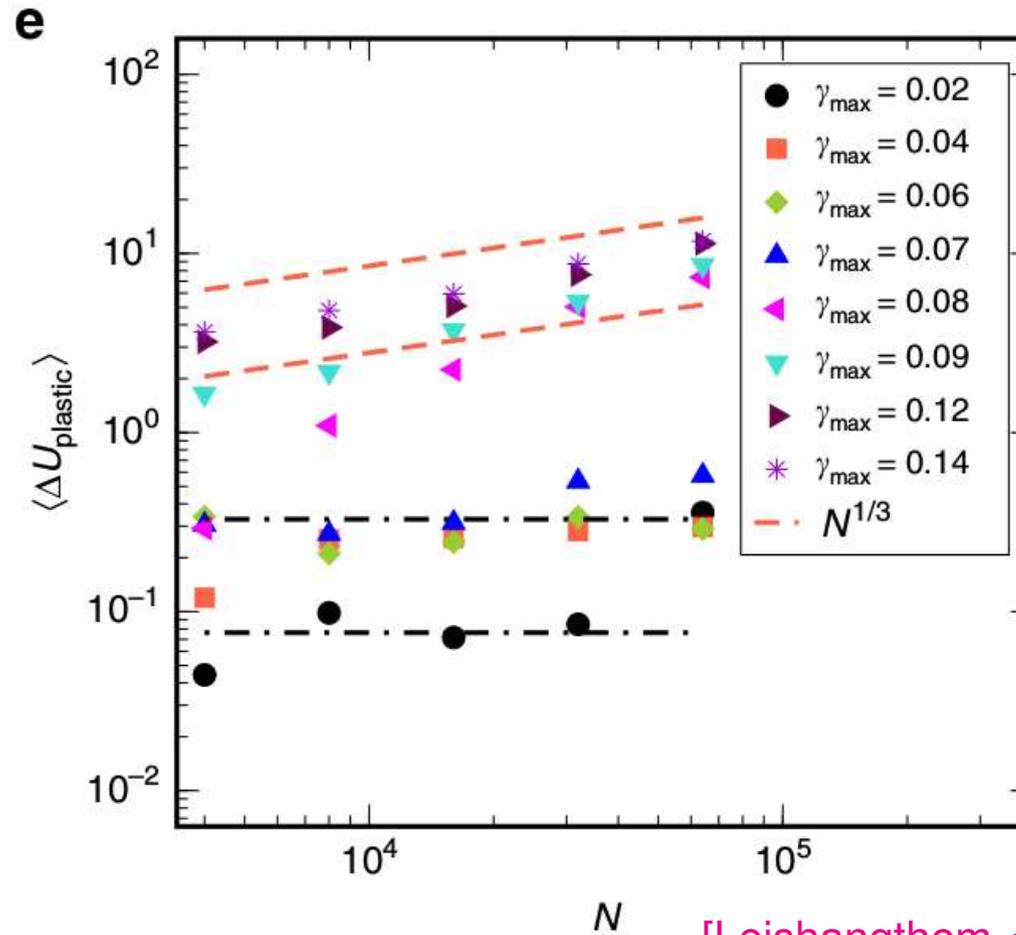
- Sharp (discontinuous?) increase of mean displacement per cycle at yielding unrelated to crossing of  $G'$  and  $G''$ . [Knowlton et al., Soft Matter '14]

# 2 - Schall's experiment



- $C(\beta)$  measures tiny (about 1 %) **anisotropy in  $S(q)$**  in diffraction plane. **Sudden change** from anisotropic (deformed elastic glass) to isotropic (plastic flow) right where  $G'$  and  $G''$  cross. [Denisov *et al.*, *Sci. Rep.* '15]
- We see **nothing at all** in  $S(q)$ , and our interpretation is **totally different**.

# 3 - Sastry's simulations

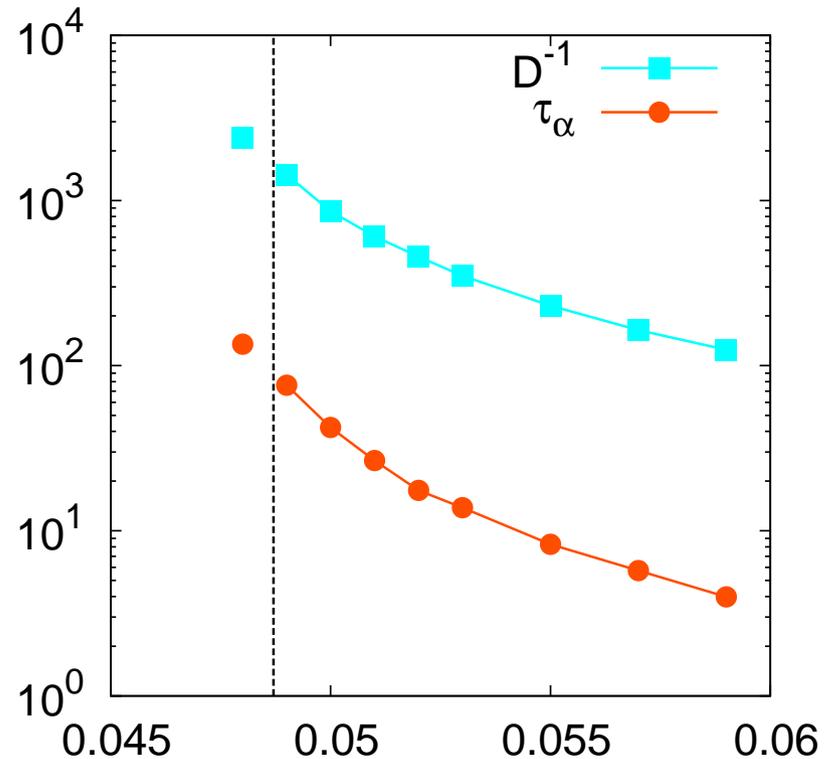
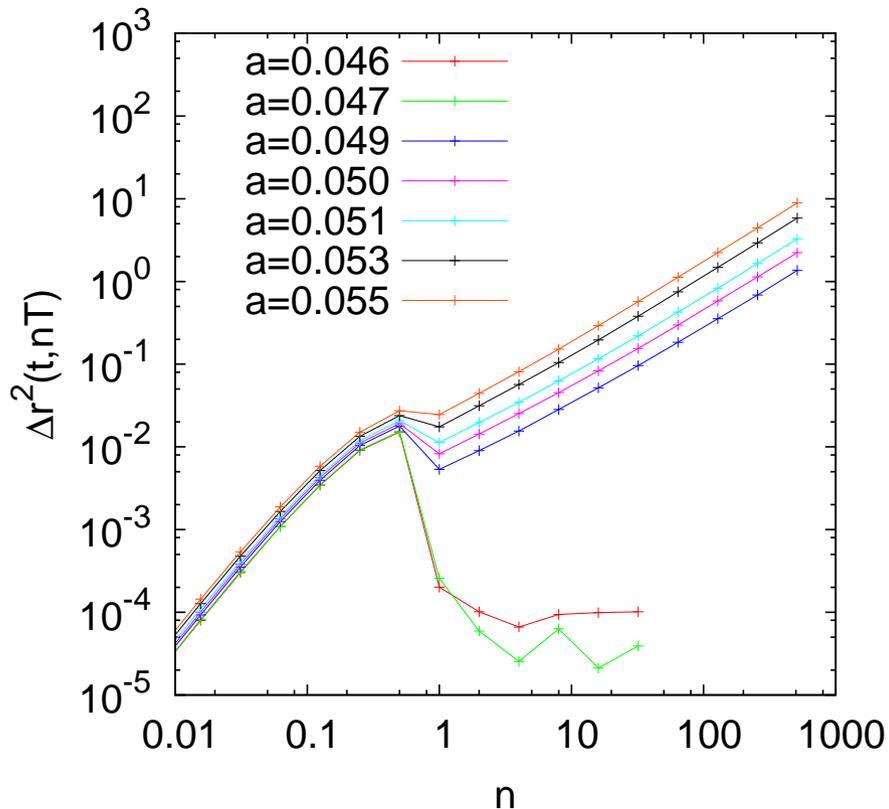


