Some comments on “The onset of the flow in disorder materials: depinning and yielding transitions”

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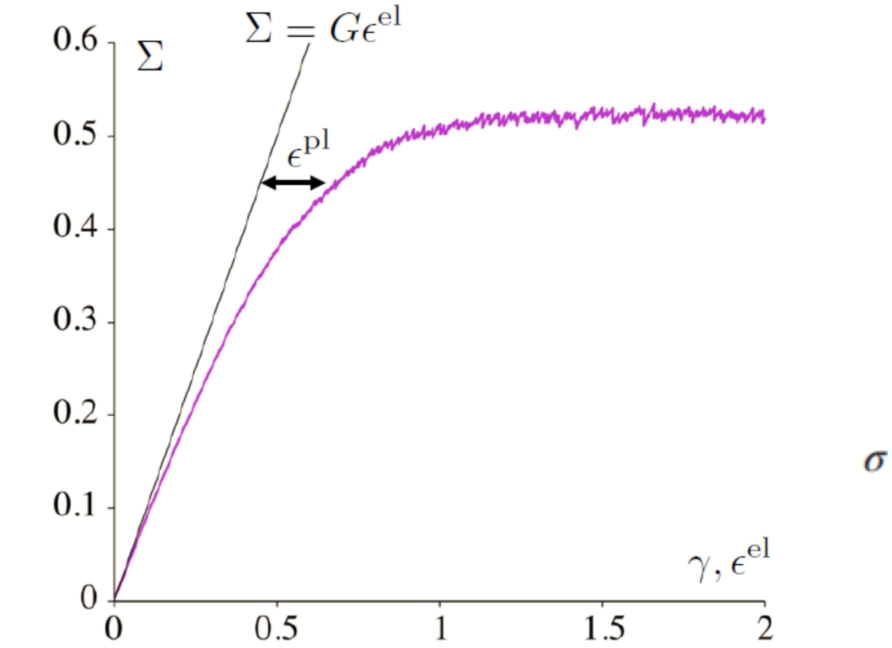
Paris, October 2018



$$\sigma = \mu\gamma, \qquad \gamma \ll 1,$$

One possibility

$$\sigma = \mu(\gamma=0)\gamma + B_2(\gamma=0)\gamma^2 + B_3(\gamma=0)\gamma^3 + \ldots .$$

H.G.E. Hentschel, S. Karmakar, E.Lerner, I. Procaccia,
Do Athermal Amorphous Solids Exist? , Phys. Rev. E **83**, 061101 (2011).

$$\Delta\sigma = \mu(\gamma)\Delta\gamma, \qquad \text{for any value of } \sigma \text{ and } \gamma, \qquad (3)$$

A. K. Dubey, I. Procaccia, C. A.B.Z. Shor, M. Singh,
Elasticity in Amorphous Solids: Nonlinear or Piece-Wise Linear?
Phys. Rev. Lett. **116**, 085502 (2016).

