The onset of the flow in disorder materials: depinning and yielding transitions

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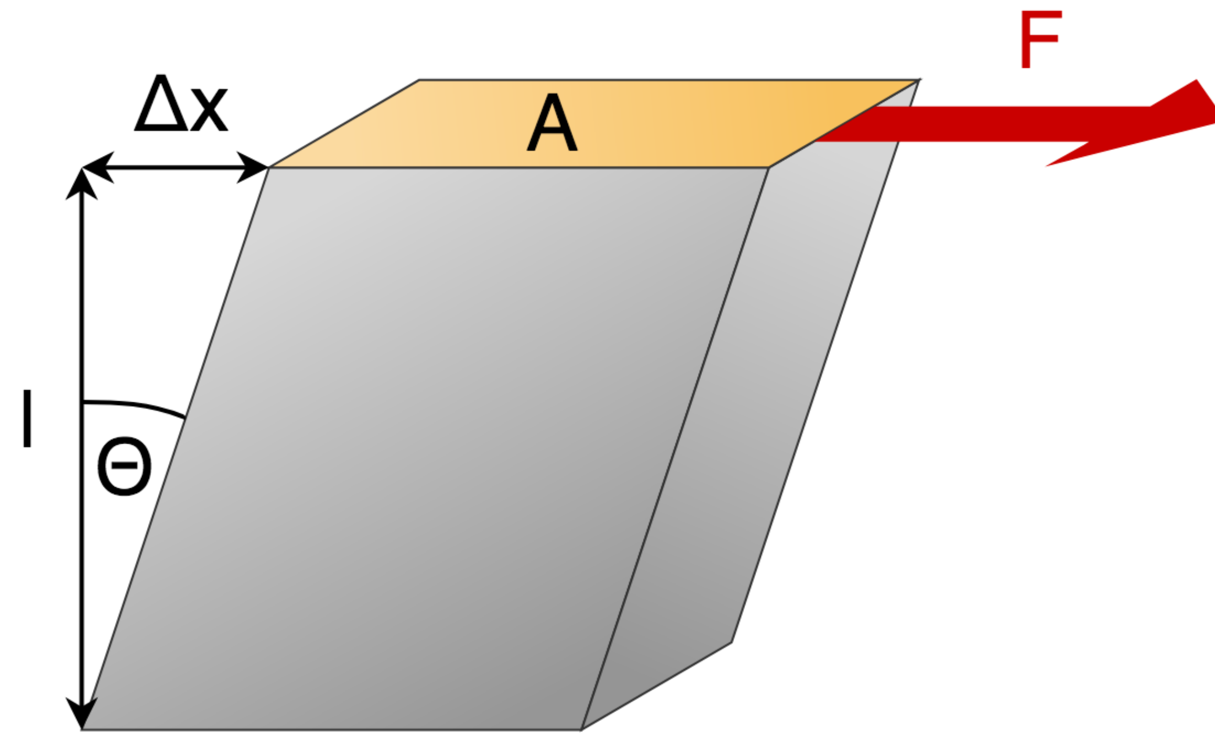
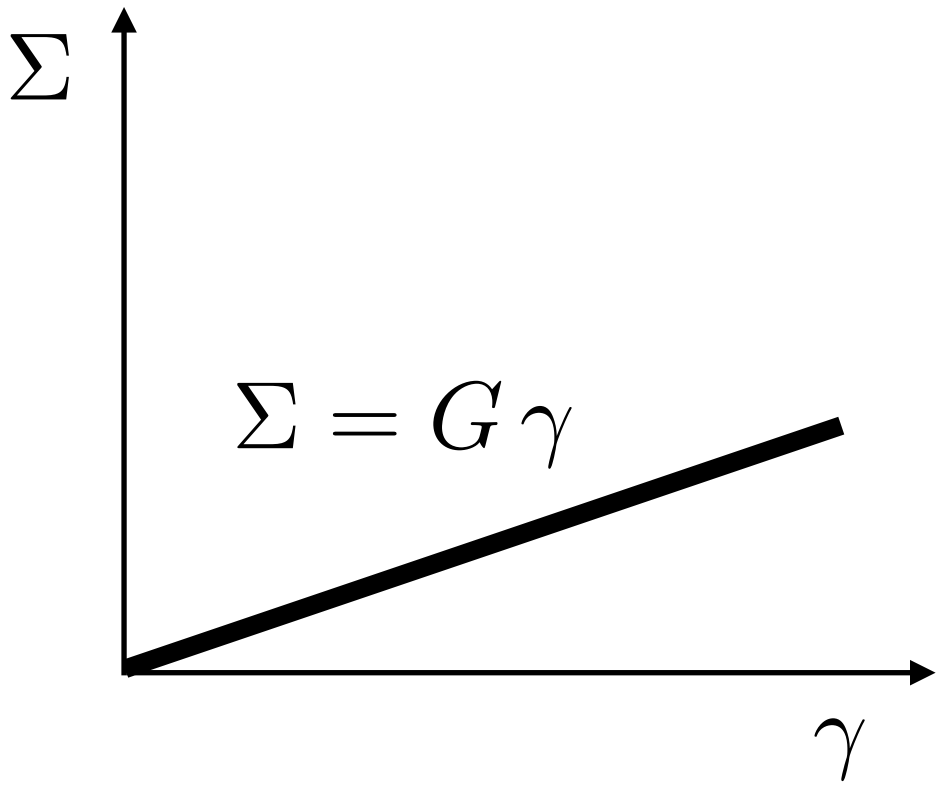
M. Ozawa, L. Berthier, G. Biroli, AR, G. Tarjus PNAS 2018
X. Cao, A. Nicolas, D. Trimcev, AR, Soft Matter 2018
J. Lin, T. Gueudré, AR, M. Wyart, Phys. Rev. Lett. 2015
J. Lin, E. Lerner, AR , M. Wyart, PNAS 2014

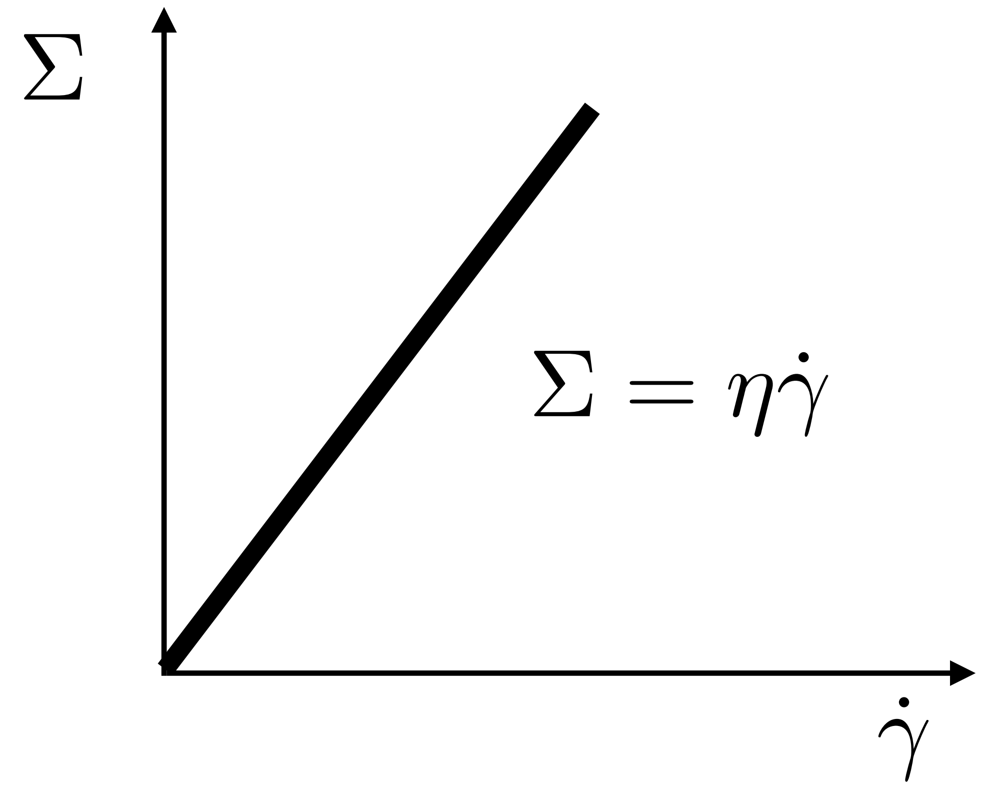
Materials under shear

$$\Sigma = \frac{F}{A}$$



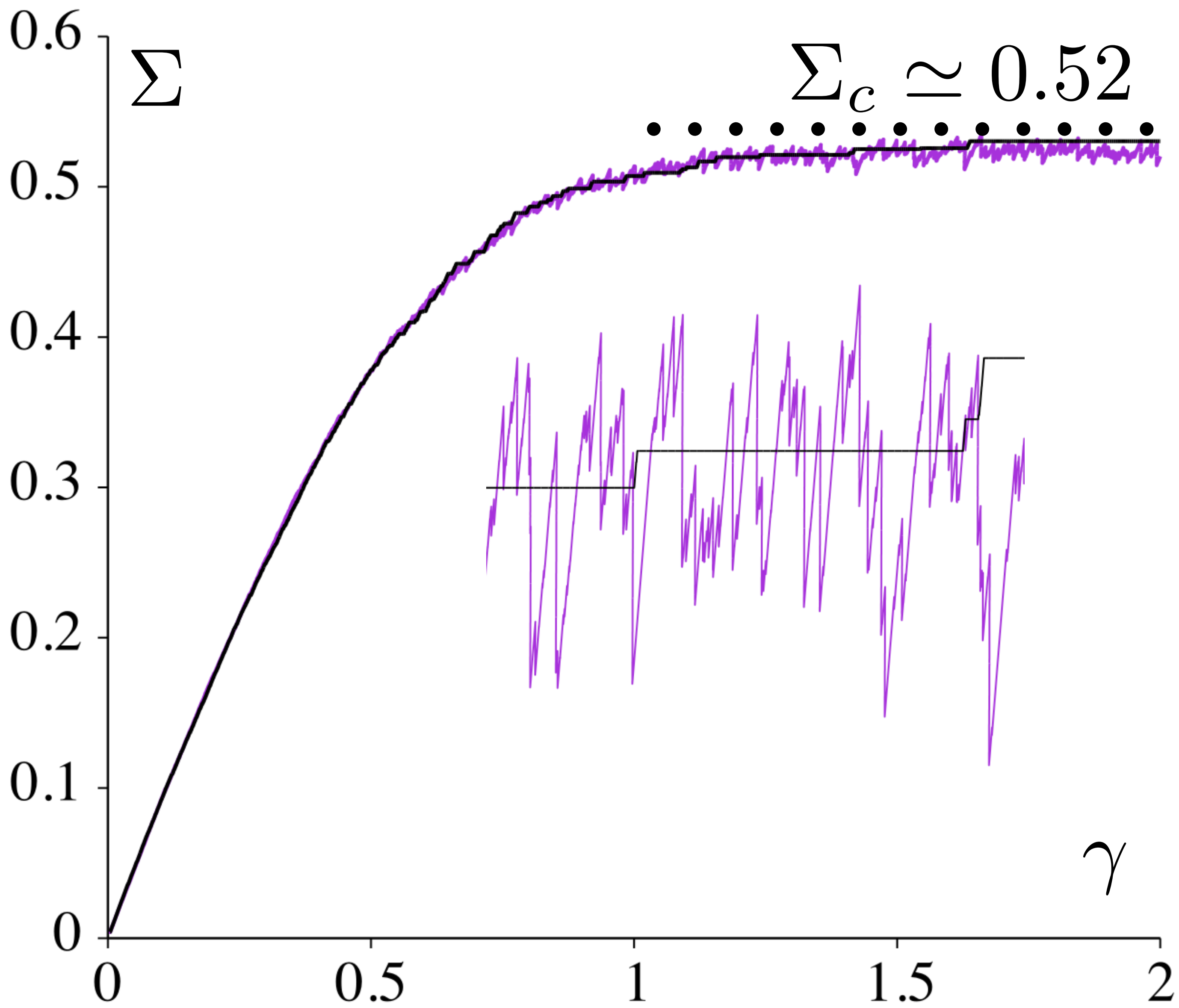
$$\gamma = \frac{\Delta X}{l}$$





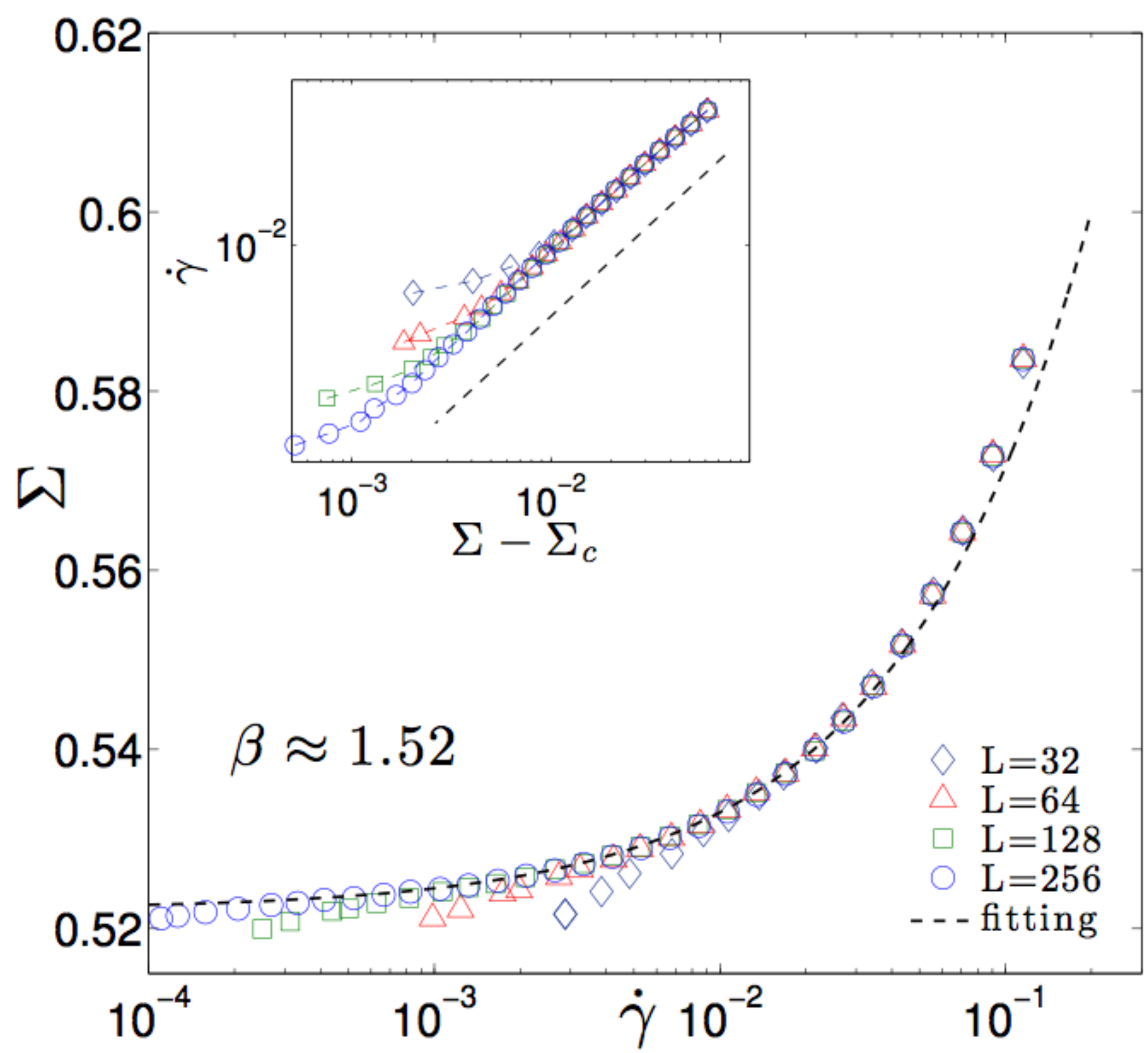
Yielding transition: dynamical melting transition

Plastic Solid



Stress/Strain Control

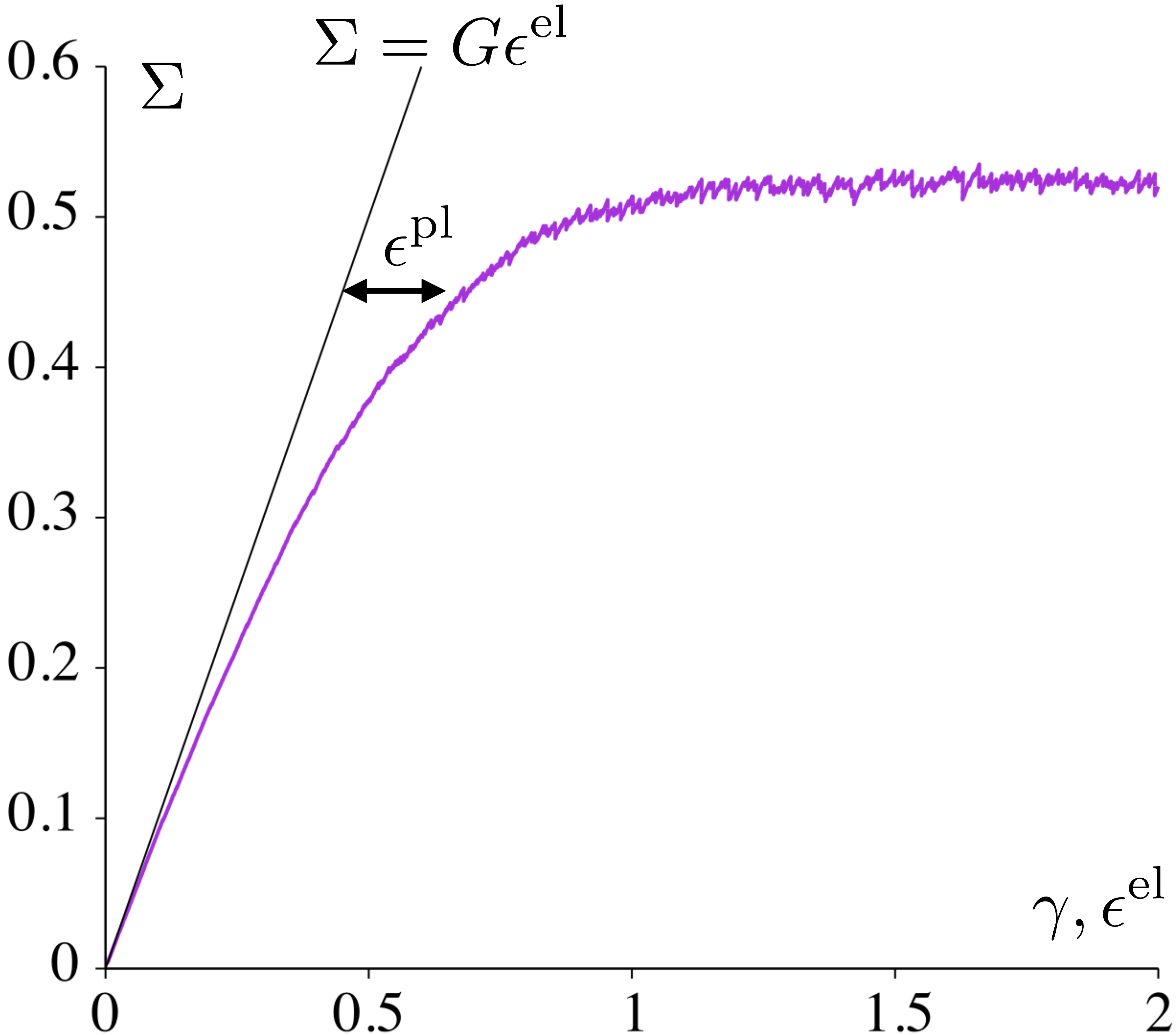
Herschel-Buckley liquid



$$\dot{\gamma} \sim (\Sigma - \Sigma_c)^{\beta}$$

Elastic, plastic strain and avalanches

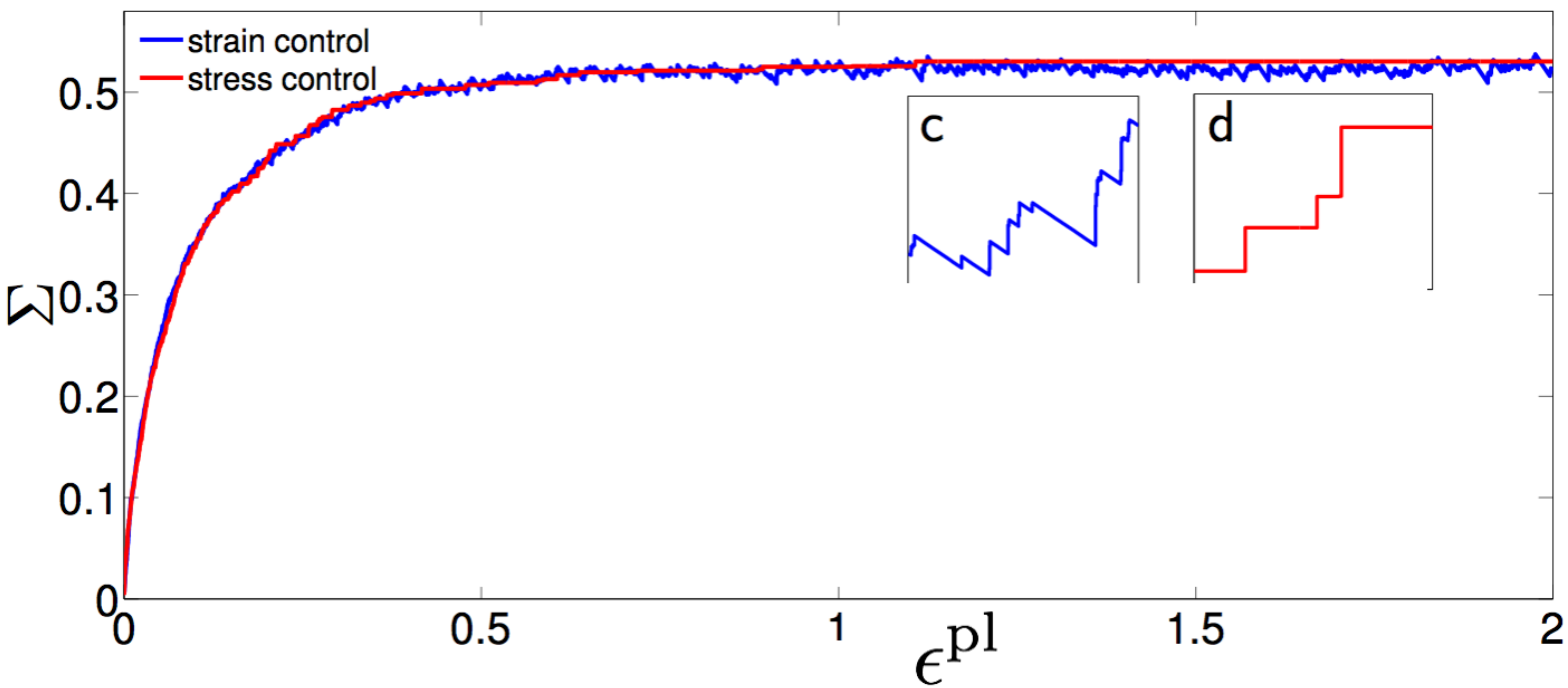
$$\gamma = \epsilon^{\mathrm{pl}} + \epsilon^{\mathrm{el}} \qquad \epsilon^{\mathrm{el}} = \frac{\Sigma}{G}$$