[Leishangthem *et al.*, Nature Comm. '15]

- Different scaling above and below the transition, which seems to confirm the **discontinuous** nature of the transition. See also [Regev *et al.*, Nature Comm. '15] for opposite conclusions!

# 4 - Periodic volume fluctuations

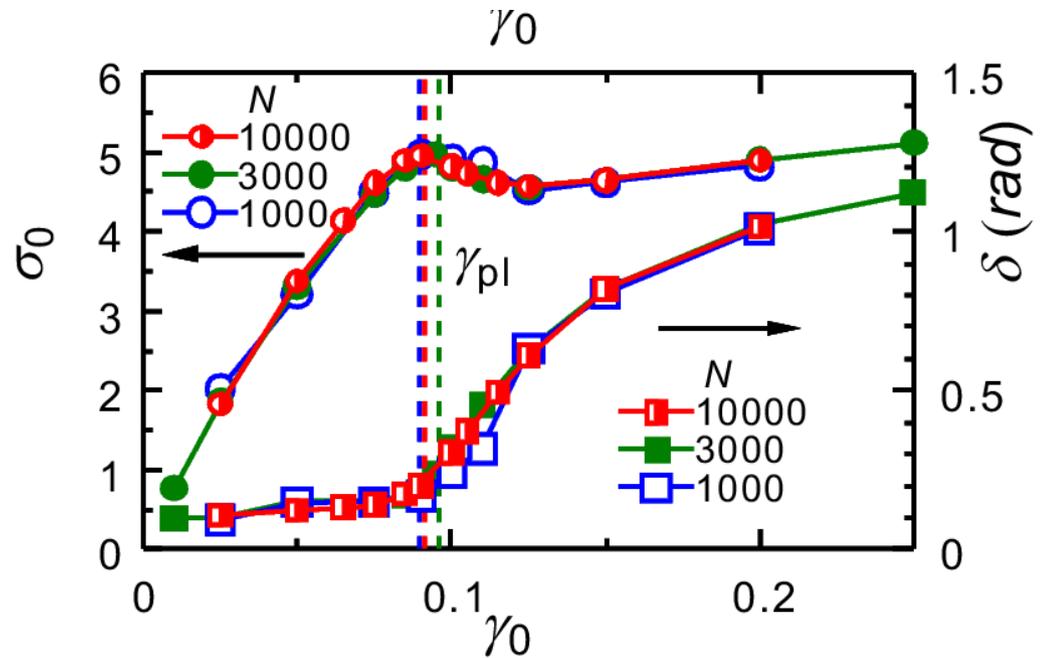
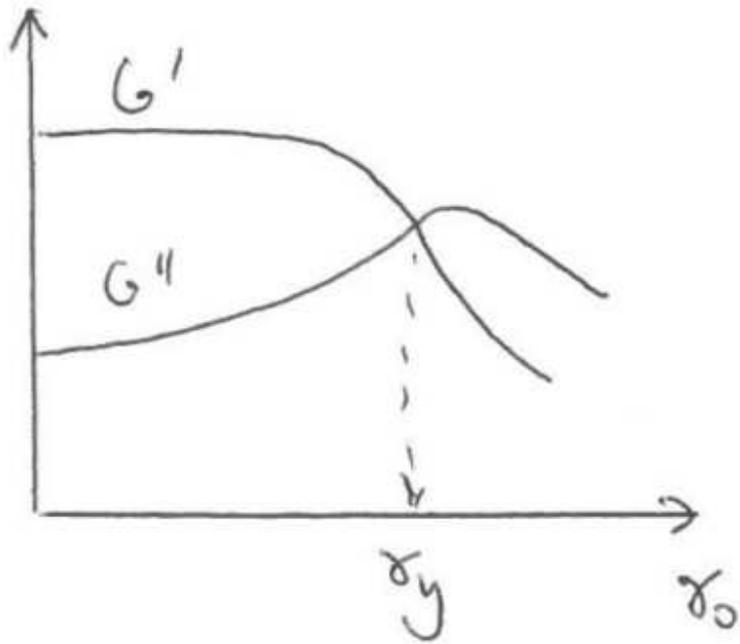
- **Abrupt** emergence of diffusive motion; **dynamical first-order transition** in actively-deforming particles. [Small versus large amplitude.]