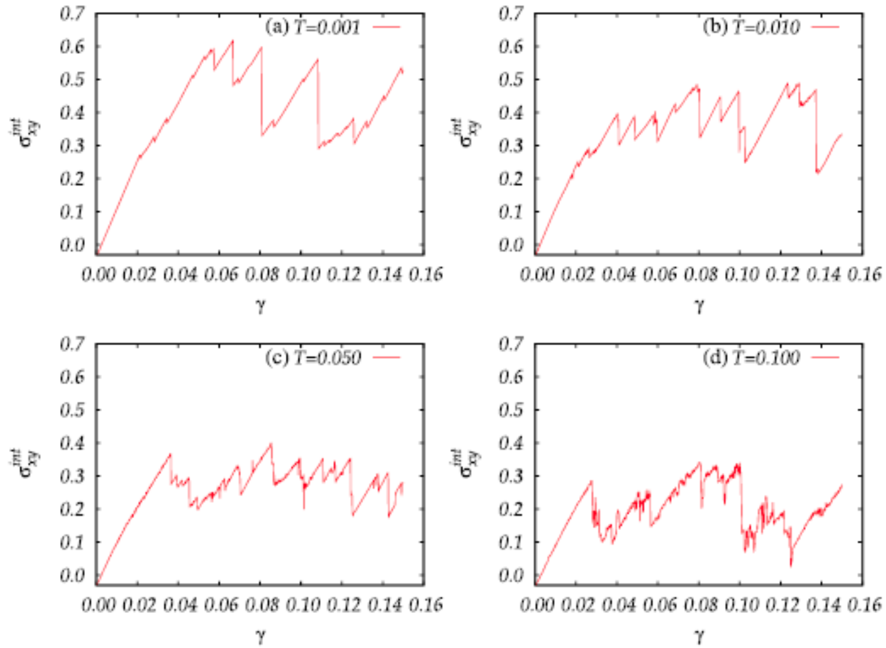


FIG. 2. Individual realizations of stress versus strain curves for systems of 1000 particles at different temperatures. Note that individual plots never attain a zero shear modulus between plastic events.

$$\mu(\gamma) = \mu_B(\gamma) - \frac{V}{k_B T}[\langle\sigma^2\rangle - \langle\sigma\rangle^2],$$

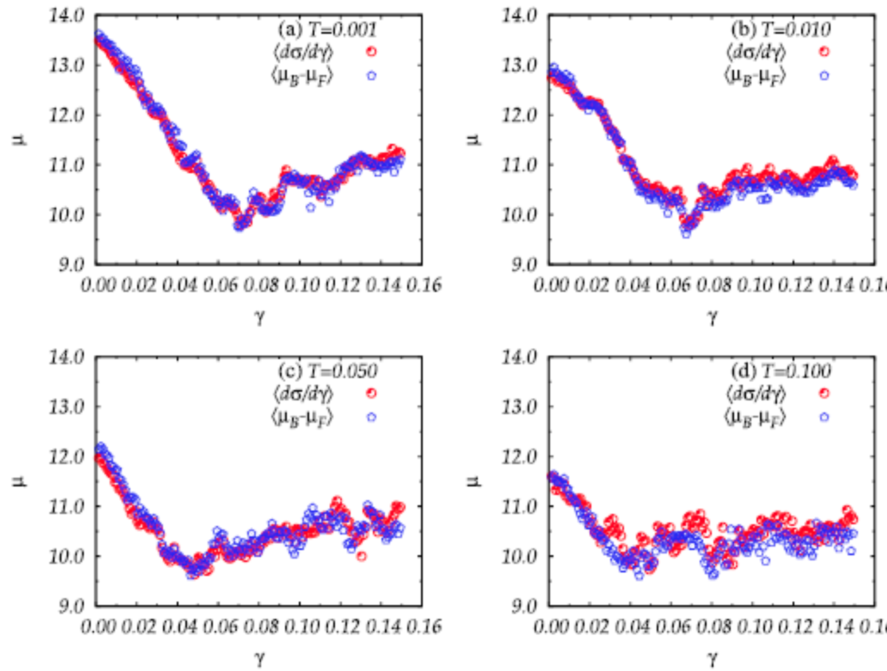


FIG. 3. Comparison of the direct measurement of quenched averages of the shear modulus from the local slope of the strain versus stress curves and the theoretical expression Eq. (4).