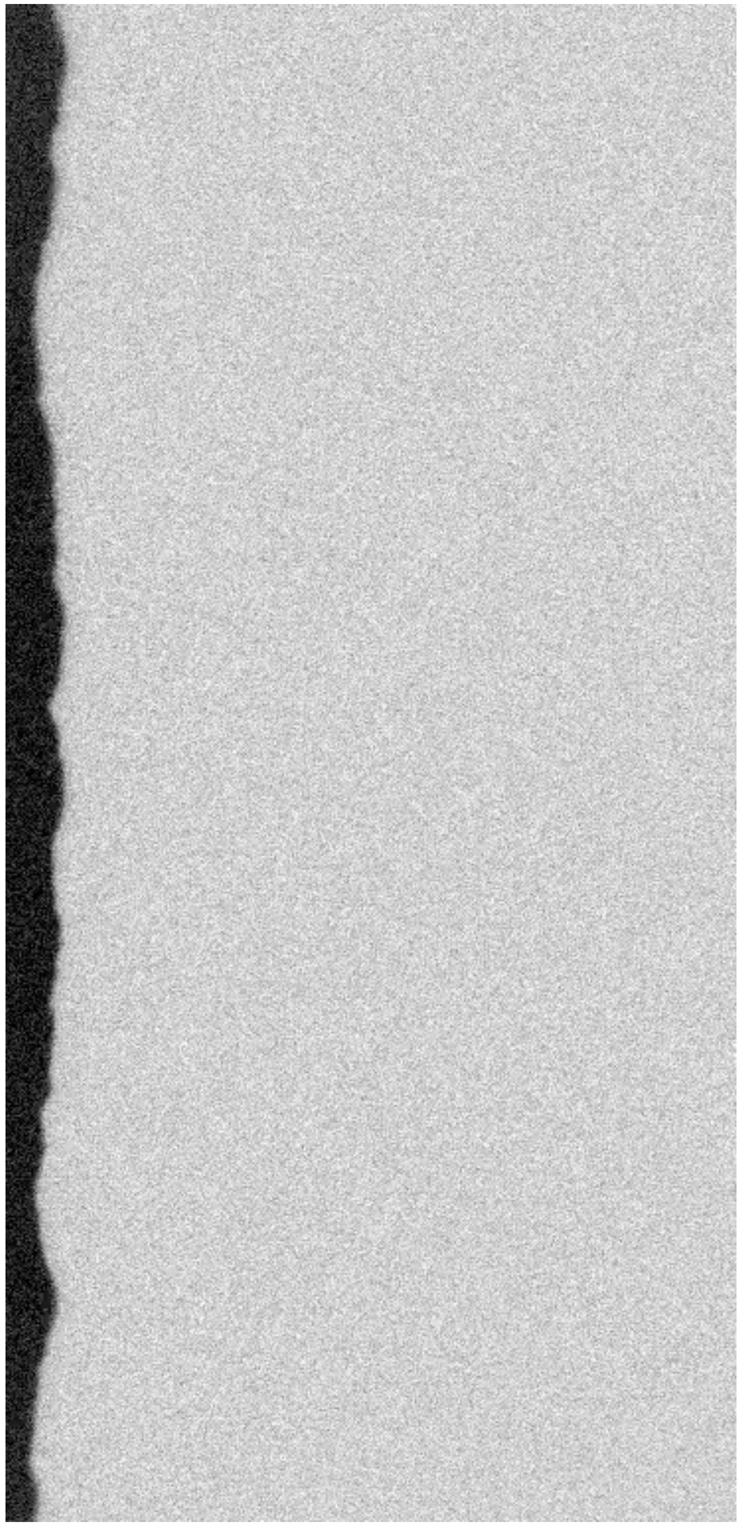
Plastic strain and avalanches

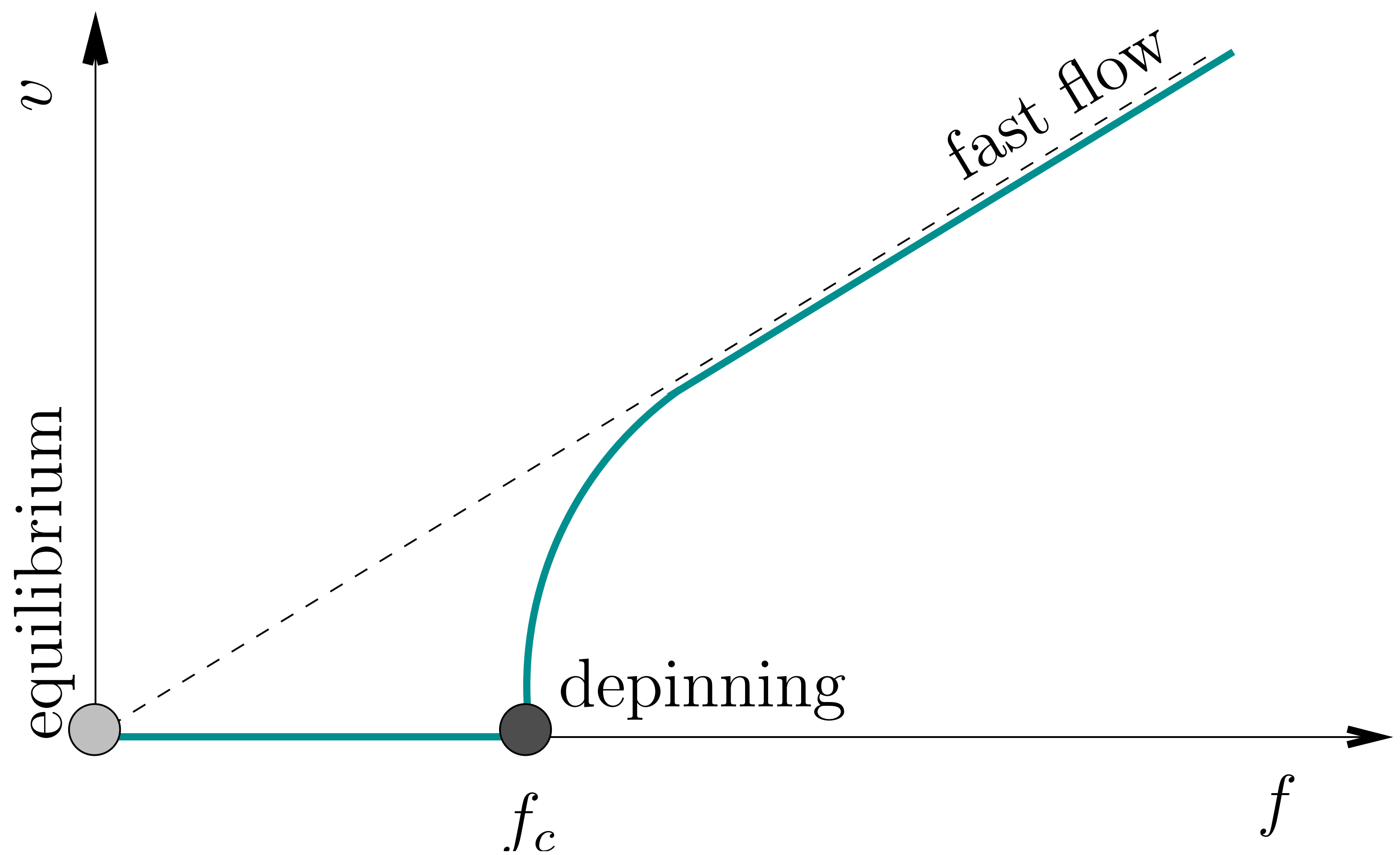
$$\gamma = \epsilon^{\rm pl} + \epsilon^{\rm el}$$

$$\epsilon^{\rm el} = \frac{\Sigma}{G}$$



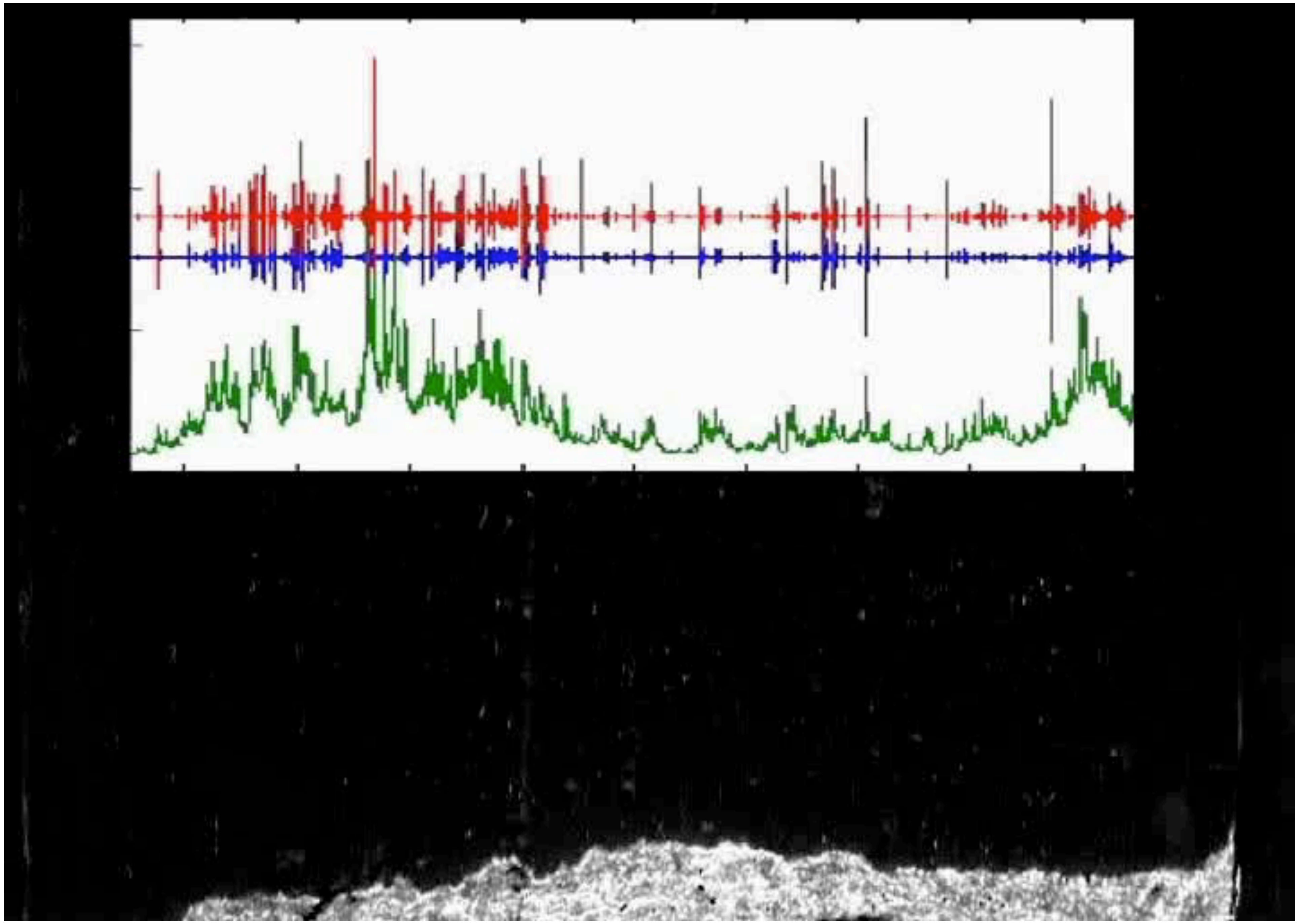
Depinning transition of an elastic interface





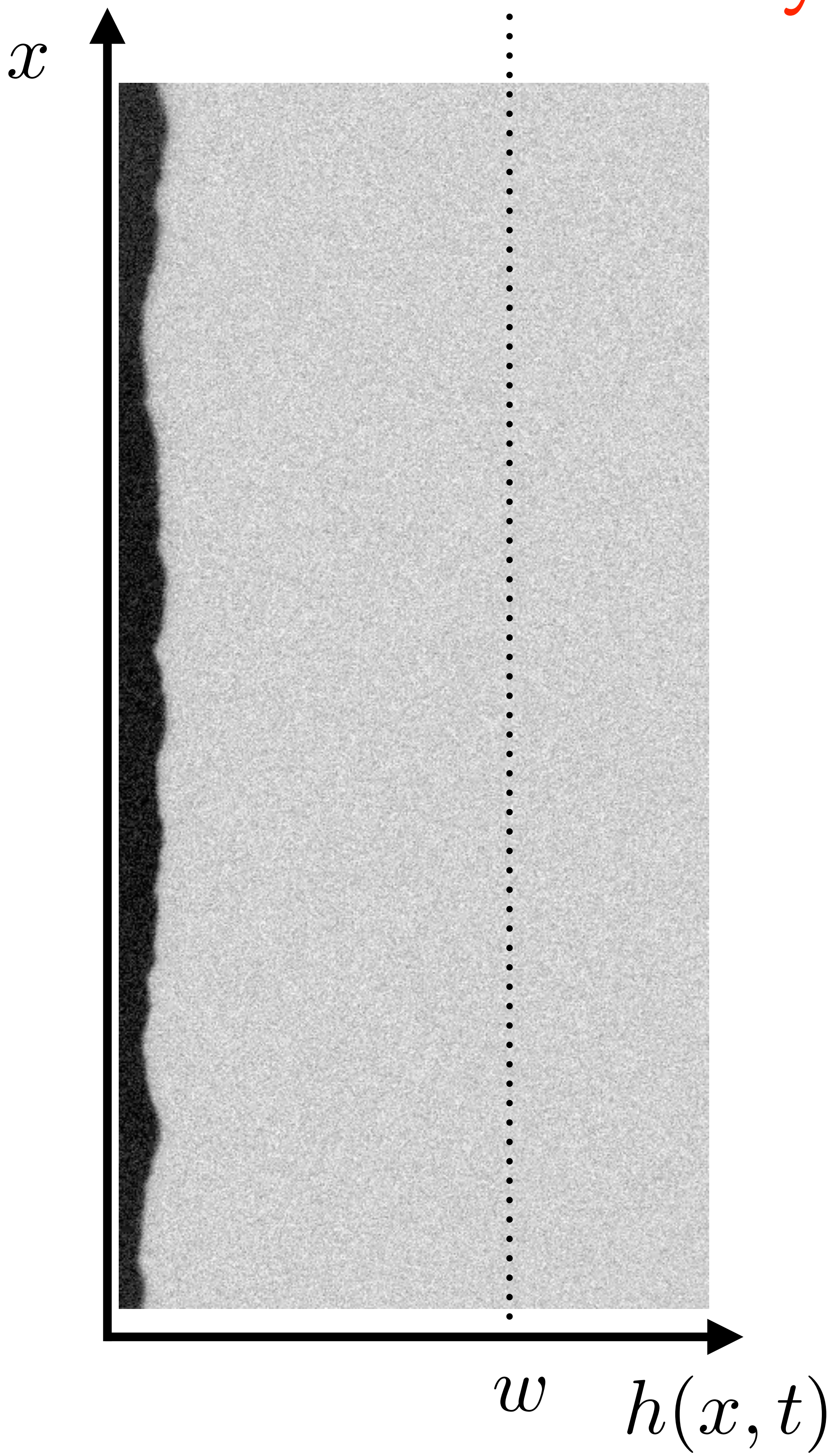
magnetic film, domain wall dynamics
by V. Jeudy & A. Mougin in Paris-Saclay

Depinning of a crack front



propagation of crack fronts in PMMA,
by S. Santucci in ENS- Lyon

Two dynamical protocols:



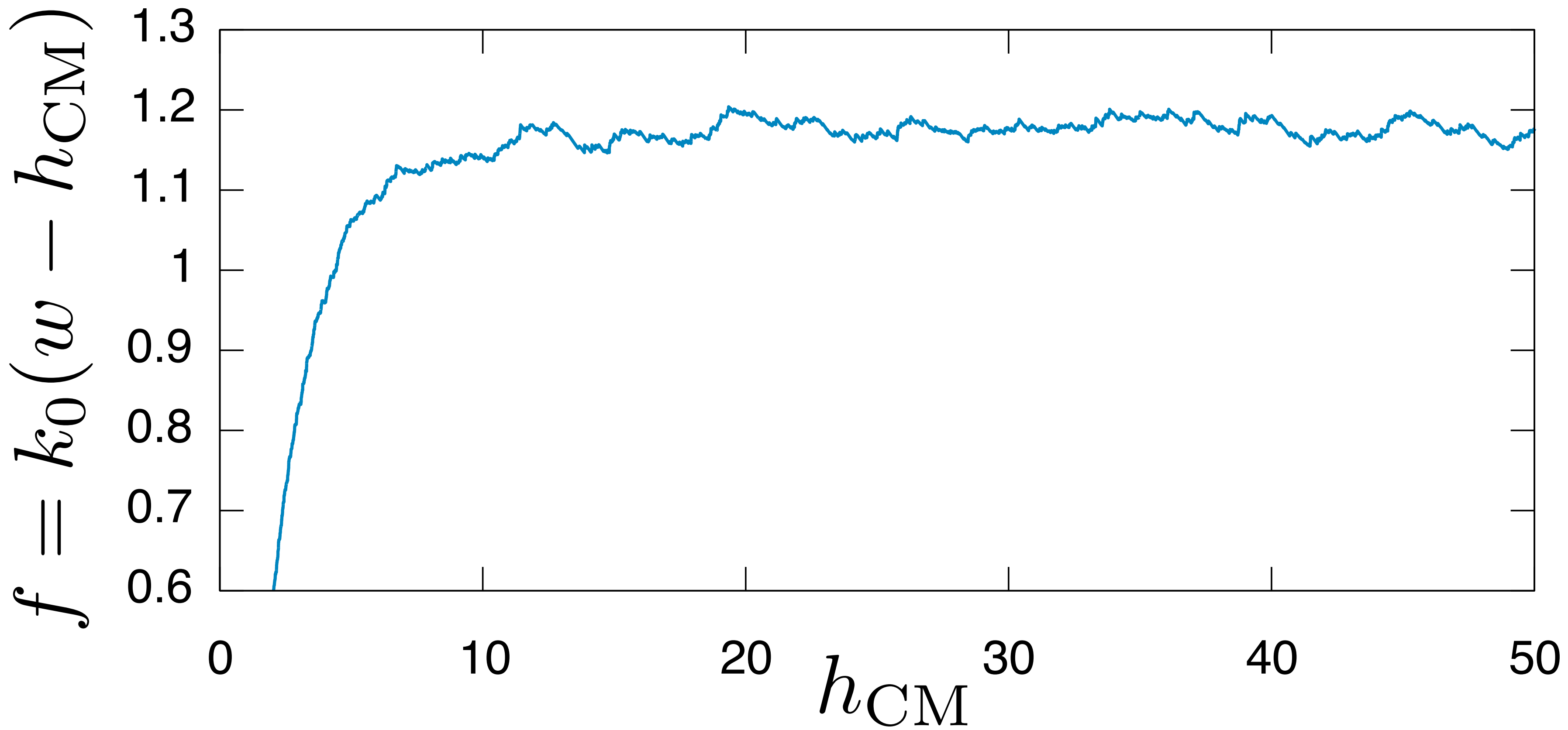
force control:

$$\partial_t h = \partial_x^2 h + f + \sigma_{x,h}^{\mathrm{dis}}$$

Loading:

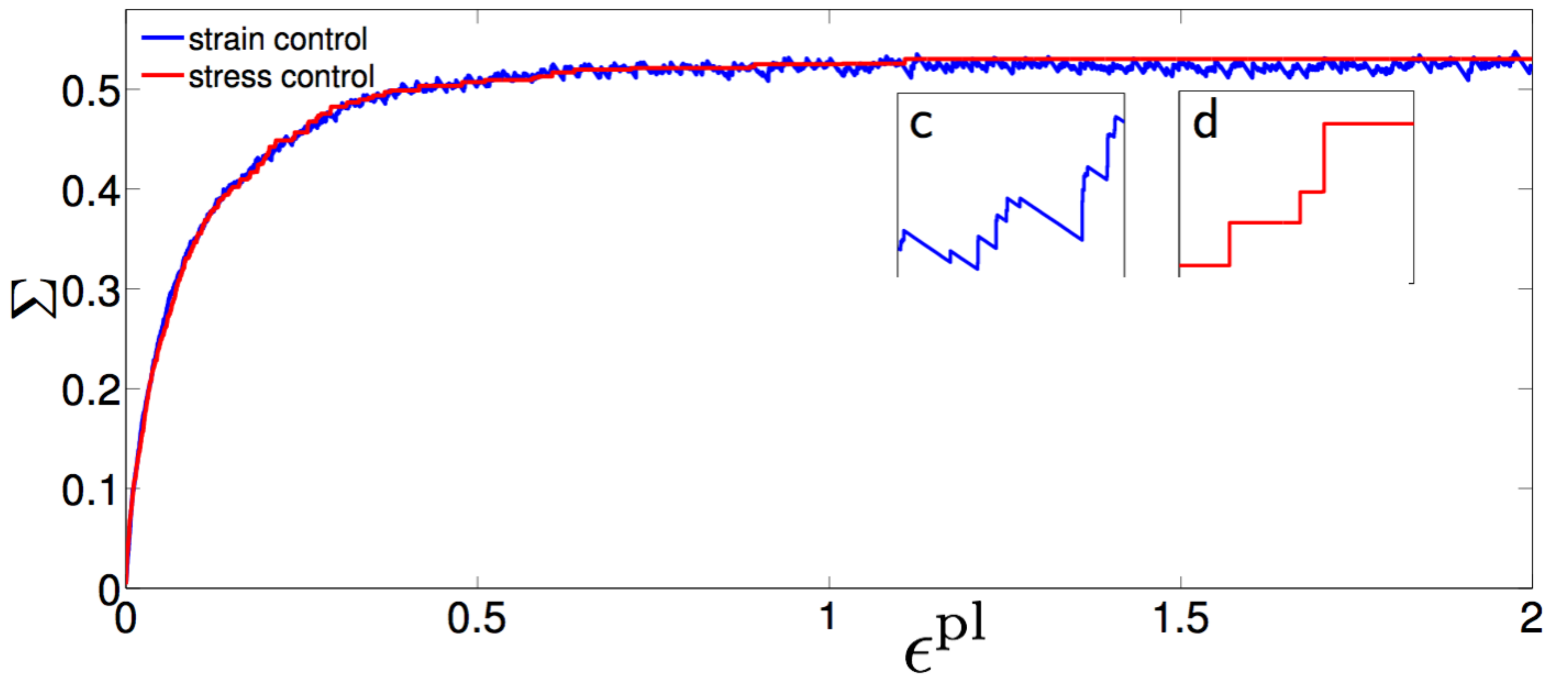
$$\partial_t h = \partial_x^2 h + k_0(w - h) + \sigma_{x,h}^{\mathrm{dis}}$$