[Tjhung & Berthier, arXiv:1607.01734]

- **Qualitatively similar** to periodic global deformation. Only difference is forcing at small rather than large scale—physics strikingly similar.

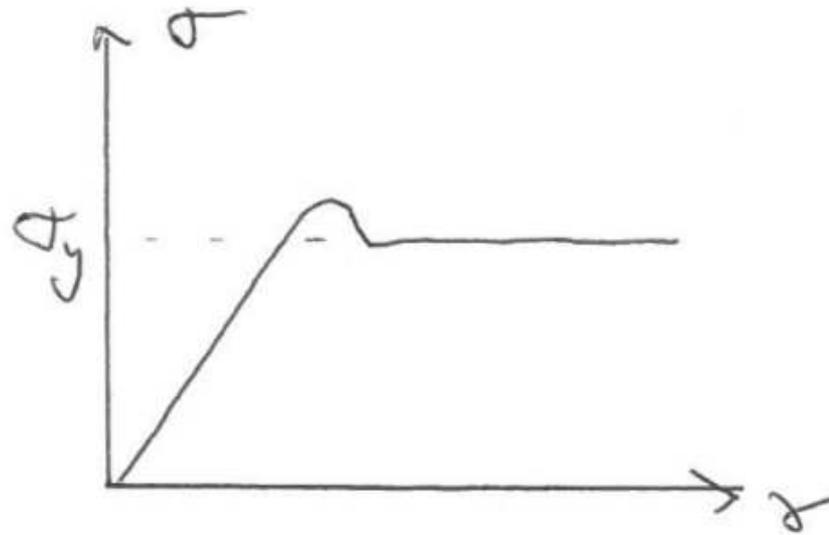
# Summary



- Right set of variables should be used.
- Two phases are **truly distinct**: unyielded phase remembers initial conditions (e.g. very stable), steady state is automatically critical (i.e. threshold and marginal).
- Transition is bound to be **discontinuous** in character, with nothing interesting on the left side. Not really 'the' yielding transition...

# A critical yielding transition?

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- **Percolation** ideas: empirical description.

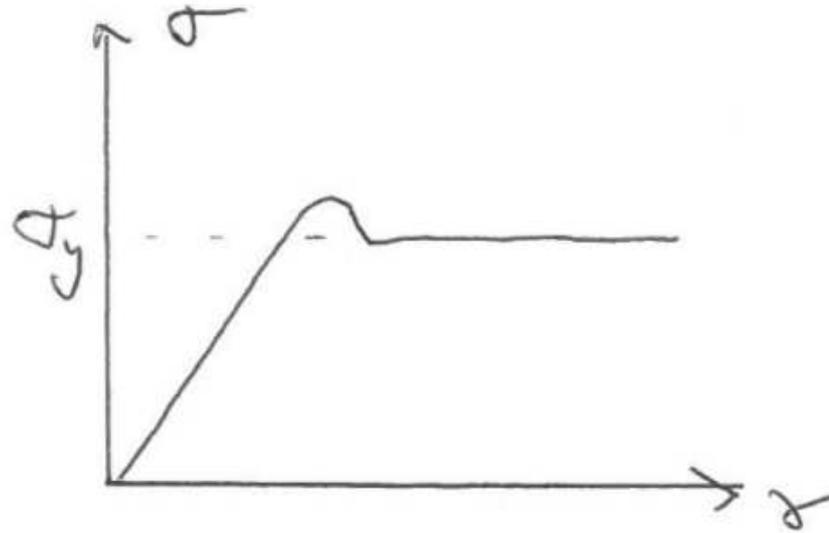
[Horbach and Chaudhuri '17, Gosh *et al.* '17]

- **Scale free** spinodal: 'mean-field' description. First-order or/and critical?

[Zamponi and Urbani '17, Procaccia *et al.* '16, Procaccia *et al.* '17]

# Elastic branch?

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- Elastic branch is **critical** everywhere.

[Lin *et al.*, PRL '15]

- Elastic branch is not critical [everybody else?, Hentschel *et al.*, PRE '15], only becomes critical when  $\gamma \rightarrow \gamma_Y$  [Procaccia *et al.* '17]

- Elastic branch becomes critical at **Gardner** point.

[Urbani & Zamponi '17, Jin & Yoshino]

# 'Well-known' facts

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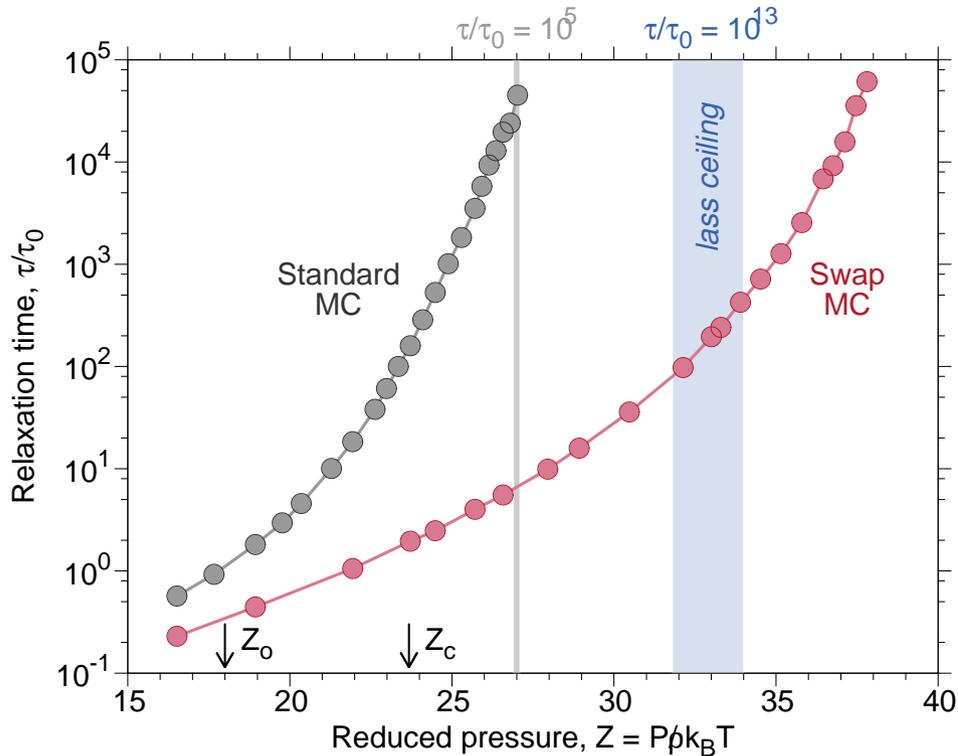
- Yielding transition is **not seen** directly through rheological observables (local dynamics, overlaps...).
- A lot of work for second order criticality and exponents in **steady state**, or equivalently for  $\dot{\gamma} \rightarrow 0$ .
- **Shear bands** are observed by accumulation of large number of discrete events, at large deformation.
- Behaviour is 'universal': foams, emulsions, glasses, chocolate, etc.

