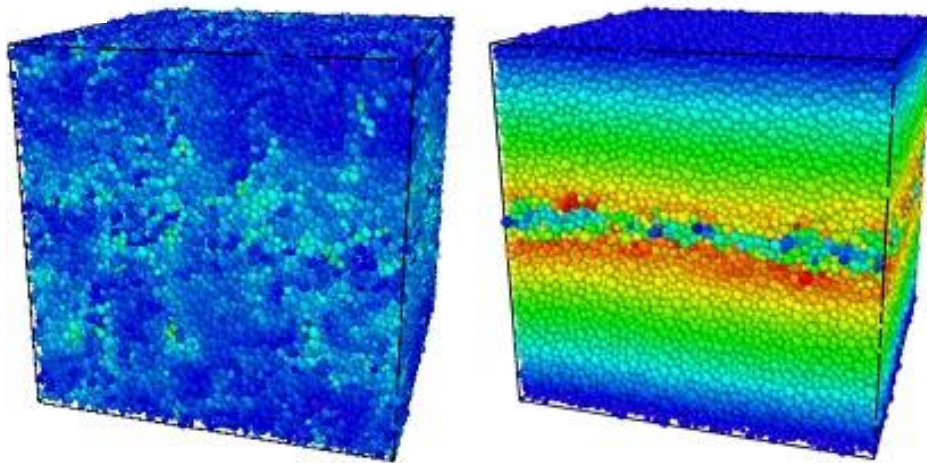


Random critical point separates brittle and ductile yielding transitions in amorphous materials



Misaki Ozawa

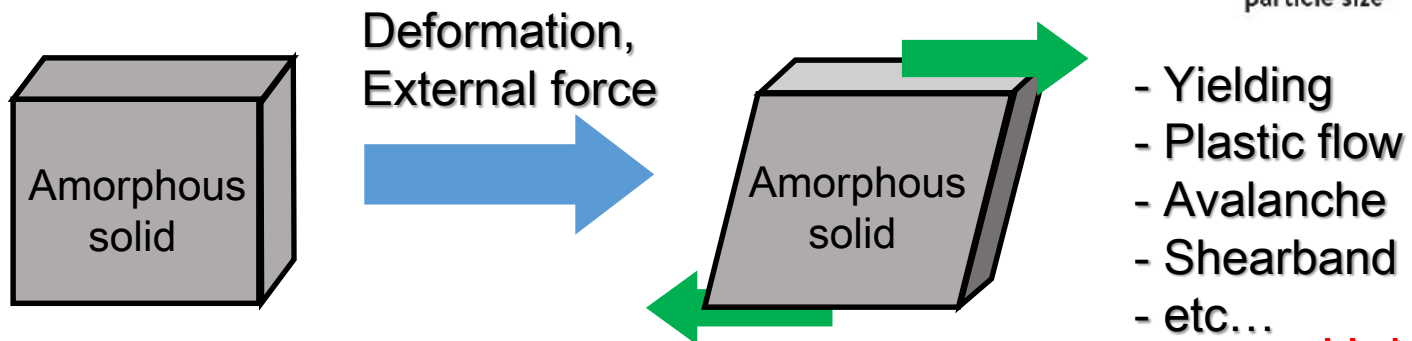
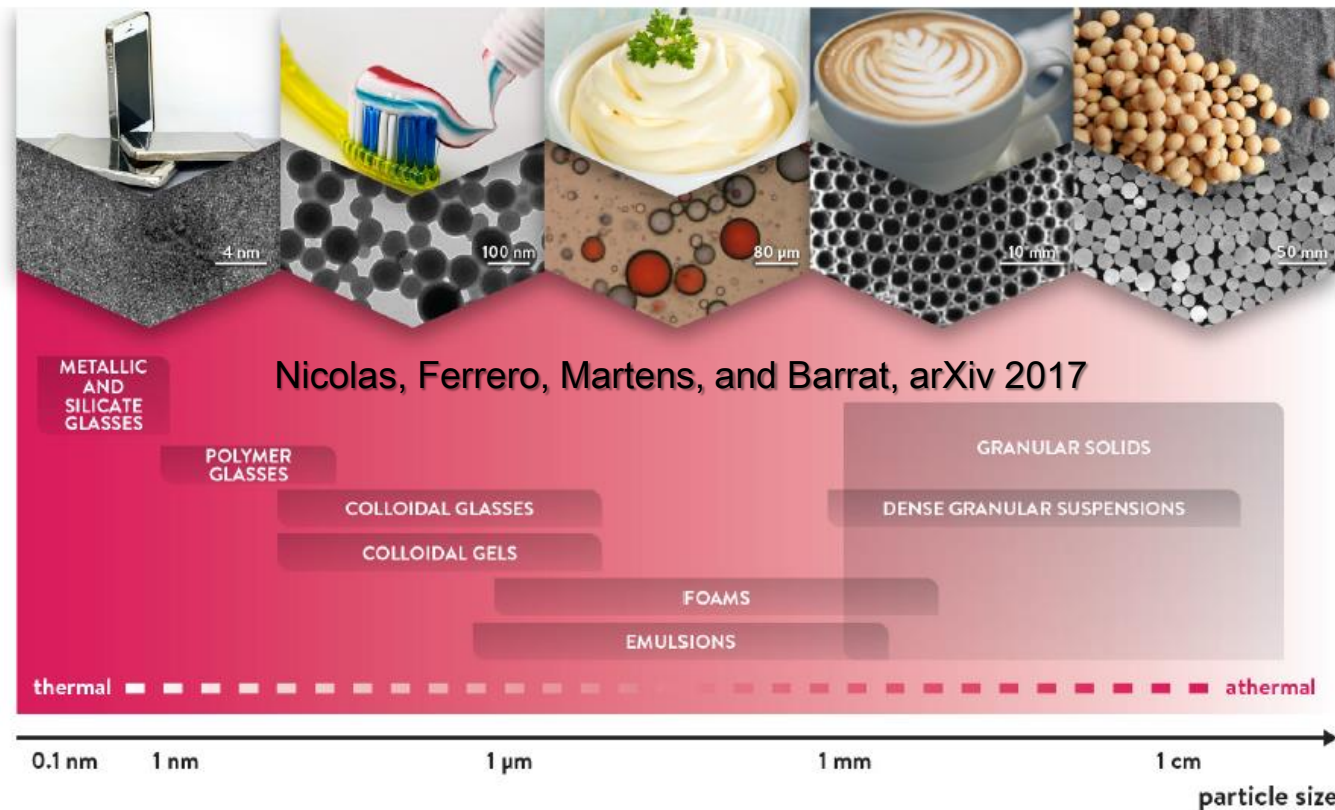
Montpellier University



Collaborators

L. Berthier, G. Biroli, A. Rosso, and G. Tarjus

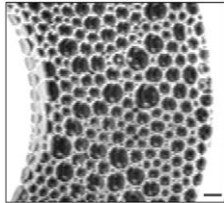
Amorphous solids



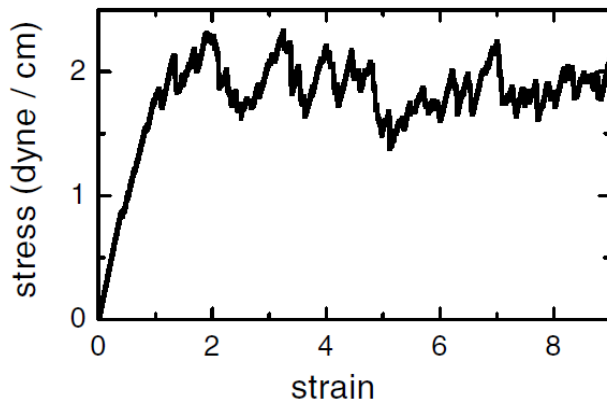
Universal!?

Stress vs. strain curves

Foams

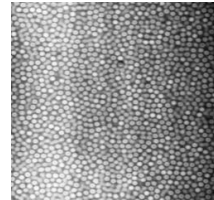


Lauridsen et al., PRL 2002

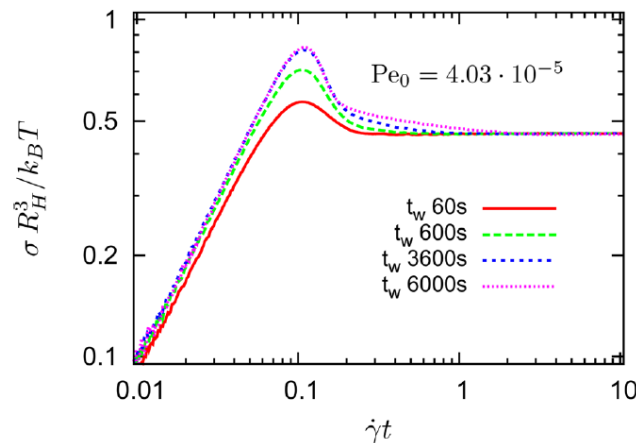


Monotonic crossover
(Ductile)

Colloids

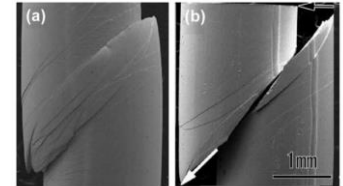


Amann et al., J. Rheol. 2013

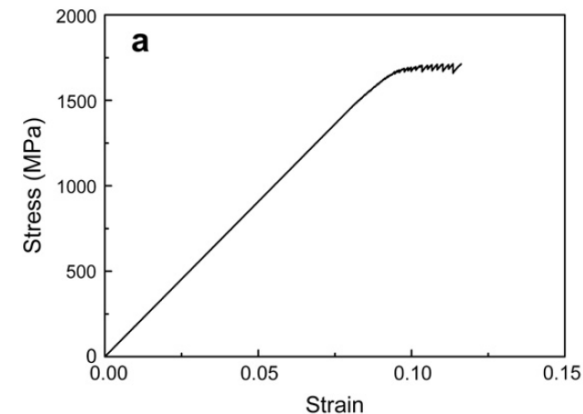


Mild stress overshoot
(Ductile)

Metallic glasses



Song et al., INTERMETALLICS 2008

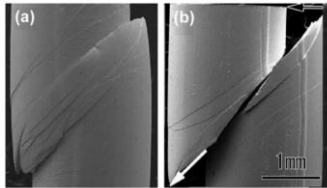


Sharp stress drop
and shearband
(Brittle)

What is the origin of different yielding behaviors? Unified picture?