Loading equivalent to strain control



$$\Sigma \Longrightarrow f$$

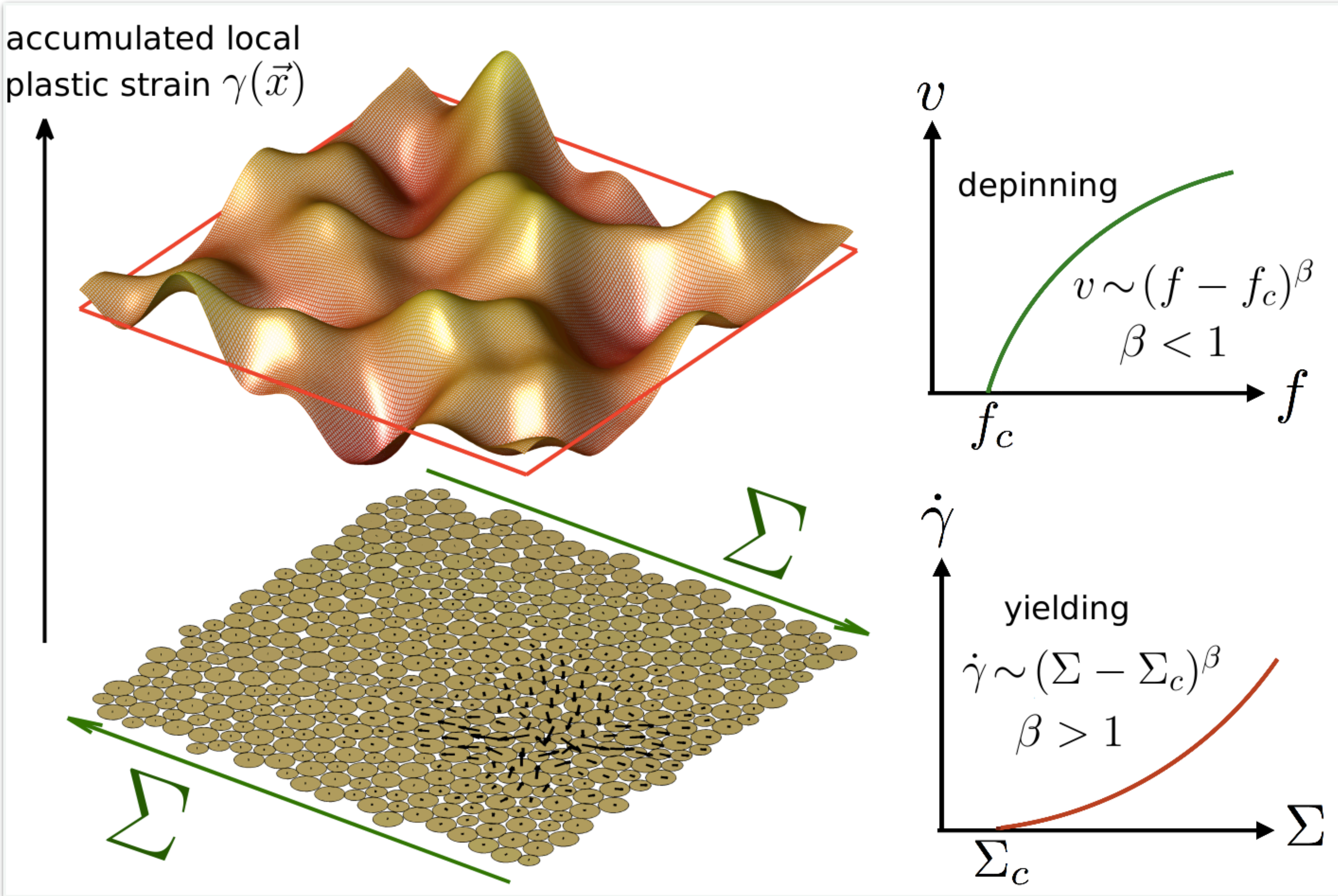
$$\epsilon^{\mathrm{pl}} \Longrightarrow h_{CM}$$



$$\gamma \Longrightarrow w$$

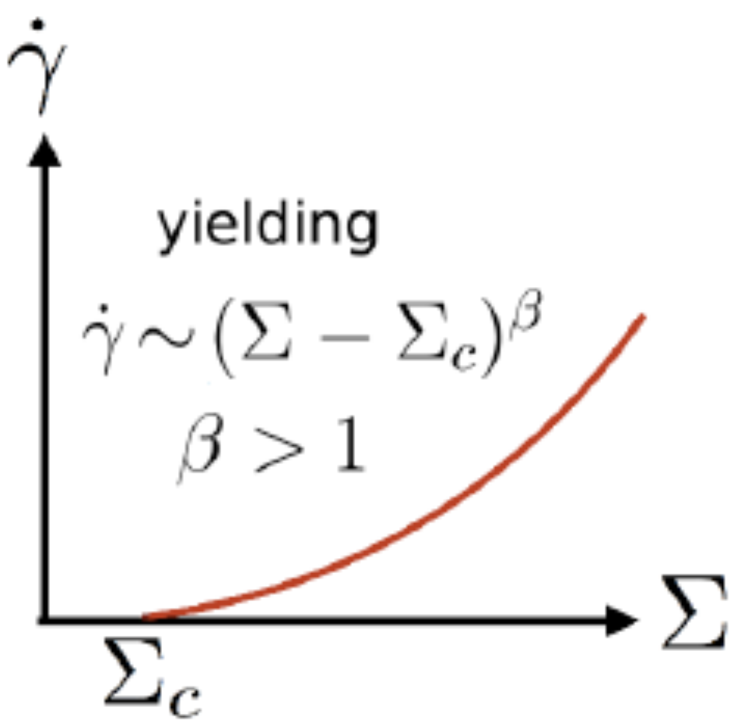
$$k_0 \Longrightarrow G = 1$$

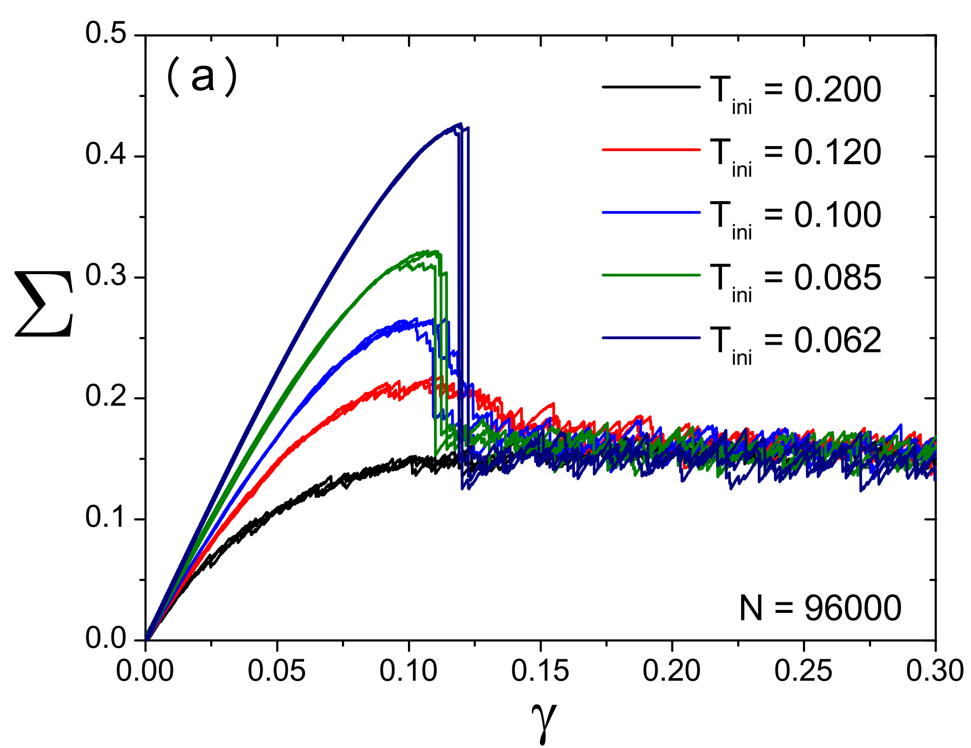
Yielding and Depinning transition

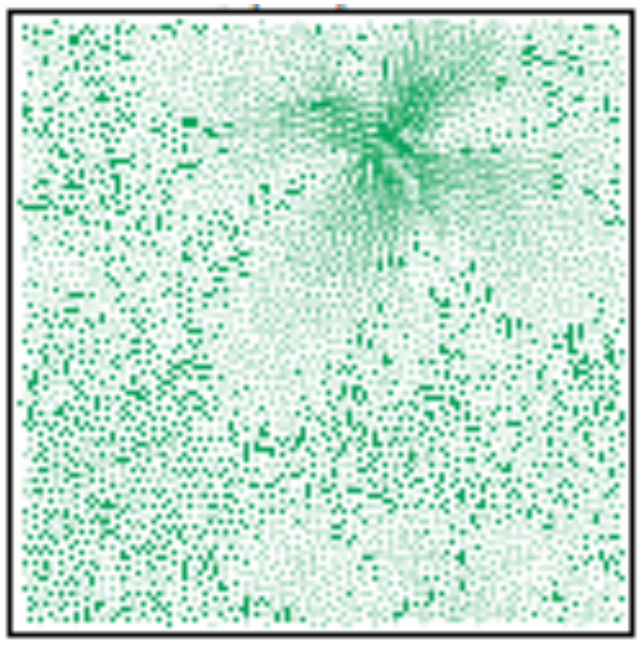


Four questions:

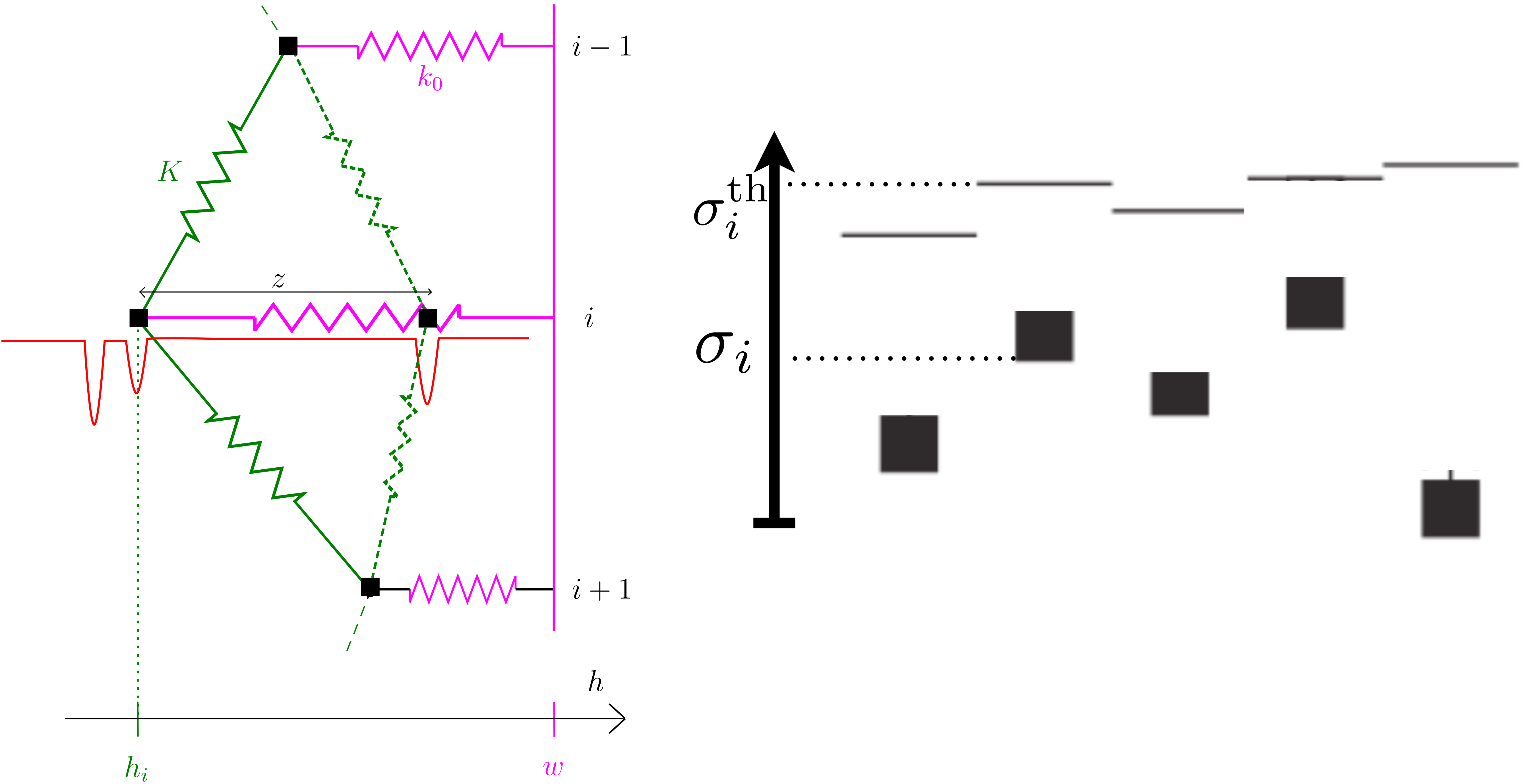
- Scaling description of yielding in the liquid phase
- Transient in solid phase 1: spanning system avalanches
- Transient in solid phase 2: failure & spinodal
- Anisotropic soft modes as failure precursors





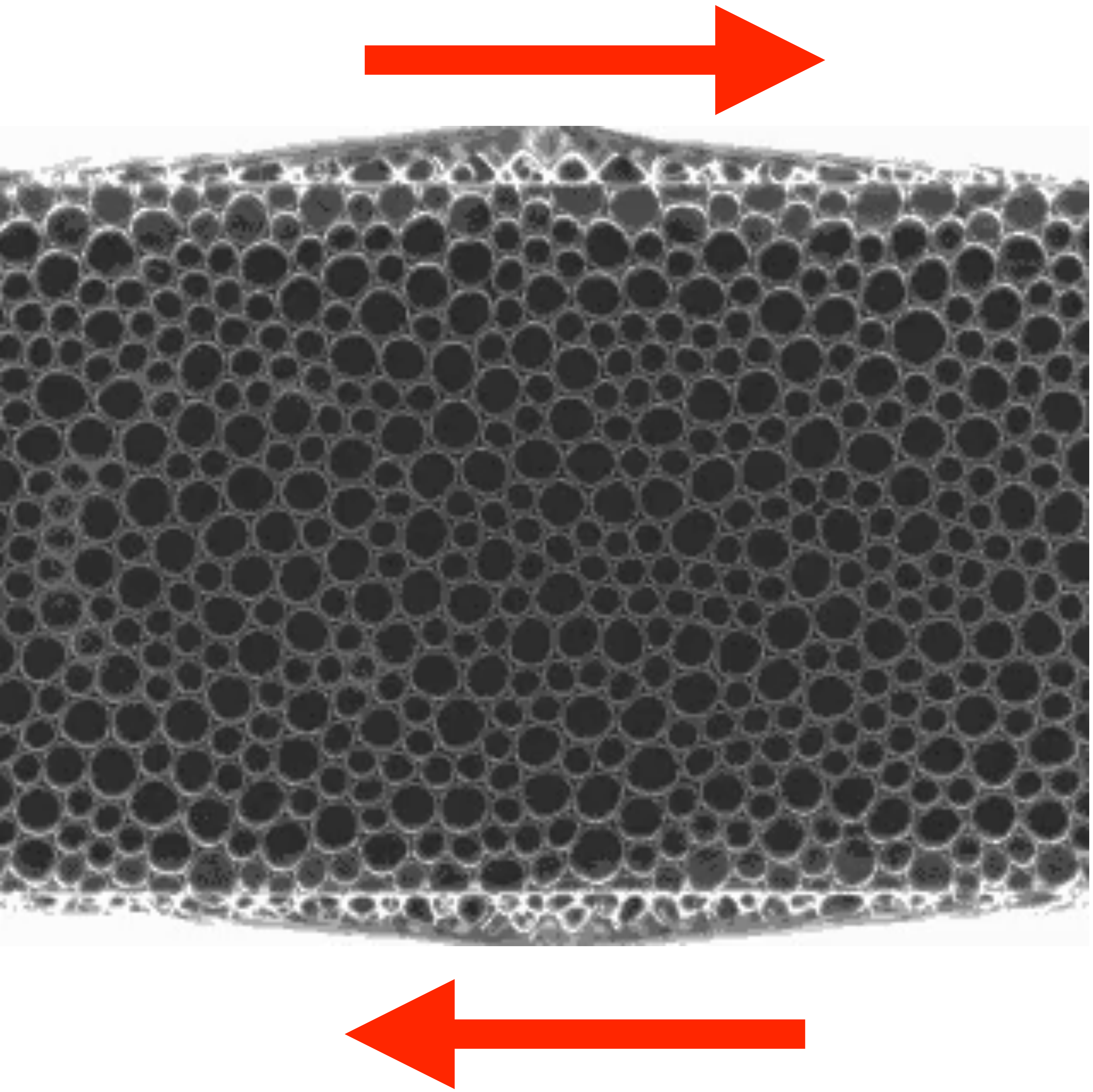


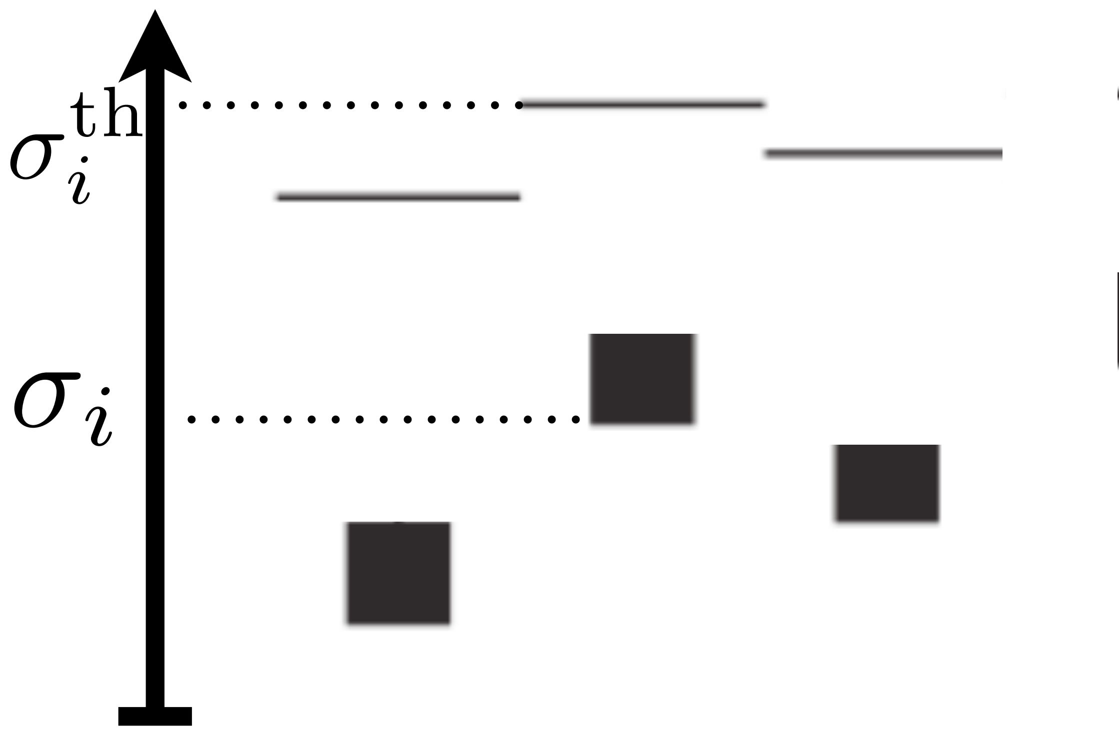
Cellular automaton for depinning



$$\partial_t h_i(t) = \underbrace{k_0(w-h_i) + K(h_{i+1}+h_{i-1}-2h_i)}_{\sigma_i} - \sigma^{th}(h_i)$$

Shear Transformations (Argon)

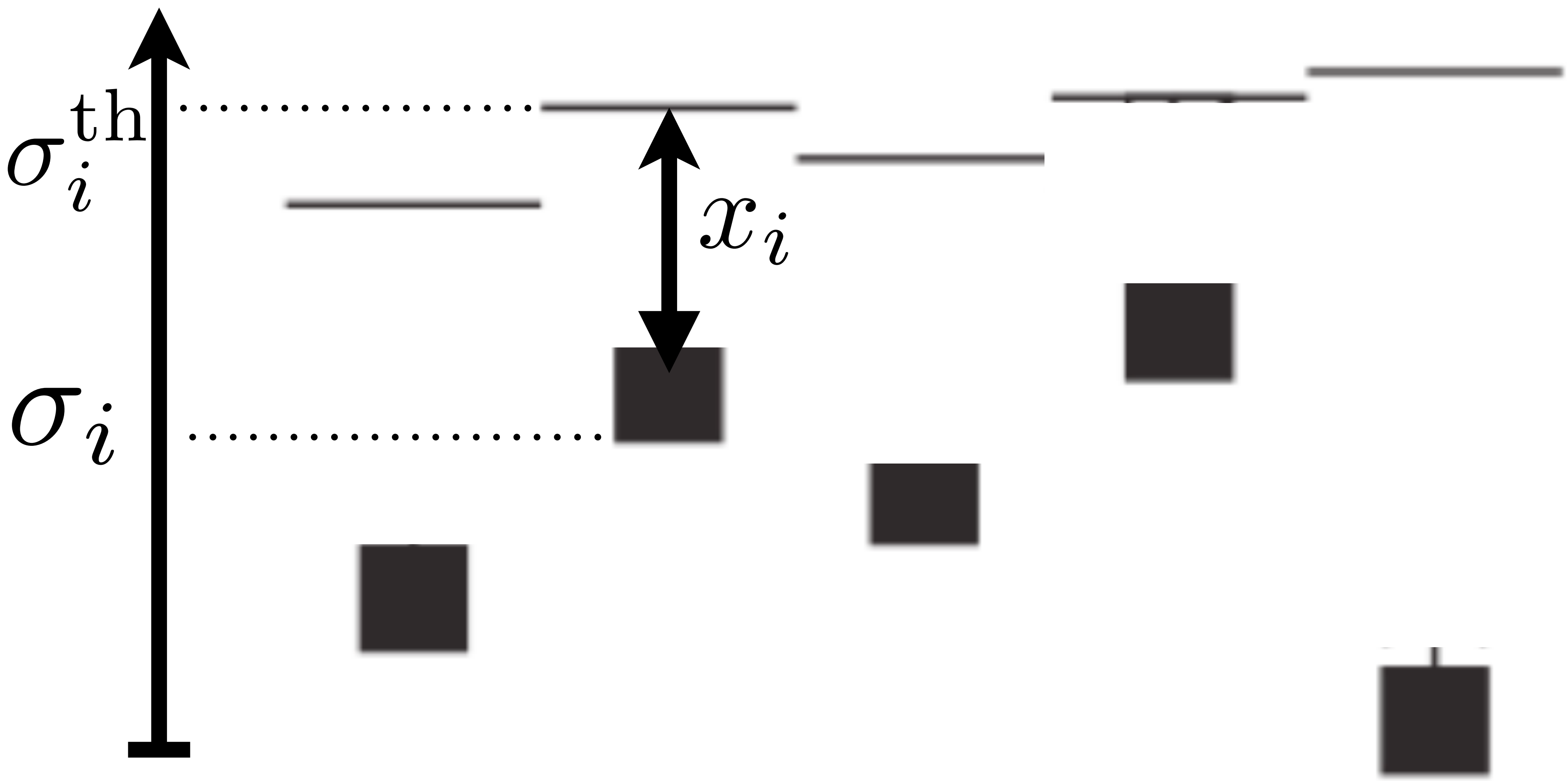




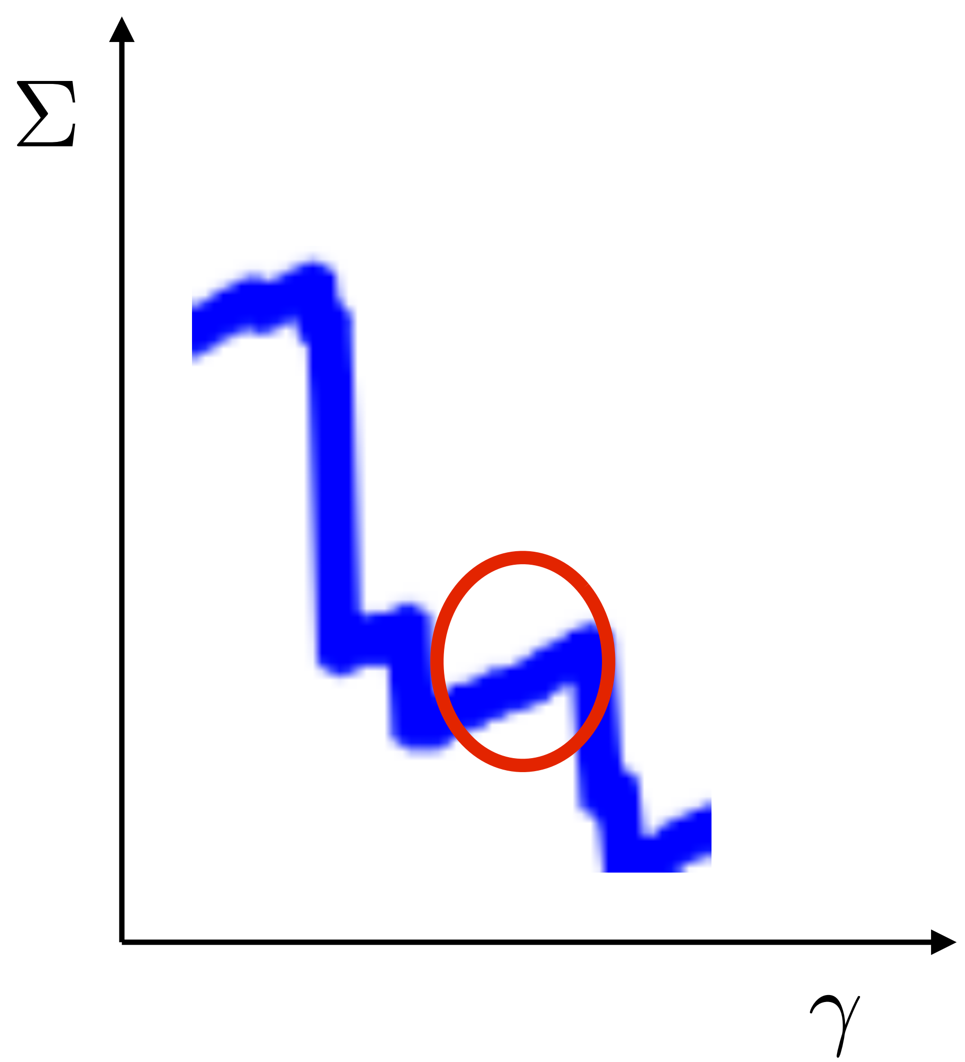
$$\Sigma = \frac{1}{N}\sum_{i=1}^{N}\sigma_i$$