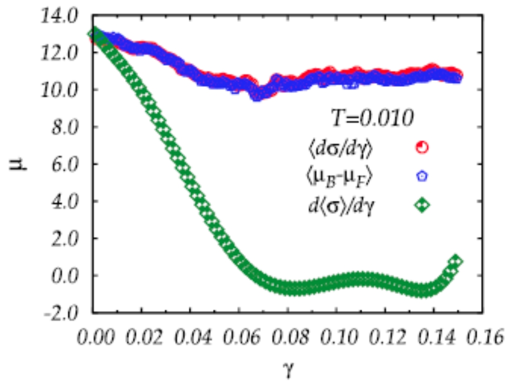


FIG. 4. An example of the comparison of annealed and quenched averages of the local slope of the strain versus stress curves and the theoretical expression Eq. (4). This example is at $T = 0.01$, but the conclusion is identical in all the tested temperatures: Except at $\gamma = 0$, the results of the annealed and the quenched averages of the stress versus strain curves differ greatly.

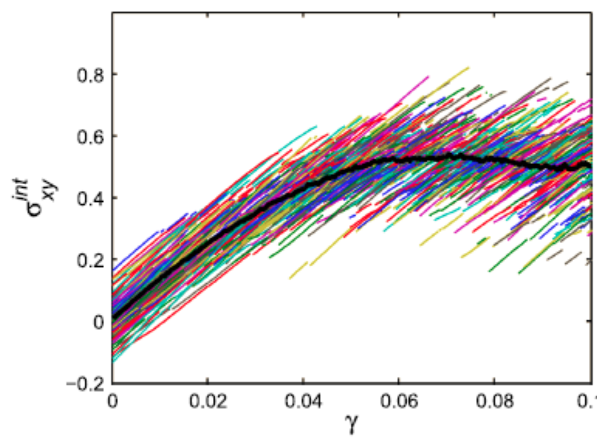


FIG. 5. The annealed stress versus strained curve (continuous black line) and the actual segments of linear response of the various realizations that were annealed to get the continuous line. Except at $\gamma = 0$, the annealed procedure does not supply the right information regarding the mechanical response.

Spinodal transition with precursor avalanches

I. Procaccia, C. Rainone and M. Singh, Mechanical Failure in Amorphous Solids: Scale-Free Spinodal Criticality, Phys. Rev. E. **96**, 032907 (2017).

G. Parisi, I. Procaccia, C. Rainone, M.Singh, Shear bands as manifestation of a criticality in yielding amorphous solids , PNAS **114**, 5577 (2017).

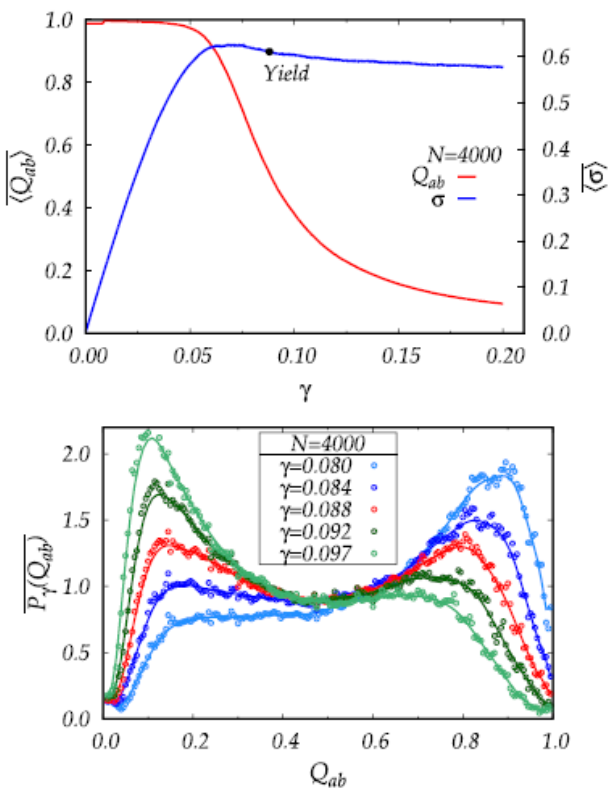


Fig. 5. (*Upper*) The order parameter $\overline{Q_{ab}}$ as a function of the strain γ superimposed on the stress versus strain curve. (*Lower*) The probability distribution function $P(\overline{Q_{ab}})$ for different values of γ in the vicinity of the mechanical yield value γ_Y.