# Computer simulations

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- Usually limited to short time scales and small systems.
- **Athermal quasi-static** simulations solve one of two timescales problem: deformation rate becomes effectively slow enough.
- Ordinary computer simulations still face the **preparation timescale** problem. Cooling rate is about **8 orders of magnitude** too fast.
- Thus, computer simulations **may not be relevant** for real glasses. Only useful for colloidal and granular materials. (Experiments on colloids **may not be relevant** for real glasses either.)
- **Glass stability** is typically not seriously taken into account in coarse-grained models & theories. Exception: mean-field!

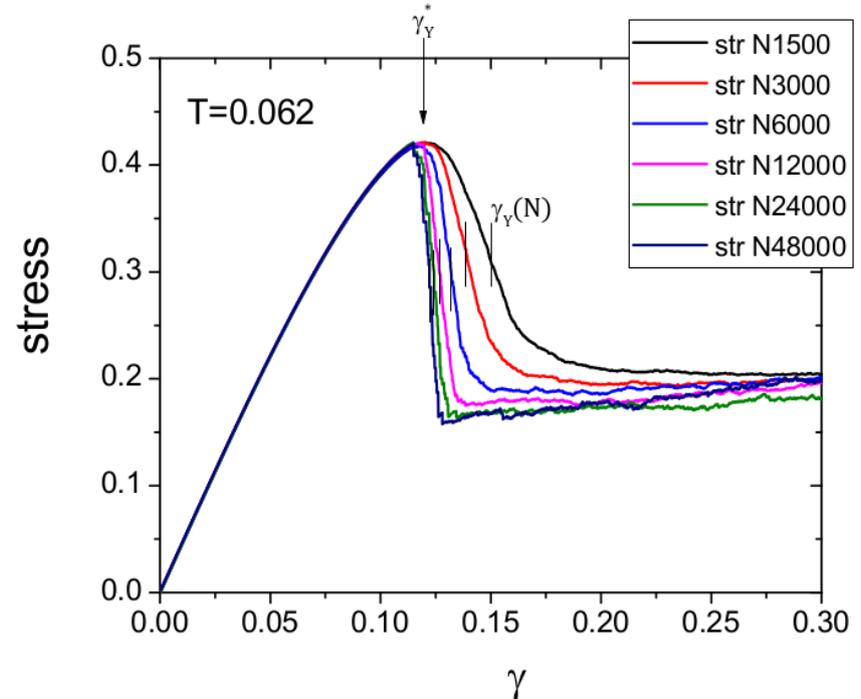
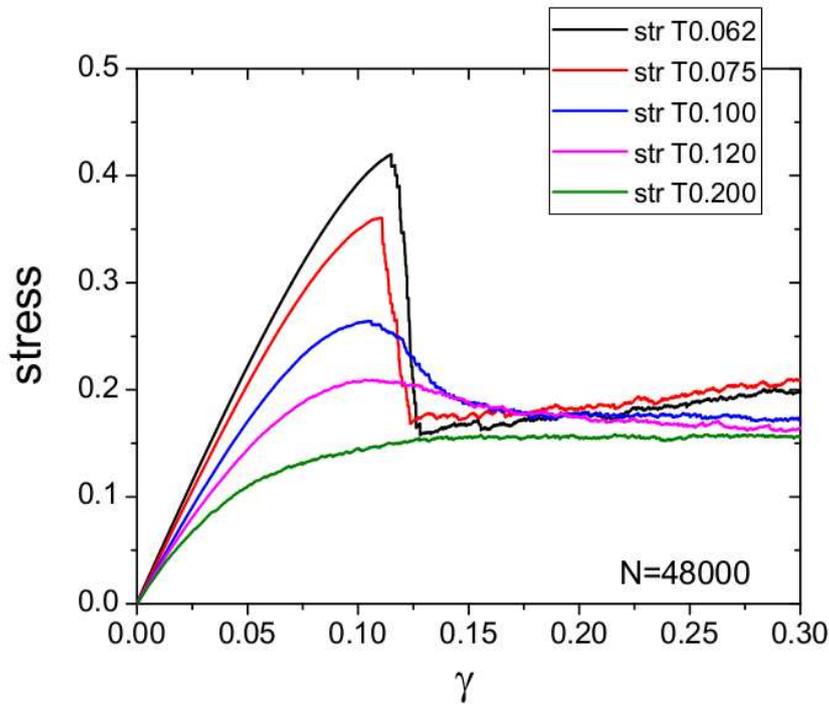
# Swap Monte Carlo



[Ninarello, Berthier, Coslovich, PRX '17]

- Simple method solves the second timescale problem: we **thermalise glasses below  $T_g$** .
- Several models thermalised below  $T_g$  (13 decades): we gain **more than 11 decades**.
- Physics: Slow **diffusion in diameter space** facilitates particle diffusion in real space.
- We can now study the **mechanical response** of glass configurations that are **experimentally relevant**: 'Well-annealed' glasses...