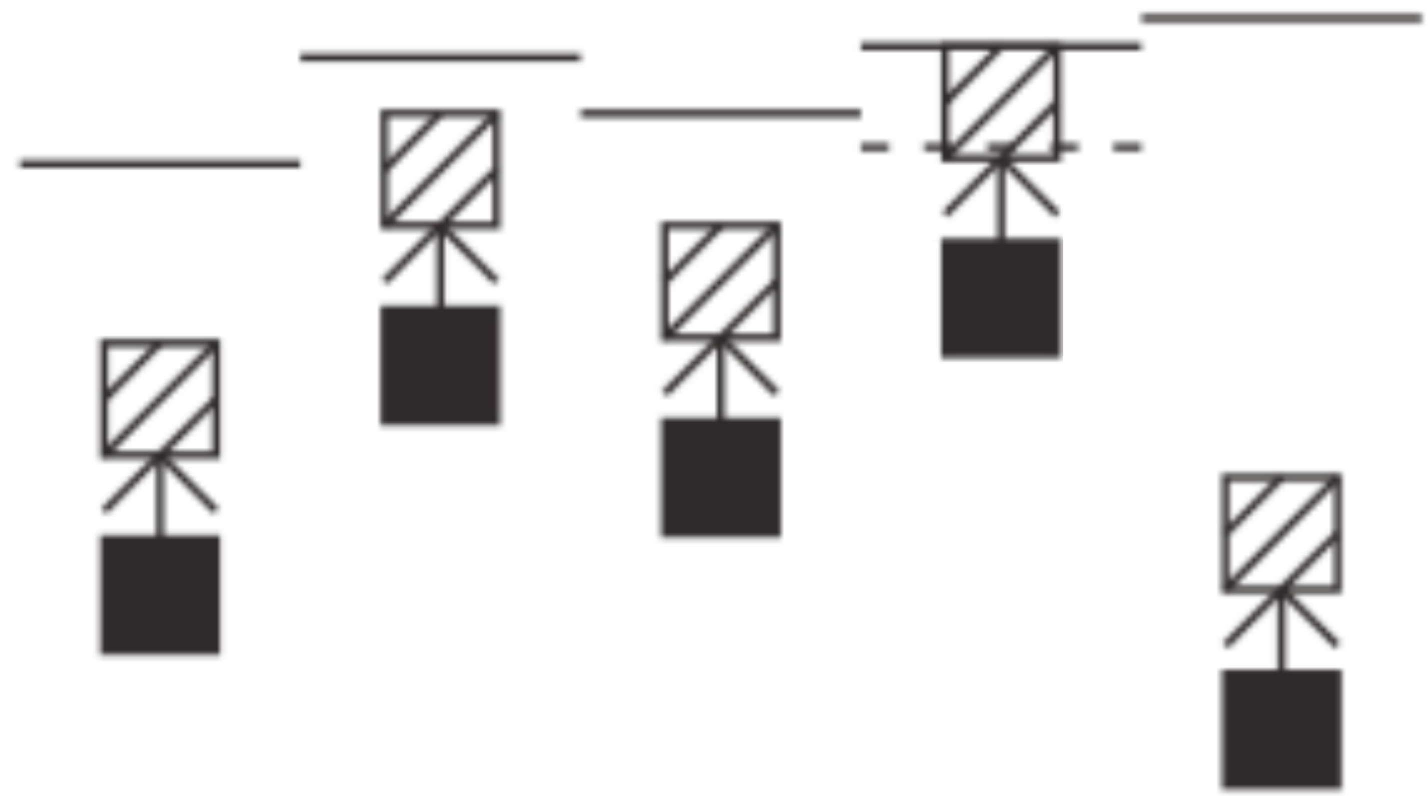
by Van Hecke group

Elasto-plastic models



The elastic part

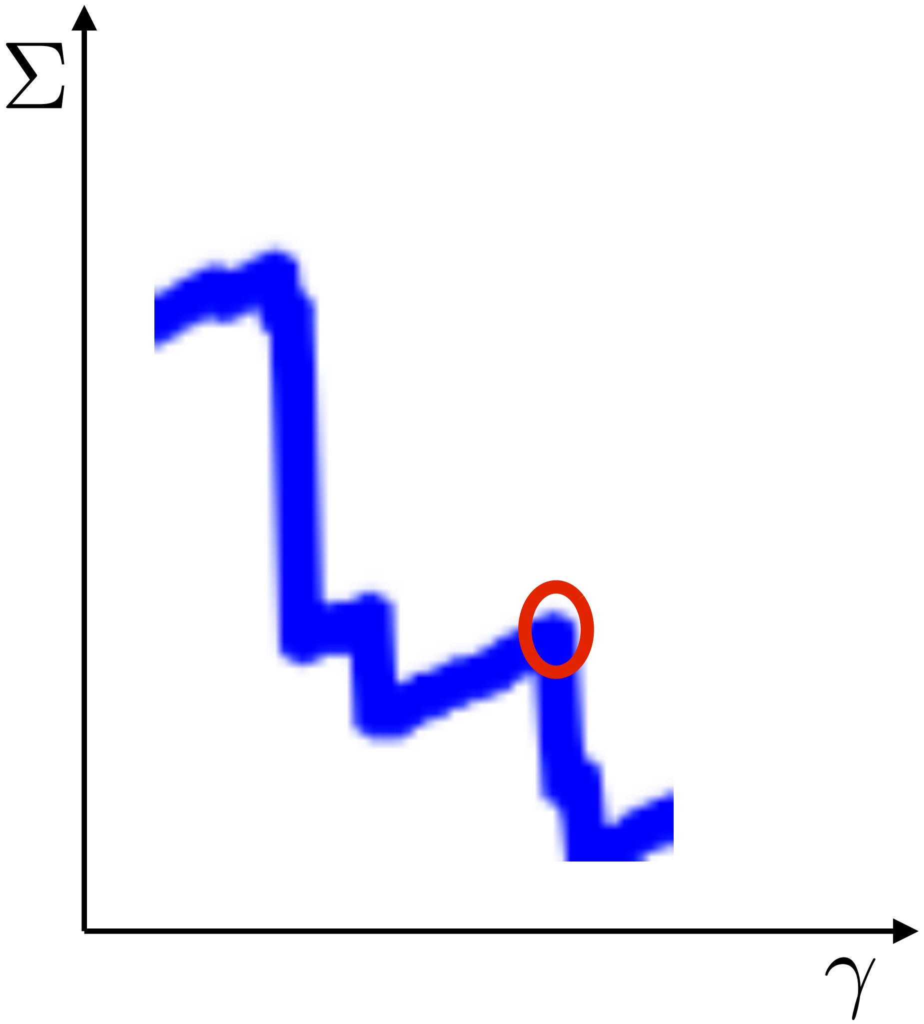




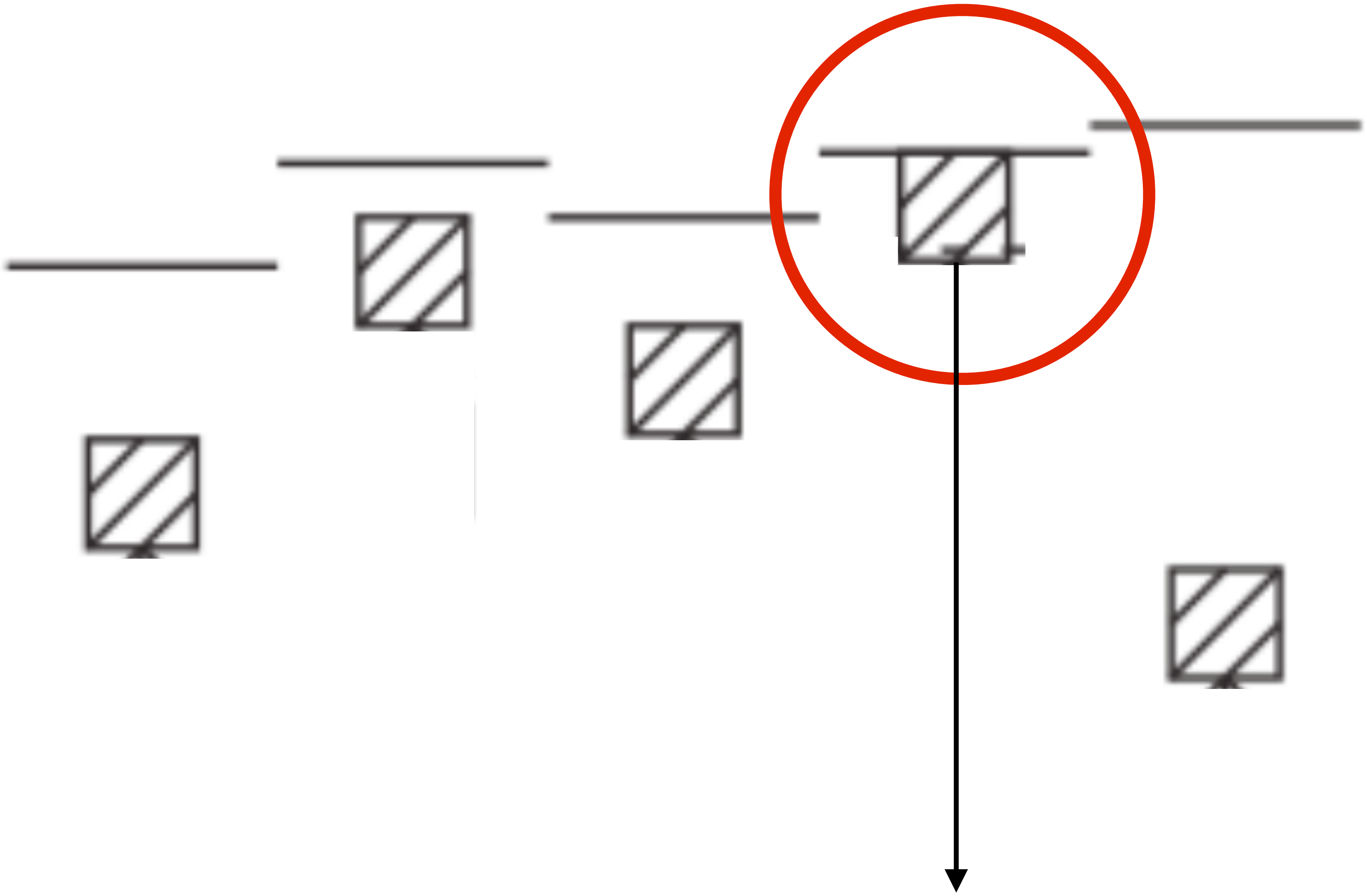
$$\sigma_i \to \sigma_i + \delta\gamma \quad \Longrightarrow \quad x_i \to x_i - \delta\gamma$$

Elastic loading up to the first instability

The plastic part



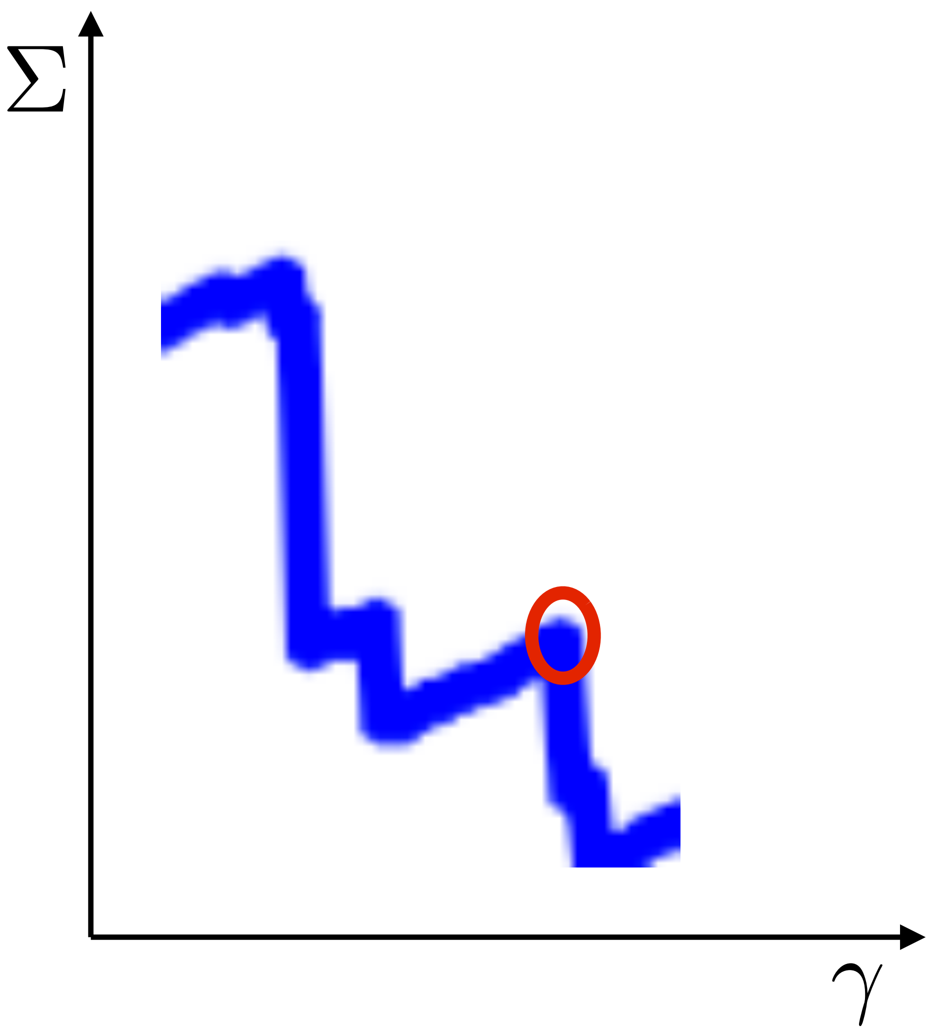
$$x_i = 0 \quad \Longrightarrow$$

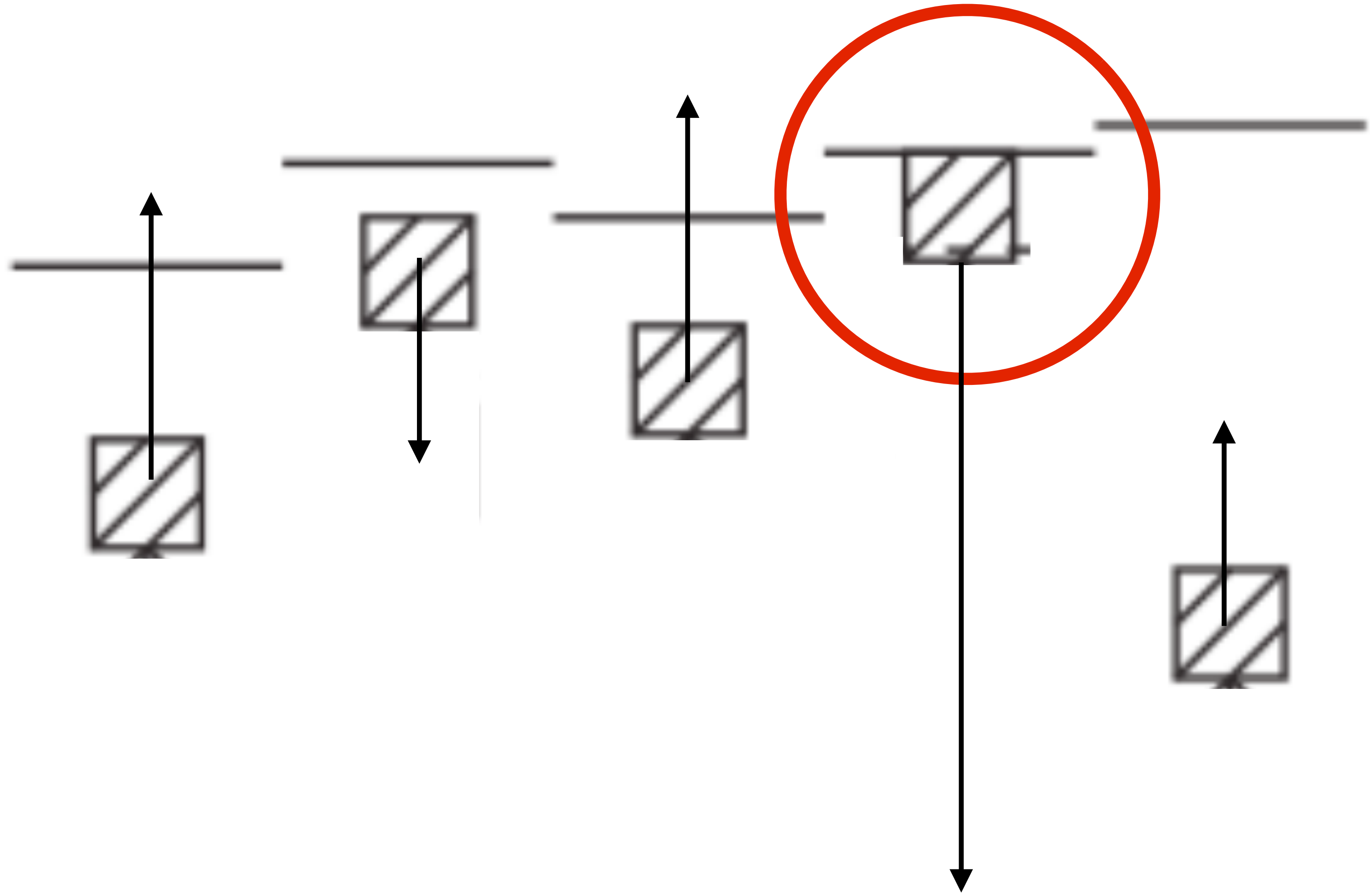


$$x_i \to x$$

(Shear Transformation)

The plastic part





$$x_i = 0 \quad \Longrightarrow \quad \begin{array}{l} x_i \to x \\ x_j \to x_j - G_{ij}\, x \end{array}$$

(stress redistribution)

Which kernel redistribution ?