Unified descriptions

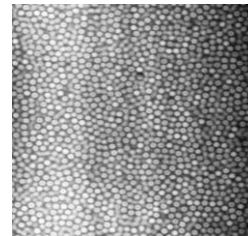
Metallic glasses



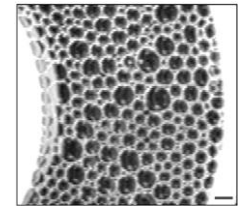
Oxide glasses



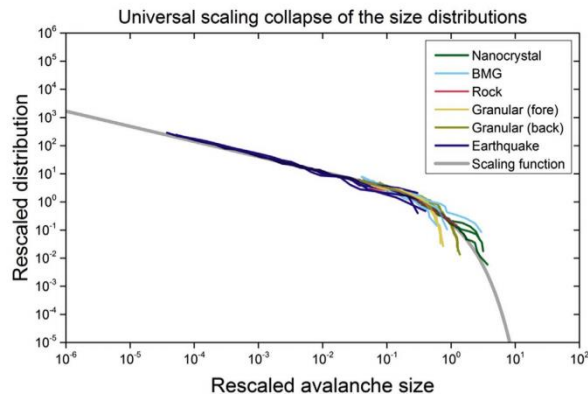
Colloids



Foams

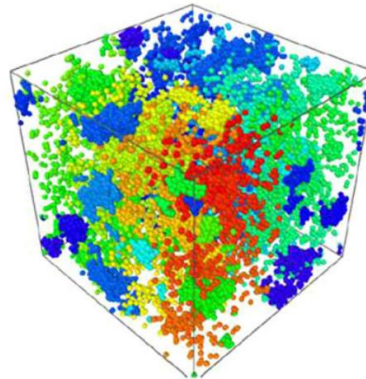


Avalanche scaling



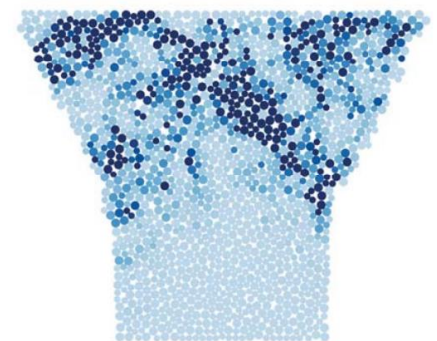
Uhl et al., Sci. Rep. 2015

Growing avalanche



Ghosh, et al., PRL 2017

Soft spots



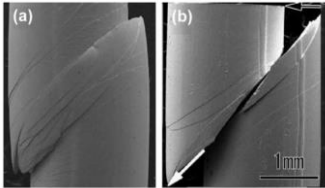
Cubuk, et al., Science 2017

Our unified picture of yielding transition

Discontinuous transition

(Brittle)

Metallic glasses



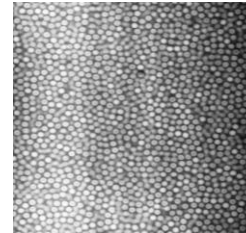
Oxide glasses



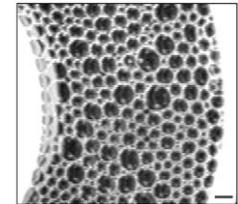
Crossover

(Ductile)

Colloids



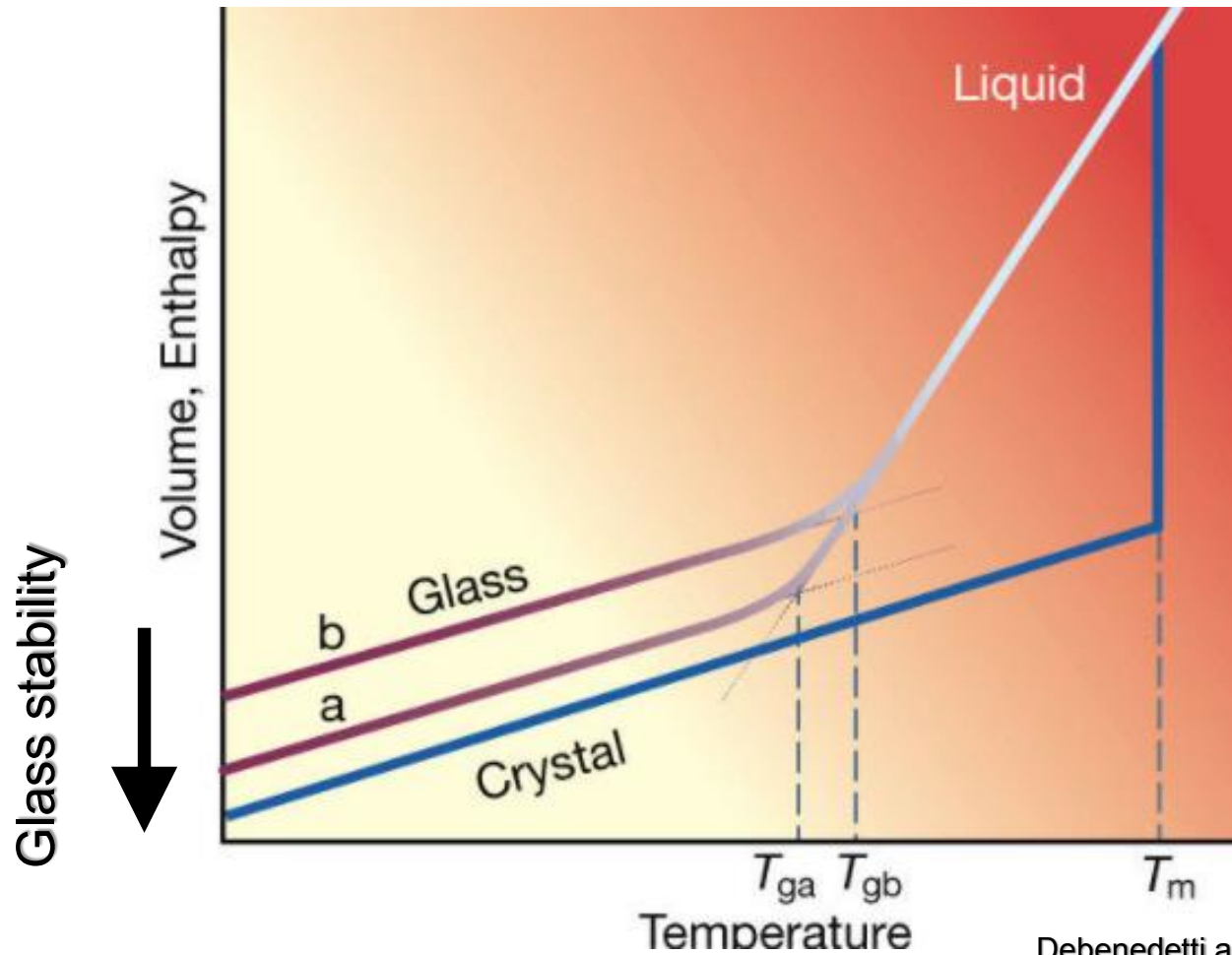
Foams



← Glass stability

Critical point

Glass stability



Debenedetti and Stillinger, Nature 2001

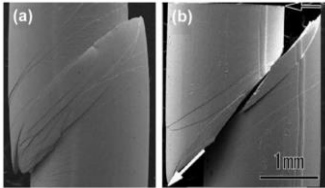
Glass stability increases with decreasing cooling rate

Our unified picture of yielding transition

Discontinuous transition

(Brittle)

Metallic glasses



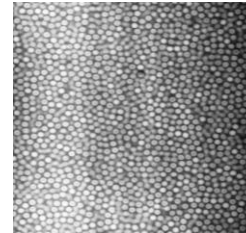
Oxide glasses



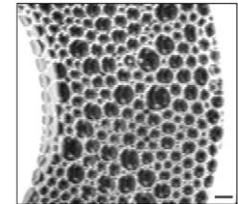
Crossover

(Ductile)

Colloids



Foams



← Glass stability

Critical point

Many papers about preparation protocols, aging, stability of glasses

Kumar, Neibecker, Liu, and Schroes, Nat. Commun. 2013

Jin, Urbani, Zamponi, and Yoshino, arXiv 2018

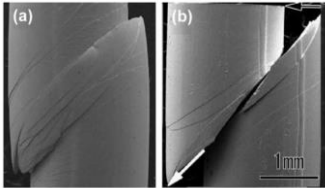
Vasoya, Rycroft, and Bouchbinder, PRAp 2016, etc...

Our unified picture of yielding transition

Discontinuous transition

(Brittle)

Metallic glasses



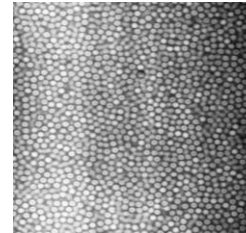
Oxide glasses



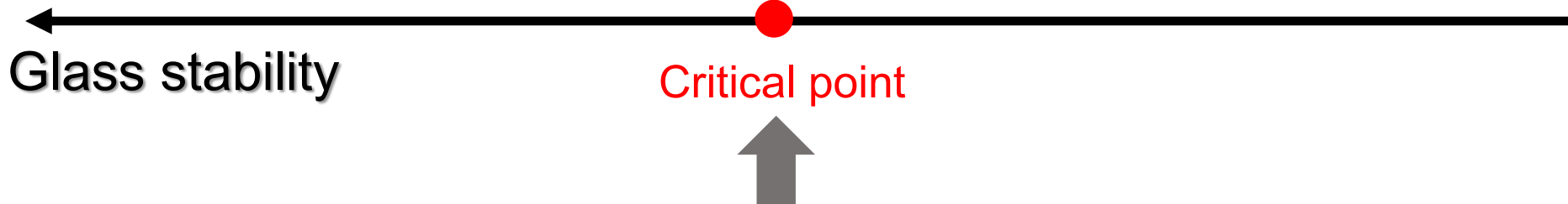
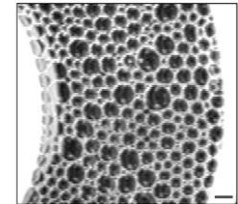
Crossover

(Ductile)

Colloids



Foams



Analogy with random field Ising model (RFIM)

Athermally ($T=0$) driven random field Ising model (RFIM)

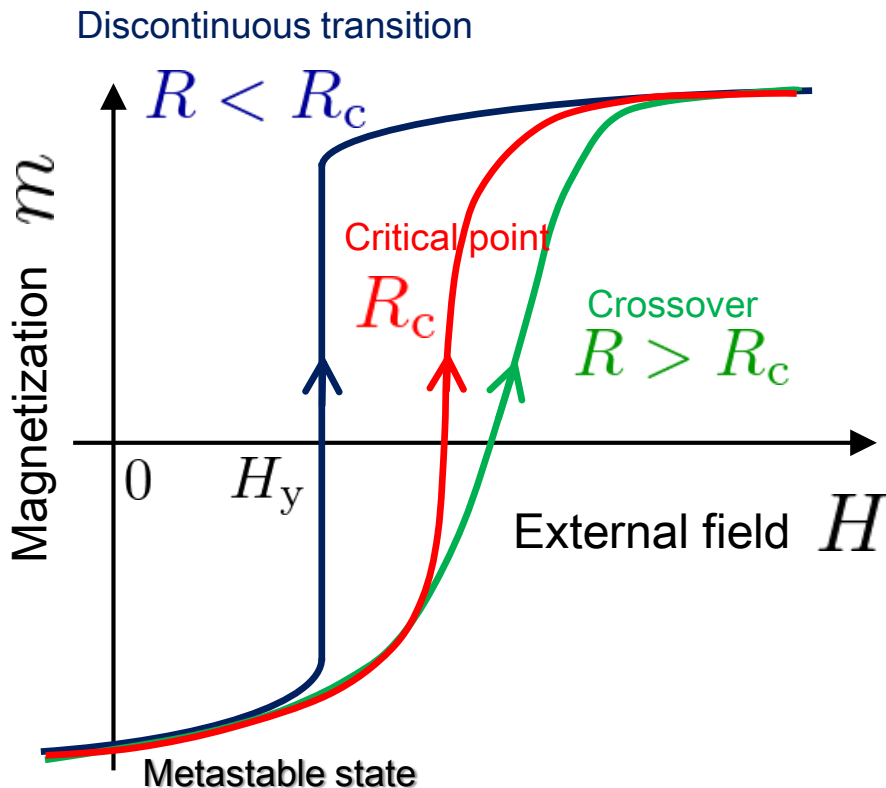
Model

$$\mathcal{H} = -J \sum_{\langle ij \rangle} S_i S_j - \sum_i (h_i + H(t)) S_i$$

Gaussian random fields

$$\overline{h_i} = 0$$

$$\overline{h_i^2} = R^2 : \text{Strength of disorder}$$



Sethna, Dahmen, and Perkovic,
The science of Hysterisis, 2005

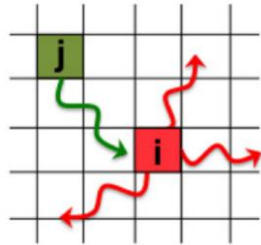
Analogy

RFIM	Yielding
Magnetization	Stress
External field	Strain
$R < R_c$	Brittle yielding
$R > R_c$	Ductile yielding
Rare droplet	Soft spot

Nandi, Biroli, and Tarjus, PRL 2016

Our strategy

■ A mean-field elastoplastic model (Talk by Alberto Rosso)

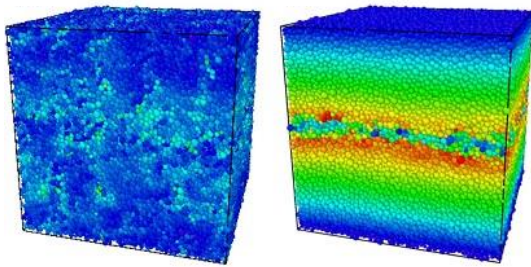


Bocquet, Colin, Ajdari, PRL 2009

Inspired by a depinning model Jagla, Landes, and Rosso, PRL 2014

Universality class of the athermally driven RFIM

■ Molecular simulations (This talk!)