$$Q_{ab} \equiv \frac{1}{N}\sum_{i=1}^{N}\theta(\ell - |\mathbf{r}_i^a - \mathbf{r}_i^b|),$$

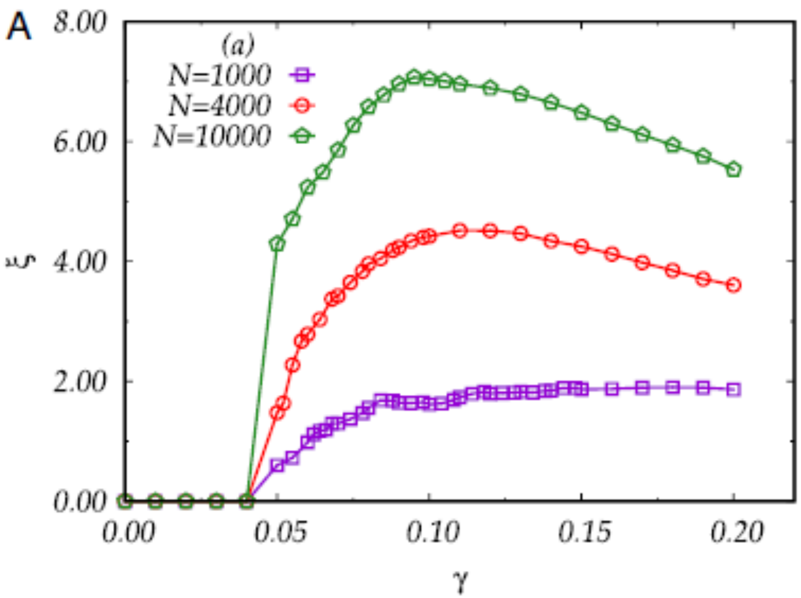
$$Q_{ab}(\mathbf{r}) \equiv \sum_{i=1}^{N}\theta(\ell - |\mathbf{r}_i^a - \mathbf{r}_i^b|)\delta(\mathbf{r} - \mathbf{r}_i^a).$$

$$G_L(\mathbf{r}) = 2G_R(\mathbf{r}) - \Gamma_2(\mathbf{r}),$$

with the definitions

$$G_R(\mathbf{r}) \equiv \overline{\langle Q_{ab}(r)Q_{ab}(0)\rangle} - 2\overline{\langle Q_{ab}(r)Q_{ac}(0)\rangle} + \overline{\langle Q_{ab}(r)\rangle\langle Q_{cd}(0)\rangle},$$

$$\Gamma_2(\mathbf{r}) \equiv \overline{\langle Q_{ab}(\mathbf{r})Q_{ab}(0)\rangle - \langle Q_{ab}(\mathbf{r})\rangle\langle Q_{ab}(0)\rangle}.$$