Depinning (2D): $G_{ij} = \frac{K}{K + k_0} \frac{1}{4}$ Positive

Eshelby (2D): $G_{ij} = \frac{\cos \theta_{ij}}{|i - j|^2}$ Positive and negative

Mean Field Models

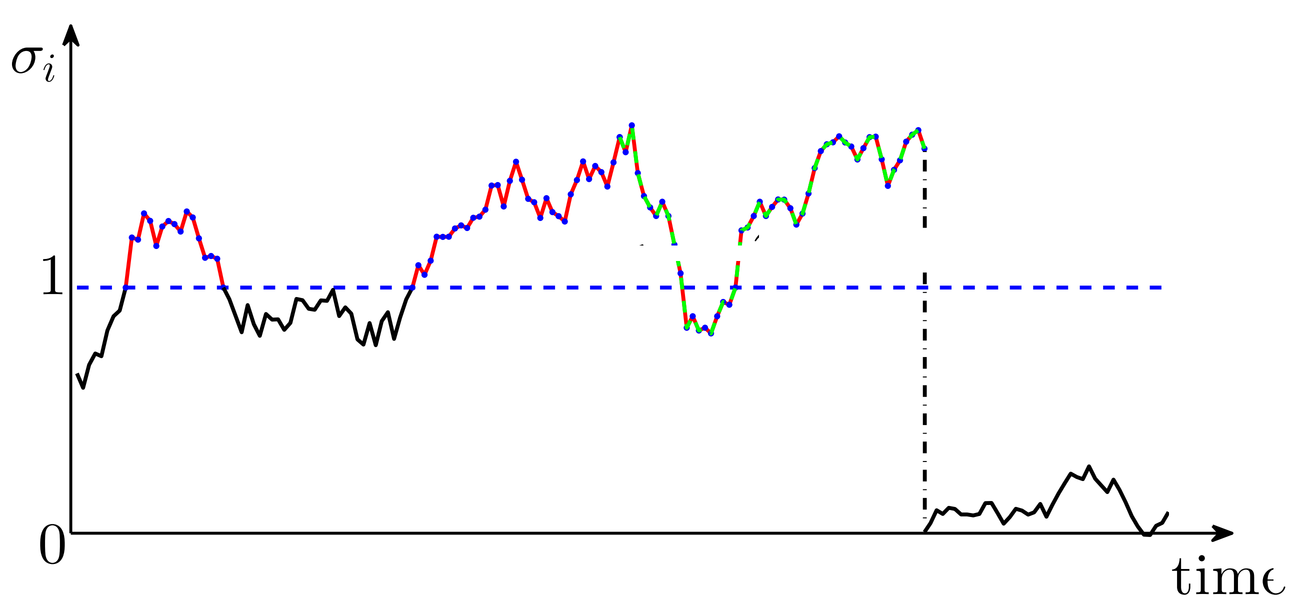
Depinning: $G_{ij} = \frac{K}{K + k_0} \frac{1}{L^d}$ Positive

Hebraud-Lequeux: $G_{ij} = \frac{\xi_j}{L^{d/2}}$ Positive and negative

Yielding/Eshelby

$G_{i \neq j}$ positive or negative

- two thresholds
- non abelianity

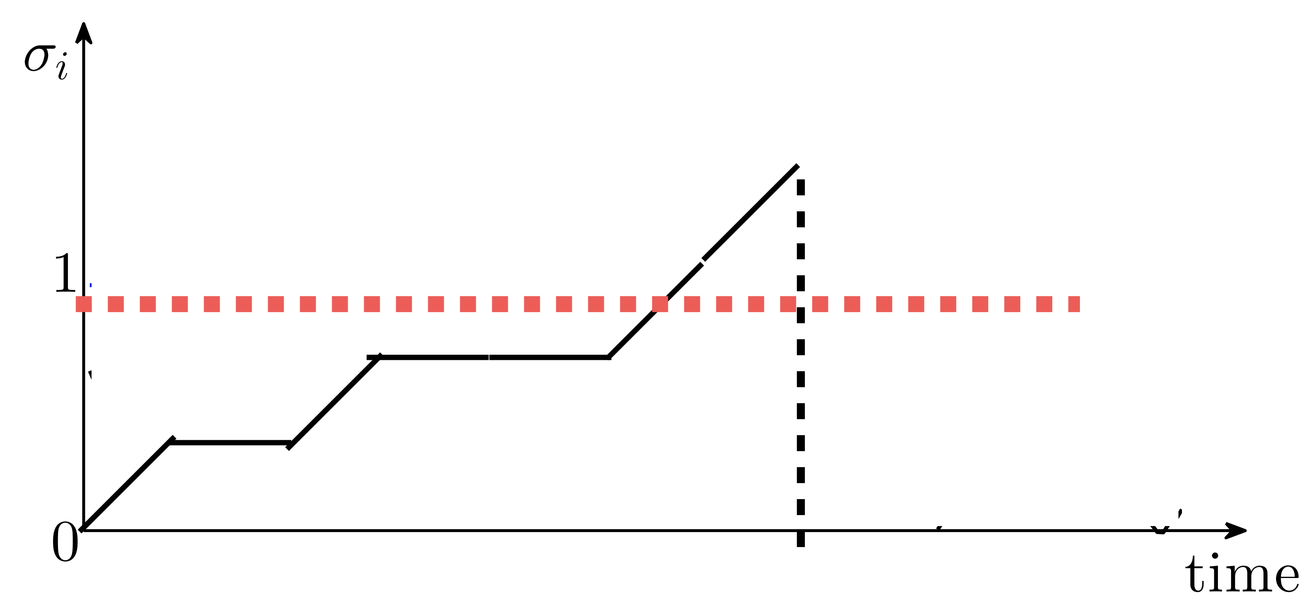


diffusion (mechanical temperature)

Depinning/Elastic

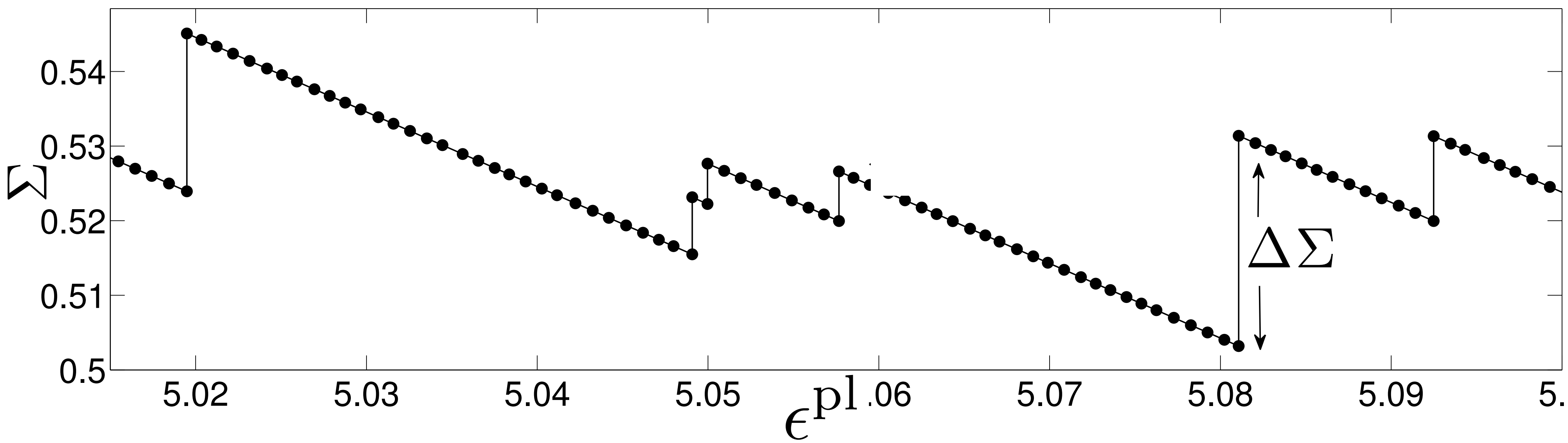
$G_{i \neq j}$ positive

- only one threshold
- abelianity



ballistic behaviour

Yielding/Eshelby Depinning/Laplacian



$$x_i = \sigma^{th} - \sigma_i \quad \Rightarrow \quad P(x)$$

Pseudo-gap in excitations

$$P(x) \to x^\theta \quad \text{when} \quad x \to 0$$

flat excitations

$$P(x) \to \text{const.} \quad \text{when} \quad x \to 0$$

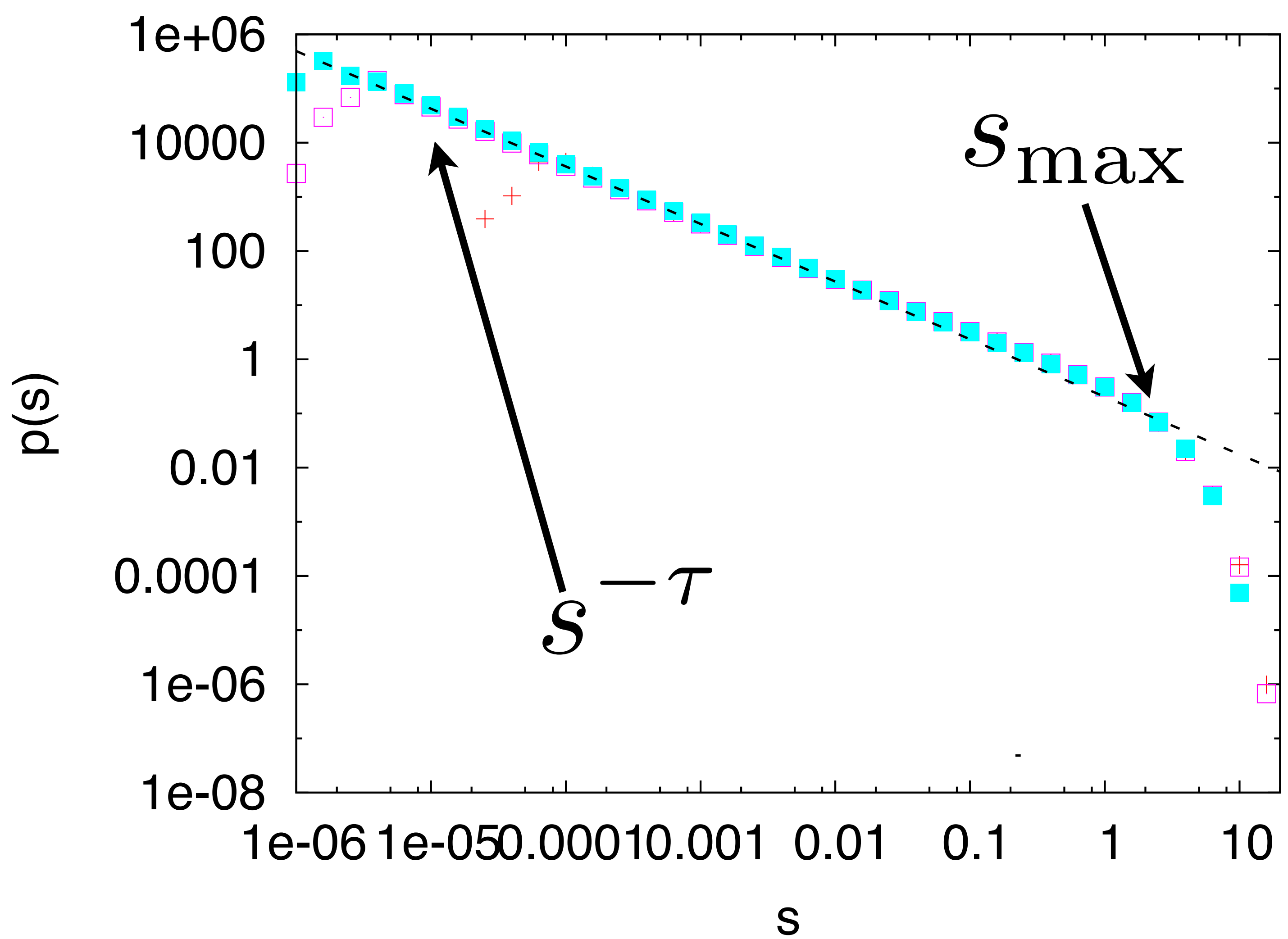
$$\int_0^{x_{\min}} P(x)dx = \frac{1}{N}$$

$$\Delta\Sigma \equiv x_{\min} \approx \frac{1}{N^{\frac{1}{\theta+1}}}$$

$$\Delta\Sigma \equiv x_{\min} \approx \frac{1}{N}$$

observed by Lerner & Procaccia and diverse microscopic simulations

Avalanche Statistics

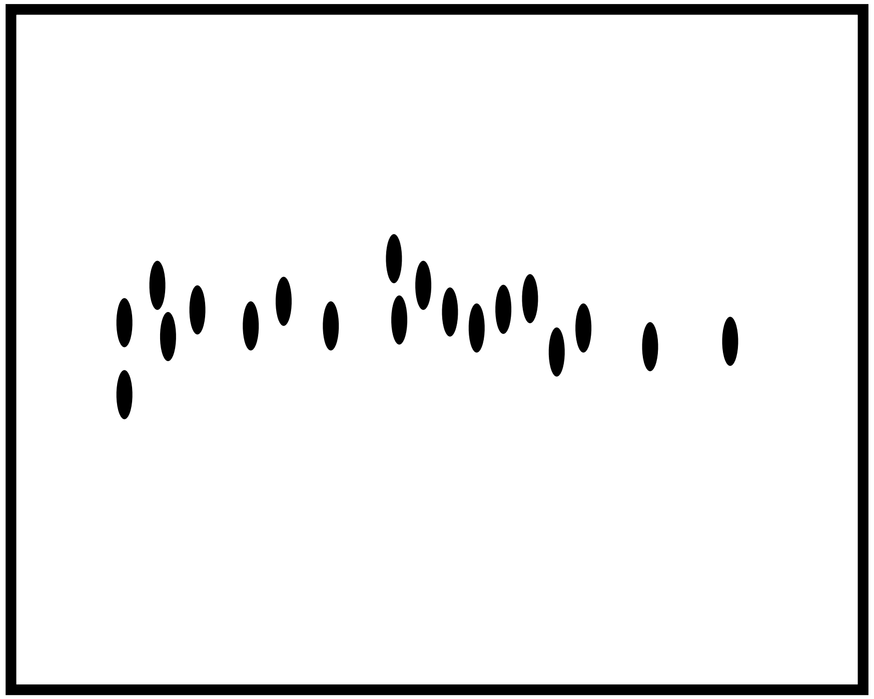


- free scale statistics: G-R exponent
- cut-off scaling: fractal dimension

Avalanches: fractal dimension

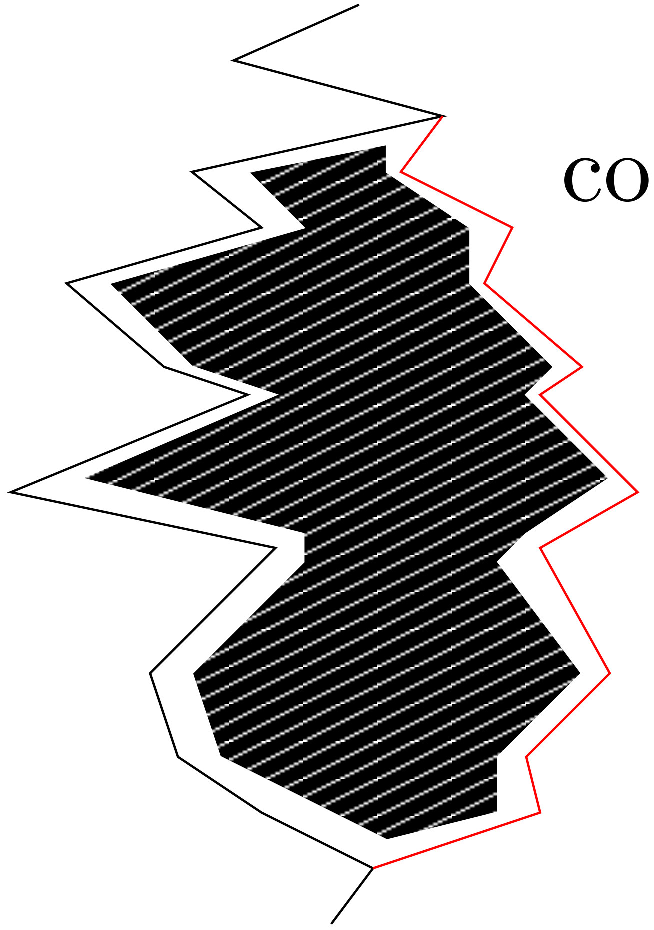
$$S_{\max} \propto L^{d_f}$$

Yielding/Eshelby



$d > d_f \approx 1$

Depinning/Elastic

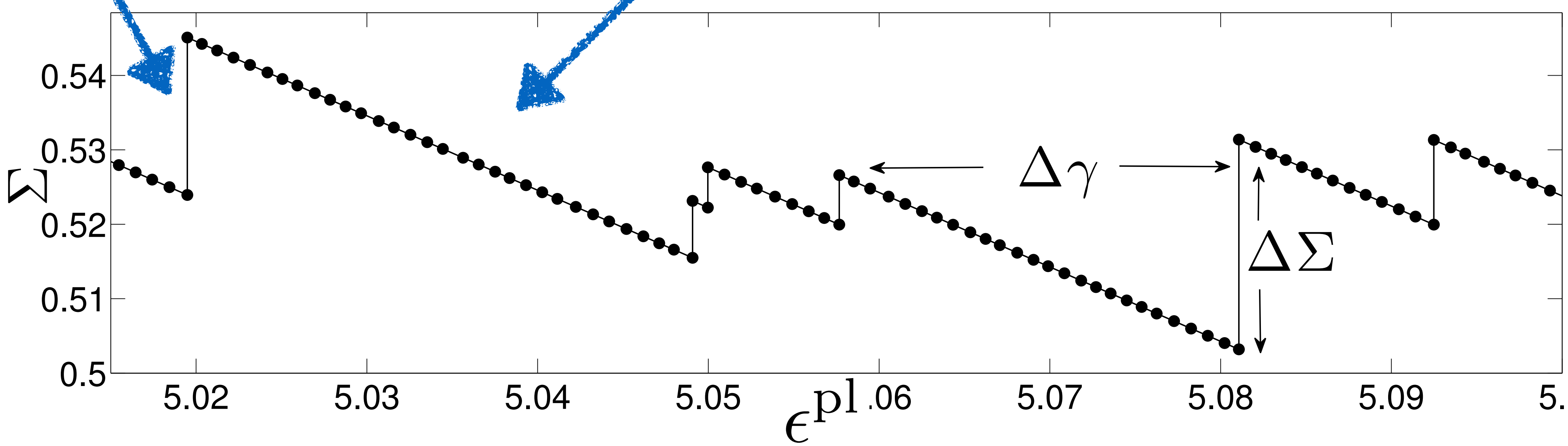


$d < d_f = d + \zeta$

Avalanche: G-R exponent

stress drive $\Rightarrow \Delta\Sigma \sim L^{-\frac{d}{\theta+1}}$

stress drop $\Rightarrow \delta\Sigma \sim L^{-d}\langle S\rangle$



Energy injected = energy dissipated

$$\langle S\rangle \sim S_{\max}^{2-\tau} \approx L^{d_f(2-\tau)}$$

$$\tau = 2 - \frac{\theta}{\theta+1}\frac{d}{d_f}$$

Yield stress fluctuation

$$\langle \Sigma_c(L) \rangle = \Sigma_c + k_1 L^{-\frac{1}{\nu}} + \dots$$

$$\delta\Sigma(L) = k_2 L^{-\frac{1}{\nu}} + \dots$$

$$\partial_t \gamma_{\vec{r}} = \int_{\vec{r}'} \mathcal{G}(\vec{r} - \vec{r}')\, \gamma_{\vec{r}'} + \Sigma + \sigma^{\mathrm{dis}}(\gamma_{\vec{r}}, \vec{r})$$

We add a tilt $\sigma_{\vec{r}}^{\mathrm{tilt}}$ of zero spatial average and defines $\tilde{\gamma}_q = \gamma_q + \mathcal{G}_q^{-1} \sigma_q^{tilt}$

In absence of disorder $\Rightarrow \frac{\partial \gamma_q}{\partial \sigma_q^{tilt}} = -\mathcal{G}_q$

$$\nu = \frac{1}{d - d_f + \alpha_k}$$

$\alpha_k = 0$ Eshelby, $\alpha_k = 2$ Laplacian

Avalanches: duration

$$T = \delta t_1 + \delta t_2 + \ldots \qquad \delta t = \frac{\tau}{\#\ \text{unstable sites}}$$

$$T \sim L^z$$

- $z > 1$ Elastic Depinning diffusion in the compact avalanche
- $z < 1$ Yielding super-balistic trip in the sparse avalanche

In Depinning we have two independent exponents $(d_f,\, z)$

How many independent exponent we expect for the Yielding transition?

Herschel-Buckley exponent

$$\xi = (\Sigma - \Sigma_c)^{-\nu}$$

$$\dot{\gamma}(\Sigma) = \frac{\Delta\,\epsilon^{pl}}{\Delta t} \approx \frac{\xi^{d_f - d}}{\xi^z} = |\Sigma - \Sigma_c|^{-\nu(d_f - d - z)}$$

$$\boxed{\beta = \nu(z + d - d_f)}$$

Critical exponents and scaling relations

exponent	expression	relations	2d measured/prediction	3d measured/prediction
θ	$P(x) \sim x^\theta$		0.57	0.35
z	$T \sim l^z$		0.57	0.65
d_f	$S_c \sim L^{d_f}$		1.10	1.50
β	$\dot{\gamma} \sim (\Sigma - \Sigma_c)^\beta$	$\beta = 1 + z/(d - d_f)$	1.52/1.62	1.38/1.41
τ	$\rho(S) \sim S^{-\tau}$	$\tau = 2 - \frac{\theta}{\theta+1}\frac{d}{d_f}$	1.36/1.34	1.45/1.48
ν	$\xi \sim (\lvert\Sigma - \Sigma_c\rvert)^{-\nu}$	$\nu = 1/(d - d_f)$	1.16/1.11	0.72/0.67

Scaling behaviour in liquid phase

- Our model - no relaxation time and steady state - displays a genuine second order phase transition

- Non abelianity and long range interactions induce a pseudo gap of the soft modes ($\theta > 0$)

- 3 scaling relations with 3 independent exponents

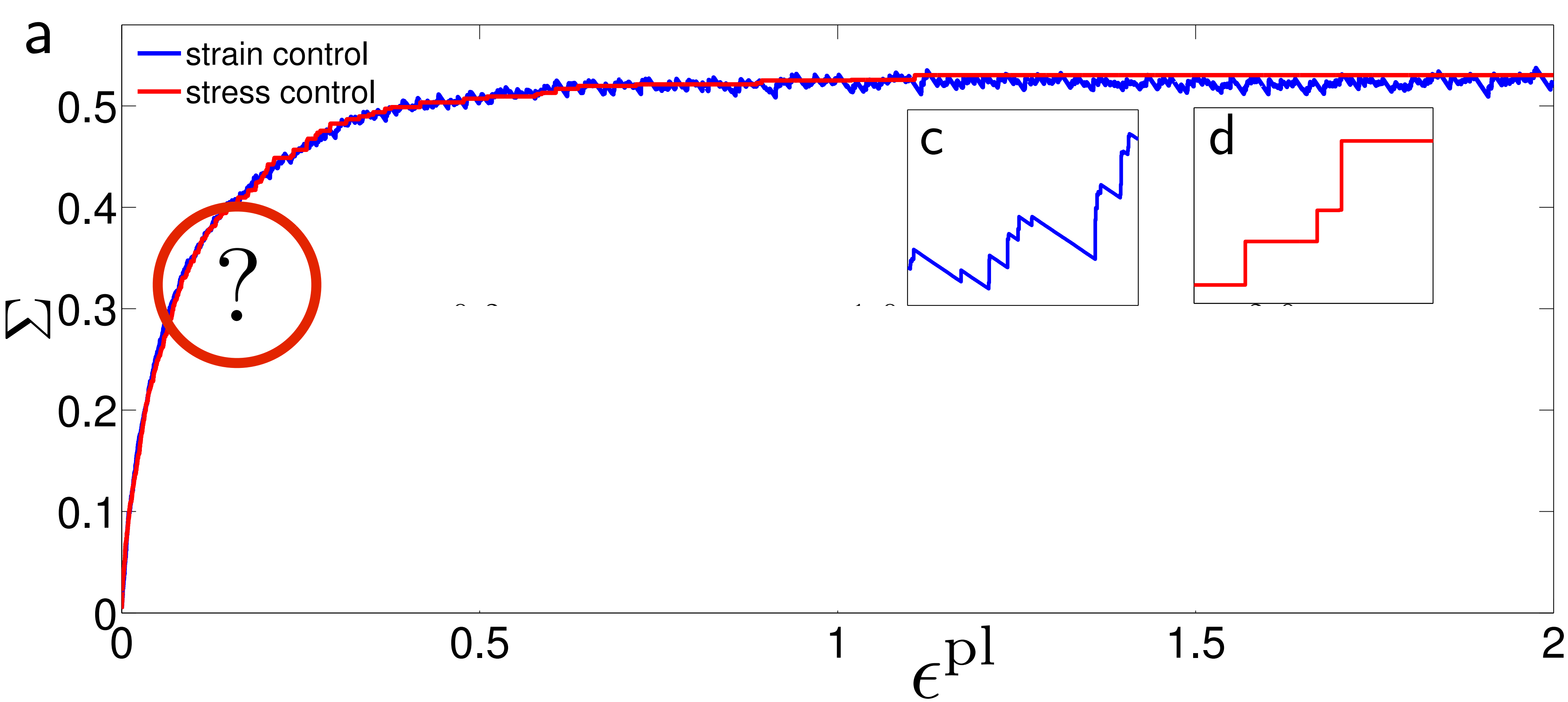
$$\nu = \frac{1}{d - d_f + \alpha_k}$$

$$\beta = \nu \left(d - d_f + z \right)$$

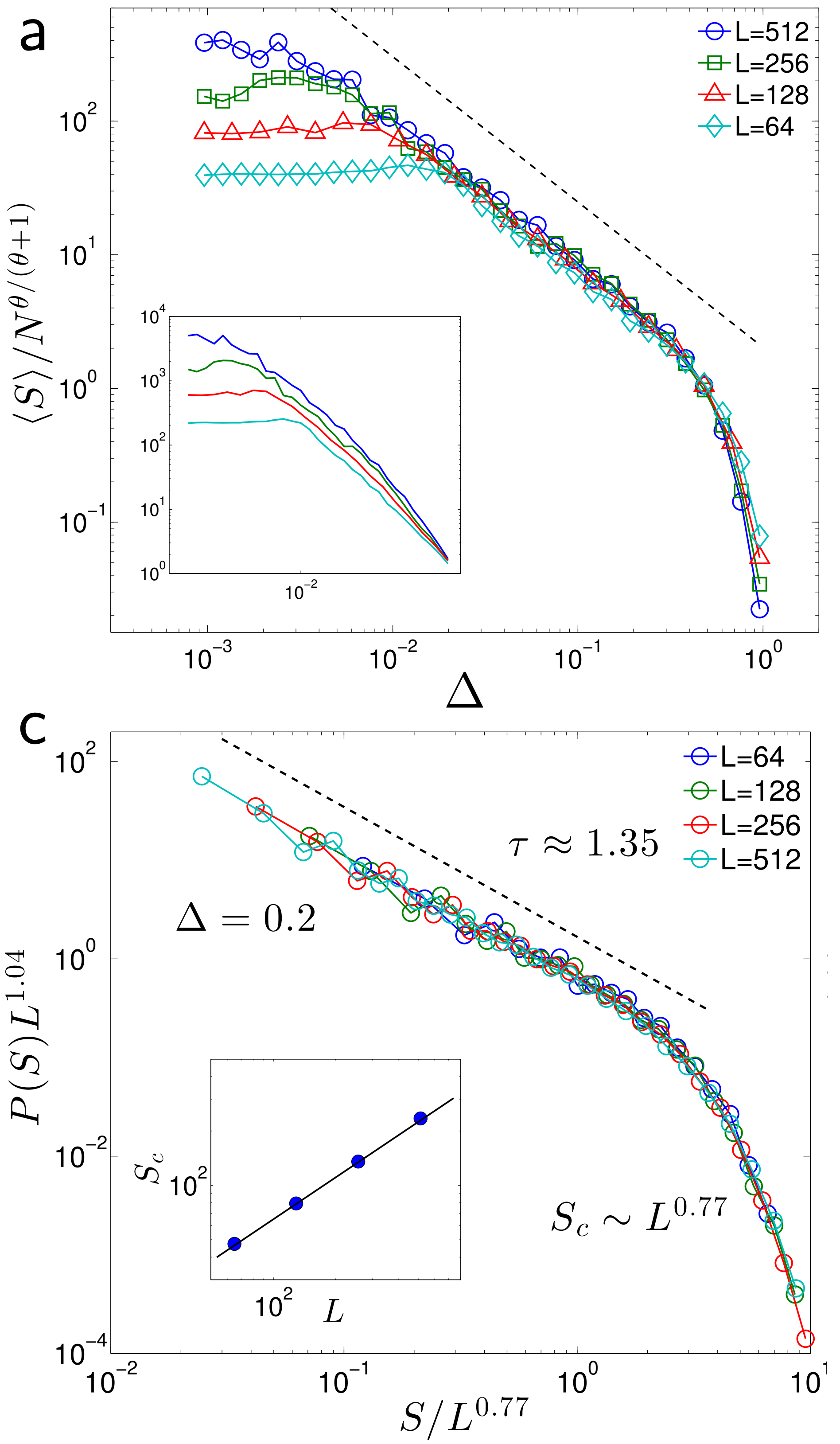
$$\tau = 2 - \frac{d_f - d + 1/\nu}{d_f} - \frac{\theta}{\theta + 1} \frac{d}{d_f}$$