Our Model: Polydisperse soft spheres $N = 1000 - 96000$

Glass preparation: Swap Monte-Carlo

Ninarello, Berthier, and Coslovich, PRX 2017

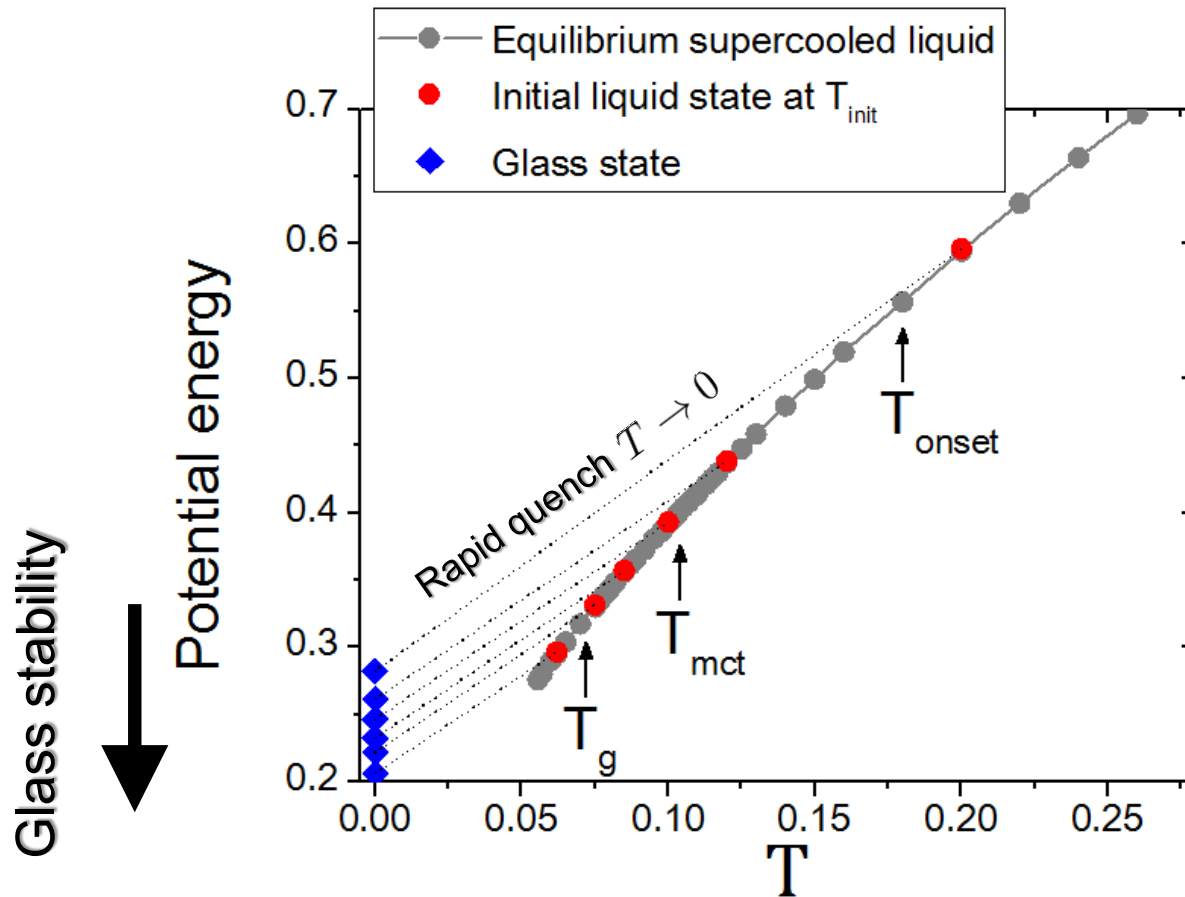
Shear protocol: Athermal quasistatic shear

Maloney and Lemaitre, PRE 2006

We will show

Critical point, rare droplet (soft spots), dimensional dependence

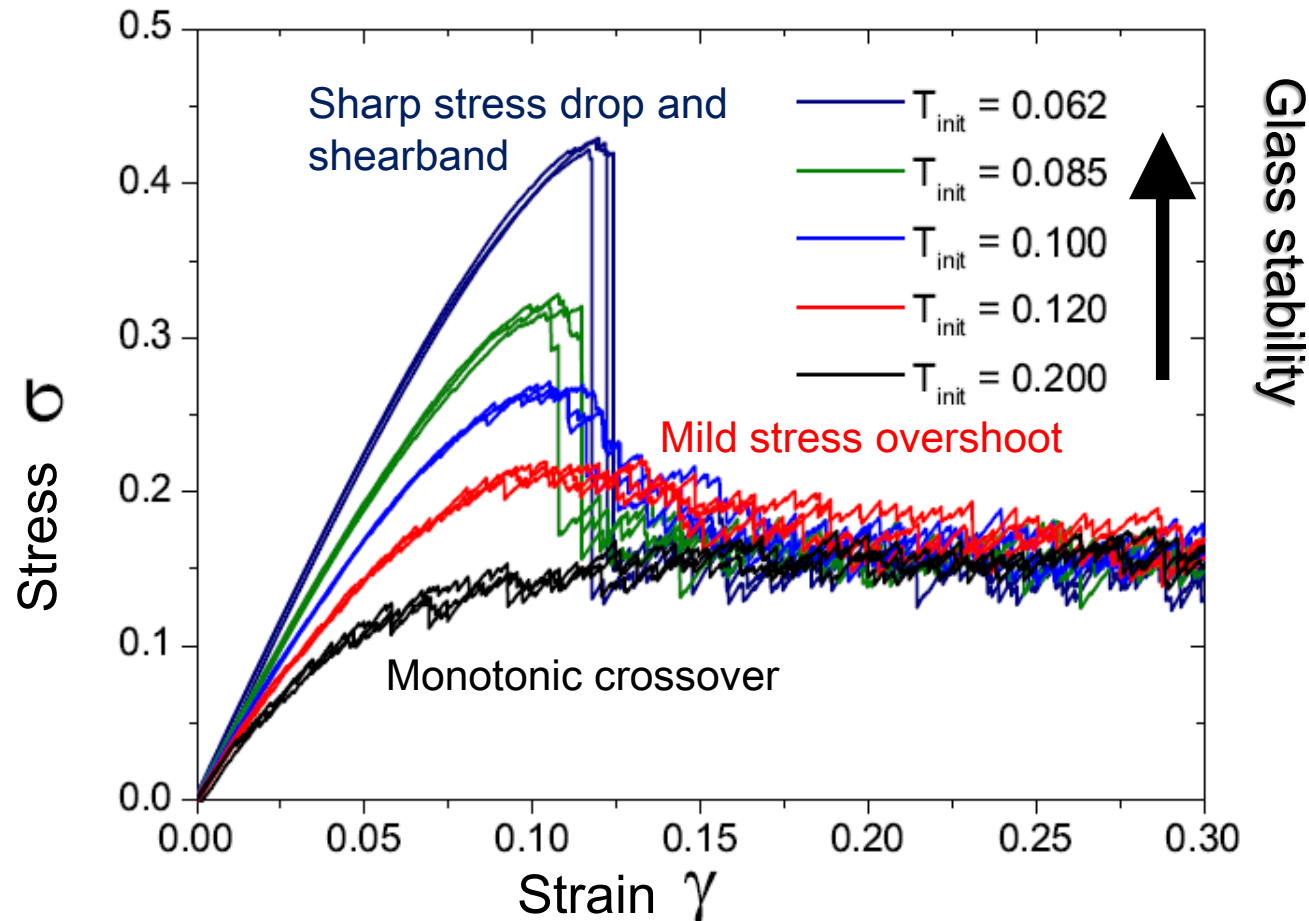
How we prepare glass samples



Glasses are generated by rapid quench of liquids at T_{init}

T_{init} characterizes glass stability (R in RFIM)

Stress vs. strain curve



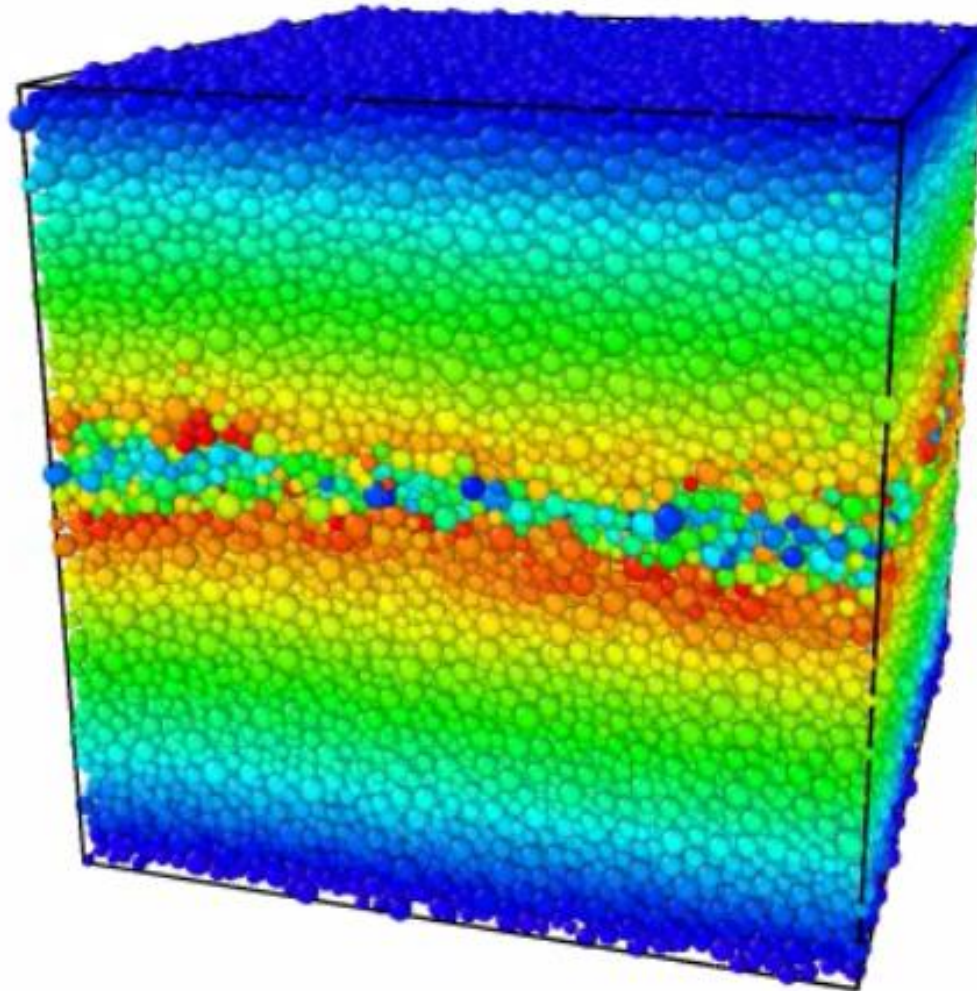
Athermal quasi-static shear simulation at $T = 0$