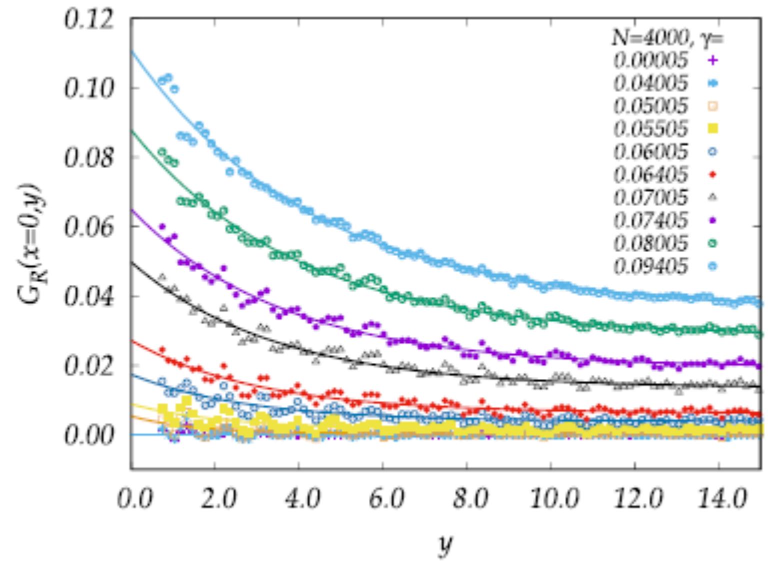


Fig. 2. The function $G_R(x=0, y; \gamma)$ for various values of γ from 5×10^{-5} to 0.09405. Note the increase in the overall amplitude of the correlator as well as the increase in the correlation length. The lines through the data are the fit function **10**.

Finite dimension effects

E.Lerner, I. Procaccia, C. Rainone, M. Singh,
[On the protocol dependence of plasticity in ultra-stable amorphous solids](), [Arxiv:1806.09134]()

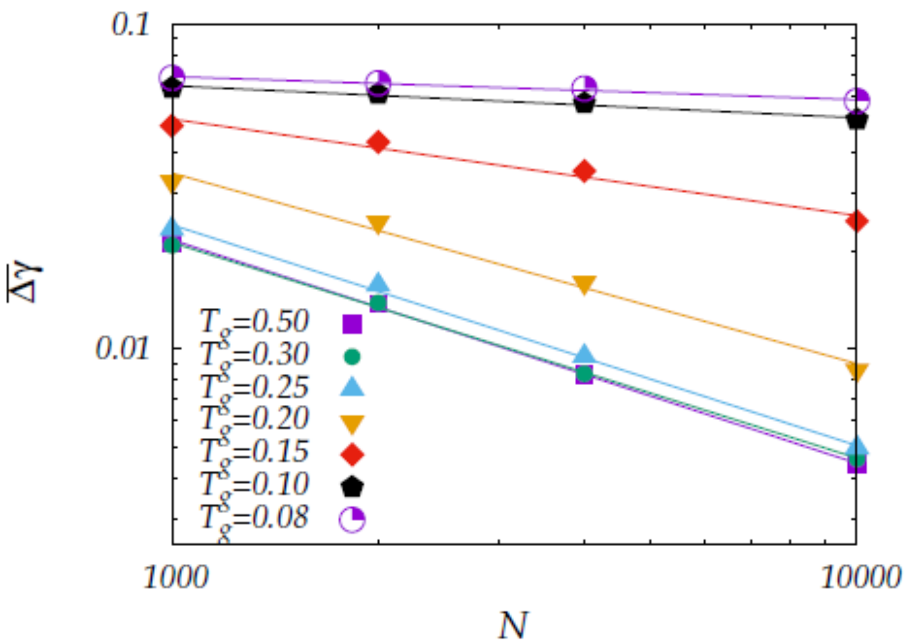


FIG. 1. System size dependence of the average $\overline{\Delta\gamma}$ interval before the first plastic event occurs. For each T_g The expected scaling N^{α} is found, but the α exponent is now dependent on the preparation temperature T_g.

This is significant due to the connection between the pseudo gap exponent and the system size dependence

$$P(x) \simeq x^{\theta}$$

$$\alpha = -\frac{1}{1+\theta}$$

So does the pseudo gap exponent depend on protocol?