Transient in the solid phase : spanning system avalanches

$$\xi = (\Sigma - \Sigma_c)^{-\nu} \quad \Longrightarrow \quad S_{\max} \sim \xi^{d_f} \approx |\Sigma - \Sigma_c|^{-\nu d_f} \quad ??$$



Spanning system avalanches (transient)



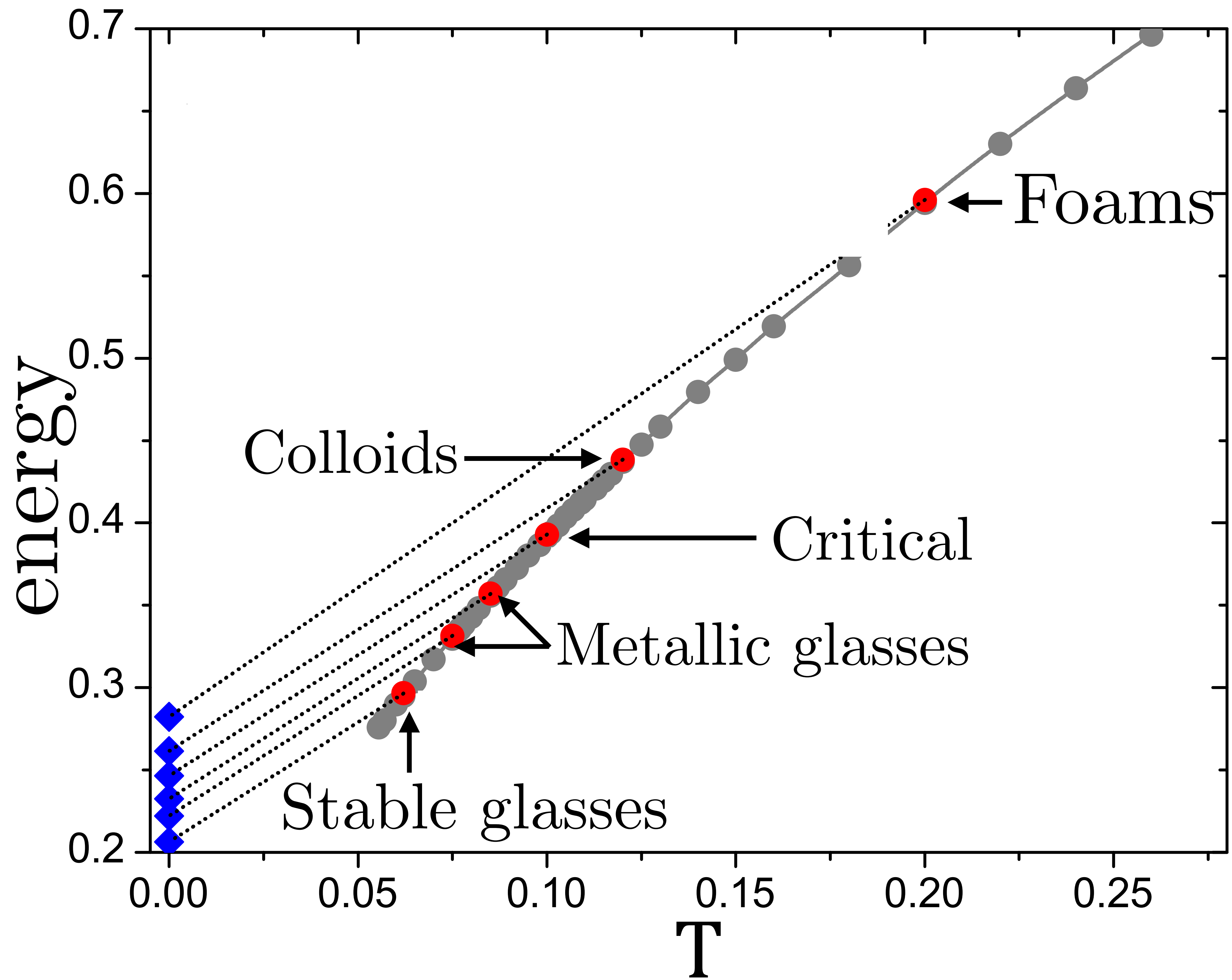
$$\Delta = 1 - \Sigma/\Sigma_c$$

$$\frac{\Delta\epsilon^{\mathrm{pl}}}{\Delta\Sigma} = \frac{\langle S\rangle}{N} N^{\frac{1}{1+\theta}}$$

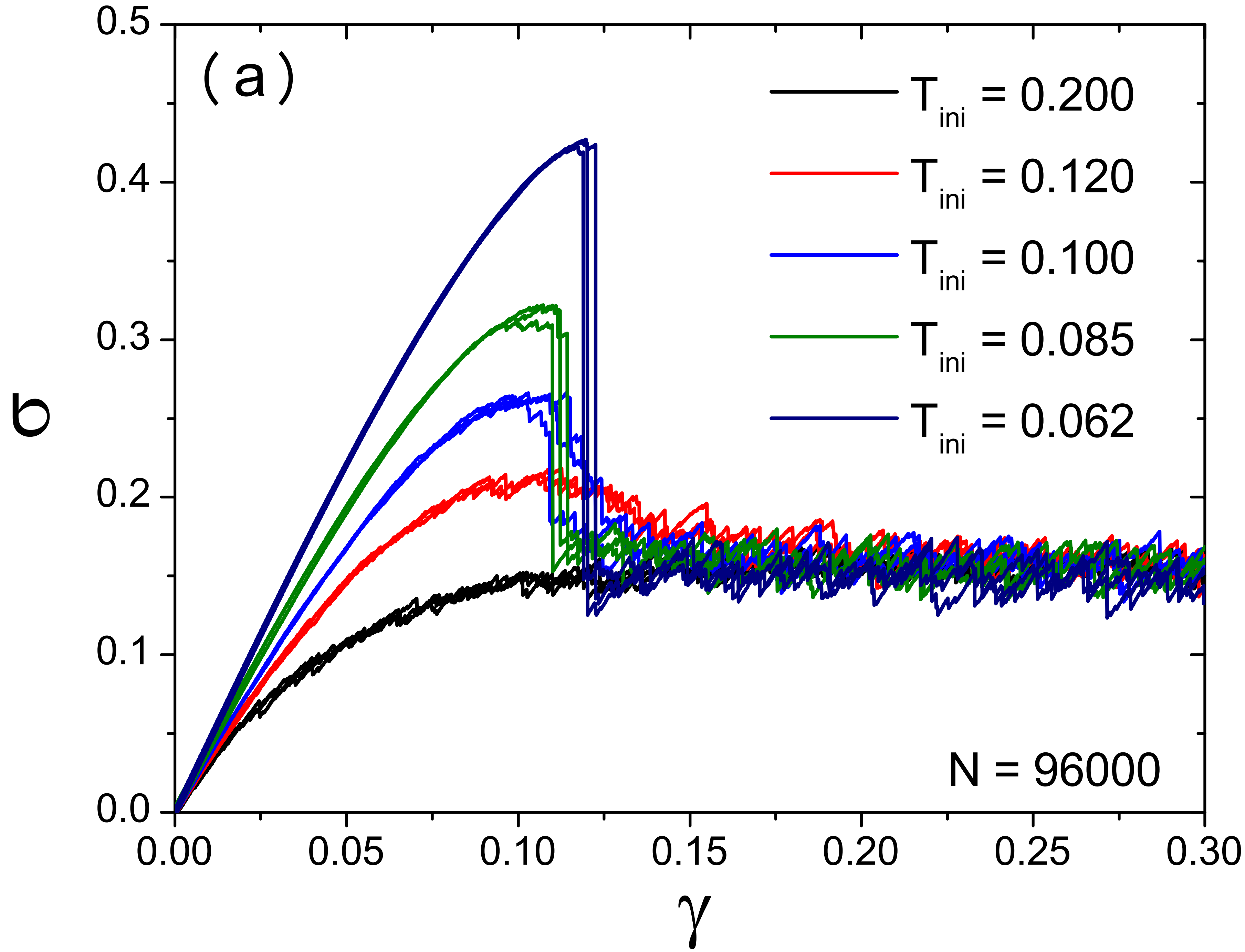
$$\Downarrow$$

$$\langle S\rangle = \frac{N^{\frac{\theta}{1+\theta}}}{\partial\Sigma/\partial\epsilon^{\mathrm{pl}}}$$

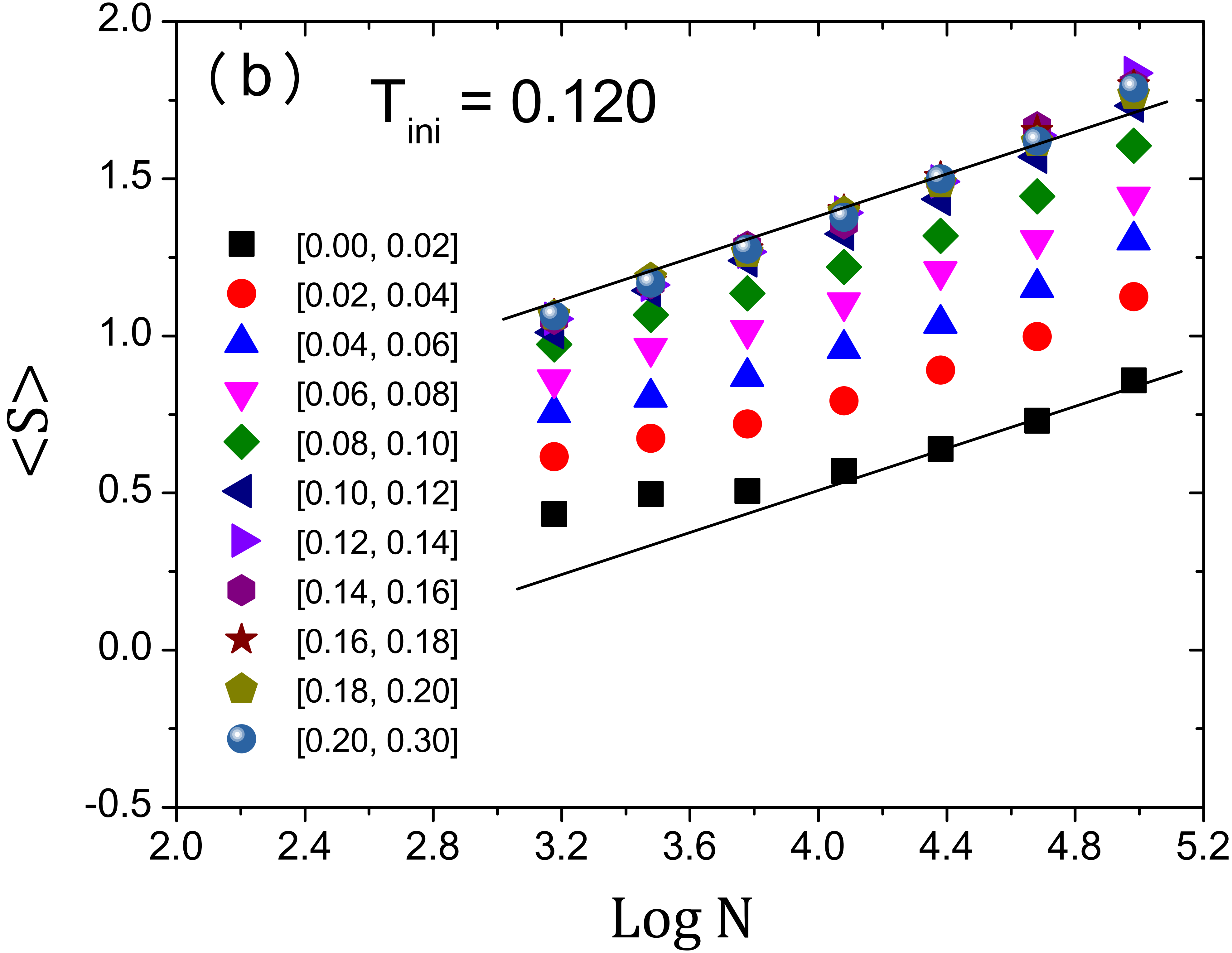
Different quenches for different materials



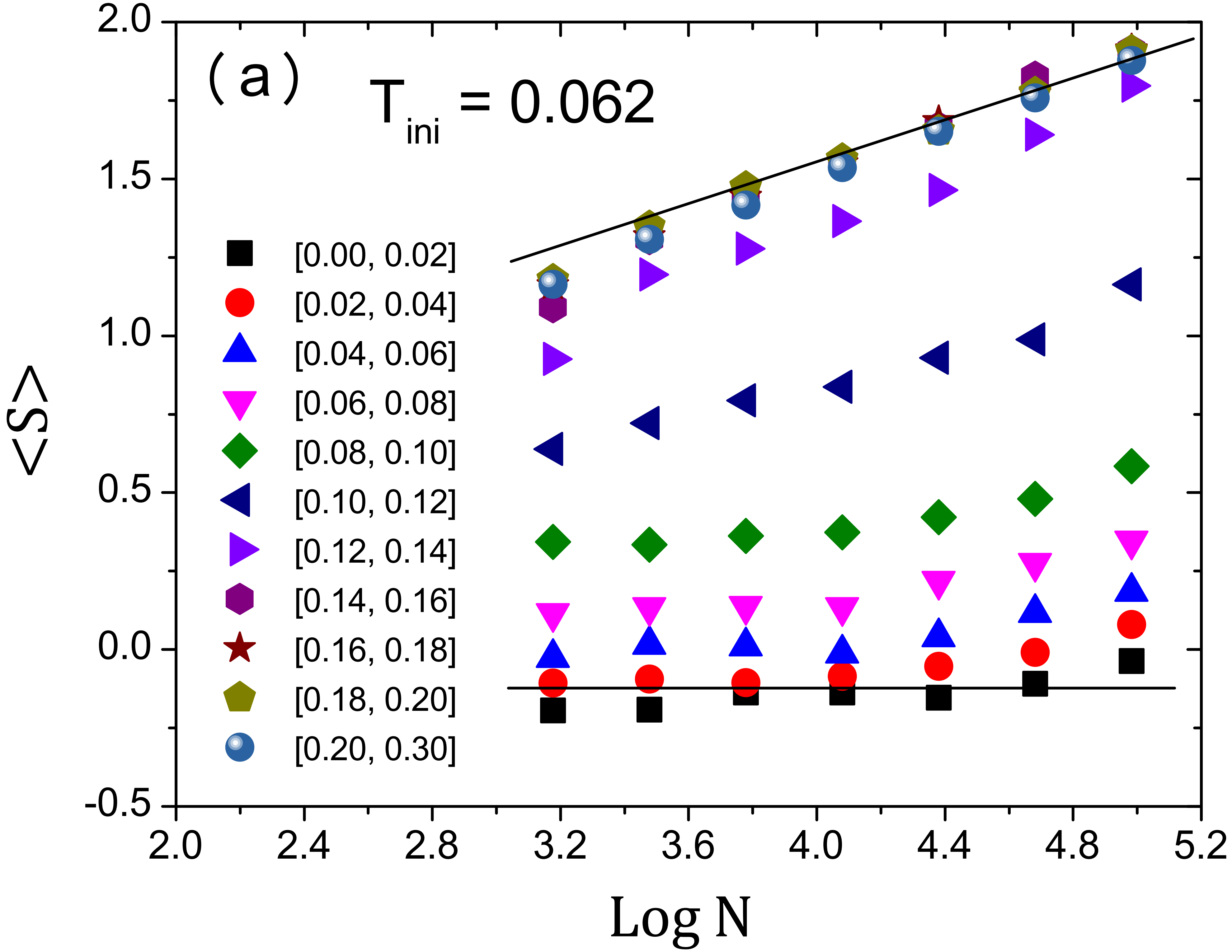
Three scenarios: monotonous, overshoot or failure



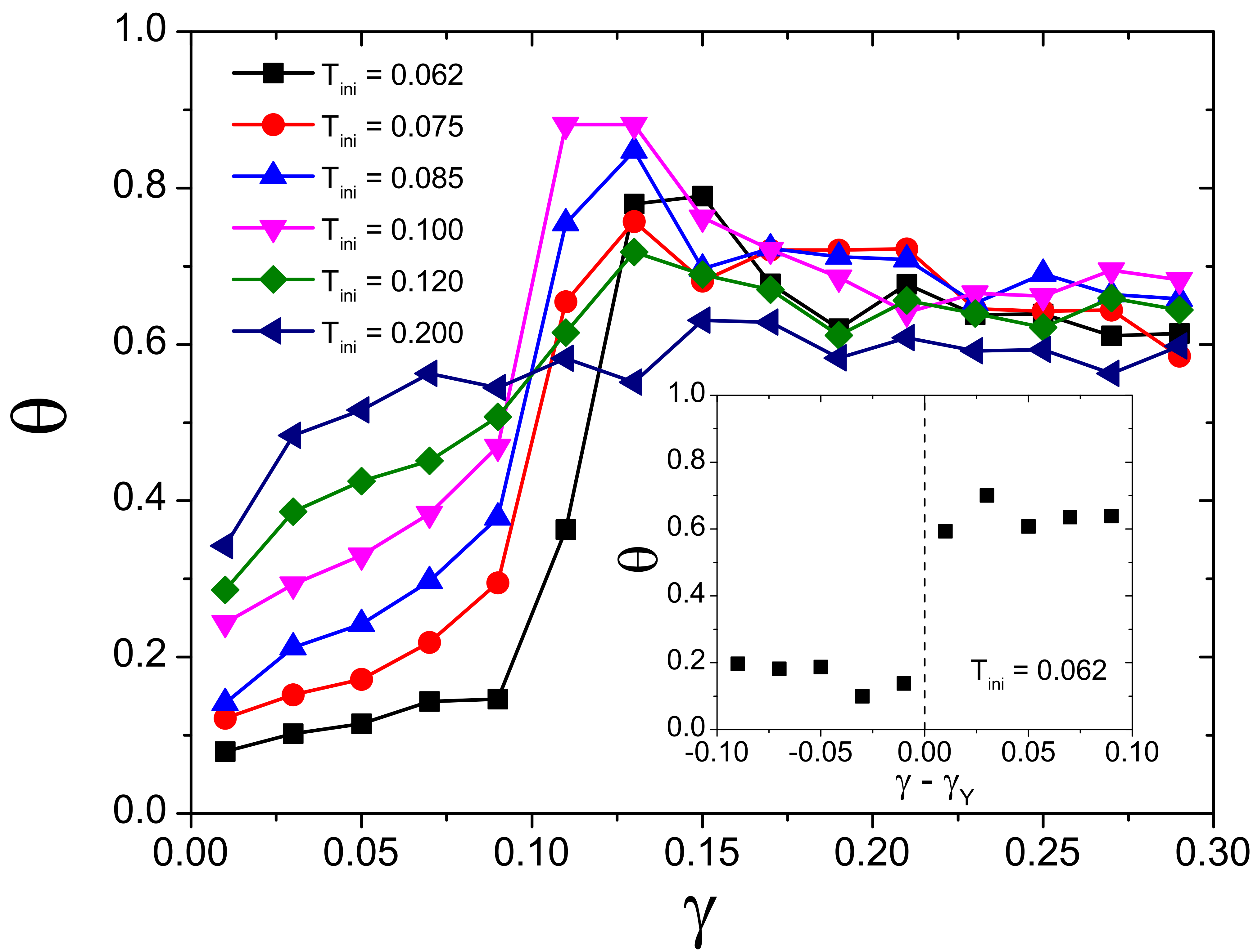
Spanning system avalanches for colloids



Much less spanning for glasses!