Essential phenomenology of yielding are covered by changing glass stability

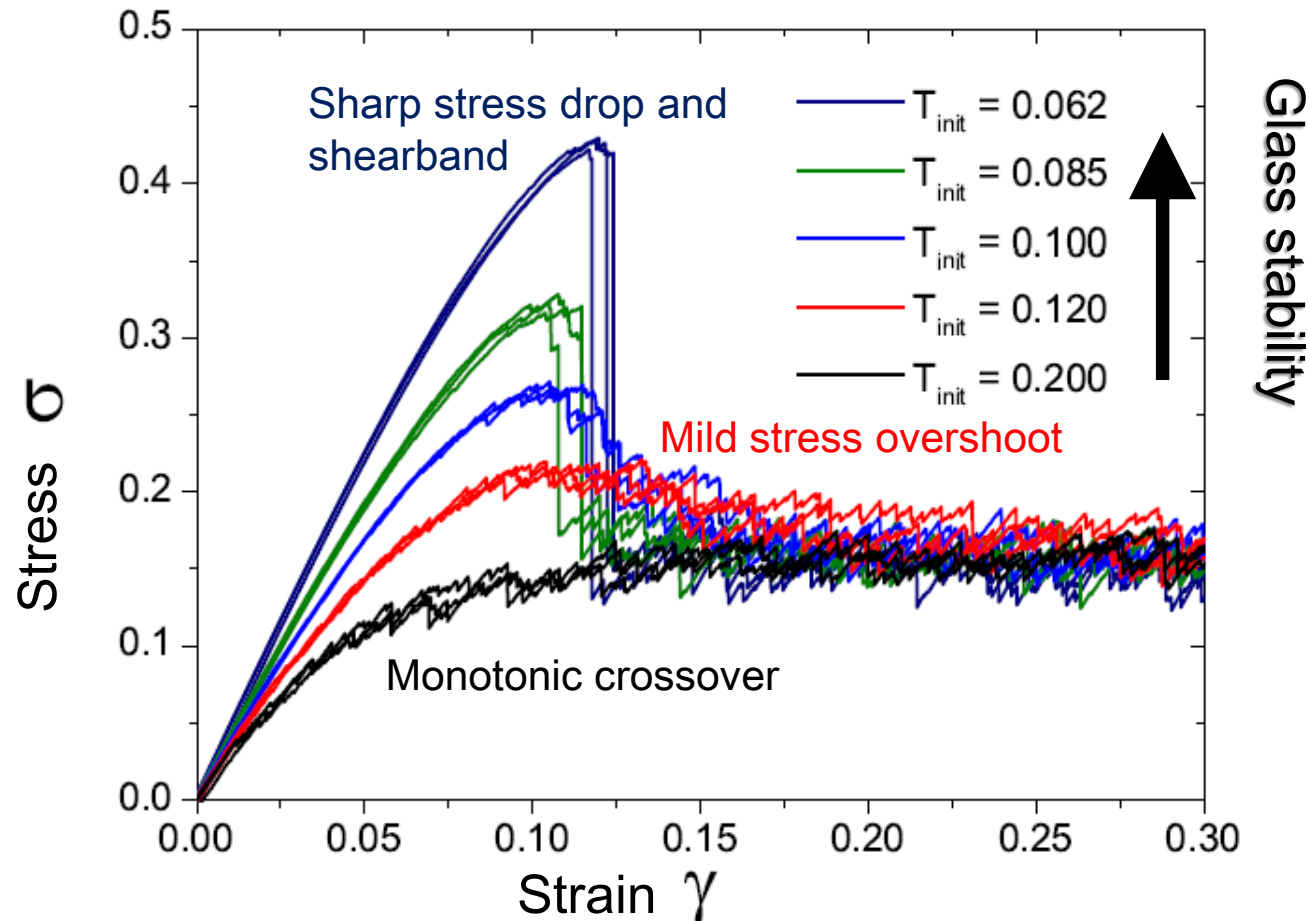
Shearband movie

$$\gamma = 122e-3$$

Non-affine displacement



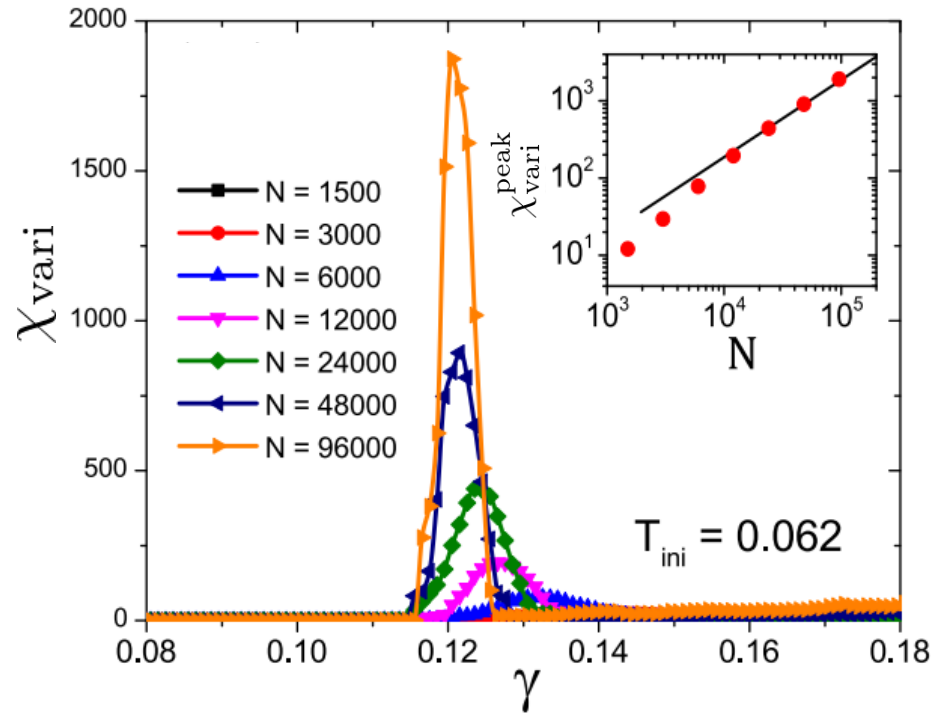
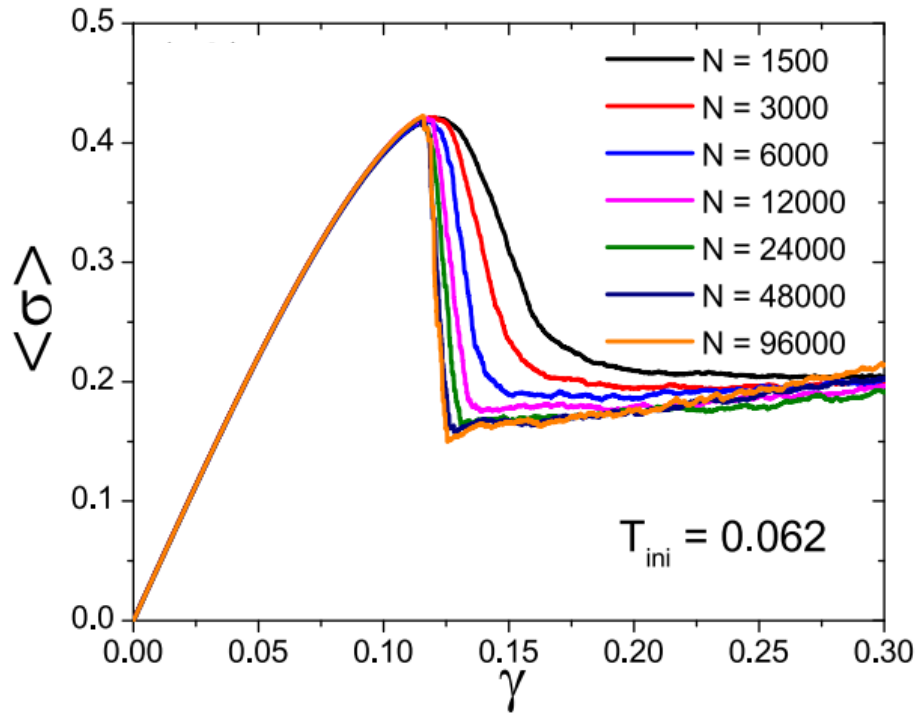
Stress vs. strain curve



Athermal quasi-static shear simulation at $T = 0$

Essential phenomenology of yielding are covered by changing glass stability

Yielding transition (Brittle)



$\langle (\dots) \rangle$: Averaging over samples

$$\chi_{\text{vari}} = N(\langle \sigma^2 \rangle - \langle \sigma \rangle^2)$$

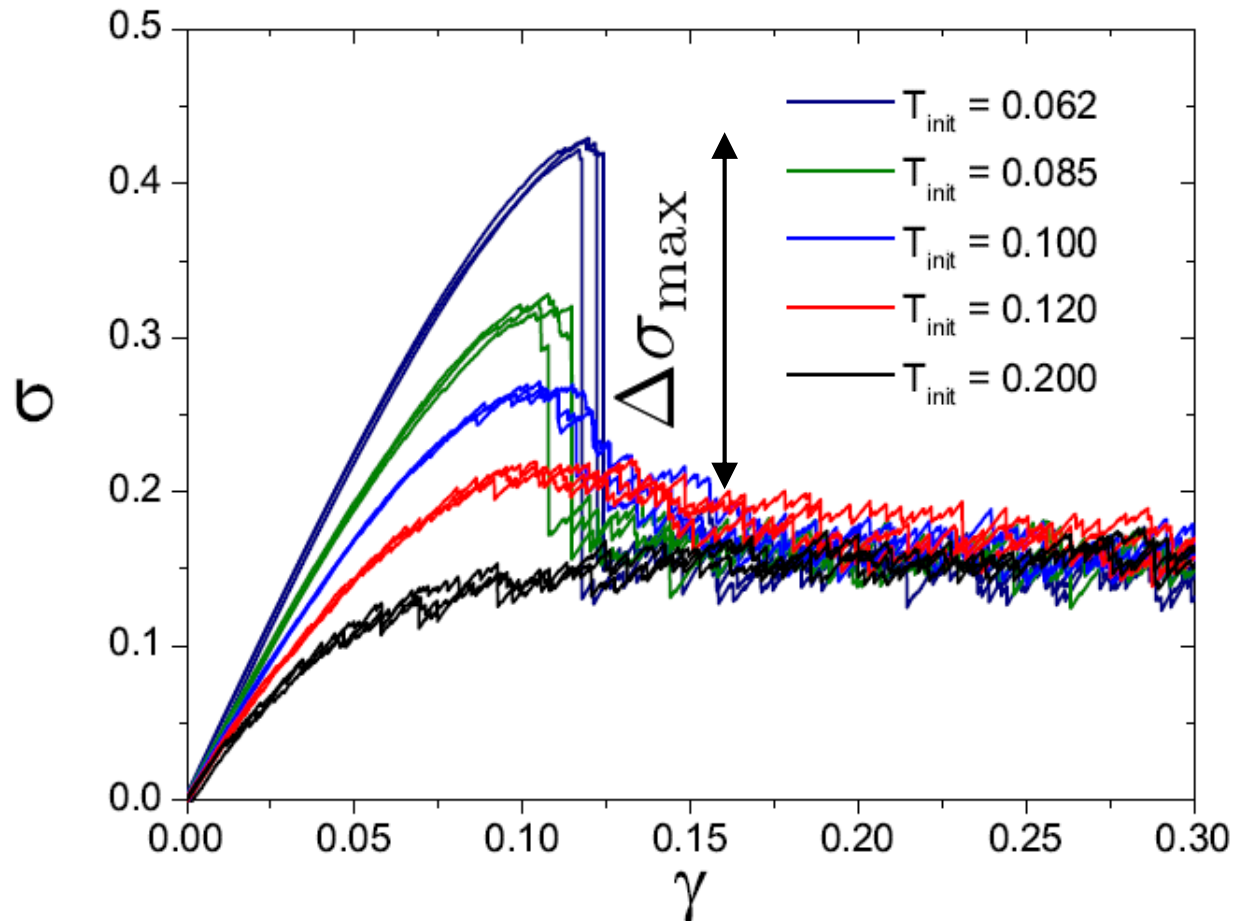
Brittle yielding is a discontinuous “transition” that survives at $N \rightarrow \infty$!