Toy model $$P(x) = c(T_g)x^{\theta}$$

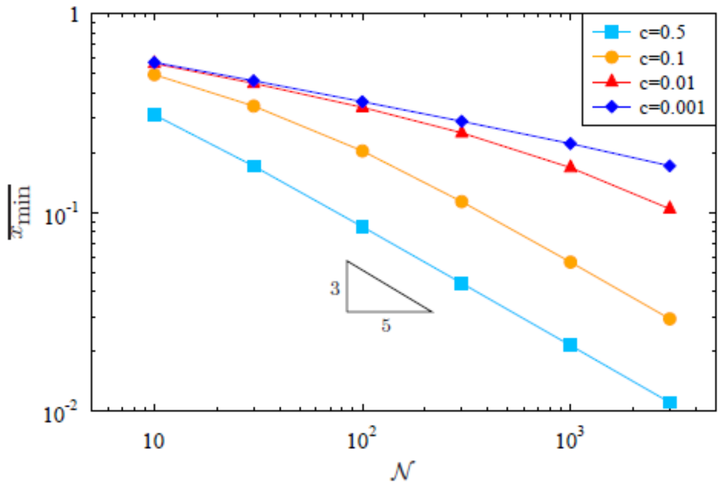


FIG. 3. As c decreases (and therefore the glass is better annealed), the asymptotic scaling $\overline{x_{\min}} \propto \mathcal{N}^{-3/5}$ is pushed to larger and larger system sizes, producing an apparent change in the scaling exponent α, which is however only a finite-size effect.

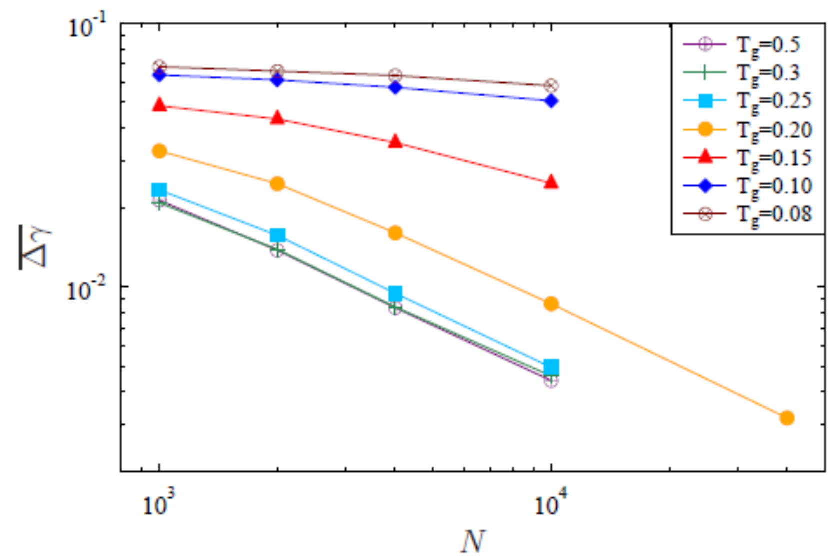


FIG. 4. System size dependence of the average $\overline{\Delta\gamma}$ interval before the first plastic event occurs, with added results for $N = 40000$. A T_g-dependent crossover in the data such as the one predicted by our toy model appears to be present.

The exponent η along the straining curve

$$\lim_{\Delta\gamma\to 0} P(\Delta\gamma, \gamma = 0) \sim (\Delta\gamma)^{\eta} \, . \qquad \langle\Delta\gamma\rangle \sim N^{\beta} \, , \quad \beta < 0 \, .$$

These scaling laws are obtained from statistically independent realizations!

Then, and only then, the theory of extreme statistics implies

$$\beta = -\frac{1}{1+\eta} \, , \quad \eta = -\left(1 + \frac{1}{\beta}\right) \, .$$

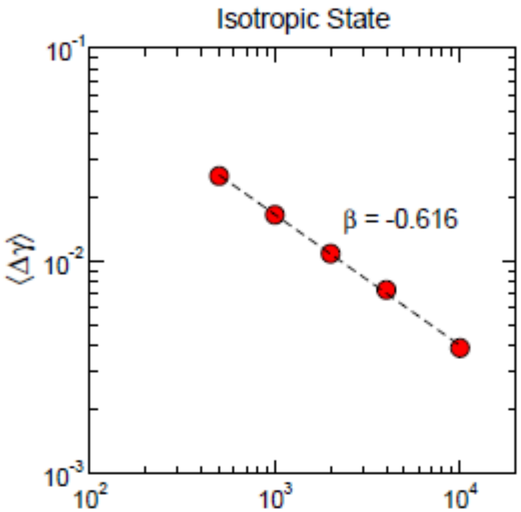
Another stable result exists for the steady state regime