Brittle jumps for theta in glasses samples

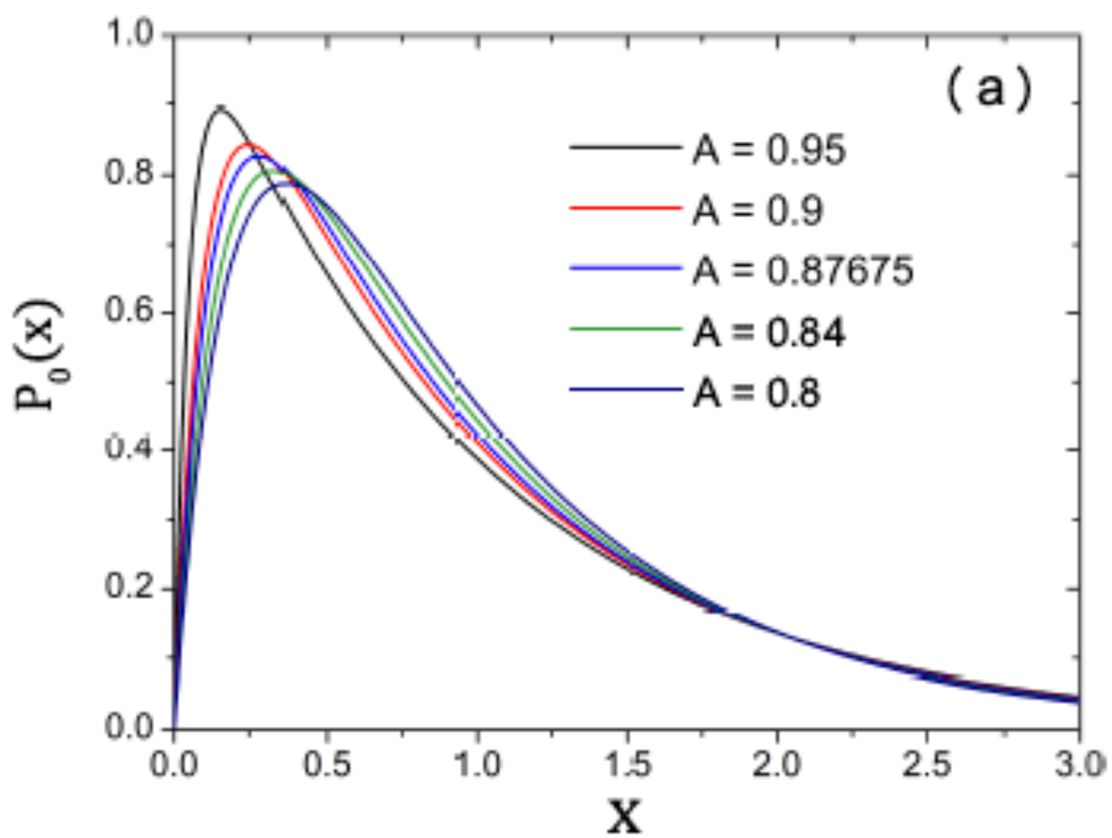


Transient: Failure & spinodal: depinning solution

$$\text{Depinning:} \quad G_{ij} = \frac{K}{K+k_0}\frac{1}{L^d}$$

$$\frac{\partial P_\gamma(x)}{\partial \gamma} = \frac{2\,K}{1-x_c P_\gamma(0)}\left[\frac{\partial P_\gamma(x)}{\partial x} + P_\gamma(0) g(x)\right]$$

Initial Condition $P_{\gamma=0}(x)$



Evolution $P_\gamma(x)$

Stability analysis

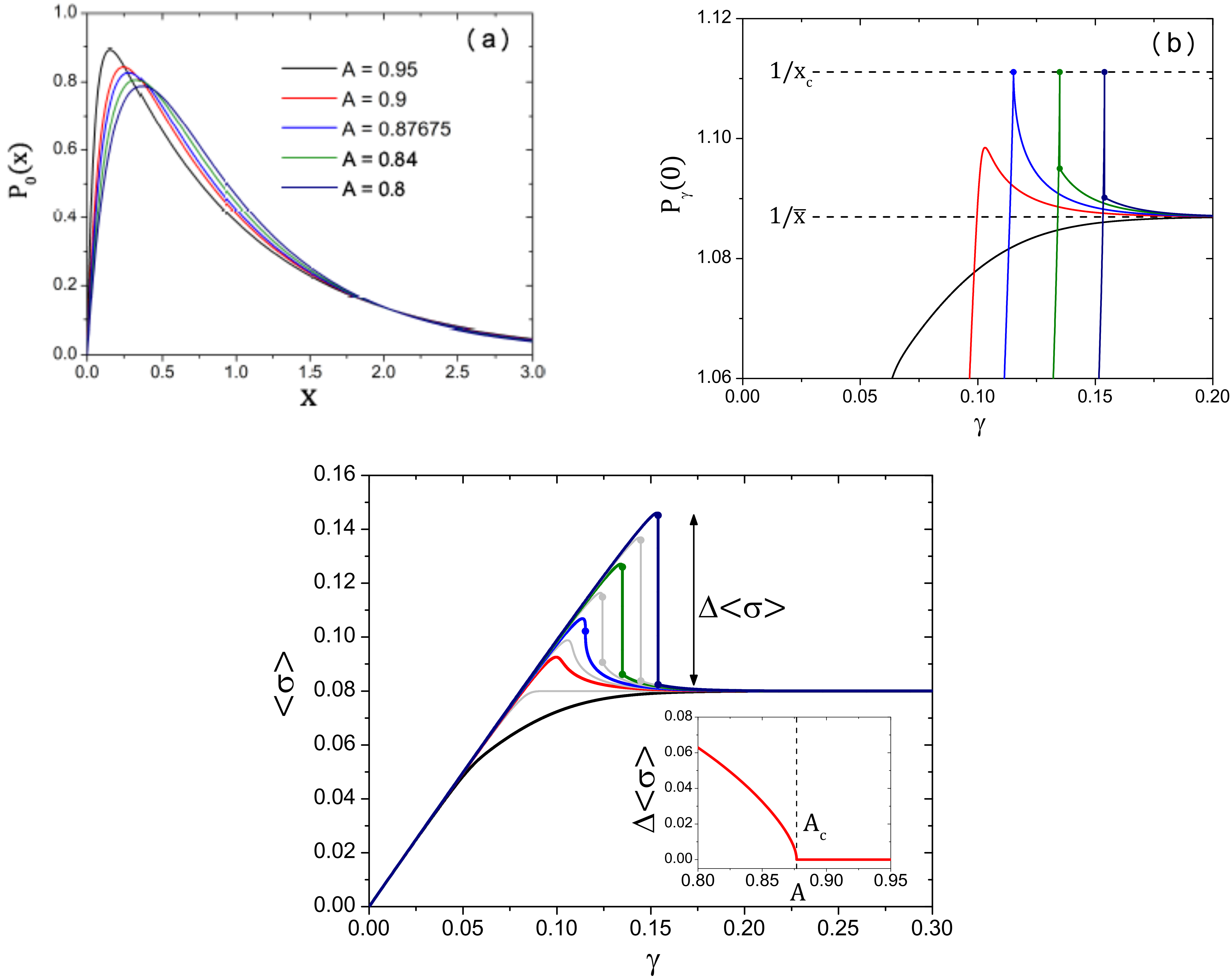
$$\text{Mean jump} \quad \bar{x} \qquad \Rightarrow \qquad x_c \frac{1}{L^d} = \frac{K\overline{x}}{K + k_0} \frac{1}{L^d} \quad \text{Mean Kick}$$

$$\int_0^{\delta x} P_\gamma(x) dx = \frac{1}{L^d} \qquad \Rightarrow \qquad \delta x = \frac{1}{P_\gamma(0) L^d} \quad \text{Mean gap}$$

$$\text{If Mean gap} < \text{Mean Kick} \qquad \Rightarrow \qquad \text{macroscopic failure}$$

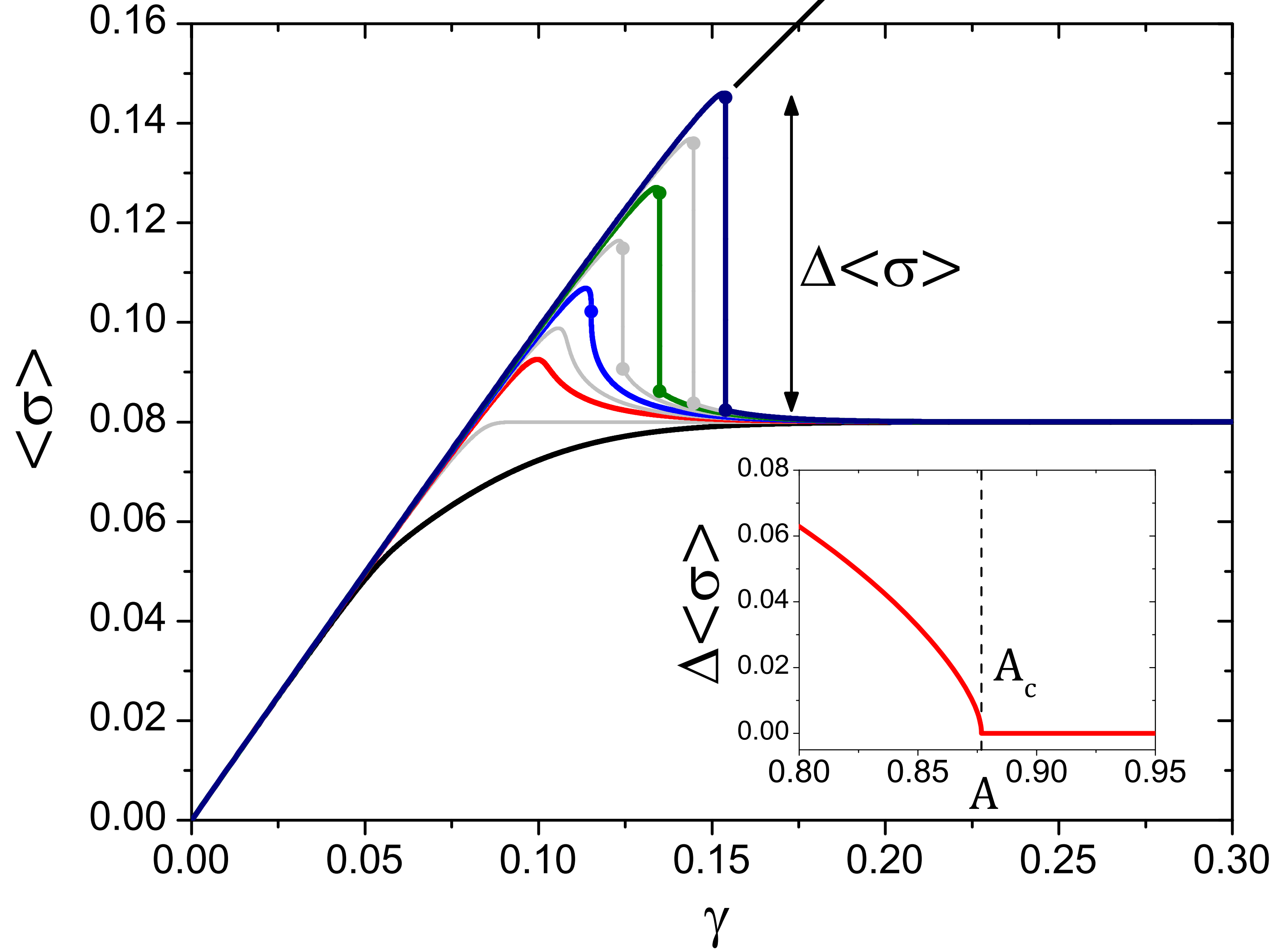
$$\text{If Mean gap} > \text{Mean Kick} \qquad \Rightarrow \qquad \text{finite avalanche}$$

$$\text{If Mean gap} = \text{Mean Kick} \qquad \Rightarrow \qquad \text{critical avalanches}$$


(a)
A = 0.95
A = 0.9
A = 0.87675
A = 0.84
A = 0.8
P0(x)
x
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
(b)
1/xc
1/x̄
Pγ(0)
γ
1.12
1.10
1.08
1.06
0.00
0.05
0.10
0.15
0.20
Δ<σ>
<σ>
γ
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
Δ<σ>
Ac
A
0.08
0.06
0.04
0.02
0.00
0.80
0.85
0.90
0.95

Spinodal transition with precursor avalanches

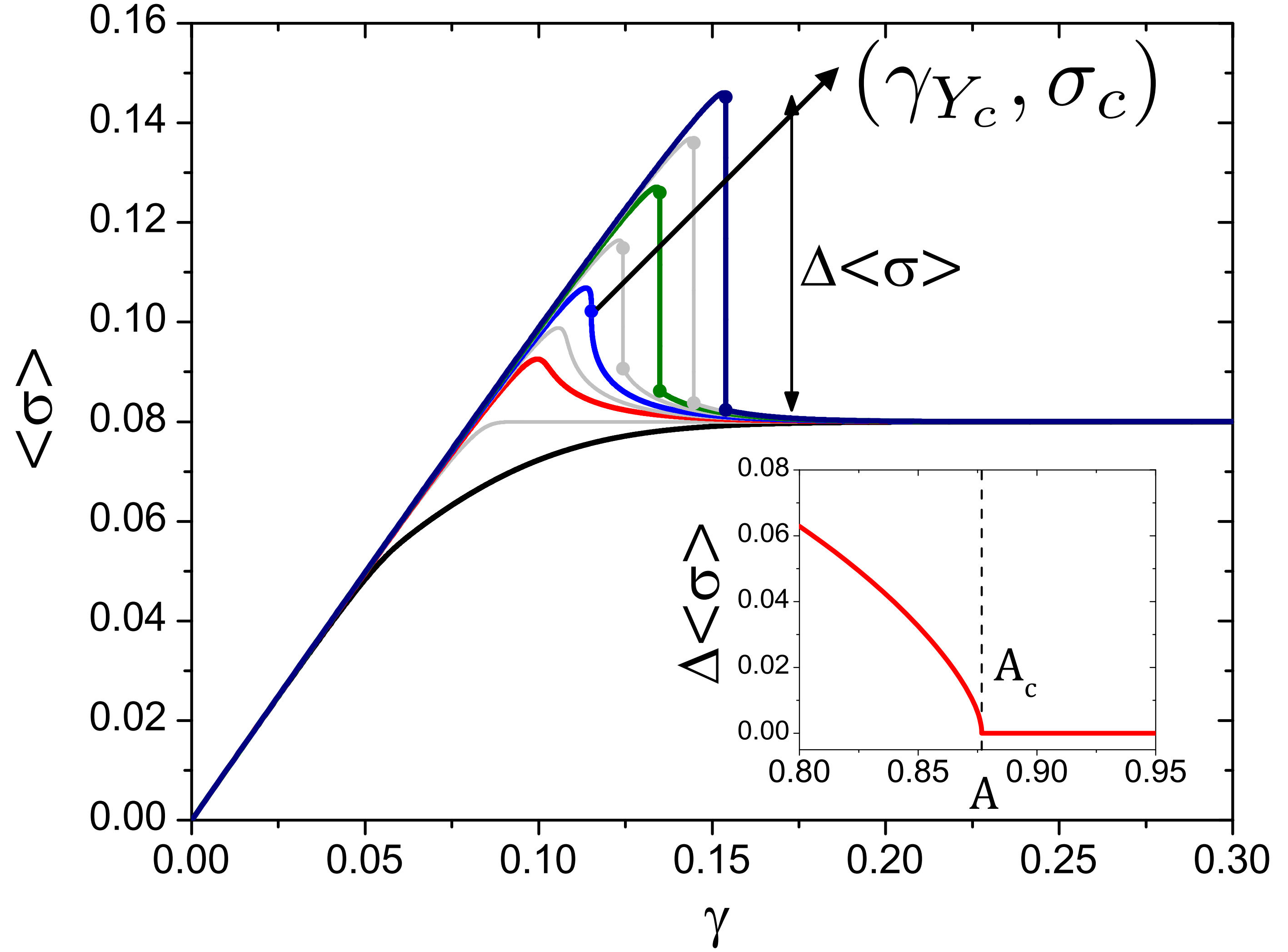
(γ_Y, σ_{sp})



$$\langle\sigma\rangle - \sigma_{sp} \propto (\gamma_Y - \gamma)^{1/2},$$

$$\mathcal{P}(S) \sim S^{-3/2} e^{-C(\gamma_Y - \gamma)S},$$

Second order transition with precursor avalanches

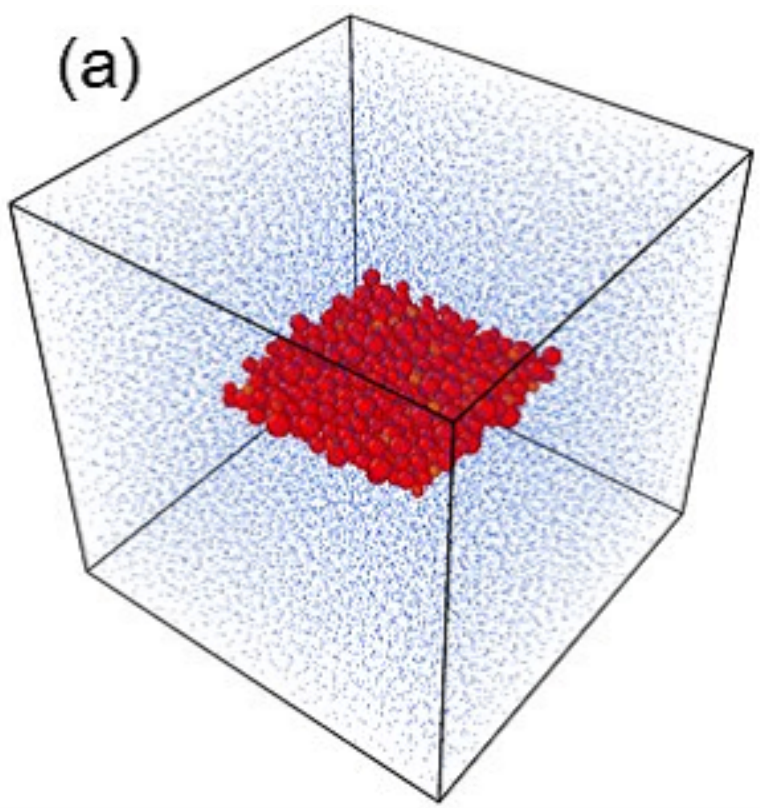


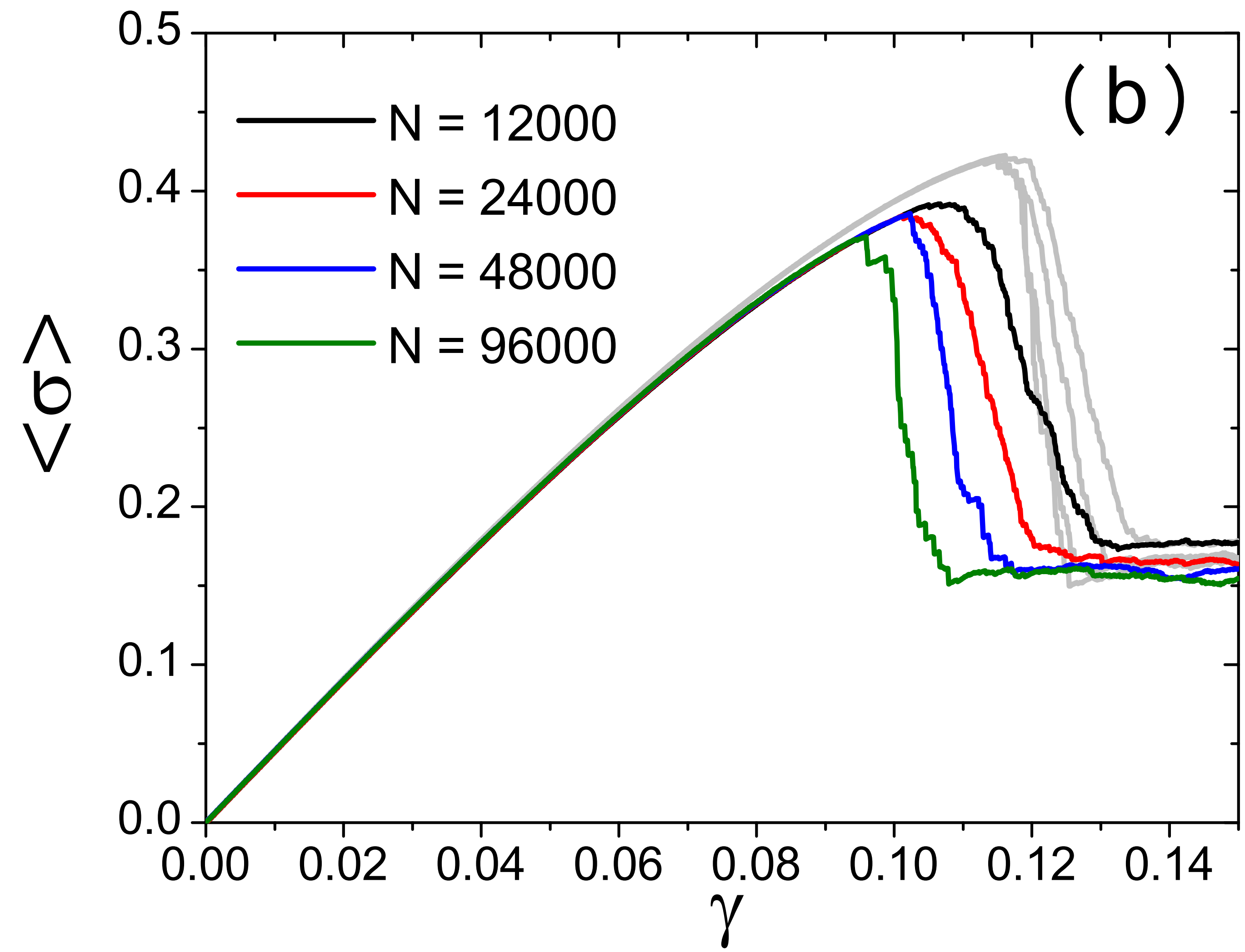
$$\langle \sigma \rangle - \sigma_c \propto \mathrm{sgn}(\gamma - \gamma_{Y_c})|\gamma - \gamma_{Y_c}|^{1/3},$$