Jaiswal, Procaccia, Rainone, and Singh, PRL 2016

Urbani and Zamponi, PRL 2017

Kapteijns, Ji, Brito, Wyart, and Lerner, arXiv 2018

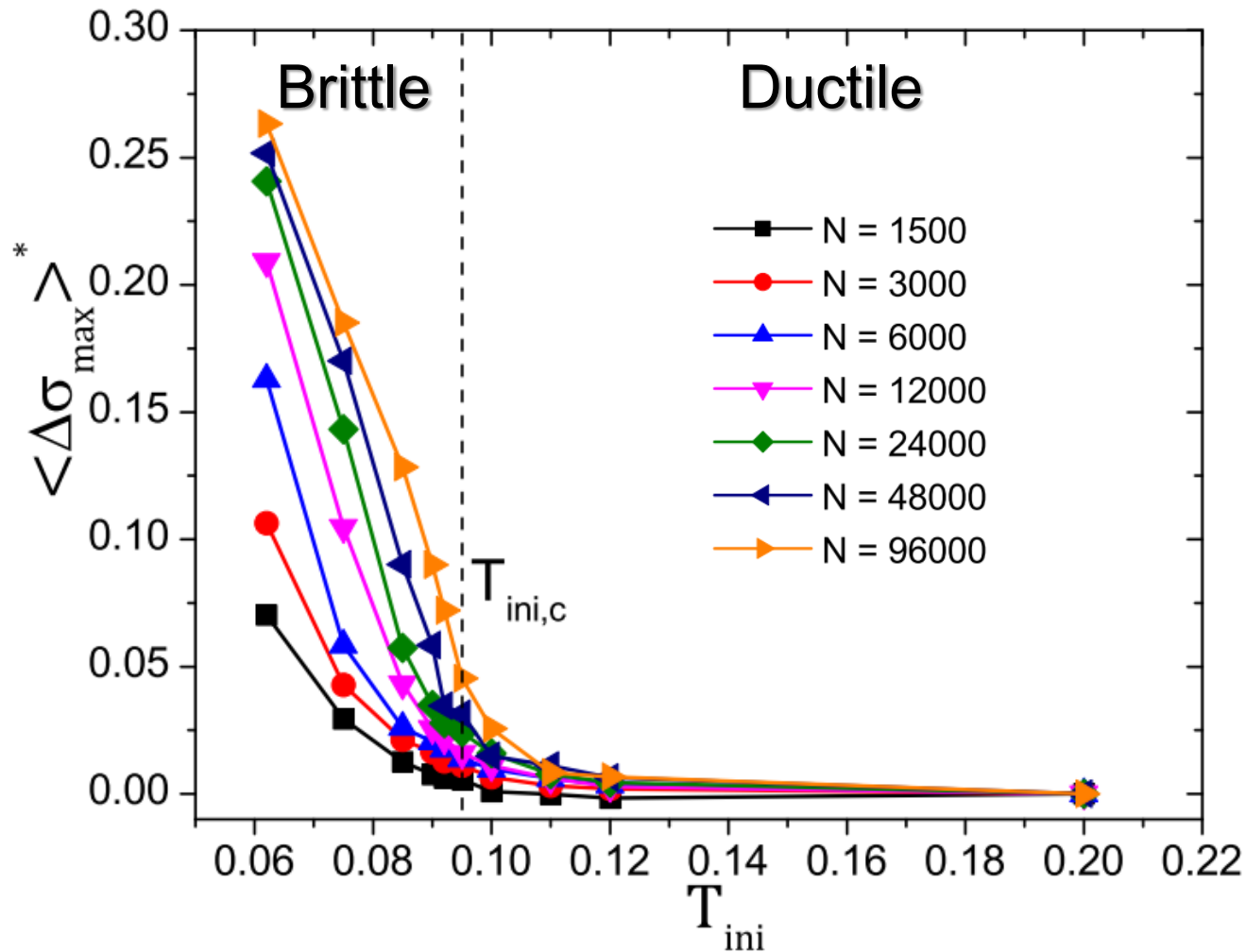
Order parameter



Control parameter: T_{init}

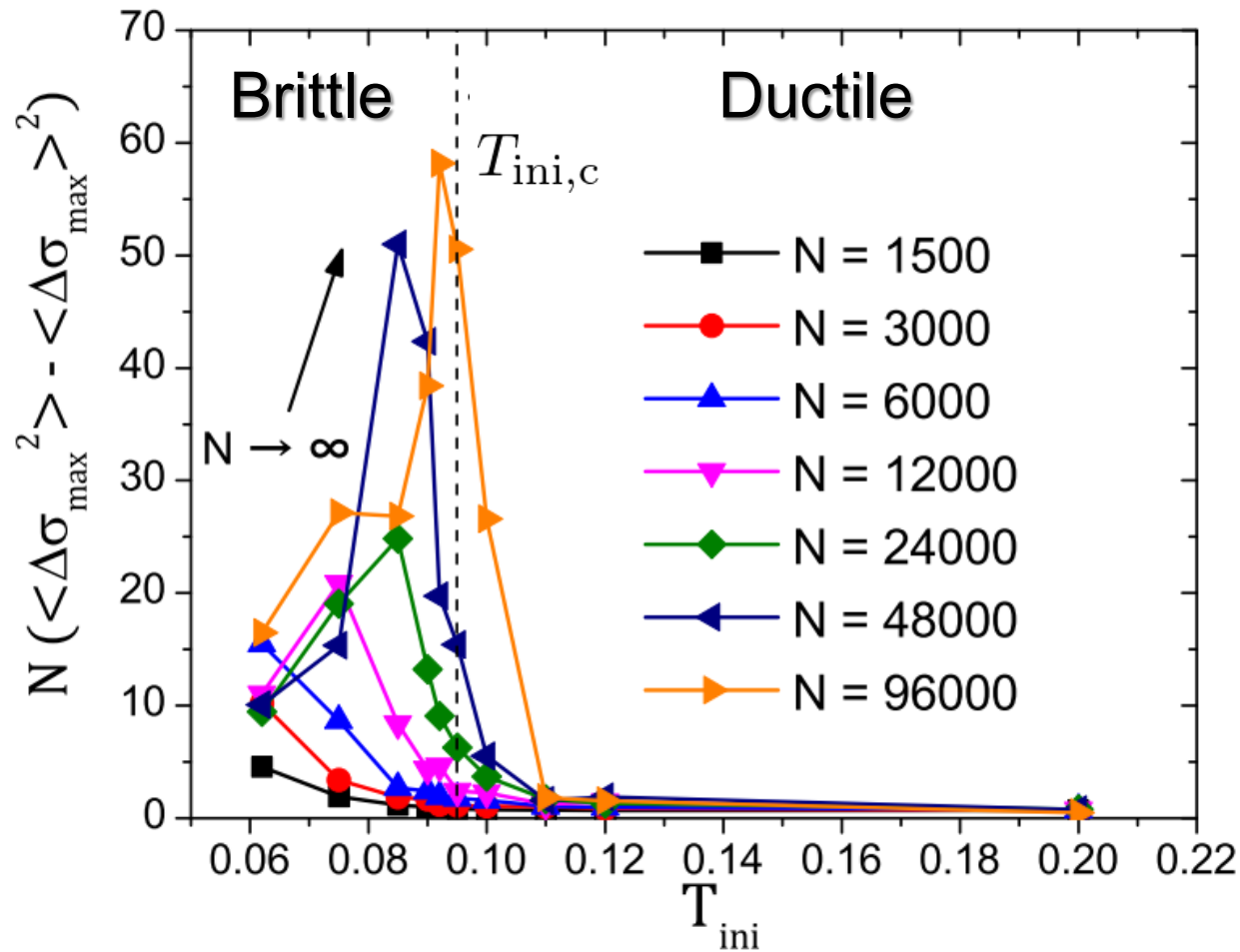
Order parameter for brittle-to-ductile transition : $\Delta\sigma_{\text{max}}$

Growth of order parameter



$\langle \Delta \sigma_{\max} \rangle$ develops below $T_{\text{ini},c}$, suggesting a critical point

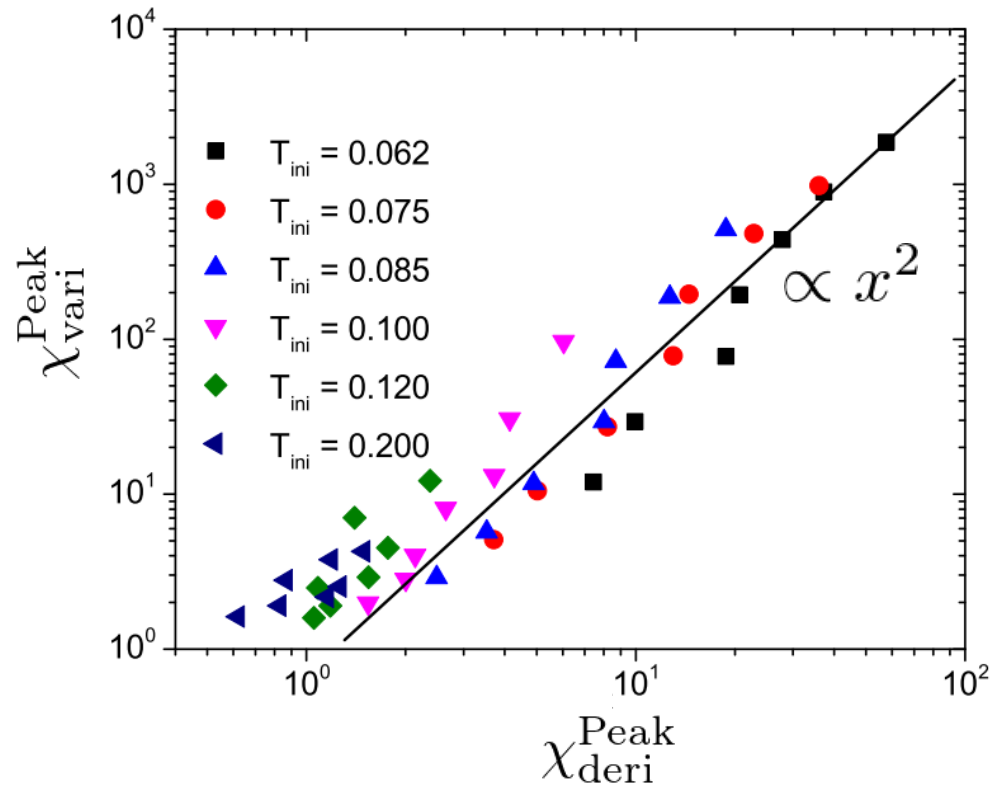
Divergence of variance



Rapid growth of the variance further supports the existence of the critical point