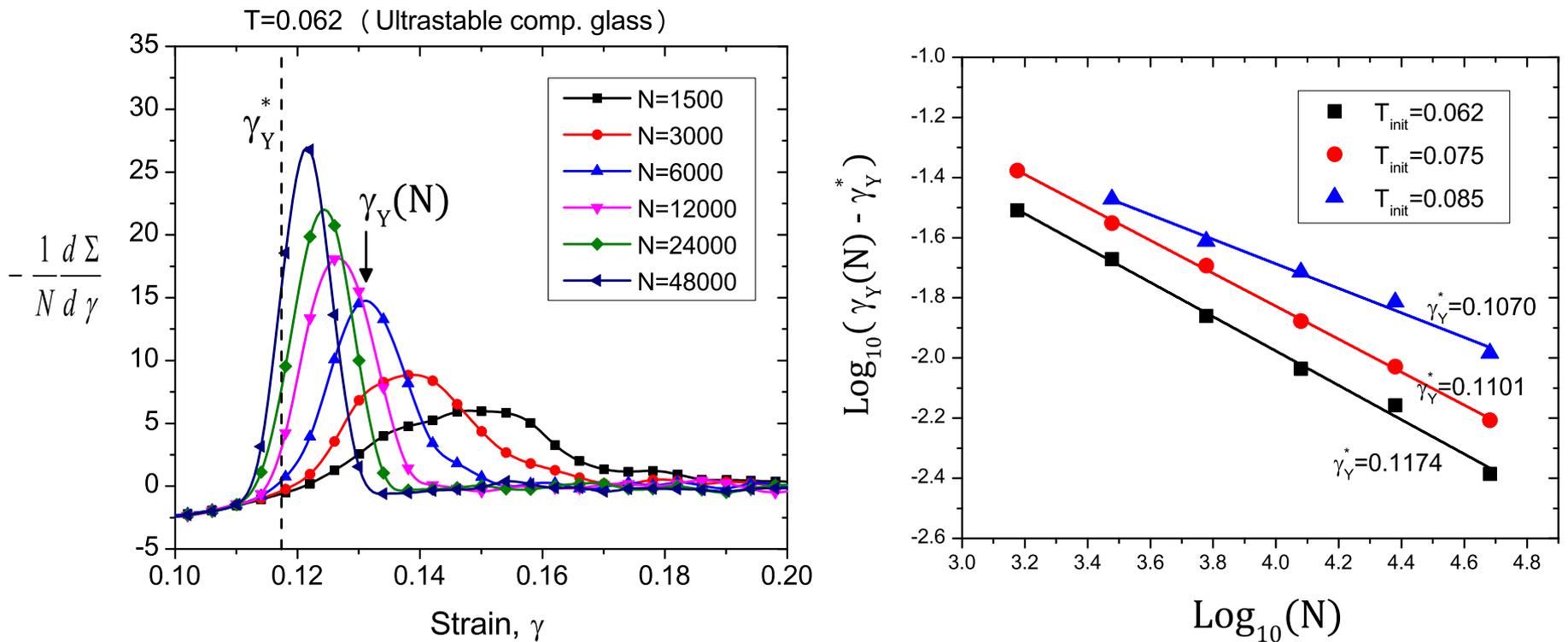
# Yielding transition exists



- For low 'enough'  $T$ , the transition becomes **easily observable** from macroscopic rheological observables.
- Becomes sharper as  $N \rightarrow \infty$ : A bona fide **phase transition**.
- No need for complicated microscopic observables. **Glass stability** changes the physics **qualitatively**.

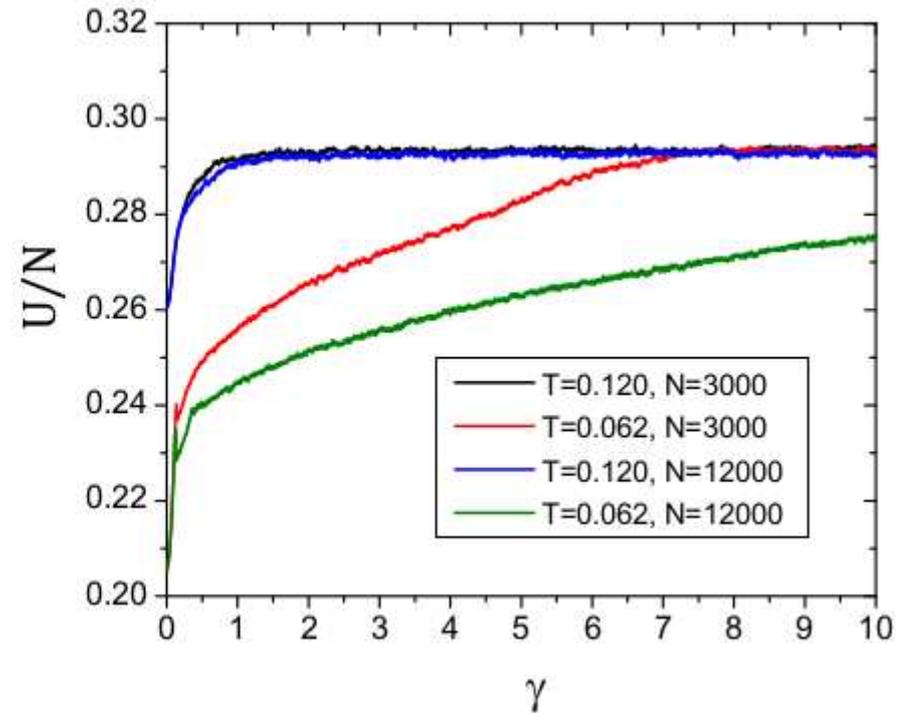
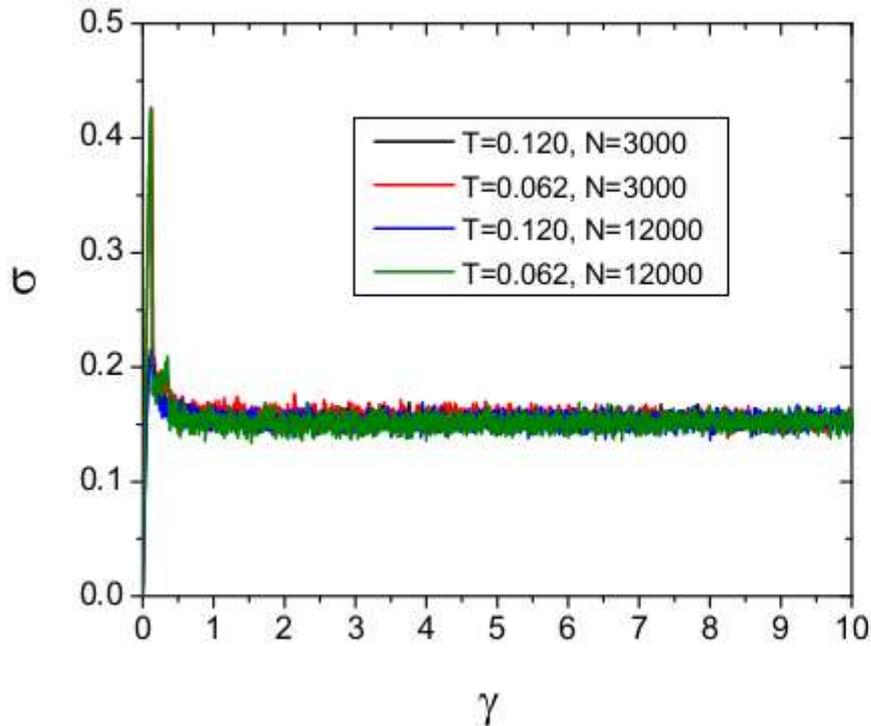
# Finite size scaling