$$\mathcal{P}(S) \sim S^{-3/2} e^{-C'|\gamma - \gamma_{Y_c}|^{4/3} S}$$

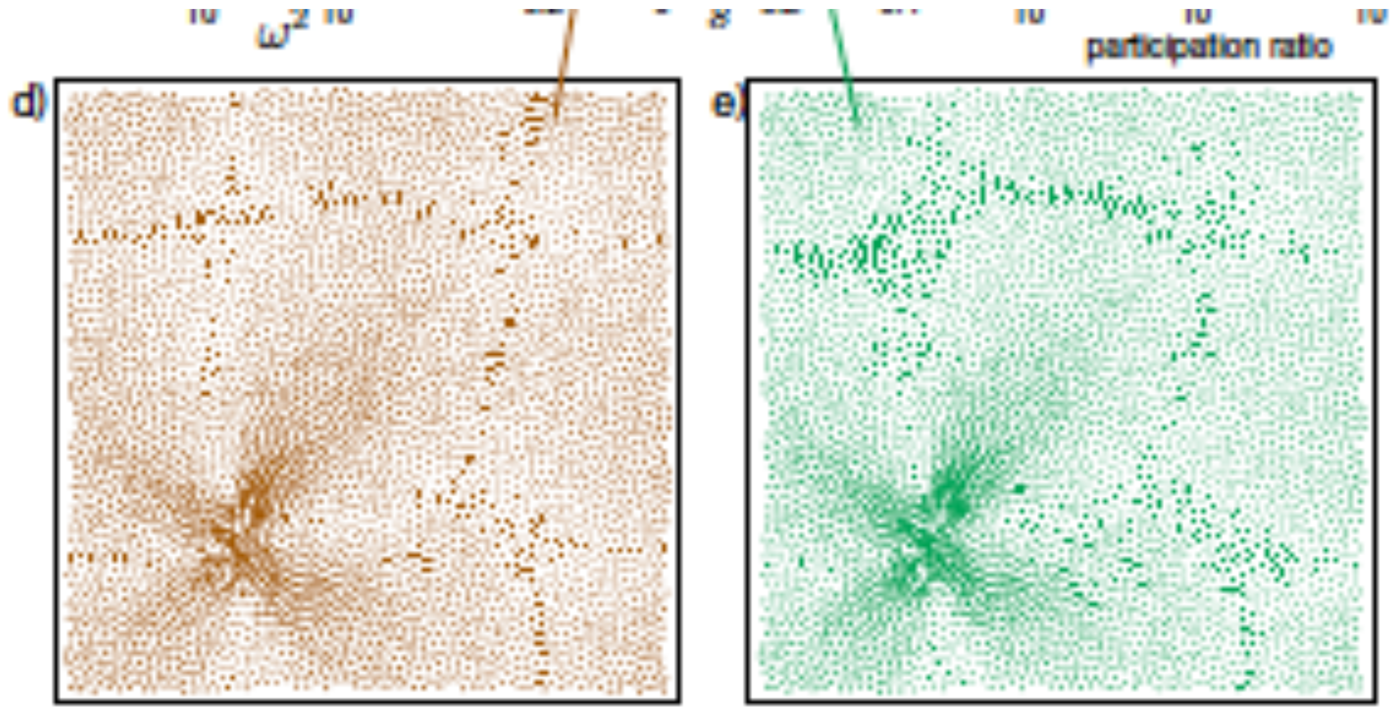
Finite dimension effects

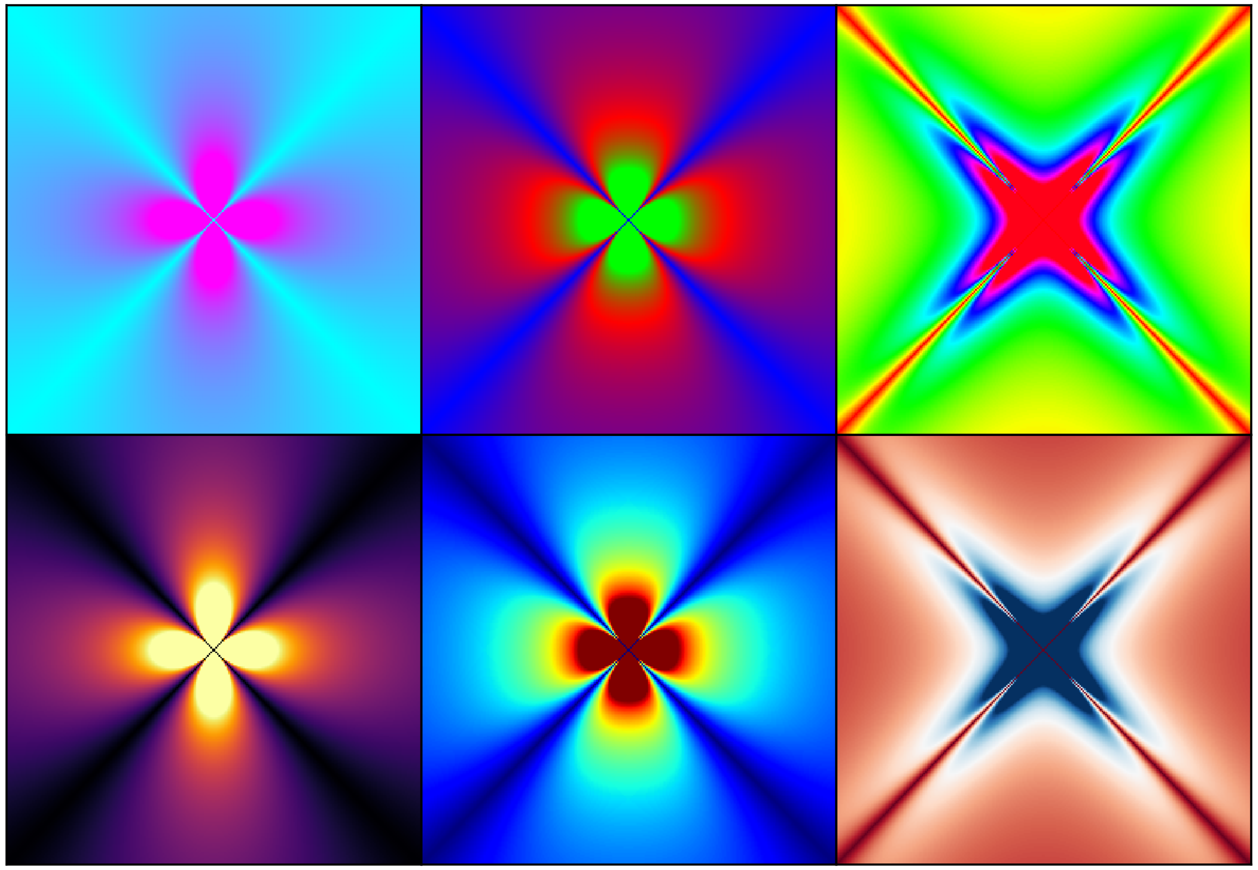
- Failure in triggered by localised shear band
- Role of rare defects
- exponents can change



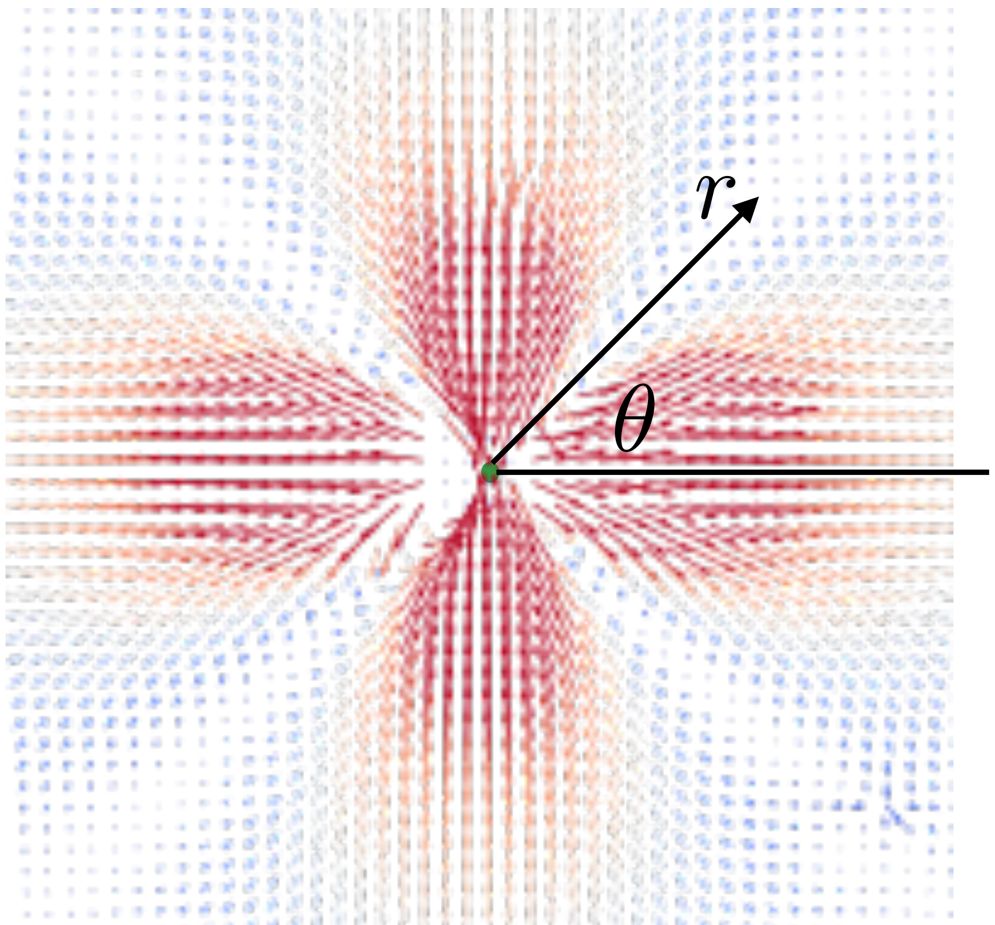


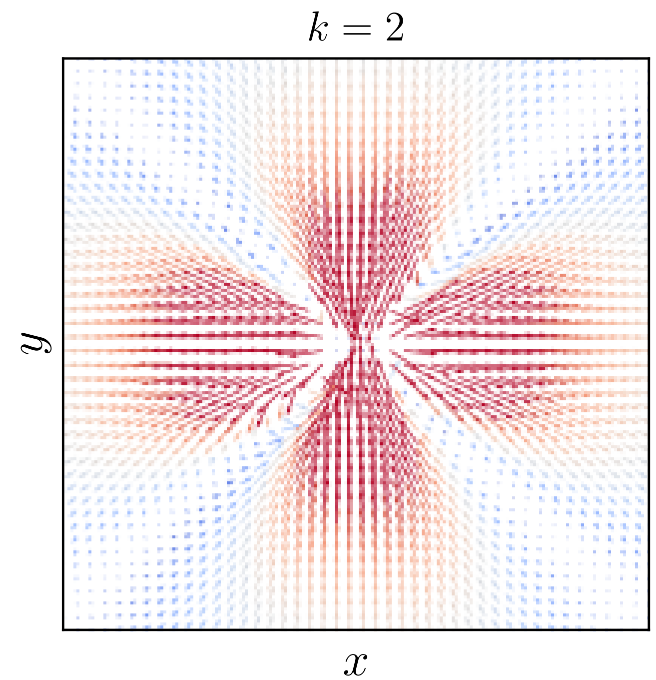
Soft modes, Shear transformations and failure

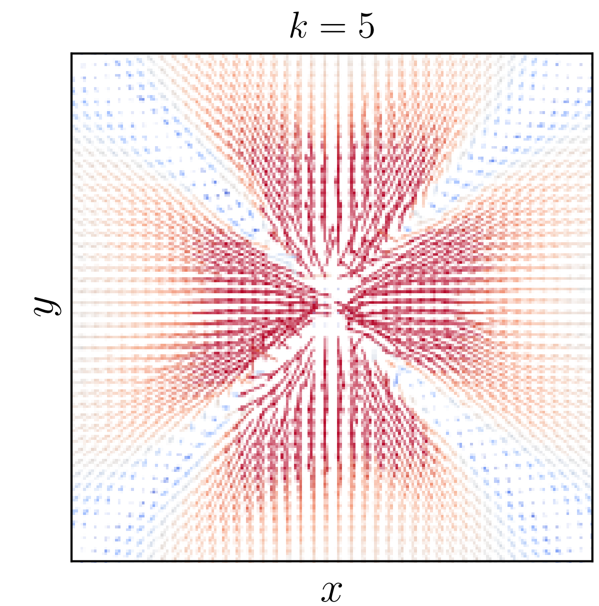


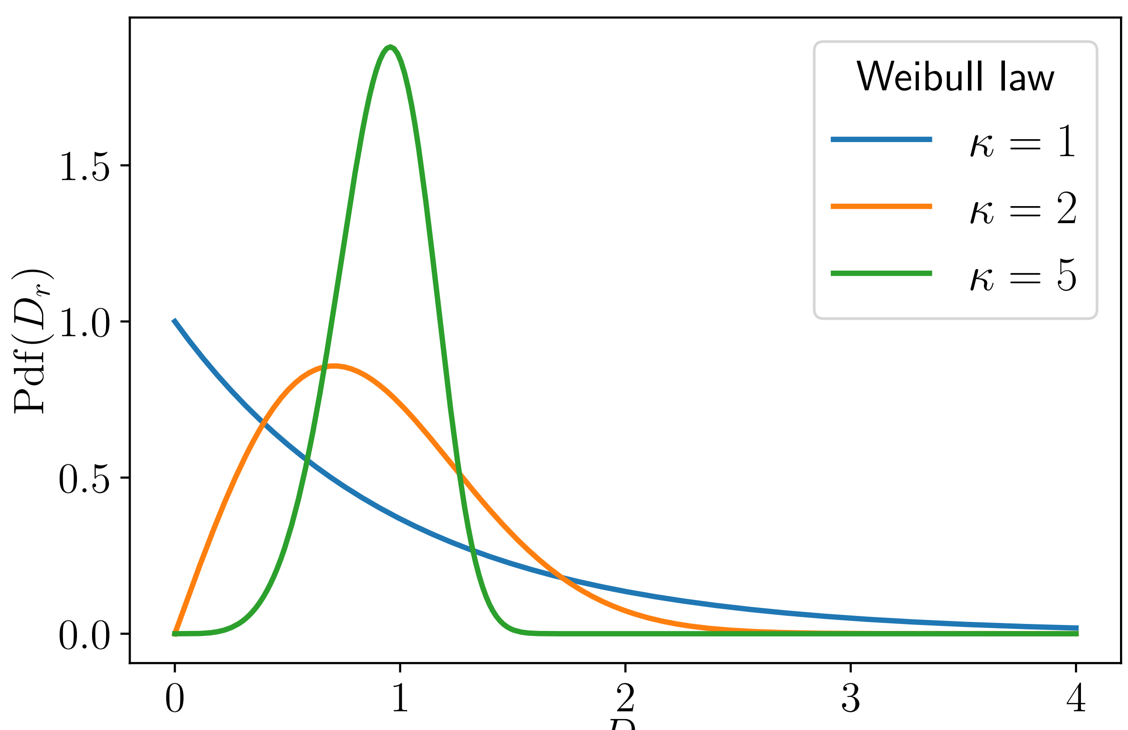


Soft modes and Anderson model

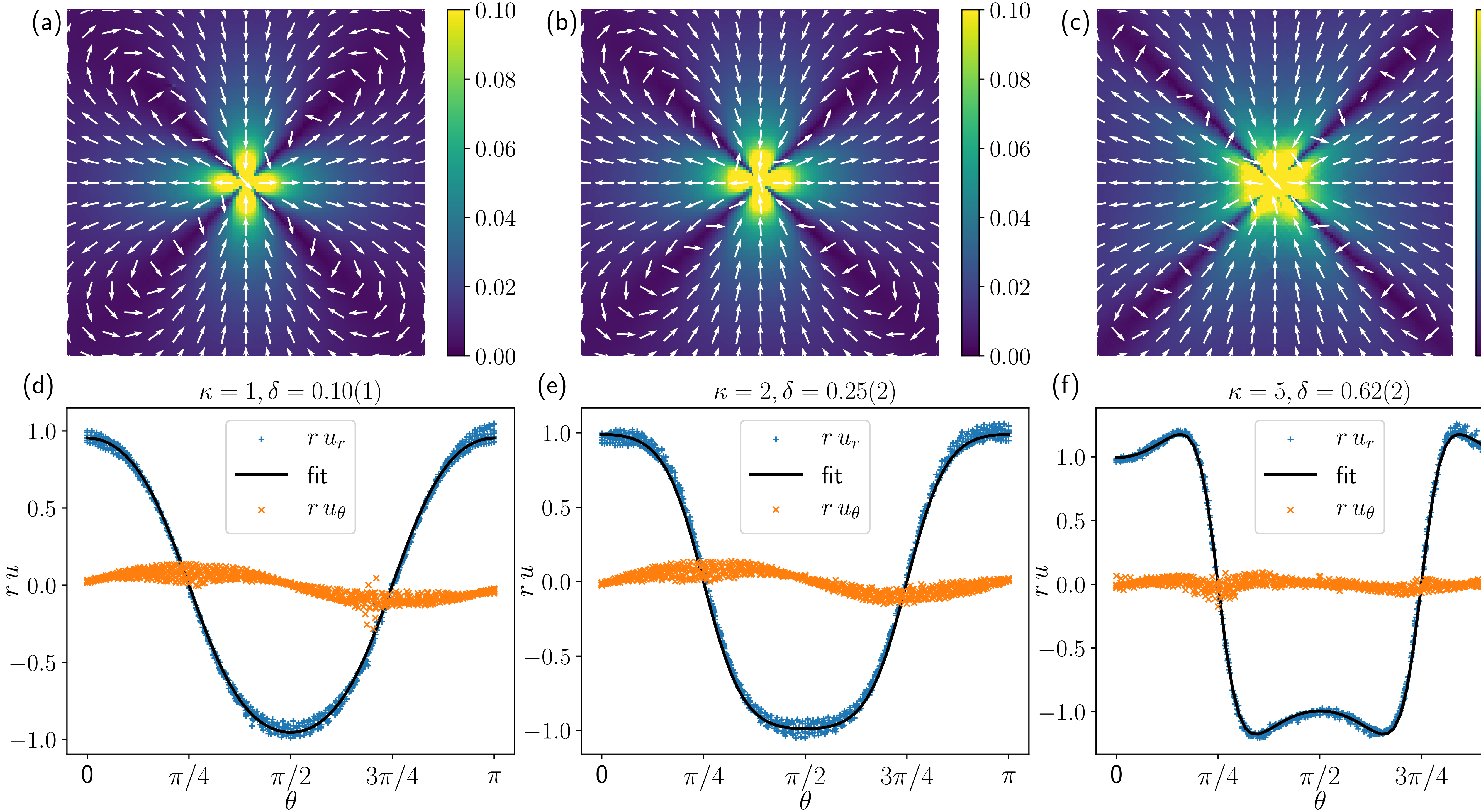






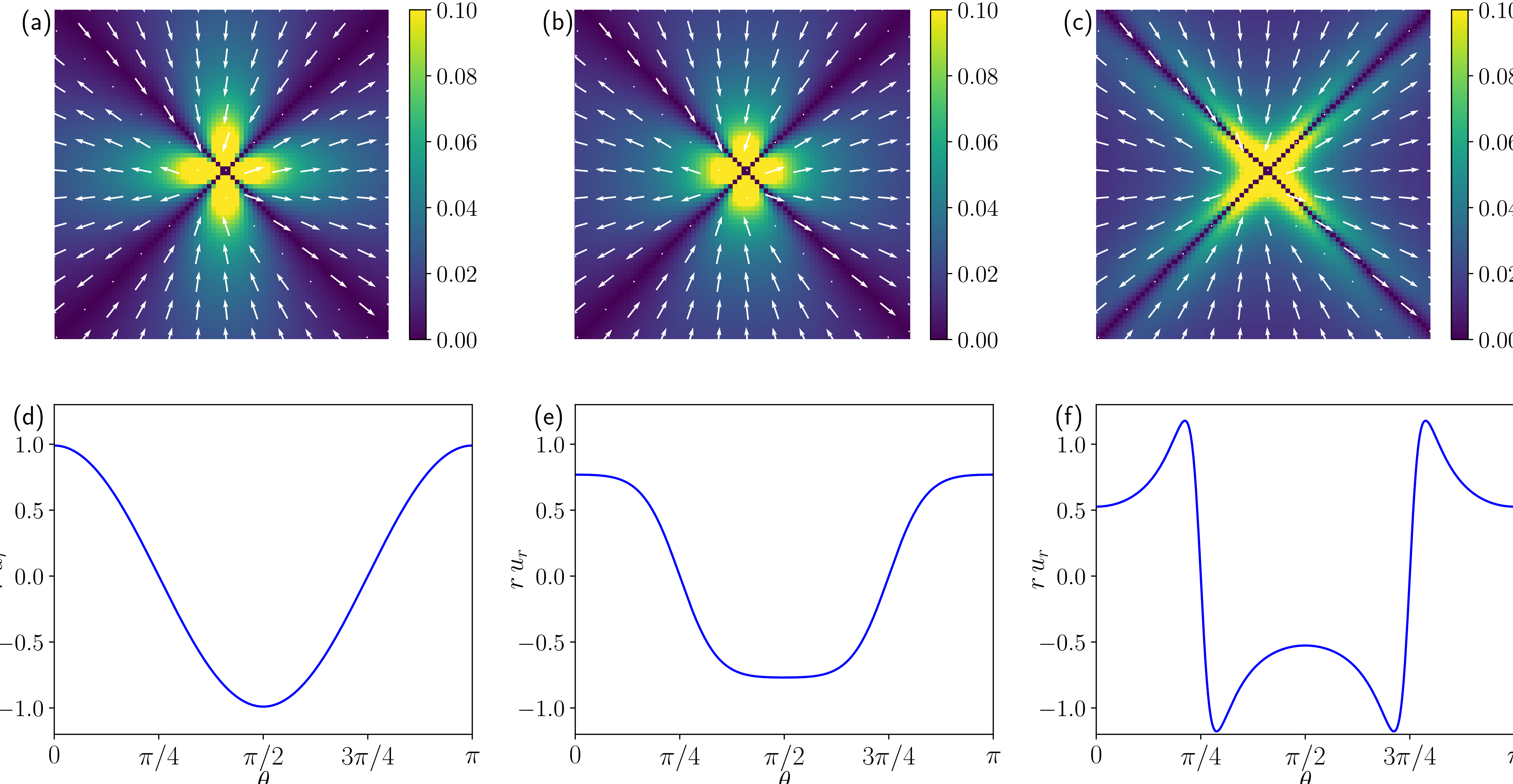


Soft modes and Anderson model



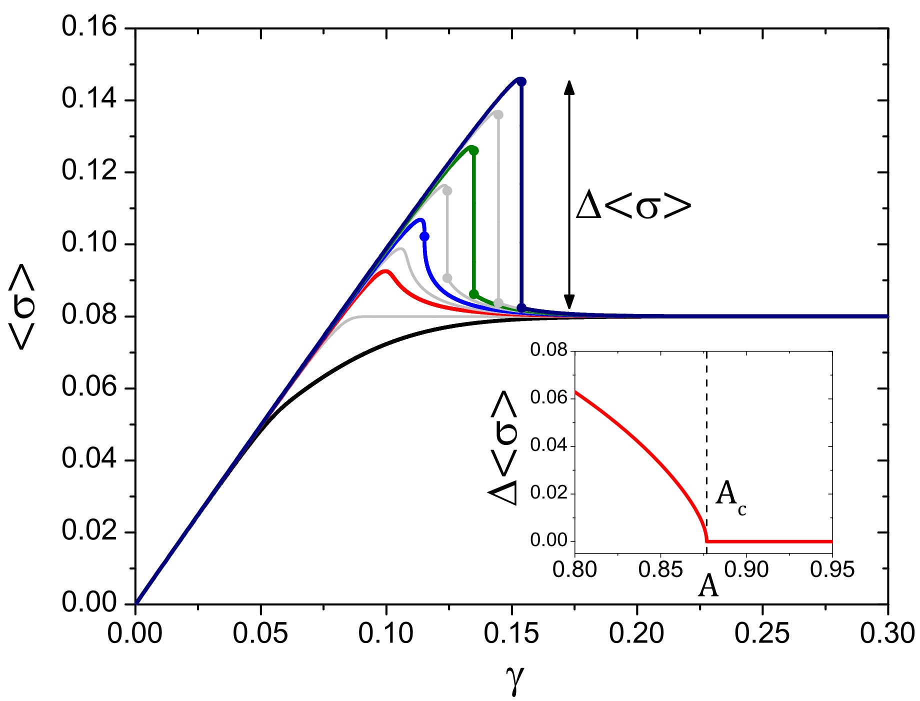
Single impurity model

$$r \cdot u_r(r, \theta) = \frac{\cos(2\theta)}{1 + \delta\cos(4\theta)} \qquad \delta = (\mu_3 - \mu_2)/(\mu_3 + \mu_2)$$



Conclusions for this transient regime:

- Failure as spinodal transition
- Large avalanches before failure (at least in MF)
- Pseudo gap evolution for different preparation



$$\langle\sigma\rangle - \sigma_{\rm sp} \propto (\gamma_Y - \gamma)^{1/2},$$
$$\mathcal{P}(S) \sim S^{-3/2} e^{-C(\gamma_Y - \gamma)S},$$

$$\langle\sigma\rangle - \sigma_c \propto \mathrm{sgn}(\gamma - \gamma_{Y_c})|\gamma - \gamma_{Y_c}|^{1/3},$$
$$\mathcal{P}(S) \sim S^{-3/2} e^{-C'|\gamma - \gamma_{Y_c}|^{4/3} S}$$