Physical meaning: Strong sample-to-sample fluctuations near $T_{ini,c}$

Comparison with RFIM



$$\chi_{\text{vari}} = N(\langle \sigma^2 \rangle - \langle \sigma \rangle^2)$$

$$\chi_{\text{deri}} = -\frac{\partial \langle \sigma \rangle}{\partial \gamma}$$

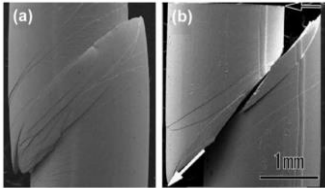
Below $T_{\text{ini,c}} \approx 0.095$ we find $\chi_{\text{vari}} \propto (\chi_{\text{deri}})^2$: the same scaling as RFIM
(Strong sample-to-sample fluctuation)

Our unified picture of yielding transition

Discontinuous transition

(Brittle)

Metallic glasses



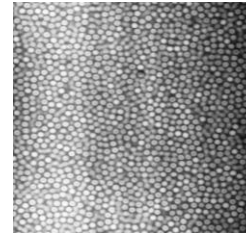
Oxide glasses



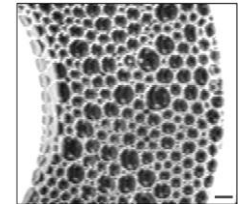
Crossover

(Ductile)

Colloids



Foams

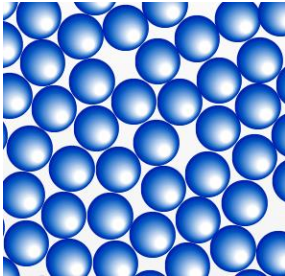


← Glass stability

Critical point

Dimensional dependence of disordered systems

Jamming transition

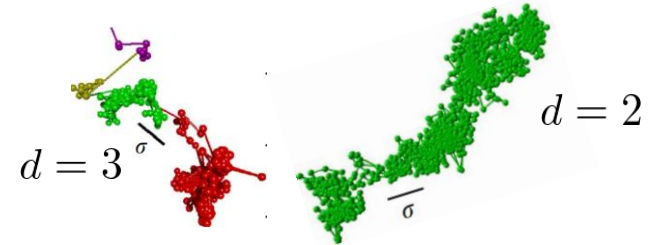


-The upper critical dimensions $d_{uc} = 2$

Goodrich, Liu, and Nagel, PRL 2012

Charbonneau, Kurchan, Parisi, Urbani, and Zamponi,
Annu. Rev. Condens. Matter Phys. 2017

Supercooled liquids and glasses



-Fundamental differences between
dynamics in $d = 2$ and $d = 3$

Flenner and Szamel, Nat. Commun. 2015

-The lower critical dimensions $d_{lc} = 2?$

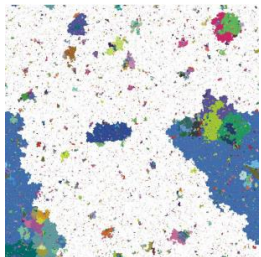
Berthier, Charbonneau, Ninarello, Ozawa, and Yaida, arXiv 2018

Random field Ising model

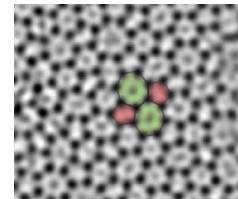
-Equilibrium $d_{lc} = 2$

-Athermally driven $d_{lc} = 2???$

Balog, Tarjus, and Tissier, PRB 2018



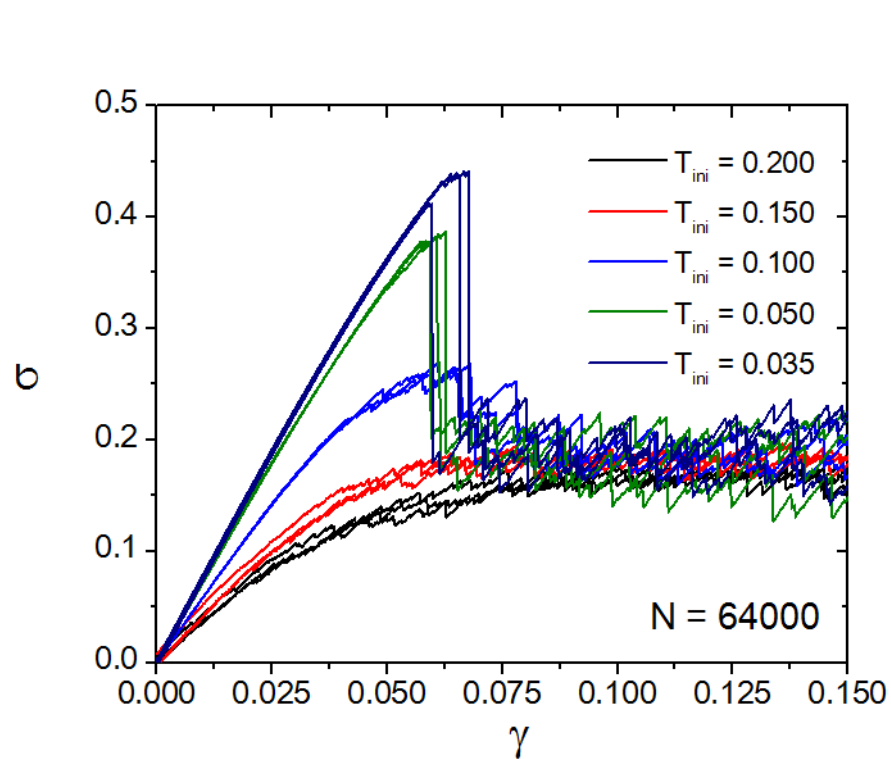
Many real 2D amorphous materials
e.g., foams, colloids, silica



Huang, et al., Science 2013

How about yielding in $d = 2$?

Yielding transition in $d = 2$

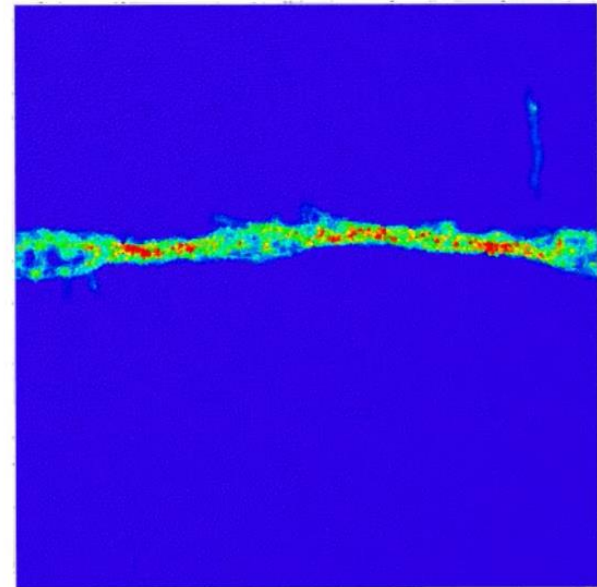


$\gamma = 66 \times 10^{-3}$

Falk and Langer, PRE 1998

Non-affine displacement

0 2

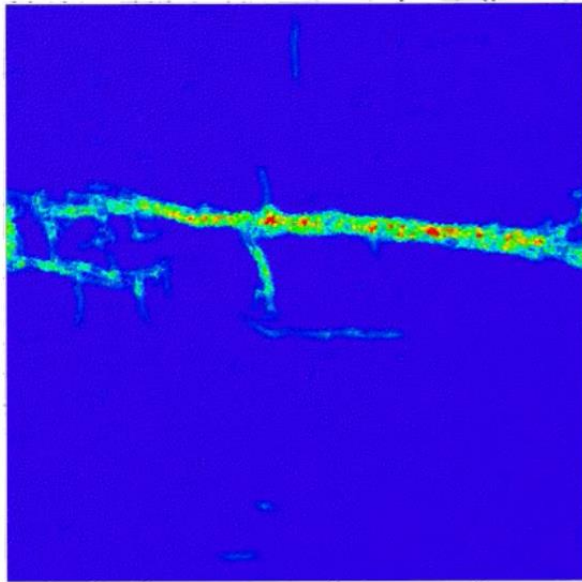


Sharp brittle yielding can be also observed in $d = 2$

Yielding transition in $d = 2$

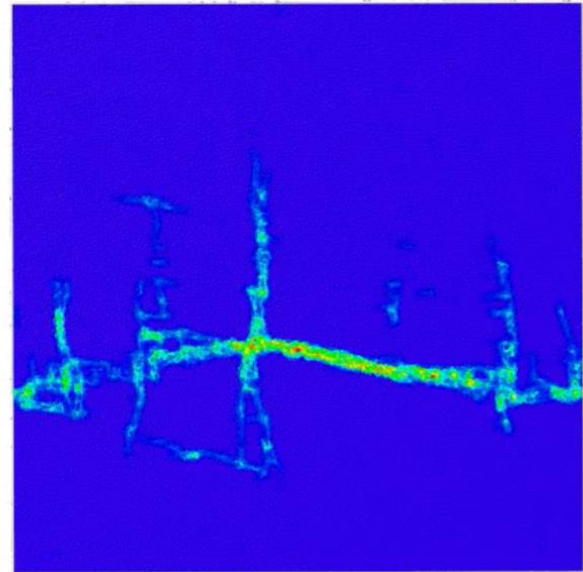
$$\gamma = 66 \times 10^{-3}$$

Non-affine displacement
0 2



$$\gamma = 71 \times 10^{-3}$$

Non-affine displacement
0 2



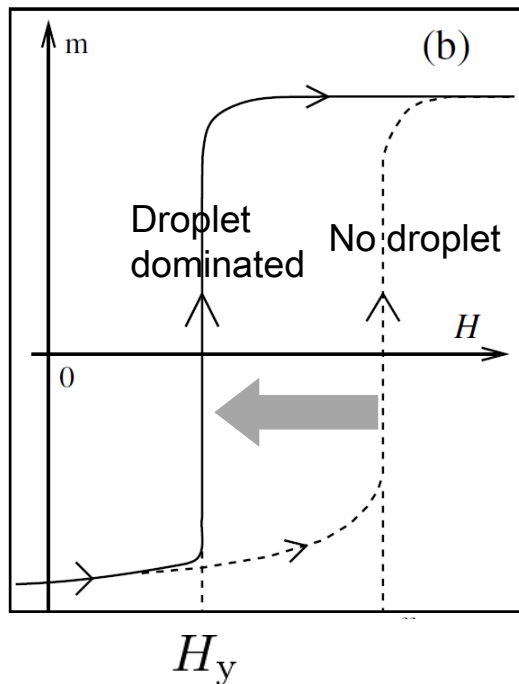
Very occasionally dirty shearbands are observed in stable glasses

Dimensional effect?