- Convergence with system size appears under control.



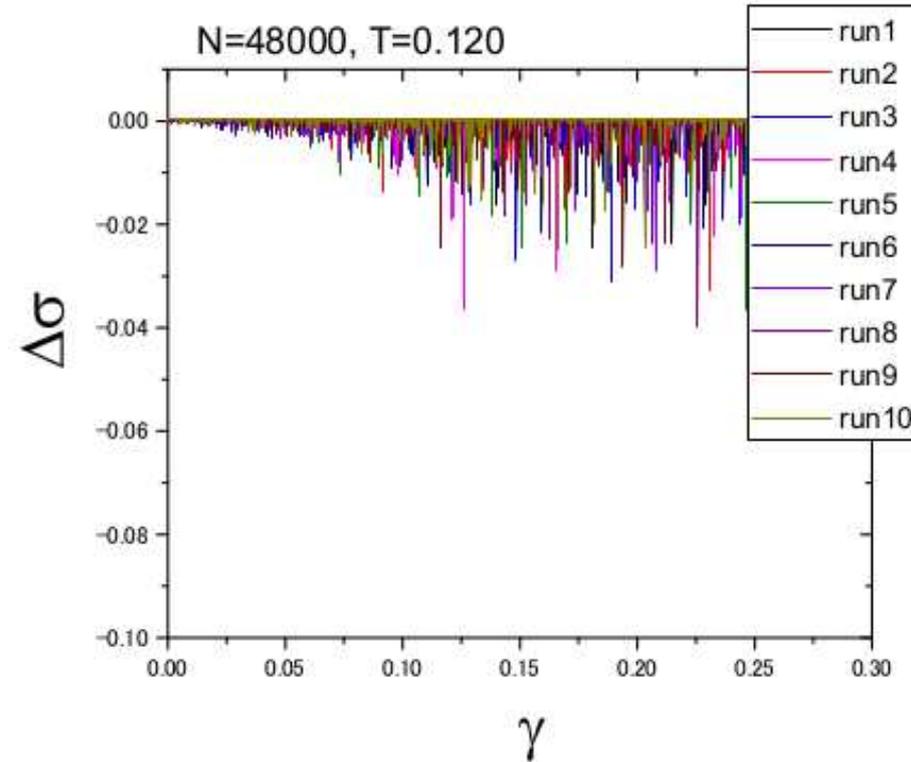
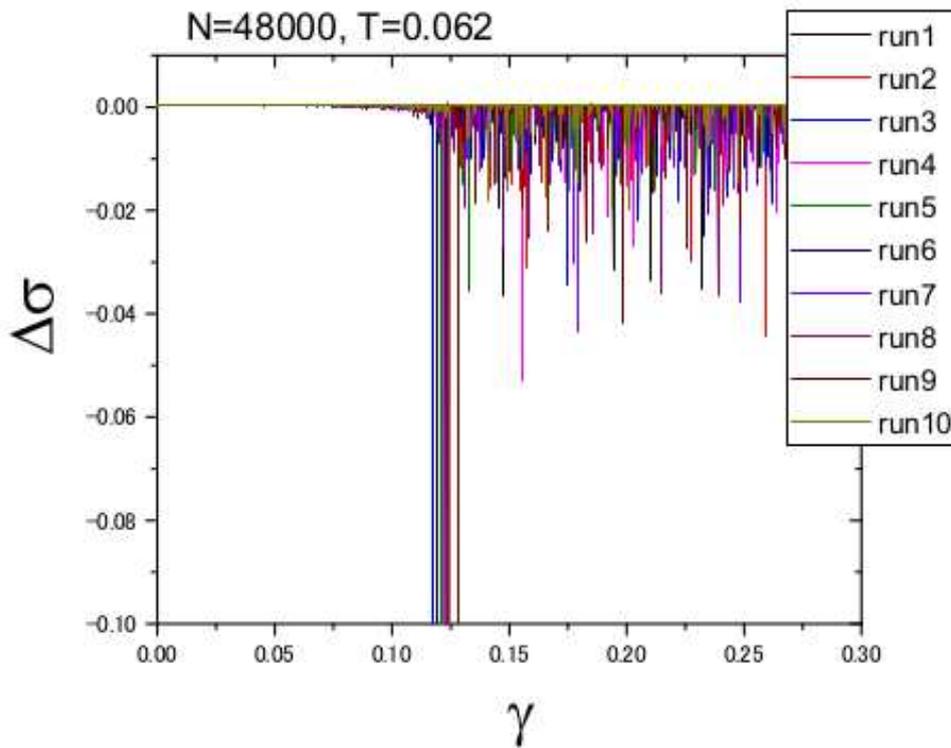
- Yielding is **not a phase transition** if  $T$  is too large. Previous simulations?
- There is a **“critical” temperature** below which discontinuity appears. We need to study that transition in more detail... In progress.