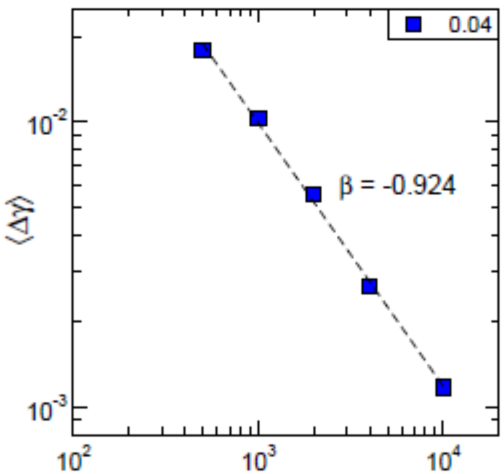
$$\langle \Delta U \rangle = \bar{\epsilon} N^{\alpha} \ , \langle \Delta \sigma \rangle = \bar{s} N^{\beta} \ , \langle \Delta \gamma \rangle = N^{\beta} \ , \qquad (4)$$

with $\alpha = 1/3$ and $\beta = -2/3$ as exact universal results.

But here extreme statistics do not apply, and the scaling relation does not exist.

So what happens in the “elastic regime”?





Is $\beta = -1$?

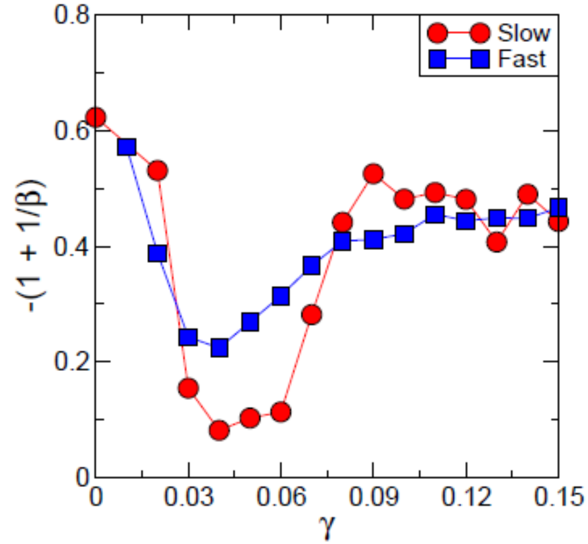
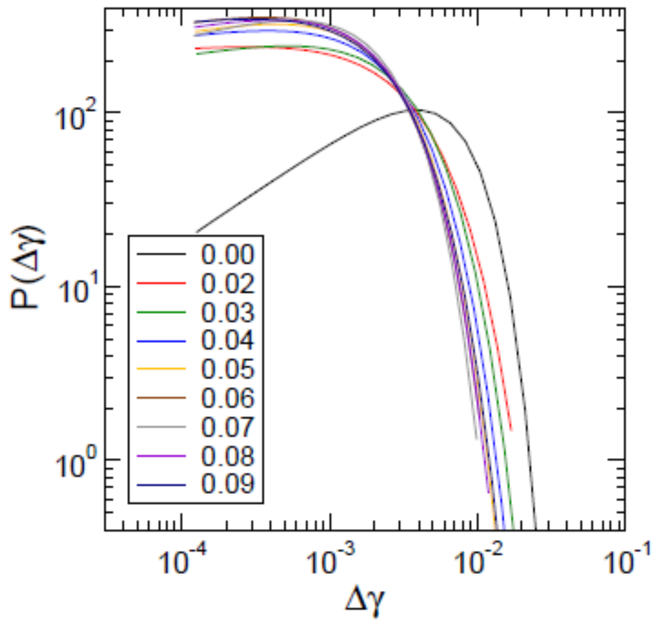