Effect of soft spot seed on yielding

Athermally driven RFIM

Rare droplet triggers transition

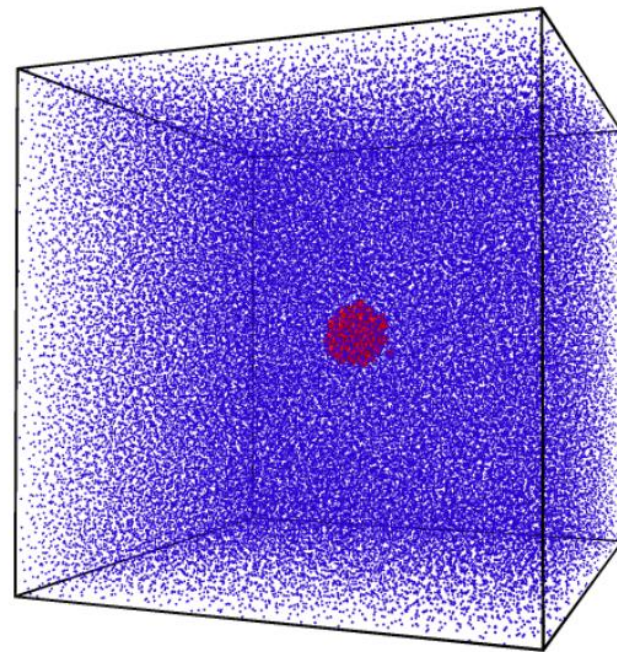


H_y is shifted by rare droplet

Nandi, Biroli, and Tarjus, PRL 2016

Yielding

Soft spot seed is rare droplet?



Inserting a soft spot seed "by hand"

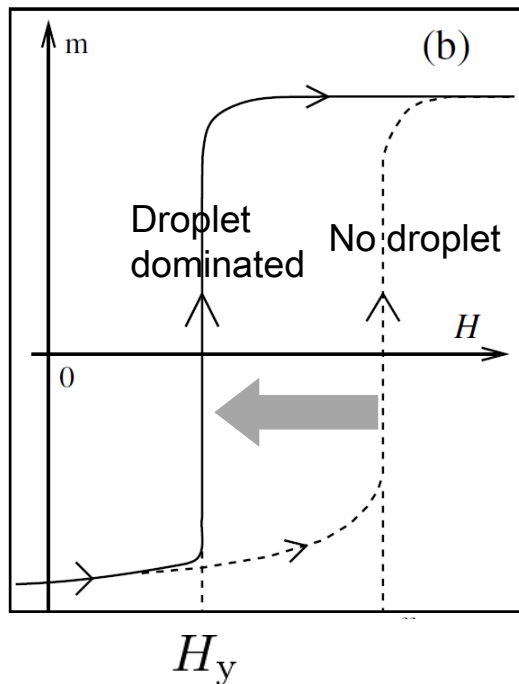
Manning and Liu, PRL 2011

Ding, Patinet, Falk, Cheng, and Ma, PNAS 2014

Effect of soft spot seed on yielding

Athermally driven RFIM

Rare droplet triggers transition

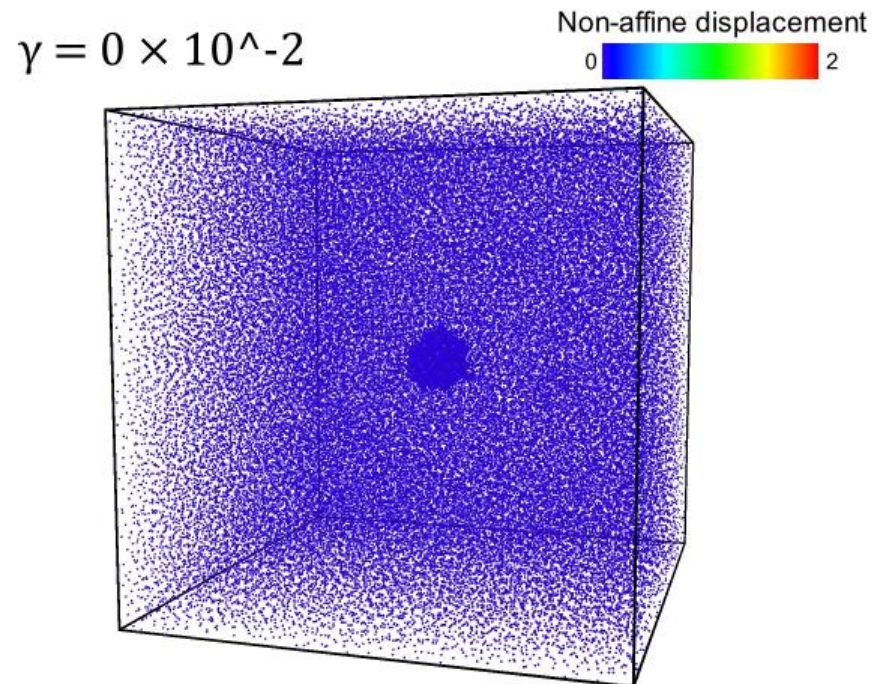


H_y is shifted by rare droplet

Nandi, Biroli, and Tarjus, PRL 2016

Yielding

Soft spot seed is rare droplet?



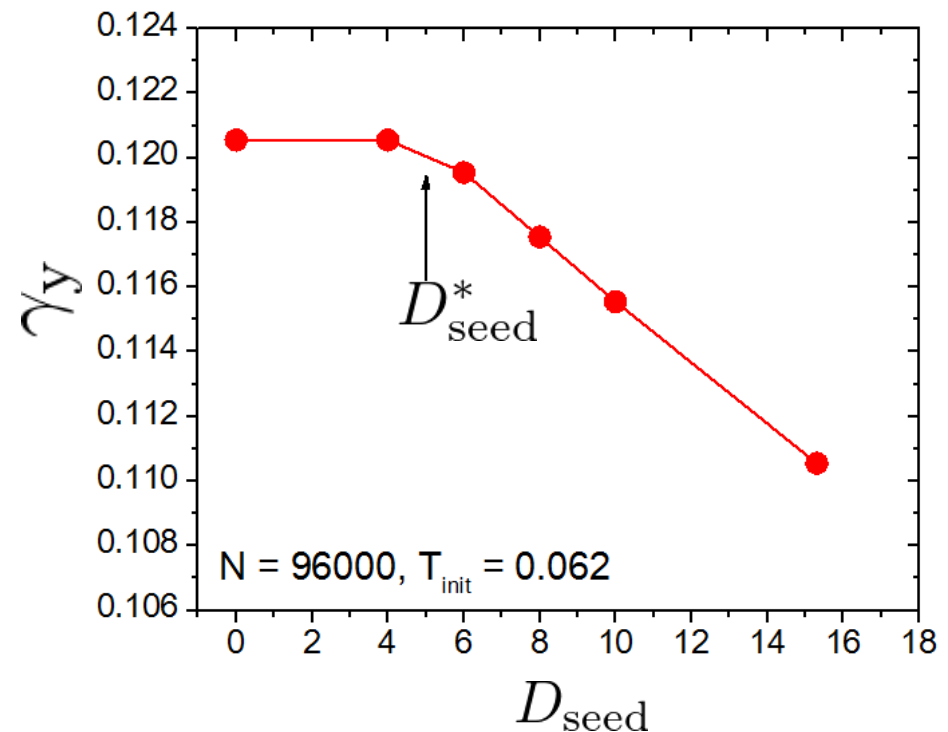
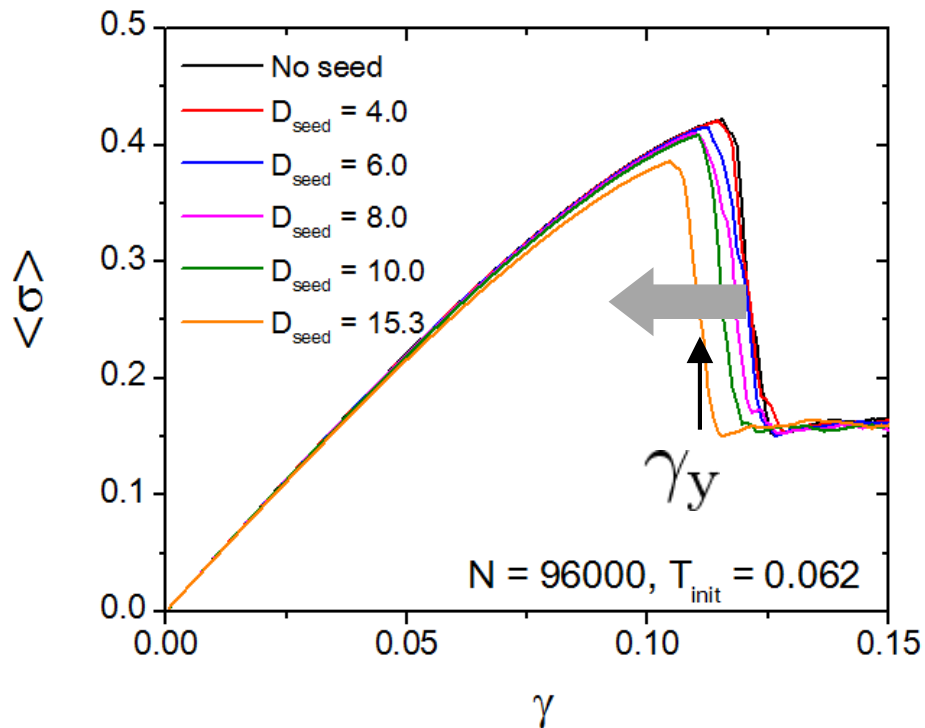
Inserting a soft spot seed "by hand"