# Steady state does not exist



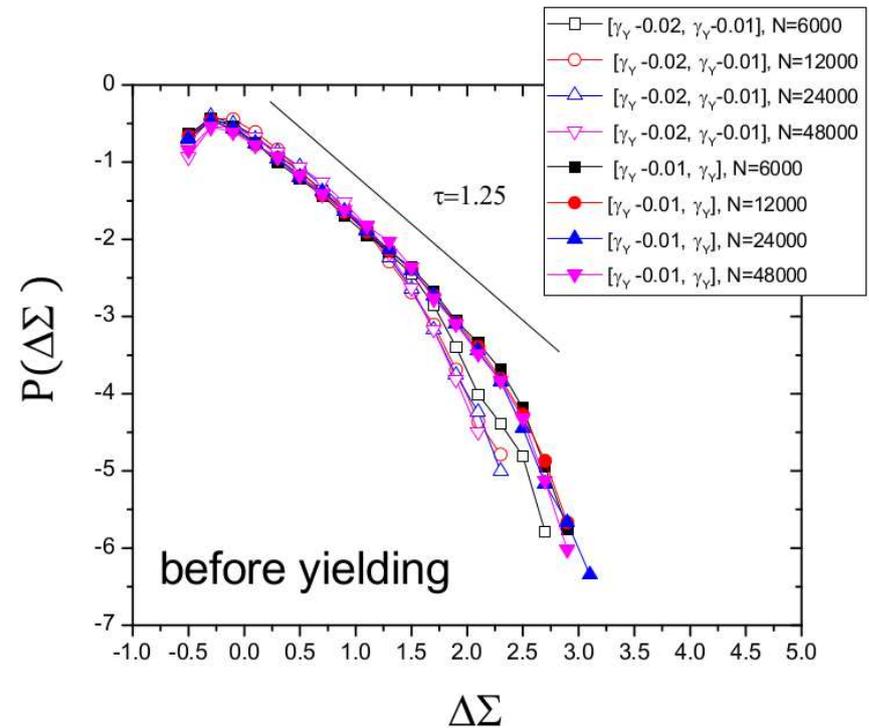
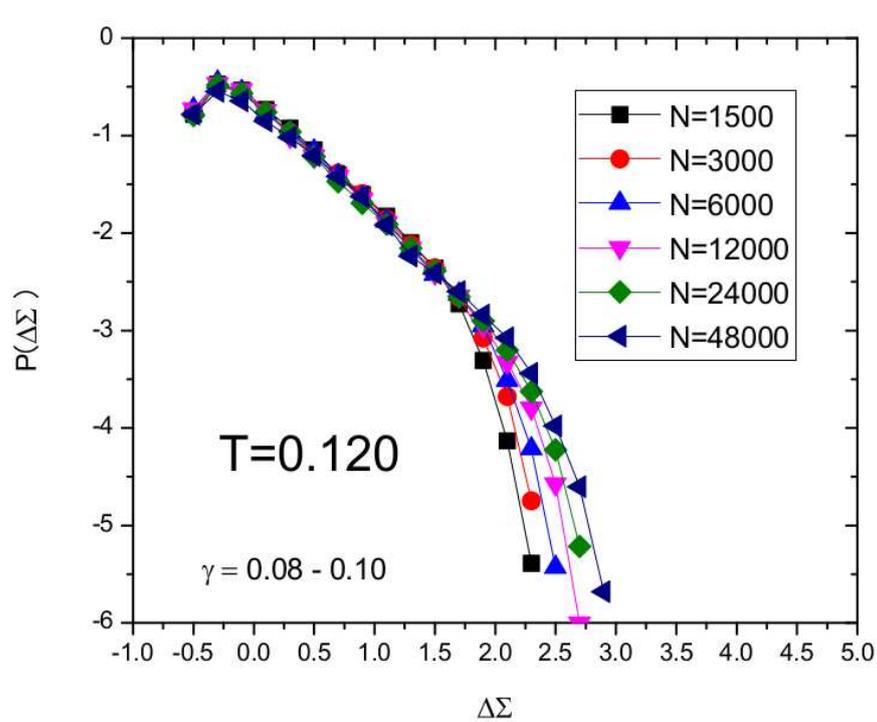
- The shear stress seems to converge.
- The energy never does. A steady state **never exists** in realistic glasses. Relevance of the heavily discussed ‘marginal’ critical point for real (hard) glasses?

# Elastic branch



- Stress and energy drops along the elastic branch apparently **disappear** at low  $T$ .
- The nature of **shear transformation zones / soft spots** changes dramatically. Amplitude of energy drops divided by 100!
- Abrupt change of critical exponents at yielding, see [Hentschel *et al.*, PRE '15]

# No marginal stability



- Distributions of stress and energy drops show no sign of criticality upon approaching yielding. **No spinodal criticality, no percolation.**
- Appearance of criticality might be due to **poor thermalisation** (simulation) or incorrect modelling (elasto-plastic models).
- Real glasses are not marginally stable.