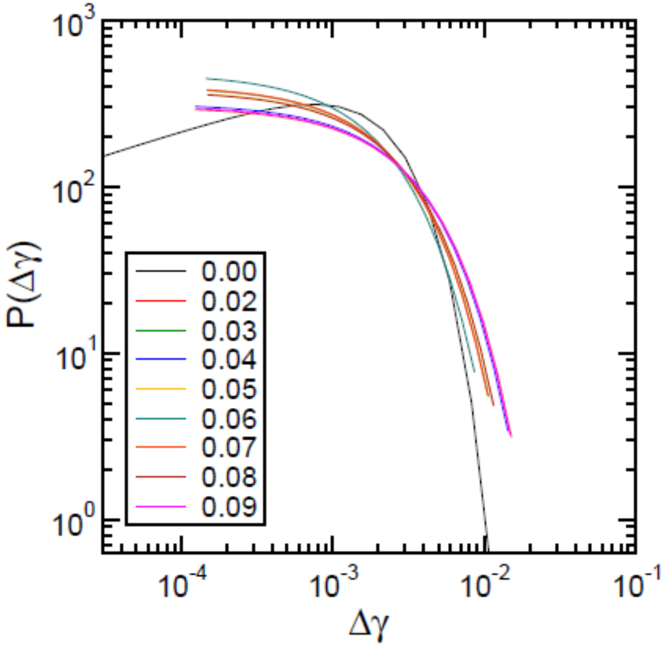
FIG. 2. A plot of $-(1+\frac{1}{\beta})$ obtained from the direct measurement of β as a function of γ. Results for the slow quench are shown in circles and for the fast quench in squares. We argue in this paper that these results suffer from severe finite size effects and in reality this figure should be replaced by Fig. 6

Since we suspect finite size effects

Slow quench



Fast quench



Result of theoretical model

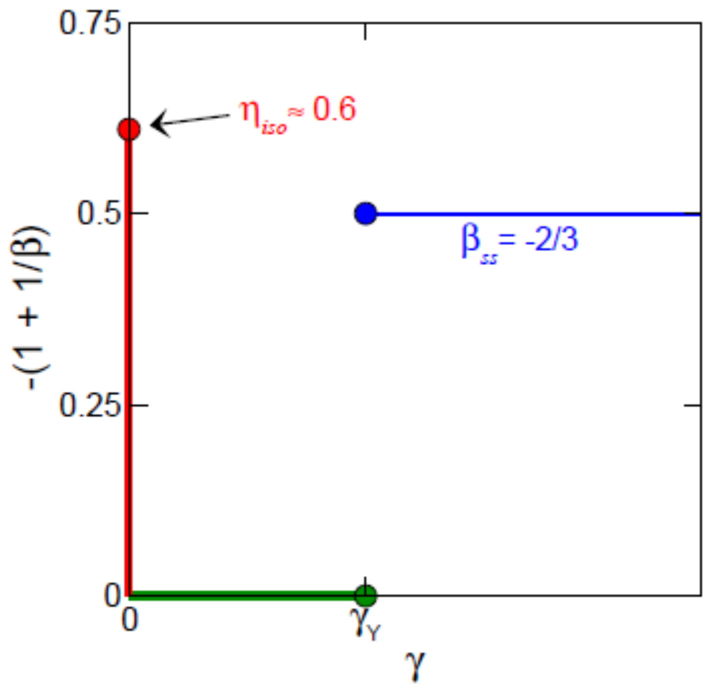


FIG. 6. Schematic presentation of the theoretical prediction for the γ dependence of $-(1+1/\beta)$. We reiterate that the value of this exponent at $\gamma = 0$ is *not* universal, whereas in the steady state it is universal