Manning and Liu, PRL 2011

Ding, Patinet, Falk, Cheng, and Ma, PNAS 2014

Effect of soft spot seed on yielding

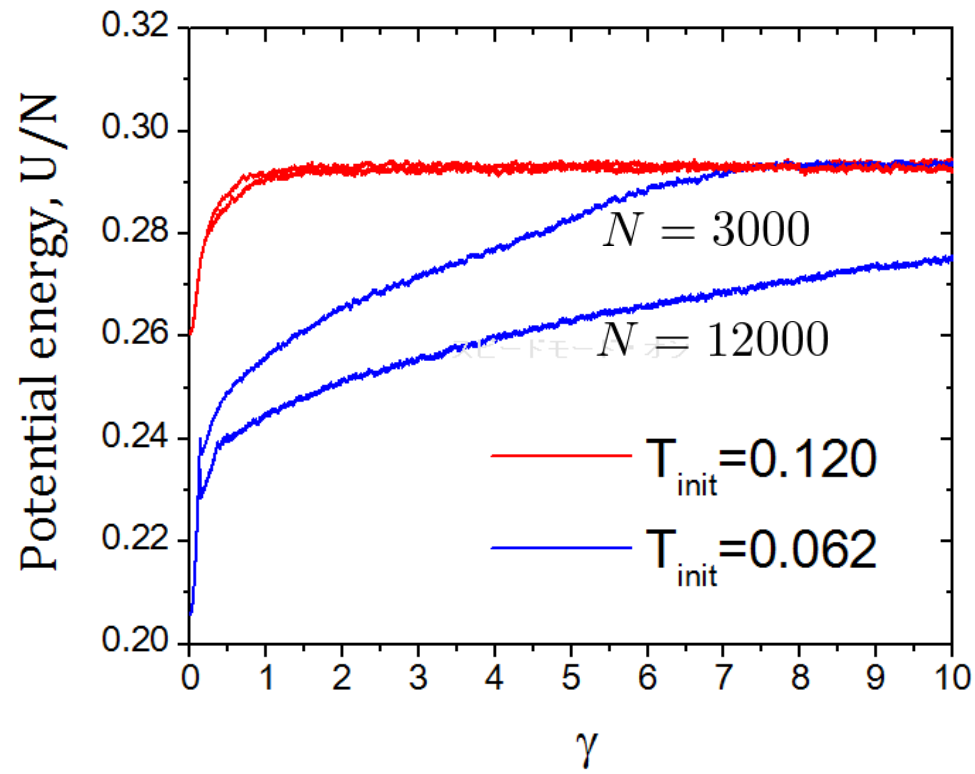
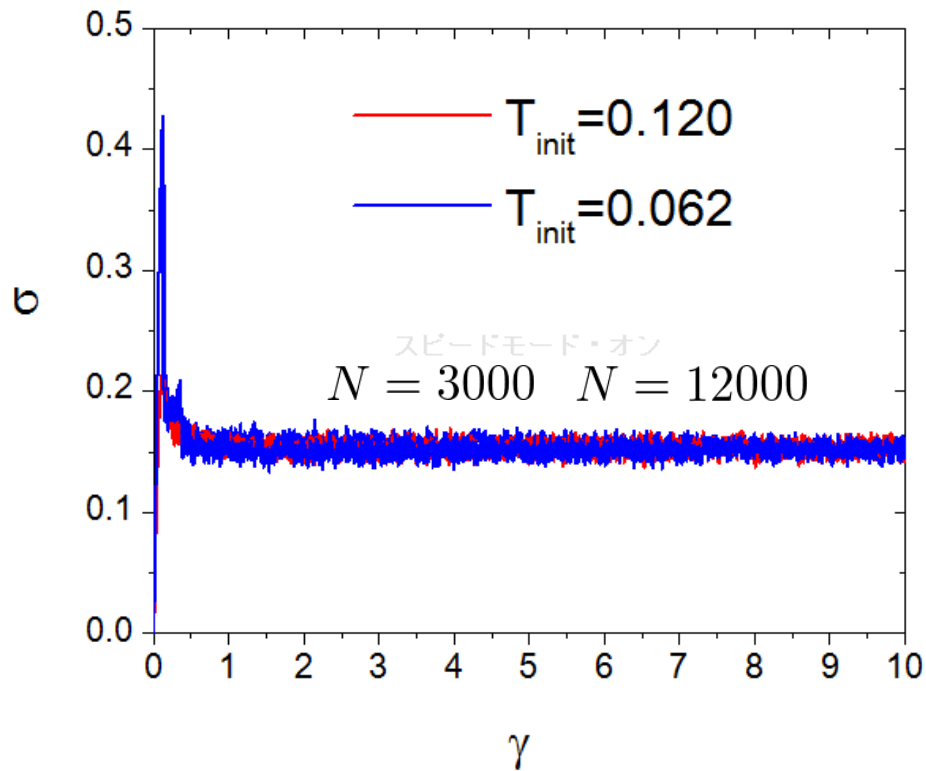
D_{seed} : Diameter of artificially inserted soft spot seed



Yielding point is decreased with increasing D_{seed} , but only above $D_{\text{seed}}^* \approx 5$

Summary and Future

- Random critical point separates brittle and ductile yielding transitions:
Numerically tested Ozawa, Berthier, Biroli, Rosso, and Tarjus, PNAS 2018
- Dimensional dependence: Shape transition in both $d = 3$ and $d = 2$
- Effect of soft spot seed on yielding: Soft spot seed shifts the yielding point
There is a threshold size
- Future: Finite temperature and shear rate
- On going big collaboration organized by L. Manning: Detailed structural analysis



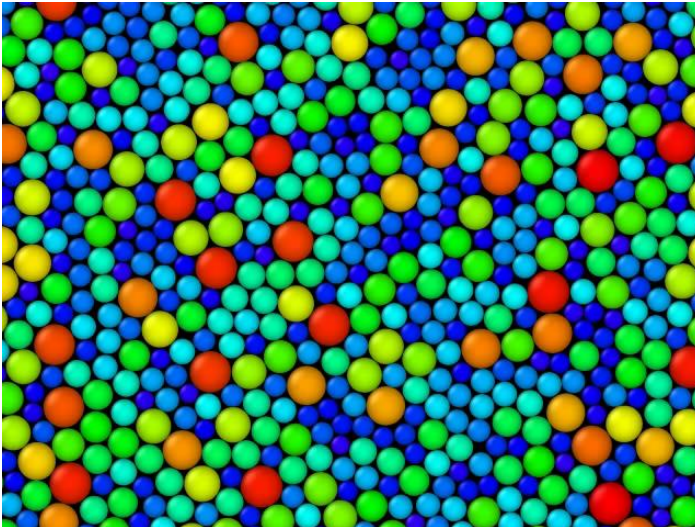
-The stress is miss reading

-Larger the system size, longer the time for the steady state

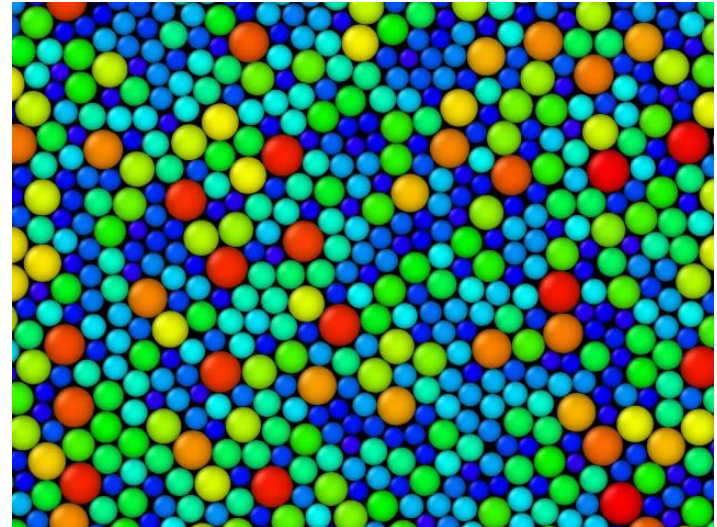
Swap Monte-Carlo simulations

Equilibrium hard disks beyond jamming: $\phi = 0.85 > \phi_J$

Normal Monte-Carlo



Swap Monte-Carlo



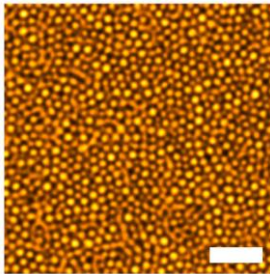
Berthier, Coslovich, Ninarello, and Ozawa, PRL 2016
Ninarello, Berthier, and Coslovich, PRX 2017

Swap MC is a very efficient thermalization technique,
producing very stable glasses

Discussion: Experimental realization

Discontinuous transition

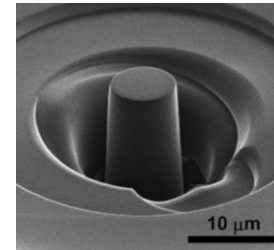
(Brittle)
Nano colloids



Hallett, Turci, and Royall,
Nat. Commun. 2018

Crossover

(Ductile)
Micro metallic glasses

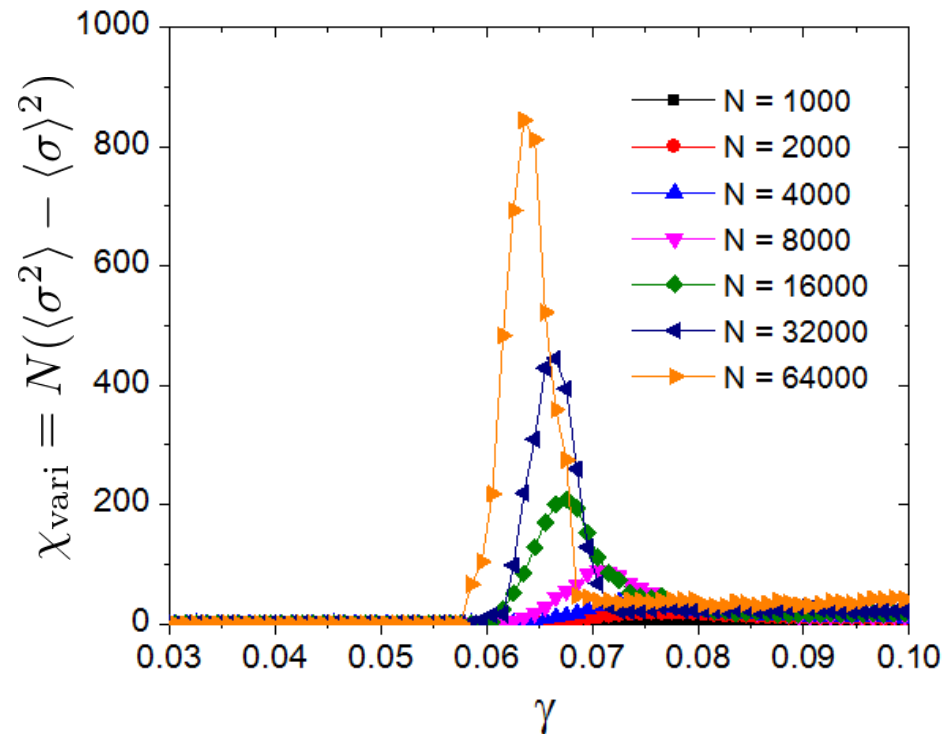
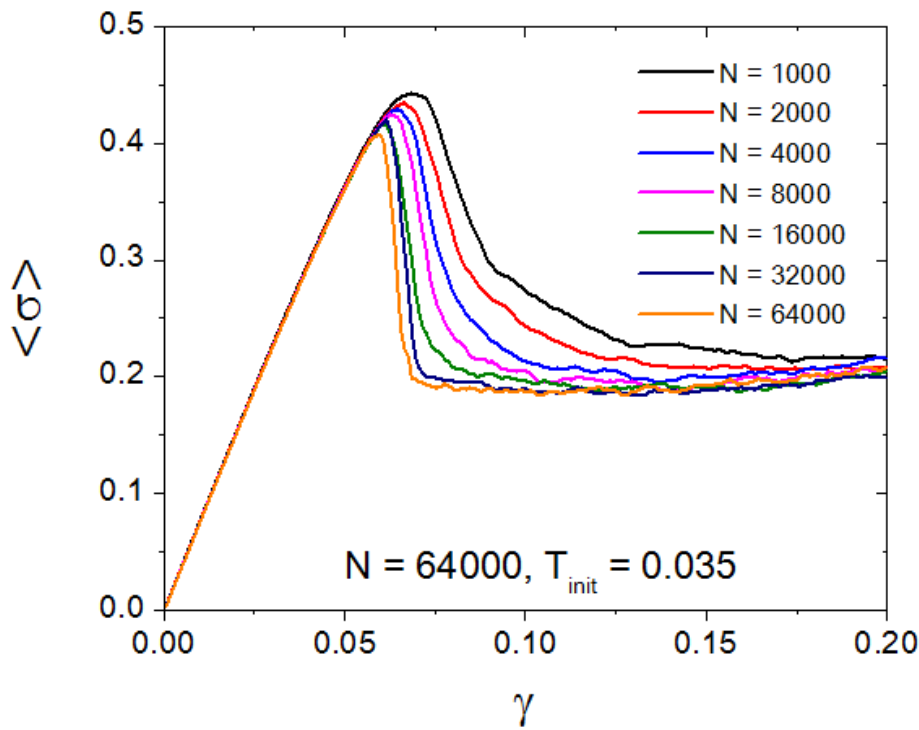


Yang, Ye, Lu, Gao, and
Liaw, JOM 2010



The critical point might be detected by some experiments

Yielding transition in $d = 2$



Yielding becomes sharper with $N \rightarrow \infty$