[Scalliet, Berthier, Zamponi, PRL '17]

# Shear bands

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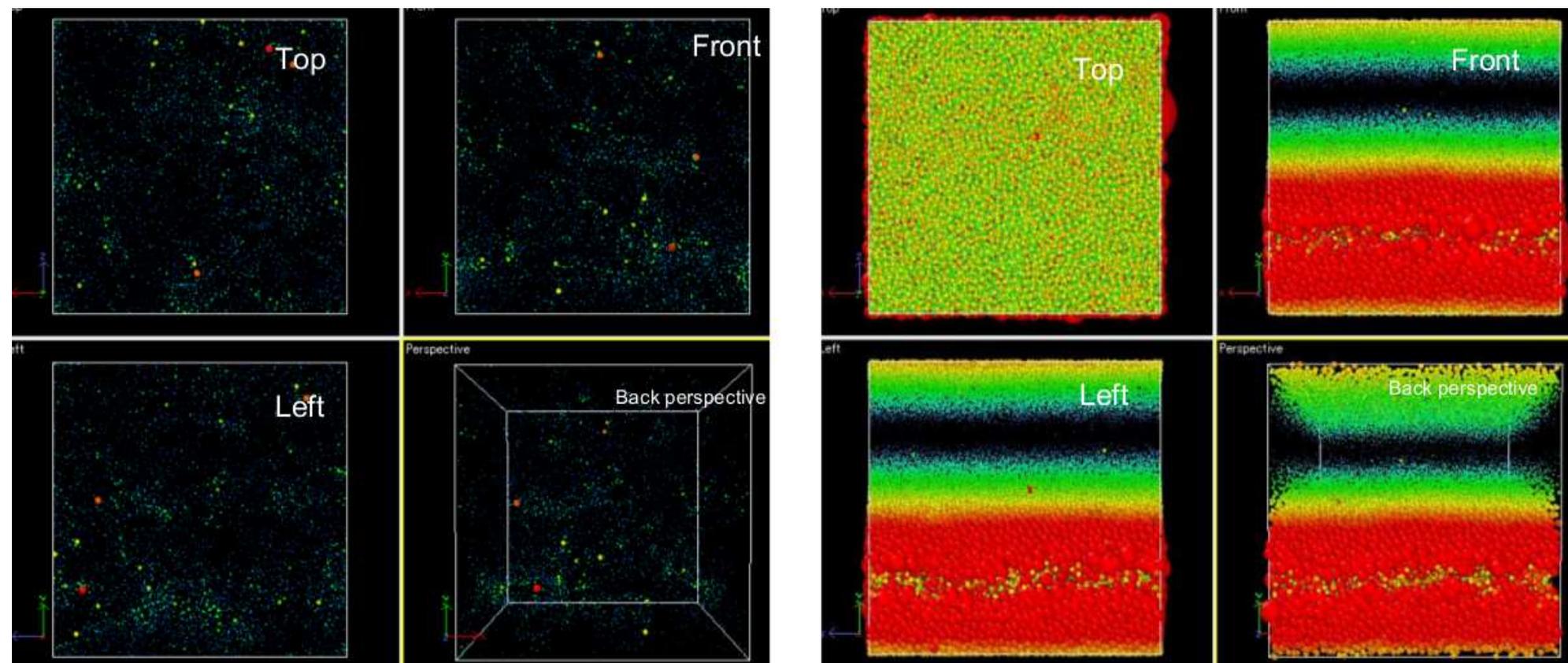
- Yielding at **high temperature** is gradual.
- Yielding at low  $T$  is **brutal**, one way or the **other**.
- No **percolation**.
- Multiple **breakings** can occur in the same sample. Looks like metallic glasses...



- The shear-band occurs within **a single energy minimization**, not through an accumulation of small events over large deformation.
- Anatomy of a single **shear band**.

# Predictability?

- Non-affine displacement at  $t - 1$  (left) and at  $t$  (right).

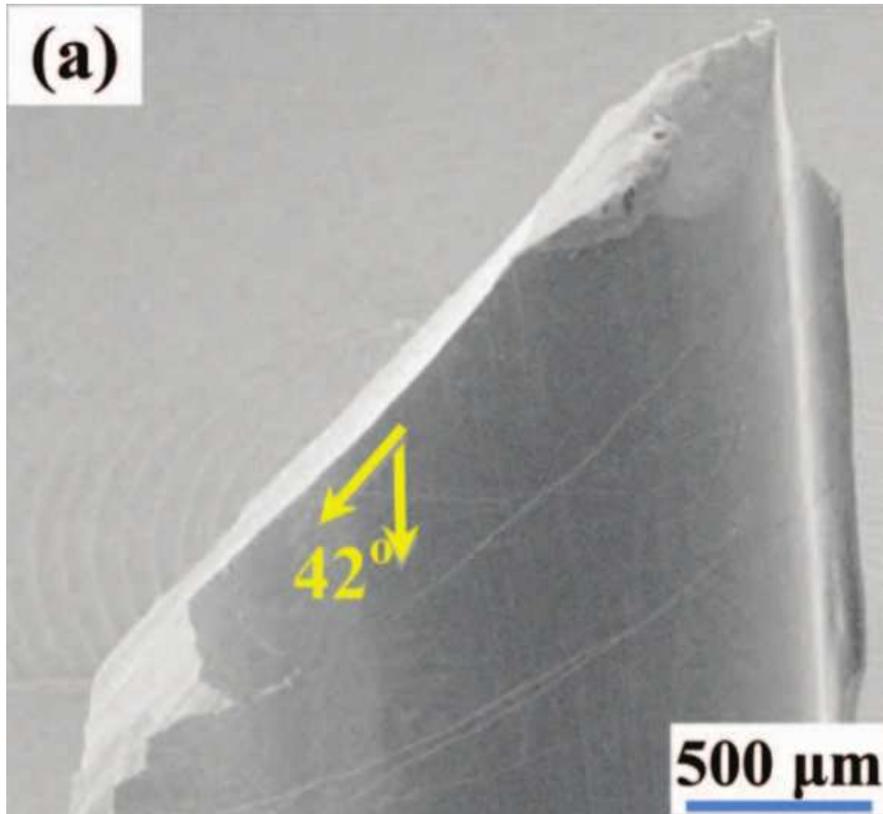


- Beuh.

# Conclusion

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- Discontinuous yielding under oscillatory shear, not sure it's useful after all.



- Sharp yielding transition exists for stable enough systems.
- Not critical, no diverging length-scale, no percolation.
- Our simulations seem appropriate for realistic glasses.